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Risk -- Mathematical models

FARMER BEHAVIOR UNDER RISK OF FAILURE

by

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This paper addresses the consequences of large, risky adjustment costs associated with moving a farmer's human capital from agricultural production. The term risky adjustment costs refers to the possibility that a farmer may have to accept involuntarily transition out of agriculture due to events beyond his control. In the absence of risk, acceptance of adjustment cost is completely the farmer's (discrete) choice. The farmer first maximizes his expected utility as if he relied only on farm revenue (including concurrent off-farm employment, if applicable); and then compares this level of farm-derived utility with that which is available elsewhere (less moving expenses, etc.). The difference between these two utility levels equals the adjustment costs corresponding to human capital.

Closer to reality, however, if the farmer chooses to continue another season, there is a chance that the ensuing revenues may be so low as to preclude future decisions to stay in agriculture. The farmer must accept transition and the accompanying costs. The critical level of revenue may be a function of household-maintenance expenditures, farm debt, and so forth. This critical level is likely to rise with farm size, but at a decreasing rate. Therefore, due to their smaller revenues, smaller farms are more likely to be at risk than larger farms.

Adjustment, or transition or transactions, costs lead to asset fixity; that is, small, or temporary, changes in the economic environment do not warrant changes in a factor's employment. A farmer's human capital fixity in particular presents important complications to conventional economic analysis, which relies on strict maximization of expected utility (or profit) derived solely from farm production.

In the presence of such risk, human capital fixity may cause the separation of, and conflict between, a farmer's objective to maximize expected utility from farming, and the farmer's objective to maximize the probability of remaining a farmer. These are, after all, sub-objectives: the farmer not only must determine how much of a commodity to produce (i.e., determine the expectation and variance of revenues), but also must decide how much to pay in order to remain a farmer in the future. These objectives diverge when the farmer has incomplete control over accepting the adjustment costs associated with leaving the farm -- when the farmer is at risk of failing.

If the costs are large, the farmer will take actions to avoid adjusting his human capital out of agriculture, and these actions may lead to what seems like inefficient farming decisions. This may entail anything from too-quickly depleting soil quality, to placing greater stress on farm labor at the expense of leisure. [1] Throughout, the paper uses the term inefficiency (or production inefficiency) in the sense that the farmer is not strictly maximizing the expected utility derived solely from farm revenues. The term is not meant to imply inefficiency in the pursuit of all sub-objectives taken together. When prices, weather, and other random events force adjustment, we term this failure. (Although the popular meaning of the term farm failure may have more to do with the immediate act of foreclosure.)

This paper attempts to provide some analytical support to the often-heard comment, "When price falls, the farmer goes out and plants more."[2] Human capital fixity may induce a peculiar set of farm decisions, causing gross production inefficiency, reflected in the term survival mode. The most noteworthy effect, caused by a farmer placing such overwhelming weight on avoiding forced adjustment, may be a downward-sloping supply curve. This may, in turn, explain over-production traps -- where the aggregate, rational-expectations equilibrium result of individuals avoiding adjustment costs is to make forced adjustment a risk to avoid. Thus, there may be justification for the government to move market equilibrium from that of over-production to that of efficient production.

If an outside observer finds a small probability of forced adjustment or failure, this does not necessarily imply the farmer is out of survival mode and simply making decisions based on maximizing farm-income-derived utility. Causality may flow the other way. A simple example in the third section demonstrates a case where there exists zero probability of failure, the farmer is rationally producing inefficiently, and supply increases with a fall in expected price. The important point is that the probability of failure is low (zero) *because* the farmer produces inefficiently.

The models presented here offer an explanation of farmer behavior that are very much like that which result from assuming safety-first objective functions. (Indeed, in the discretely random world of section III, the model implies safety-first behavior.) In models of these types, the farmer, at least implicitly, purchases assurance against suffering adjustment with foregone efficiency in producing utility from any season's farm income.[3]

The models and ideas in this paper relate to asset fixity. In order to avoid confusion, since most agricultural economists (to varying degrees of partisanship) have opinions on the question of asset fixity, the first section presents this paper's view of fixity, especially as it pertains to human capital. The approach here is simple: asset fixity is the rational decision-maker's response to adjustment costs. The second section presents a conceptual model of farm decisions with human capital fixity. It demonstrates that factors are used inefficiently in terms of strict maximization of farm-based utility. Those factors that are purchased prior to the production process are under-utilized in the sense that their marginal products are higher than what would be optimal under strict maximization. Those that can be utilized without immediate cash expenditure ("mined" or "borrowed off of"), such as soil quality and farm household labor, are over-utilized. Commodity production may be increased or decreased, relative to the case of no adjustment costs, depending on the degree of complementarity of factors. Government policies that reduce farm revenue volatility bring farmer decisions closer to strict maximization, and, hence, may either increase or decrease supply.

The problem presented in the second section is fairly general; therefore, particularizations are useful for the sake of going to the essence of the problem. The third section presents a specific model as illustration of the concepts developed in the second. This specific example is presented in two parts: the first analyzes the farmer's use of a single input (his own effort), and the second demonstrates an over-production trap that might arise in aggregate equilibrium.

The fourth section turns to the practical application of the ideas developed in this paper. We analyze empirically corn production in Illinois using the model of how risky adjustment costs affect input choices. The study focuses on the affects of farm size and other structural variables on farmer divergence from strict profit maximization.

I. Asset Fixity

There is a commonly-accepted notion that supplies of agricultural commodities are relatively inflexible in an environment of declining real prices, but more flexible, more responsive, to increasing real prices. This kinky characteristic of supply, often termed irreversibility, rationalizes federal farm policies, especially those that lessen price swings and support farm incomes. A widespread justification for this characteristic of supply is that productive assets, most notably physical capital, are more quickly accumulated in the farm sector in response to price and income increases than they are disposed of in response to declines in farm income. That is, productive capacity is fixed, or trapped, once introduced into farming operations. [4] Given the rationality of agents, asset fixity arises because of adjustment costs. And, although asset fixity is a widely-held belief, explanations for asset fixity -- that is, the sources of adjustment costs -- are less than generally accepted.

Two major conceptual sources have been hypothesized regarding the fixity of physical assets. They are not mutually exclusive. One may regard the first source of adjustment costs as lying in the lumpy nature of the production process itself. Galbraith and Black, for example, hypothesize that the existence of large fixed costs associated with reorganizing the farm operation make short run adjustment unprofitable. Changes in the economic environment would tend to induce changes in productive capacity only if they were of sufficiently high magnitude and of sufficiently long duration.

On the other hand, one may follow the more general views of G. Johnson and Edwards, and regard the second source of adjustment costs as lying in the discrete difference between the on-farm value of assets and their alternative, off-farm value. Or put in another way, there is a large divergence of the opportunity cost of *not* using the asset from the salvage value of the asset. Low salvage value may arise from several things: transportation costs, specificity of capital to the farm operation, and limited information regarding quality of the item -- the "lemons" principle of Akerlof (1970). Whatever the cause of low salvage values, once acquired productive capacity changes only with discrete (perhaps large) changes in the on-farm-use value.

One may consider both explanations of asset fixity as similar. In the first, adjustment costs are found in the discrete nature of productive units; in the second, adjustment costs are found in the lumpy nature of alternative values of assets. Both explanations similarly yield asset fixity, and hence imply similar inelastic behavior of supply over small or temporary price changes. They may be usefully distinguished, however, for the practical purpose of empirically locating adjustment costs. [5]

A farmer's human capital is subject to similar adjustment costs. From one perspective, one may view a farmer's human capital as a discrete unit. Although a farmer's labor may be divisible in part between on-farm and off-farm employment, his farm-specific capital by its very nature is lumpy. Or from another perspective, one may view the salvage value of human capital (the opportunity cost of being a farmer) as less than the on-farm value (the opportunity cost of not being a farmer). The discrepancy between a farmer's salvage value and his on-farm value may arise from (at least) two important things: first, the specificity of capital makes it worth little elsewhere; and, second, the farmer places a personal, or psychic, premium on earning an income from farming (or, conversely, he may discount exclusive off-farm employment).[6] In money dollars (accounting for moving costs, etc.), a farmer may *seem* to have a high salvage value, but in utility, derived from these non-farm dollars, the salvage value is low.

There are two major distinctions of interest between a farmer's human capital and other factors subject to adjustment costs. First, a farmer -- the decision maker who judiciously employs physical factors in the optimal production of his welfare -- is indistinguishable from his human capital. A person's human capital cannot be bought or sold; the person only obtains rents for his capital. This seems like an obvious point: a farmer cannot employ or dispose of his human capital in farm production without employing or disposing of himself as a farmer.

Second, there are certain minimal expenditures necessary in order to maintain human capital in farming. The farm family must eat, clothe itself, and otherwise purchase the material items it needs to endure happily alongside its neighbors.[7] In addition there may exist other current fees requisite for future production to take place: minimal debt service, minimal use of certain publicly-provided goods (e.g., water), insurance, and so forth. When deciding to continue farming or not, these minimum expenditures may be considered variable costs. Once the decision to farm is made, they are fixed. If these commitments are expected not to be met, the farmer certainly chooses not to farm.

The adjustment, however, of a farmer's human capital out of agriculture is not always an active decision. Such forced adjustment occurs when farm production does not cover the minimal, necessary costs. In other words, there may arise from time to time economic or natural environmental conditions that effectively make infinite the opportunity cost of farming; the farmer fails and moves on. Without this risk of forced adjustment, or failure, the farmer would simply compare the expected utility of farming with the utility of leaving farming and make the optimal discrete decision to continue. It is this risk that leads to seemingly inefficient production decisions.

II A model of farmer behavior

This section turns to a general model of farmer behavior when there exists a risk of forced adjustment. The pertinent characteristics of this model are generally discussed in the previous section; to summarize:

a) Human capital, in the form of farm control and management, is unresponsive to market signals, due to large adjustment costs of finding, taking, and enjoying alternative employment. These cost may be of the conventional pecuniary sort, or of the psychic kind.

b) Controlling or managing a farm necessarily implies that there are associated with human capital some forms and quantities of physical assets. One cannot be a farmer without a farm.

c) Human capital, unlike physical capital or other resources, needs considerable expenditures on maintenance in a specific job. That is, there are fixed costs to employing oneself in agriculture. If these costs are not covered, due to incompletely controllable events, the farmer must leave farming.

The first two characteristics do not imply that the returns to physical assets are necessarily too low from a simple farm-derived utility maximizing standpoint. For a risk-neutral farmer, for example, marginal value products are optimally set equal to marginal costs. From a purely financial accounting view, returns are equal to what the larger marketplace might suggest -- given that a farmer will always be a farmer.[8] The third characteristic, however, implies that a farmer is not assured of always being a farmer; the farmer may purchase assurance of remaining in agriculture (or, more simply, surviving) by inefficiently using factors of production.

A model of a farmer's behavior, when he is at risk of leaving farming, is inherently intertemporal. The farmer must trade-off the amount of utility he gets in any year from farming with the probability that, because of some particularly bad year (and not covering minimal expenditures), he must leave farming. Intertemporal models may lead to intractable complications; therefore, the following mathematical model makes certain assumptions, in addition to other specifications, that retain the essence of the broad problem without losing the ability to analytically represent what is going on. Specifically, in what follows, only one factor -- the farm manager -- is subject to adjustment costs; random events, that might occur from year to year are independent over time; the alternative utility that the farmer receives off the farm, if he fails, is some constant value; and once failed the farmer leaves agriculture forever. These assumptions produce several analytically attractive characteristics of the results. Farm decisions are constant over time; the optimal amount of expected farm-derived utility is likewise constant; the adjustment cost of moving out of agriculture is simply the difference (also constant) between optimal yearly expected farm-derived utility and off-farm utility. Furthermore, the farmer's objective function can be written in terms of yearly expected farm-derived utility, off-farm utility, and the probability of failure.

The model

A farmer yearly produces a commodity, the per-acre amount of which is denoted by y, by combining certain physical factors. The factors of production are of two types: those which must be yearly purchased out of cash revenues prior to realization of actual production and price received, and those which may be utilized in the year but paid for in the indeterminate future. The per-acre level of the first type is denoted by x, and examples of this type include hired labor, fertilizer, etc. The per-acre level of the second type is denoted by k, and examples of this type include land quality, farm household labor, machinery, etc. The per-acre production function in a given year is generally given by

$$y = y(x, k, \varepsilon) \quad ;$$

where ε represents some random effect on output y, such as weather. The number of acres produced is given by A, which is take here to be a constant. The farmer faces every year an unknown price p, a random variable ($p > 0$, $E[p] = \mu$), with some time-invariant probability density function given by $g(p)$. The constant per-unit cost of the immediately purchased factors x is given by w, and that of the other factors k is given by i.

Failure is defined as earning some available cash income that does not cover the minimal expenditures in a year necessary to farm, f; that is, failure is defined by

$$A[py - wx] < f \quad .$$

If the farmer fails, the farmer leaves farming and earns some sure utility level I each year thereafter.

For ease of presentation, we assume for the remainder of this section that production is certain and price the only random element of concern to the farmer. This particular assumption is relaxed in the more specific example following in the next section.

The yearly probability of failure, π, is given by $\pi(x,k) = \int_{p_d}^{0} g(p)dp$; where $p_d y - wx = \frac{f}{A}$. The utility from farming in any year is given by

$$U(p,x,k) = U[A(py - wx - ik)] \quad ,$$

and the expected utility from farming is simply

$$\overline{U}(x,k) = E_p[U(p,x,k)] \quad .$$

Finally, the constant rate by which the farmer discounts future expected utility (either from farming or not farming) is given by β.

Table 1 presents possible future events that the farmer must account for if he chooses farming. From this one can easily represent the farmer's objective function as

$$V(x,k) = \frac{\overline{U}(x,k) - I}{1 - \beta[1 - \pi(x,k)]}$$

Let k^* and x^* satisfy the first order conditions of maximizing the objective function:

$$\frac{\partial y}{\partial k} = \frac{iE[U']/E[U'p]}{1 + \vartheta}$$

$$\frac{\partial y}{\partial x} = wE[U']/E[U'p]\frac{1 + \vartheta E[U'p]/E[U']}{1 + \vartheta}$$

$$\vartheta = V\beta g(p_d)p_d/[AyE(U'p)]$$

Here marginal utility is given by U'. ϑ may be interpreted as a measure of the influence of human capital fixity on farm production decisions.

Compare these first order conditions to that of strict maximization of $E[U]$:

$$\frac{\partial y}{\partial k} = iE[U']/E[U'p]$$

$$\frac{\partial y}{\partial x} = wE[U]/E[Up]$$

Note that since $\vartheta > 0$, the farmer appears to be over-utilizing k based on conventional marginal conditions. Further, it is reasonable to suppose that the critical price defining failure, p_d, is small relative to expected price. If p_d is such that $E[Up] > E[U]p_d$, then it would appear that the farmer is under-utilizing x.[9]

Now suppose that there is an improvement in the rest of the economy relative to that of the agricultural sector As adjustment costs $(\overline{U} - I)$ grow insignificant, $\vartheta \rightarrow 0$, and the farmer behaves as if he were simply maximizing the utility derived solely from farming $(\overline{U})$. And similarly, if there is an improvement in the probability distribution of commodity price, $g(p_d) \rightarrow 0$, farm decisions move toward production efficiency. We may interpret $g(p_d)$ as a measure of the degree to which a farmer can marginally influence the probability of failure via production decisions. Regardless of potential adjustment costs, if there is no influence at the margin, $g(p_d) = 0$, then again the farmer acts as if he were simply maximizing farm-derived utility.

Note that with the general representations of $V(\,)$ and $g(\,)$ one cannot immediately determine the effect of changing farm size on the optimal choices of factors. Although p_d decreases with an increase in A, decreasing ϑ, $g(p_d)$ may decrease or increase. In addition V increases with an increase in farm size, positively affecting ϑ. It i, however, more likely that if p_d is small relative to μ, then $g(p_d)$ decreases with farm size. The effect of decreasing f, and thus decreasing the probability of failure, would similarly be to bring k^* and x^* into line with productive efficiency.

To be more specific, suppose the farmer is risk neutral. The first order conditions may then be written as

$$\frac{\partial y}{\partial k} = \frac{i}{\mu}/(1 + \vartheta) \qquad (1a)$$

$$\frac{\partial y}{\partial x} = \frac{w}{\mu}\frac{1 + \vartheta\frac{\mu}{p_d}}{1 + \vartheta} \tag{1b}$$

$$\vartheta = V\beta g(p_d)p_d/(A\mu y) \tag{1c}$$

As we have noted above, relative to $\vartheta = 0$, the marginal product of k is set lower and the marginal product of x is set higher. (This is true given $p_d < \mu$.) What happens to the actual levels of k and x depends of the degree of substitutability. For example, take k and x to be representative of single factors. From a point of zero adjustment costs ($\vartheta = 0$) the following show the effect on factor decisions due to an increase in adjustment costs:

$$\frac{\partial k}{\partial \vartheta} = \frac{-1}{\Delta}[y_{xx}\frac{i}{\mu} - y_{xk}\frac{w}{\mu}(1 - \frac{\mu}{P_d})]$$

$$\frac{\partial x}{\partial \vartheta} = \frac{-1}{\Delta}[y_{kk}\frac{w}{\mu}(1 - \frac{\mu}{P_d}) - y_{xk}\frac{i}{\mu}]$$

where Δ is the determinant of the matrix of second partial derivatives of y with respect to x and k, which we take to be negative definite.

These effects are of ambiguous sign. One can say, however, that if y_{xk}, then $\frac{\partial k}{\partial \vartheta} > 0$ and $\frac{\partial x}{\partial \vartheta} < 0$ (although this will hold for some x and k such that y_{xk} is positive but sufficiently close to zero).

The effect on total product is given by

$$\frac{\partial y}{\partial \vartheta} = y_k\frac{\partial k}{\partial \vartheta} + y_x\frac{\partial x}{\partial \vartheta}$$

Here again the effect is of ambiguous sign. In this case, however, if $y_{xk} < \frac{w}{i}y_{kk}$, then $\frac{\partial y}{\partial \vartheta} > 0$.

III A specific model

We now turn to a simple model of a farmer's behavior under risk of forced adjustment. This example is presented in two parts. In the first, a farmer avoids risk by expending a greater amount of his effort in the production of a cash income in order to increase the likelihood of covering the minimal necessary expenditures to remain a farmer. The purpose here is to show the possibility of a backward-bending supply curve; the discrete jumps in supply that may occur at various level of expected price; and the conditions under which a reduction in price variance reduces supply and improves productive efficiency.

The second part of this example turns to analyzing market equilibrium. A stable, long-run equilibrium is defined where there is zero probability of forced adjustment, and where expectations are rational. This example shows that rational-expectations equilibria can arise where farmers are permanently in survival mode, producing inefficiently. Further, if a survival-mode equilibrium exists with an inelastic demand curve, then there also exists another rational-expectations equilibrium with productive efficiency. This condition of two equilibria is an example of an over-production trap. That is, due to over-production, market conditions are such that over-production is optimal for individual farmers; and that, if in concert farmers reduced production to that of a conventional equilibrium, no individual would have incentive to expand. With an elastic demand curve there may exist a survival-mode equilibrium, but this cannot strictly be called a trap, since only a single rational-expectations equilibrium exists.

For ease of presentation, suppose there is simply one farmer in the market producing some level of commodity, y, out of effort, e, and receiving some level of price, p. Production is random, and, therefore, so will be price. Let the utility function from farming be the simple sum of the goods consumed out of farm revenues, py, over some minimal expenditure f, and leisure time: $U = (py - f) + (1 - e)$. Random production is given by $y = 2e^{1/2}\varepsilon$; where ε is a random term taking on two values (associated say with bad and good weather), $\varepsilon = \varepsilon_l$ or $\varepsilon = \varepsilon_h$ $(\varepsilon_l < \varepsilon_h)$, with equal probability. The demand curve is of constant elasticity $p = ay^{-b}$.

Therefore random revenues can be written as

$$py = a(2e^{1/2})^{-b}\, 2e^{1/2}\varepsilon^{1-b}$$

Hence, this problem with output and price random can be reduced to a simpler conceptual problem with only one source of randomness. It is conceptually easier to redefine price as having an expected value of $\mu = 2y^{-b}$ with multiplicative error of $u = \varepsilon^{1-b}$, where $E[u] = 1$. Now the new random term u can take on two values associated with the two values of ε: $u = u_l$ or $u = u_h$ $(u_l < u_h)$. The competitive farmer acts as if he cannot influence expected price; the farmer views revenues as $py = \mu u 2e^{1/2}$. The failure condition is where $py < f$, or where $e < (\frac{f}{2\mu u})^2$.

To summarize the previous discussion of the more general model, the farmer's objective function is given by

$$V = \frac{U(e) - I}{1 - \beta(1 - \pi)}$$

where $U(e)$ is the expected farm-derived utility, I the alternative utility of leaving farming, β the personal discount rate, and π the probability of failure. Now in this simple model we may set, without loss of generality, the non-farm utility level to zero, $I = 0$. The objective function may then be represented as

$$V = \begin{cases} \mu 2e^{1/2} + (1 - e) - f & \text{if } e^{1/2} < \dfrac{f}{2u_h\mu} \\[2ex] \dfrac{\mu 2e^{1/2} + (1 - e) - f}{1 - .5\beta} & \text{if } \dfrac{f}{2u_h\mu} \le e^{1/2} < \dfrac{f}{2u_l\mu} \\[2ex] \dfrac{\mu 2e^{1/2} + (1 - e) - f}{1 - \beta} & \text{if } e^{1/2} > \dfrac{f}{2u_l\mu} \end{cases}$$

Figure 1 shows one possible set of values of the function V. The concave lines show the values of the objective function for given levels of π. The heavily-drawn lines show the objective function taking π into account. The heavily-drawn lines represent the set of possible choices of effort open to the farmer. For the case shown in the figure, optimal choice of effort is where

$$e = \frac{f}{2u_l\mu}^2$$

Here there is no chance of failure, but the farmer is in survival mode and away from the point of productive efficiency, which is at $e = \mu^2$.

Optimal effort is given in table 2, where the functional representation of effort is conditional on regions in which expected price may fall. Optimal supply over expected price is graphically illustrated in figure 2. One should note that the farmer may not choose to be on a portion of the supply curve where $\pi = \frac{1}{2}$, instead either choosing a supply where $\pi = 0$, or where $\pi = 1$. The downward sloping portions of the supply curve are the regions of expected price where the farmer is in survival mode.

The affect of eliminating price variability, abstracting from equilibrium effects, is illustrated in figure 2. Consider a expected price of $\tilde{\mu}$. Eliminating variance ($var(u) \to 0$) yields an optimal supply of $y = 2\tilde{\mu}^2$.

Now consider the market equilibrium. A stable, long-run equilibrium is an expected market price, μ_e, such that the number of farmers is constant (i.e., $\pi = 0$), and where

$$\mu_e = a[y_e*]^{-b}$$

$$y_c* = 2e_c*^{1/2}$$

$$e_e* = e*(\mu_e)$$

Stability implies that equilibrium price falls along the segments cd and dj on the supply curve in figure 2. The most interesting case is where the farmer is in survival mode, but $\pi = 0$; that is, on segment cd of figure 2.

For a survival mode equilibrium to exist, expected price must be such that

$$\mu_e < [\frac{f}{2u_l}]^{1/2} .$$

In this case

$$\mu_e = a^{\frac{1}{1-b}} [\frac{f}{u_l}]^{\frac{-b}{1-b}} .$$

For an inelastic demand curve ($b > 1$), the conditions for survival survival-mode equilibrium are given by

$$\frac{f}{u_l} < 2^{\frac{1-b}{1+b}} a^{\frac{2}{1+b}} .$$

In fact, for all cases of demand, the above condition is simply that which provides for the standard (non-survival-mode) equilibrium. That is, if a survival-mode equilibrium exists, then a conventional one does also. A conventional equilibrium exists where

$$\mu_e = (2a)^{\frac{1}{1+b}} .$$

Of course, an elastic demand may also yield a stable, survival-mode equilibrium, but if one does not exist, then a conventional one would not exist as well. (The inequality above is reversed.) This leads to the idea of an over-production trap. For both inelastic and elastic demands, survival-mode equilibrium may exist. But only in the former case is one justified in using the term over-production trap. In the latter case, survival mode arises due only to the objectives of farmers. In the elastic-demand case, survival mode exists because of the objectives of farmers *and* the accident of expectations consistent with survival mode. Farmers are trapped by their rational expectations; without altering farmers' objectives, a conventional equilibrium may be attained.

IV Empirical Application

The preceding sections have discussed the influence of human-capital asset fixity on production decisions. This section turns to the practical matter of empirically addressing farm production based on the conceptual model above. First, one should note that, even under risk-neutrality as defined by a constant marginal utility of realized income, the usual assumptions underlying the use of cost and profit functions are inapplicable in this case, since the marginal-product-equals-price rule does not hold. Therefore, even if one used only disaggregated data, the conventional correlations of cost shares, for example, with factor prices would not represent production technology as standard application of duality theory would suggest. The difficulty is that optimally marginal products are set to effective prices, which are unobserved. These effective prices are the observed prices adjusted by other factors reflecting the influence of adjustment costs and the probability of failure.

In the standard application of duality, there are estimable equations for a production function (or cost, or profit) function and each marginal condition. Supposing that there are n choice variables with associated prices, the standard application would have $n + 1$ equations from which to estimate the parameters representing the production technology. This is possible because one may solve n choice variables for n prices, all of which can be observed. In the non-standard case we describe above, however we can only solve for the choice variables in terms of the n prices and 2 price-adjustment factors, $\frac{1}{1 + \vartheta}$ and $\frac{1 + \vartheta \frac{\mu}{p_d}}{1 + \vartheta}$ from equations (1a) and (1b).

Nevertheless, the marginal rates of technical substitution between inputs with common adjustment factors, are dependent only on observable prices. Therefore, at best $n-2$ choice variables can be solved in terms of observable prices and two inputs associated with different adjustment factors. And one can obtain $n-1$ equations from which to estimate production function parameters.

This is not surprising, since this model introduces at least two additional unknowns into the choice problem. Under risk-neutrality, besides the parameters defining technology, the true adjustment costs of leaving farming are unobserved as is the true probability of failure as a function of farm decisions. (In the more general problem there would be other unknowns as well, such as the parameters defining the expected utility function.) In order to estimate the production function parameters, unconditioned on adjustment costs and the failure probability, we must allow the influence of these two additional unknowns to be reflected through some similar number of observables.

At this point we should note at least one generalization on the model presented in the above sections. Instead of merely two types of factors, one that must be paid for immediately and one that need not be, there may exist factors that exhibit a mix of these two attributes. That is, only a fraction of the cost of such inputs appears in the equation that indicates failure. Admitting such factors introduces yet another unknown -- the fraction of costs that must be paid for today; and thus the number of estimable equations is further reduced. One could go so far as to admit that each input may have its own adjustment factor, reducing the available equations to estimate the technology to one.

To illustrate empirically the conceptual and theoretical models above, and to test certain hypotheses that arise from the theory, we turn to an analysis of corn production in Illinois. This study estimates a per-acre production function, assuming risk-neutrality, and making use of the plausible restrictions on parameter estimates. Furthermore, we test whether the adjustment factors driving a wedge between marginal products and prices, move in the direction implied by our model and intuition. Specifically, we wish to see if input choices approach productive efficiency as farm size increase and over time (as farming grows more integrated into the larger economy). Additionally, we are interested in what degree government programs affect the deviation of input choice from productive efficiency.

We choose four inputs (in per acre amounts) to a Cobb-Douglas production function. Following the conceptual model, the inputs are separated into those that must be paid for immediately, fertilizer and hired labor, and those that may go unpaid, farm family labor and physical capital, represented by machinery use. In order to reduce the scope of the problem, we assume that in the estimates of a corn production function, one may ignore the possible influence of other crops -- both in the usual joint-production sense, and, more importantly, in the sense of choosing a portfolio; and moreover, one may restrict the number of separately chosen inputs to the four mentioned. The data are taken from *Summaries* of Illinois Farm Business Records from 1971 to 1979. The data are averages of farm characteristics, production levels, factor expenses, and labor employed for farms in particular size ranges and regions. Price data are from the *Summaries*, *Agricultural Statistics*, and the *Commodity Yearbook*.

We represent yield by Y, and input levels by X_i; where $i = 1$ denotes fertilizer, $i = 2$ hired labor, $i = 3$ family labor, and $i = 4$ machinery. The W_i represent prices associated with the inputs, and the α_i represent associated elasticities to be estimated. We assume risk neutrality of farmers and that output price is independent of output, which is reasonable given the level of disaggregation of the data. The April quote of the November futures contract for corn is taken to proxy expected output price, and we represent this expected price by μ. We also include as shift variables the amount of tillable acreage (Z_1) to account for a scale effect, and the average soil quality of the farm (Z_2). (See the *Summaries* for a definition of this variable.) Finally, note that the marginal conditions for optimization are

$$\frac{\mu Y}{W_i X_i} \alpha_i = \rho_1 \quad i = 1,2$$

$$\frac{\mu Y}{W_j X_j} \alpha_j = \rho_2 \quad j = 3,4$$

where $\rho_i, i = 1,2$ represents the unknown adjustment factors associated with the two types of inputs.

We may represent the production function as

$$\ln Y = k + \sum_{i=1}^{4} \alpha_i \ln X_i + \sum_{j=1}^{2} \beta_j \ln Z_j$$

Applying the marginal product restrictions within each input group we can specify the system of equations

$$\ln Y = \alpha_0 + \alpha_1 \ln X_1 + \alpha_2 \ln \frac{W_1 X_1}{W_2} \alpha_3 \ln X_3 + \alpha_4 \ln \frac{W_3 X_3}{W_4} + \beta_1 \ln Z_1 + \beta_2 \ln Z_2 + u_1 \qquad (2a)$$

$$\frac{W_2 X_2}{\mu Y} = r_1 \frac{W_1 X_1}{\mu Y} + u_2 \qquad (2b)$$

$$\frac{W_4 X_4}{\mu Y} = r_3 \frac{W_3 X_3}{\mu Y} + u_3 \qquad (2c)$$

where $r_1 = \frac{\alpha_2}{\alpha_1}$, and $r_3 = \frac{\alpha_4}{\alpha_3}$.

Of particular interest is how the adjustment factors (ρ_i) are influenced by non-input variables. In order to investigate this, we assume that the adjustment factors may be represented in the following manner

$$\rho_1 = \frac{\mu Y}{W_i X_i} \alpha_i = a_1 + b_1 S + c_1 T + d_1 D + u_{1i} \quad i = 1,2$$

$$\rho_2 = \frac{\mu Y}{W_j X_j}\alpha_j = a_2 + b_2 S + c_2 T + d_2 D + u_{2j} \quad j = 3,4$$

where S represents farm size in gross acreage, T a time index, and D the per cent tillable land under program diversion.

From the discussions in the previous sections, we expect that as farm size grows the adjustment factors approach one, falling ($b_1 < 0$) in the case of fertilizer and hired labor ($i = 1,2$), and increasing ($b_2 > 0$) in the case of family labor and machinery ($j = 3,4$). The time index we expect to reflect an overall improvement in alternatives for farm family labor, growing integration of the farm sector and the larger economy, and other structural changes that promote stricter profit-maximizing (that is, $c_1 < 0$ and $c_2 > 0$). The program variable, D, is meant to reflect farmer's response to government payments. We may identify three effects from government programs. First, programs may reduce the risk of failure and thereby promote efficiency. Second, programs may increase the opportunity cost of leaving farming, at the same time reducing failure risk, and thereby promote inefficiency. And third, programs may increase the effective output price through target prices.

Although there are three distinct effects associated with government programs, we may be able to detect which has the greater influence by examining the pair of coefficients d_1 and d_2. If the first effect predominates, then the sign of the coefficient on D is expected to be positive for family labor and machinery ($d_2 > 0$), and negative for fertilizer and hired labor ($d_1 < 0$). If the second effect predominates, then the signs on this coefficient will be reversed ($d_1 > 0$ and $d_2 < 0$). Finally, if the third effect predominates, then the signs on both coefficients will be negative for both equations ($d_1 < 0$ and $d_2 < 0$).

Using the first order condition, we may add four additional equations to the system

$$\rho_1 = \frac{\mu Y}{W_1 X_1} = \frac{a_1}{\alpha_1} + \frac{b_1}{\alpha_1}S + \frac{c_1}{\alpha_1}T + \frac{d_1}{\alpha_1}D + \frac{u_{11}}{\alpha_1} \quad (3a)$$

$$\rho_1 = \frac{\mu Y}{W_2 X_2} = \frac{a_1}{\alpha_2} + \frac{b_1}{\alpha_2}S + \frac{c_1}{\alpha_2}T + \frac{d_1}{\alpha_2}D + \frac{u_{12}}{\alpha_2} \quad (3b)$$

$$\rho_2 = \frac{\mu Y}{W_3 X_3} = \frac{a_2}{\alpha_3} + \frac{b_2}{\alpha_3}S + \frac{c_2}{\alpha_3}T + \frac{d_2}{\alpha_3}D + \frac{u_{23}}{\alpha_3} \quad (3c)$$

$$\rho_2 = \frac{\mu Y}{W_4 X_4} = \frac{a_2}{\alpha_4} + \frac{b_2}{\alpha_4}S + \frac{c_2}{\alpha_4}T + \frac{d_2}{\alpha_4}D + \frac{u_{24}}{\alpha_4} \quad (3d)$$

Unfortunately, this system involves a several non-linear restrictions on the parameters, therefore, a two-stage estimation procedure is employed. First, use equations (2b) and (2c) to obtain estimates of the ratios of elasticities $\frac{\alpha_2}{\alpha_1}$ and $\frac{\alpha_4}{\alpha_3}$, say $\hat{r}_1$ and $\hat{r}_3$. Use those estimates in equation (2a) to obtain

$$\ln Y = \alpha_0 + \alpha_1[\ln X_1 + \hat{r}_1 \ln \frac{W_1 X_1}{W_2}] + \alpha_3[\ln X_3 + \hat{r}_3 \ln \frac{W_3 X_3}{W_4}] + \beta_1 \ln Z_1 + \beta_2 \ln Z_2 + u_1 \quad (4)$$

Then impose linear restrictions on the four equations given by (3a) - (3d); for example, $a_2' = \hat{r}_1 a_1'$ and $b_4' = \hat{r}_3 b_3'$, etc. The second step is to estimate the system given by equation (4) and the four equations given by (3a) - (3d), with the linear restrictions.

The results of the estimation process are reported in Table 3. The estimated elasticities are all of expected sign and of plausible magnitude. One result particularly to note is the large difference between the elasticity of family labor and that of hired labor. Another result to note is the sign and magnitude of the coefficient on acreage under corn, indicating a positive scale effect. The per-acre production function, however, exhibits decreasing returns.

The coefficients on farm size in the adjustment-factor equations support the conclusions that can be drawn from the conceptual model. Over farm size the adjustment factor falls for fertilizer and hired labor and increases for family labor and machinery.

The coefficients on the time index suggest that, for a given farm size and degree of program participation, production decisions have been deviating further from pure maximization. These results are somewhat surprising in light of accepted wisdom that agriculture is in transition to greater integration in the rest of the economy. First, it must be remembered that the data are for a fairly short period of time, between 1971 and 1979. Second, the results for the time variable are supported at least in part by the recent work of Vasavada on the measurement of excess inputs. Using a dynamic adjustment model, he concludes that for aggregate levels of labor and capital, surpluses have shown a marked tendency to decline over a longer period of time (since 1948). During the 1970s, however, his results demonstrate a decline followed by an upswing in input-surplus indices. Troughs occur in 1972 for capital and 1974-1975 for labor.

The coefficients on the program variable are of opposite signs for the two adjustment-factor equations, and support the conclusion that programs exacerbate the deviation from production efficiency.

V Conclusions

The analytical and empirical results of this paper offer some insight into farmer behavior under risk of being forced to leave farming. A farmer cannot purchase insurance against such a risk, both because of problems of access to adequate credit, and because there exists no formalized market for insurance. The farmer, therefore, seeks to mitigate against this risk by deviating in his production decisions from what is optimal from a simple expected-profit maximizing case. Production factors with immediate cash outlay tend to have higher effective prices than without the risk, since part of their cost must be measured in the contribution to increasing the probability of failure. The marginal products of these factors are set higher than observed prices would optimally warrant. Conversely, factors that may be delayed in cash expenditure tend to have lower effective prices for the opposite reason, and their marginal products are set lower than observed prices warrant.

Factors of the last type are of particular interest, since their contribution to aggregate capacity may be of greatest significance. Farm-operator labor, or farm family labor, tends to be over-utilized at the expense of non-monetary alternatives. (In the particularized model above, this non-monetary alternative is the farmer's leisure time.) Increasing monetary alternatives -- that is, improving opportunities for off-farm income -- would encourage the farmer to use operator labor in a similar manner to hired labor (aside from quality considerations). Physical capital owned by the farmer is treated in the same way. Although the empirical study concentrates only on machinery in the analysis, physical capital (more importantly perhaps) also includes aspects of land quality and long-term productivity. The conceptual analysis suggests that during periods when farmers face higher probability of failure and human-capital adjustment costs are larger, farmers would tend consciously to increase the deterioration of their land resources.

This last point underlines a feature of the agricultural economy that is of disturbing significance for farm policy. When output prices fall, the farmer sees both an increase in the probability failure and a decrease in the expected utility derived solely from farming. If the larger, non-farm economy remains unchanged, the farmer also experiences a fall in the adjustment cost of leaving agriculture; and this would tend to lessen any further deviation from efficient production that a greater probability of failure would promote. Unfortunately, downturns in agriculture are likely to be worse during downturns in the larger economy. Opportunities for off-farm income, if one wishes to remain in farming, are likely to be reduced, and, more importantly for the potential of a timely reduction in aggregate capacity, possibilities for alternative employment outside agriculture are likely to be few. This feature would tend to increase adjustment costs at the same time the probability of failure is increasing, exacerbating the deviation from efficient farm production decisions.

This paper also offers an empirical investigation of corn production in Illinois. This is done in order both to demonstrate the applicability of the conceptual model, and to substantiate certain conclusions that can be drawn regarding the degree of deviation from simple profit-maximization. Conventional estimation of dual functions, such as those of cost and profit, is unwarranted in the presence of risky adjustment costs. Nevertheless, the theory does admit certain restrictions to an estimable system of supply and factor demands, from which one can use output- and input-price data in the estimation of production technology. These restrictions are a subset of those allowed under the assumptions of strict profit-maximizing (or cost-minimizing). In addition to prices as explanatory variables, levels of a certain number of representative inputs (in this study, two) must be used with a corresponding reduction in the number of estimable equations.

If one is willing to make certain assumptions regarding the proportions by which observed input prices must be adjusted to equal effective prices, then these proportions, or adjustment factors, may serve as additional estimable functions of exogenous (structural) variables. The estimation results indicate that larger farms deviate less from production efficiency than do smaller farms, but that over the period of time studied farmers have been moving further from setting marginal products equal to observed prices. Furthermore, the results indicate that government programs may tend to counter the trend toward efficiency, which would accompany traditional expansion of average farm size, rather than to alleviate the effects on production decisions of avoiding failure.

Footnotes

1/For example, Thompson, Gwynn, and Sharp report the survey of married farm women in Yolo County, California, and remark: "The increased participation by women on smaller farms was found to result from the need for the entire family to use its total resource for survival rather than to a greater opportunity for women to participate on small farms."

2/This eyebrow-raising comment was made by Senator Tom Harkin of Iowa in critical remarks at a conference on Farm Policy sponsored by Resources for the Future, Washington, D.C., December, 1984.

3/In a more general sense, models of risk-averse agents imply the purchase of stability in exchange for inefficient profit maximization.

4/For a broad view of asset fixity in U.S. agriculture, see G. Johnson and Quance, eds. Of particular interest for this present paper is the chapter in that volume on labor by Chennareddy and Jones. M. Johnson and Pasour may be read as a reminder not to confuse adjustment costs of moving inputs with irrationality of decision makers. Finally, one may find differing, specific definitions of asset fixity in empirical studies. For example, Chambers and Vasavada define asset fixity in terms of putty-clay technology (which they reject for a aggregate U.S. production function).

5/For example, the tomato harvester is a very lumpy asset; once acquired its productive capacity is not subject to incremental changes that might otherwise be profitable as price changes. By examining the physical nature of what a tomato harvester actually is, one concludes that it is a fixed asset -- at least over a range of output prices below that which induced its purchase. On the other hand, the capital embodied in the tomato harvester is specific; and, even if the machine were physically divisible, its off-farm-use value is negligible. (That is, the capital embodied in the machine is not divisible between farming and non-farming.) By examining the difference between the harvester's on-farm value and its off-farm, or salvage, value, one similarly concludes that it is a fixed asset -- without actually knowing what the machine is.

6/For an empirical analysis of the influence of psychic costs on rural-urban migration, see Deaton, Morgan, and Anshell. In her extremely interesting discussion, "Farm Family Displacement and Stress," Boss remarks, "We see a lot of resistance to change in farm families even when it appears that the change would be a positive step in the long run. This resistance is found in all people, not just farm families. Most of us do not like change, and we resist it despite the handwriting on the wall. One has to admire the *irrational* perseverance of these farm families. They keep trying in the face of a reality that clearly tells them it will not work. (p. 74, emphasis added)" As economists, we would not say irrational, but at worst rational actions based on ill-conceived perceptions of true benefits and costs. The farm family may not have experience with certain acts that might (or so some would say) make it happier. It is not so much that the family ignores the clear failure of the present strategy, but that a better alternative is far from obvious.

7/Pertinent to this notion of minimal expenditures may be items rarely found in production economics texts. For example, Boss lists certain barriers to coping with change, one of which is most often ignored in conventional economic analyses of farm decisions: "...[T]here is the problem of the farmer's 'machismo.' Here I consciously refer to the man on the farm who too often tries to keep up with the other men in the acquisition of tractors, livestock, acres, number of silos, and bigger and better trucks. (p. 74)" Similar to this catalogue of "machismo," one might also add more feminine items that contribute to minimal expenditures. Again, as economists, we would restate this following Akerlof's theory of social custom (Akerlof 1980), or by hypothesizing that keeping up with others is a rule of thumb that has proven profitable in the past.

but not immediately obviously harmful today.

8/There may still exist a question of socially-optimal scale for farms. This additional subtlety to the problem we leave for future research.

9/At this point we should point out that the specification of survival -- covering minimal expenses -- is stylized in one important way. Minimal expenses, f, is a constant, and specifically not an increasing function of three items: farm size, input use, or past failure to cover all expenses (i.e., some portion of past ik). Although relaxing the current simplification would complicate matters and presentation, to make f a function of the first two items would not alter the basic implications of the model as long as f increases over these items at a decreasing rate. The third item is conceptually more important. If the farmer delays payment on some portion of the cost of inputs k, one expects that portion to contribute to higher minimal expenditures in the future (or perhaps decreased output), and thus a higher future probability of failure for all levels of inputs. This would tend to blur the distinction between x and k in the active avoidance of failure. Nevertheless, as long as the farmer does not have to pay a price of delaying payment of more than his discount rate, and at failure time all delayed payments outstanding are forgotten, then k would be a more attractive input than x using the simple criterion of minimizing probability of failure.

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DISCOUNT FACTOR	PROBABILITY: π	$\pi(1-\pi)$	$\pi(1-\pi)^2$	$\pi(1-\pi)^3$
1	U_e^F	U_e^N	U_e^N	U_e^N
β	I	U_e^F	U_e^N	U_e^N
β^2	I	I	U_e^F	U_e^N
β^3	I	I	I	U_e^F
β^4	I	I	I	I
.	.	.	.	.
.	.	.	.	.

$$U_e^F = \mathbb{E}(U \mid P < P_d)$$

$$U_e^N = \mathbb{E}(U \mid P \geq P_d)$$

Table 1: Possible Future Utility Levels

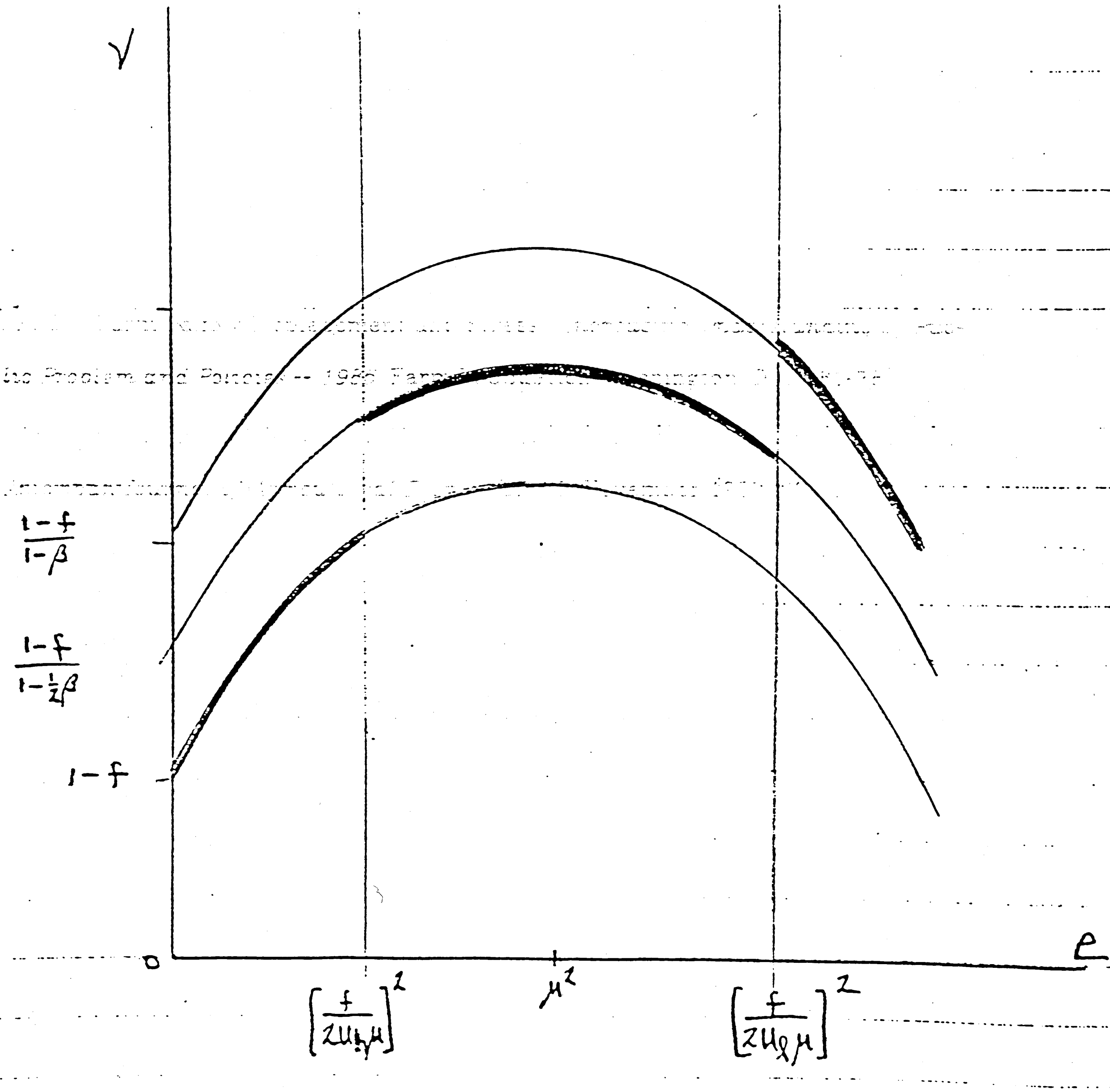

Figure 1: Values of the Objective Function over Effort

Table 2: Optimal Effort Conditional on Regions of Expected Price

e^*	Condition μ
μ^2	$\mu \geq \frac{f}{2u_l\mu}$
$\left[\frac{f}{2u_l\mu}\right]^2$	$\frac{\frac{f}{u_l} - (\frac{f}{2u_l\mu})^2 + 1 - f}{1-\beta} > \frac{\mu^2+1-f}{1-\frac{1}{2}\beta}$ $\mu < \frac{f}{2u_l\mu}$
μ^2	$\frac{\mu^2+1-f}{1-\frac{1}{2}\beta} > \frac{\frac{f}{u_l} + (\frac{f}{2u_l\mu})^2 + 1 - f}{1-\beta}$ $\frac{f}{2u_l\mu} \leq \mu < \frac{f}{2u_l\mu}$
$(\frac{f}{2u_h\mu})^2$	$\frac{\frac{f}{u_h} + 1 - (\frac{f}{2u_h\mu})^2 - f}{1-\frac{1}{2}\beta} > \mu^2+1-f$ $\frac{\frac{f}{u_l} - (\frac{f}{2u_l\mu})^2 + 1 - f}{1-\beta} < \frac{\mu^2+1-f}{1-\frac{1}{2}\beta}$ $\mu < \frac{f}{2u_h\mu}$
μ^2	$\mu^2+1-f > \frac{\frac{f}{u_h} - (\frac{f}{2u_h\mu})^2 + 1 - f}{1-\frac{1}{2}\beta}$ $\mu^2+1-f > \frac{\frac{f}{u_l} - (\frac{f}{2u_l\mu})^2 + 1 - f}{1-\beta}$ $\mu < \frac{f}{2u_h\mu}$

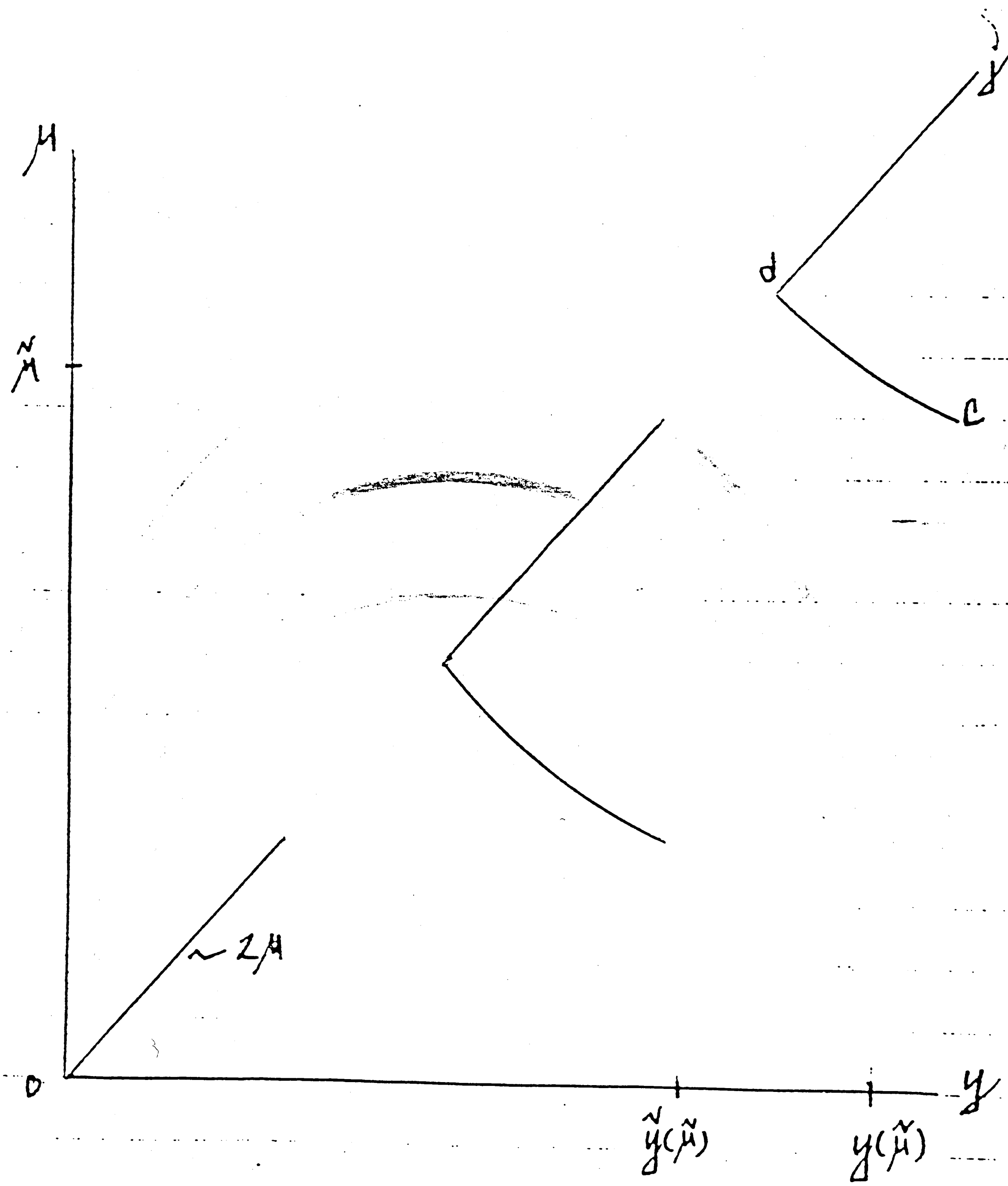

Figure 2: Optimal Supply over Expected Price

$$r_1 = \frac{\alpha_2}{\alpha_1} = 0.19008 \quad (25.173)^*$$

$$r_3 = \frac{\alpha_4}{\alpha_3} = 1.7534 \quad (35.076)$$

$$\alpha_0 = 2.9123 \quad (7.9106)$$

$$\alpha_1 = 0.11647 \quad (2.5523)$$

$$\alpha_3 = 0.14045 \quad (5.2308)$$

$$\beta_1 = 0.30280 \quad (5.2067)$$

$$\beta_2 = 0.00658 \quad (10.039)$$

$$\frac{a_1}{\alpha_1} = r_1 \frac{a_1}{\alpha_2} = 17.702 \quad (25.344)$$

$$\frac{b_1}{\alpha_1} = r_1 \frac{b_1}{\alpha_2} = -0.01008 \quad (-10.372)$$

$$\frac{c_1}{\alpha_1} = r_1 \frac{c_1}{\alpha_2} = 0.54715 \quad (3.4693)$$

$$\frac{d_1}{\alpha_1} = r_1 \frac{d_1}{\alpha_2} = 0.20314 \quad (2.2141)$$

$$\frac{a_2}{\alpha_3} = r_3 \frac{a_2}{\alpha_3} = 6.3354 \quad (11.755)$$

$$\frac{b_2}{\alpha_3} = r_3 \frac{b_2}{\alpha_3} = 0.01333 \quad (17.736)$$

$$\frac{c_2}{\alpha_3} = r_3 \frac{c_2}{\alpha_3} = -0.26217 \quad (-2.2199)$$

$$\frac{d_2}{\alpha_3} = r_3 \frac{d_2}{\alpha_3} = -0.36563 \quad (-5.4540)$$

$\alpha_2 = 0.02214$ $\qquad$ $\alpha_4 = 0.2463$

*(t-stats)

Table 3. Coefficient Estimates

r_1	$=$	$\frac{\alpha_2}{\alpha_1}$	$= 0.19008$	(25.173)*
r_3	$=$	$\frac{\alpha_4}{\alpha_3}$	$= 1.7534$	(35.076)
α_0			$= 2.9123$	(7.9106)
α_1			$= 0.11647$	(2.5523)
α_3			$= 0.14045$	(5.2308)
β_1			$= 0.30280$	(5.2067)
β_2			$= 0.00658$	(10.039)
$\frac{a_1}{\alpha_1}$	$=$	$r_1 \frac{a_1}{\alpha_2}$	$= 17.702$	(25.344)
$\frac{b_1}{\alpha_1}$	$=$	$r_1 \frac{b_1}{\alpha_2}$	$= -0.01008$	(-10.372)
$\frac{c_1}{\alpha_1}$	$=$	$r_1 \frac{c_1}{\alpha_2}$	$= 0.54715$	(3.4693)
$\frac{d_1}{\alpha_1}$	$=$	$r_1 \frac{d_1}{\alpha_2}$	$= 0.20314$	(2.2141)
$\frac{a_2}{\alpha_3}$	$=$	$r_3 \frac{a_2}{\alpha_3}$	$= 6.3354$	(11.755)
$\frac{b_2}{\alpha_3}$	$=$	$r_3 \frac{b_2}{\alpha_3}$	$= 0.01333$	(17.736)
$\frac{c_2}{\alpha_3}$	$=$	$r_3 \frac{c_2}{\alpha_3}$	$= -0.26217$	(-2.2199)
$\frac{d_2}{\alpha_3}$	$=$	$r_3 \frac{d_2}{\alpha_3}$	$= -0.36563$	(-5.4540)

$\alpha_2 = 0.02214$ $\alpha_4 = 0.2463$

*(t-stats)

Table 3. Coefficient Estimates