HOUSEHOLD INVENTORIES AND MARKETED SURPLUS IN SEMI-SUBSISTENCE AGRICULTURE

Mitch Renkow
North Carolina State University

Abstract

A model of semi-subsistence agriculture recognizing the ability of farm households to hold inventories of home-grown staple foods is developed. The model's solution suggests a method for empirically distinguishing between food security and arbitrage motives for holding inventories. A formula for computing marketed surplus response is derived.

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I. Introduction

A notable feature of nearly all developing countries is that most agricultural households produce a significant portion of the staple foods which they consume. This is the case across a wide cross-section of geographical locations, levels of technological advancement, and land tenure arrangements. How these "semi-subsistence" households allocate output of staple foods between home consumption and market sales has been the subject of considerable analysis by economists, one which has important implications for the determination of aggregate market supply, food disappearance patterns, and the attendant nutritional consequences for rural and urban dwellers.

A major focus of this research has been measuring the response of marketed supply (or "marketed surplus") to changes in prices and other exogenous variables. Most analyses begin by positing an identity which sets marketed surplus equal to the difference between output and consumption. Differentiating this identity then yields an expression for the price elasticity of marketed supply as a function of the price and income elasticities of consumption and the price elasticity of total output.

The point of departure for this paper lies in the omission of household storage of staple foods from existing work on semi-subsistence households. Implicitly, all work to date has assumed that households costlessly store exactly the amount allocated to home consumption over the period between harvests. Arguably,

this is a relatively harmless simplification, especially if the marginal cost of storage is low. A more serious conceptual problem, however, is that existing models take consumption, output, and sales as occurring simultaneously. Thus, a change in the price of a commodity which is produced by the household affects marketed surplus through its impact on both contemporaneous consumption and production. This is not normally the case; rather, the output from which marketed surplus is drawn is generally predetermined and exists in the form of currently held inventories and/or recent harvests. Stocks on hand and expected future output may be expected to influence marketing decisions, but only through wealth effects on consumption. As will be demonstrated below, highly restrictive assumptions on the ability of households to hold inventories and on the formation of price expectations are required in order to justify the analytical approach which has previously been used.

Two factors may be surmised as primarily motivating semi-subsistence households to hold inventories of staple foods. First, households might want to minimize their reliance on local markets for the satisfaction of basic food needs and hold stocks of food as a contingency against potential supply disruptions over which they have no control. Second, inventories of home-produced staples might result from profit-seeking behavior if household decision-makers perceive opportunities to take advantage of intra-seasonal price movements for a particular storable commodity. Subject to limitations on on-farm storage capacity

and storage costs, such arbitrage opportunities may well influence the timing of market sales.

In the next section a simple model of the economic behavior of semi-subsistence farm households is developed. The model follows in the tradition of the household-firm literature (Jorgensen and Lau; Barnum and Squire) but is distinguished from previous work by its recognition of the ability of households to store important consumption items (specifically, staple foods). The model's solution highlights the importance of expected future prices as an argument of both inventory and commodity demand functions, and suggests a method of empirically distinguishing between different motives for holding stocks. In the final section, an expression for the own-price response of marketed surplus is derived and examined against comparable expressions derived from earlier models. The paper concludes with a brief discussion of how the model developed in the paper is being implemented empirically.

II. An Agricultural Household Model with Storage

Consider a representative semi-subsistence household that produces a single storable food commodity which is either consumed or traded for one other (composite) commodity. Utility is derived solely through the consumption of the two goods and leisure,[1] with the one-period household utility function given by

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[1] The model can easily be expanded to include vectors of storable and non-storable commodities, cash crop production, and production of non-storable foods. The "bare-bones" model presented

4

(1) $U_t = U(X_{1t}, X_{2t}, X_{Lt})$,

where $U(\cdot)$ is a twice-differentiable, continuous, quasi-concave function. The household is assumed to maximize the expected (discounted) value of a stream of current and future utilities up to the end of the current cropping cycle. Each cropping cycle is composed of T+1 periods $(0,...,T)$, extending from harvest to harvest, and cycles overlap in the sense that period T of one coincides with period 0 of the next.

At the beginning of each cropping cycle the quantity Q_0 is harvested. In each period, the household sells the quantity M_t at price P_{1t}; consumes an amount X_{1t} of the produced commodity; and purchases an amount X_{2t} of the alternate commodity at price P_{2t}. The portion of marketable surplus not sold or consumed in a given period must be stored. Disappearance of the produced commodity in period t is governed by the stock identity[2]

(2) $I_t - I_{t+1} + Q_t = X_{1t} + M_t$,

where I_t and I_{t+1} are carryin and carryout inventory levels and Q_t is output of the produced commodity in period t. For simplicity, output is assumed to depend only on one variable input (labor) and one fixed factor (land):

(3) $Q_T = Q(L_0,...,L_{T-1};A)$,

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here is sufficient to illustrate the salient points, however.

[2]Storage losses are ignored here because they do not substantively affect the behavioral implications of the first order conditions. Assuming proportional losses has an effect identical to lowering the household's subjective rate of time preference.

where L_t denotes total labor used by the household in period t (both family and hired) and A is the household's fixed quantity of land. By definition, $Q_t = 0$ for $t = 1,\ldots,T-1$.

Assume that the one-period cost of holding inventories is given by

(4) $\qquad C(I_{t+1},X_{1t}) = \tfrac{1}{2}f^{-1}(I_{t+1} - g_0 - gX_{1t})^2, \quad > 0 \quad f,g \geq 0.$

This specification -- discussed in Holt, et al. and Belsley, and more recently employed by Wohlgenant -- posits inventory costs as being composed of two offsetting components. The first is the physical cost of holding stocks, which is increasing in I_{t+1}. The second is the convenience yield derived from having stocks on hand with which to satisfy household consumption demand. In the production oriented inventory literature this latter component typically refers to the opportunity cost of stock-outs and back-ordering. Here the emphasis is on consumption, but the logic is nonetheless the same.

In the context of semi-subsistence agriculture, it seems likely that the cost of stocking out rises more steeply than the physical cost of holding inventories. This would certainly be the case if a market for the stored commodity were absent, as then stocking out would mean doing without. Alternatively, the existence of covariate production risk over a large geographical area -- a feature characteristic of rainfed agriculture in many locations -- would tend to cause the marginal cost of stocking out to be high, even in the presence of complete markets. In this event, an individual household's poor harvest would tend to
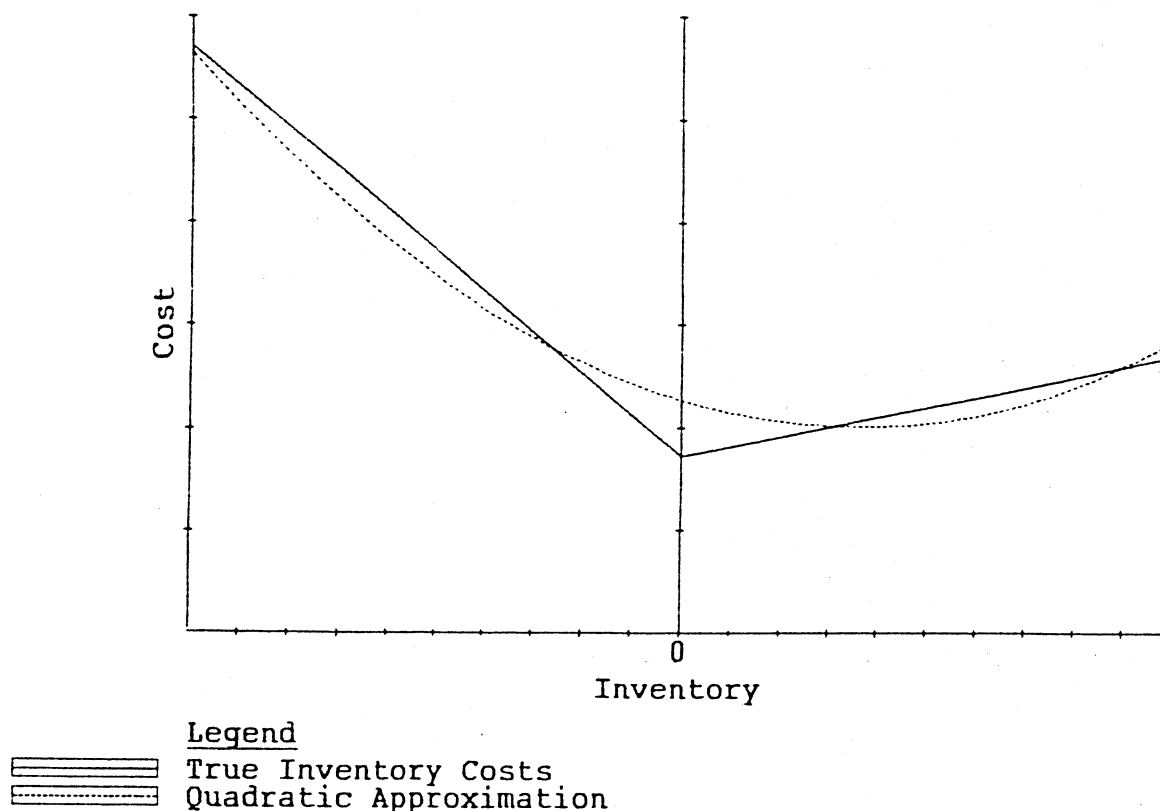
Figure 1. Quadratic Approximation of Inventory Costs

be correlated with diminished aggregate supply (and attendant

higher prices).

In light of the preceding discussion, equation (4) may be

regarded as a reasonable approximation to the household's true

storage costs. This is demonstrated in Figure 1.   Inventory

holdings are plotted on the x-axis and costs on the y-axis. For

positive inventory holdings, storage costs rise in proportion to

the quantity held. To the left of the y-axis the cost of not

having stocks on hand to meet demand rises more steeply. The

quadratic specification of equation (4) -- shown as a dotted line

-- approximates the "true" inventory cost function.

Sources of cash income for the household include sales of

agricultural output, off-farm labor earnings (at a wage P_{Lt}), and exogenous non-wage income (Y_t). Households are also assumed to be able to borrow (B_t) at a one-period interest rate r. Household expenditures consist of commodity purchases, storage costs, production costs, and loan repayments. Thus in each period the household faces a budget constraint given by

$$(5) \quad P_{1t}M_t + P_{Lt}(F_t - L_t) + Y_t + B_t = P_{2t}X_{2t} + C(\cdot,\cdot) + (1+r)B_{t-1},$$

where F_t is family labor. It is assumed that markets exist for all commodities and labor, that hired and family labor are perfect substitutes, and that the household is a price taker in these markets, thus insuring that the model is recursive (Barnum and Squire). To complete the model, a time constraint states that in each period the total time available to the household (T^*) is devoted to either leisure (X_{Lt}) or labor:

$$(6) \quad T^* = X_{Lt} + F_t.$$

Equations (1)-(6) above define an optimization problem to be solved in each period by the household. To obtain a solution to this problem, first solve (2) for M_t and (6) for F_t. Next, substitute these and (3) into the budget constraint and form the Lagrangean

$$(7) \quad \text{Max } E_t \sum_{s=t}^{T} b^{s-t} \{U(X_{1s},X_{2s},X_{Ls}) + \lambda_s [P_{1s}(Q_s + I_s - I_{s+1} - X_{1s})$$
$$+ P_{Ls}(T^* - X_{Ls} - L_s) + B_s + Y_s - P_{2s}X_{2s} - C(I_{s+1},X_{1s})$$
$$- (1+r)B_{s-1}]\}, \qquad Z = (X_{1t},X_{2t},X_{Lt},I_{t+1},B_t,L_t)$$

Here $b = (1+r)^{-1}$ is the discount rate (assumed constant) and E_t denotes a mathematical expectation conditional on information Ω_t

available at time t. Ω_t is assumed to include all current and past values of the control variables in Z and prices. Differentiating (7) with respect to the control variables yields the following first order conditions for any period t:

(8) $\qquad U_{1t} - \lambda_t (P_{1t} + C_x) = 0$

(9) $\qquad U_{2t} - \lambda_t P_{2t} = 0$

(10) $\qquad U_{Lt} + \lambda_t P_{Lt} = 0$

(11) $\qquad bE_t (\lambda_{t+1} E_t P_{1t+1}) - \lambda_t (P_{1t} + C_I) = 0$

(12) $\qquad E_t (\lambda_t - \lambda_{t+1}) = 0$

(13) $\qquad b^{r-t} E_t P_{1t} (\partial Q_t / \partial L_t) - P_{Lt} = 0$

plus the budget constraint. λ_t is the marginal utility of period t income and C_I and C_x denote the partial derivatives of the inventory cost function with respect to I_{t+1} and X_{1t}.

Equations (8)-(10) yield the standard result that at the optimum the marginal rate of substitution between any pair of goods (including leisure) will equated to the ratio of the prices of those goods. Note however that the full price of the storable commodity includes the marginal convenience yield (C_x) associated with it. The sign of C_x will be ambiguous, depending on current demands for consumption and inventories of the produced commodity, and the parameters describing convenience yields (g and g_e).

Equation (13) yields another standard result, namely that the optimal allocation of labor into the production process is such that the (expected) value of its marginal product equals the wage rate. This result also highlights the recursiveness of the model in that none of the demand-side control variables appear in

(13).

Given the assumed inventory cost function discussed above, equations (11) and (12)[a] imply that optimal inventory holdings will be a linear function of the current and (discounted) expected value of the next period's price for the stored commodity and the household's demand for the commodity in the current period. Substituting (4) and (12) into (11) and re-arranging:

(14) $I_{t+1} = g_0 + f(bE_t P_{t+1} - P_{t}) + gX_{1t}$.

This result makes it clear that the expected price of the stored commodity, $E_t P_{t+1}$, will appear as an argument in the household's Marshallian demands for goods and leisure. The reason is straightforward: the effect on inventory demand of a ceteris paribus change in $E_t P_{t+1}$ will lead to a change in either M_t or X_{1t} (or both) and consequently alter the composition of the commodity bundle consumed by the household.

Equation (14) is important for another reason. It provides an empirically tractable way of distinguishing between motives for holding inventories discussed above. The parameters f and g may be regarded as measures of the strength of the arbitrage and food security motives for holding inventories discussed earlier. Estimating (14) either as a single equation or as part of a system allows one to test if either of these motives are empirically insignificant.

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[a] The result that $E_t \lambda_{t+1} = \lambda_t$ is derived somewhat differently by Browning, et al. in the context of a life-cycle model of labor supply. Here, it hinges on the assumption that the utility rate of time preference equals the rate of interest on borrowed funds.

III. Marketed Surplus Response Revisited

By explicitly considering inventory-holding as an element of the household's overall allocation problem, the model developed above represents a significant departure from previous analyses of semi-subsistence households. Inventories add an inter-temporal dimension to the problem, calling attention to the effect of price expectations on both inventory demand and consumption demand. The model also treats the timing of the household's economic activity more carefully than previous work. In particular, the assumption that production, consumption and sales occur simultaneously has been abandoned. Instead, the present analysis recognizes that the output from which marketed surplus is drawn by the household is predetermined, existing in the form of currently held inventories and new production. This has implications for marketed surplus response which are taken up presently.

Differentiating (2) with respect to P_{1t} (and noting that $dQ_t/dP_{1t} = dI_t/dP_{1t} = 0$)

$$(15) \qquad \frac{dM_{1t}}{dP_{1t}} = - \frac{dX_{1t}}{dP_{1t}} - \frac{dI_{t+1}}{dP_{1t}}.$$

From equation (14)

$$(16) \qquad \frac{dI_{t+1}}{dP_{1t}} = \frac{\partial I_{t+1}}{\partial E_t P_{1t+1}} \cdot \frac{dE_t P_{1t+1}}{dP_{1t}} + \frac{\partial I_{t+1}}{\partial P_{1t}} + \frac{\partial I_{t+1}}{\partial X_{1t}} \cdot \frac{dX_{1t}}{dP_{1t}}$$

$$= -f(1 - b \cdot \frac{dE_t P_{1t+1}}{dP_{1t}}) + g \cdot \frac{dX_{1t}}{dP_{1t}}.$$

Thus the response of marketed surplus to a contemporaneous price change is given by

$$(17) \qquad \frac{dM_t}{dP_{1t}} = -(1+g) \cdot \frac{dX_{1t}}{dP_{1t}} + f(1 - b \cdot \frac{dE_t P_{1t+1}}{dP_{1t}})$$

Since the effect on expected future price of a change in the
current price will in general be positive and no greater than
unity, the sign of dM_t/dP_{1t} depends on the price responsiveness
of consumption.

Note that the first order conditions imply that the Marshal-
lian demand for the produced commodity may be written as $X_{1t} =$
$X(P_{1t}, P_{2t}, E_t P_{1t+1}, W_t)$ where $W_t = Y_t + [b^{\tau-t} E_t (P_{1\tau} Q_\tau) - \Sigma P_{Lt} L_t +$
$P_{1t} S_t] + P_{Lt} T^* = Y_t + \Pi_t + P_{Lt} T^*$. W_t is an expression for the
household's period t wealth which includes exogenous income,
expected net revenue from production, the value of stock on hand
$(S_t = I_t + Q_t)$, and the value of household time. The effect of a
change in P_{1t} on current consumption may then be written as

$$(18) \quad \frac{dX_{1t}}{dP_{1t}} = \frac{\partial X_{1t}}{\partial P_{1t}}\bigg|_{d\Pi=0} + \frac{\partial X_{1t}}{\partial \Pi_t} \cdot \frac{d\Pi_t}{dP_{1t}}$$

$$= \frac{\partial X_{1t}}{\partial P_{1t}}\bigg|_{d\Pi=0} - X_{1t} \cdot \frac{\partial X_{1t}}{\partial W_t} + (S_t + b^{\tau-t} E_t Q_\tau \cdot \frac{dE_t P_\tau}{dP_{1t}}) \cdot \frac{\partial X_{1t}}{\partial W_t}.$$

This analysis bears a strong resemblance to Strauss' in its
explicit recognition of wealth effects induced by a price change.
In addition to the usual substitution and income effects of a
conventional Slutsky equation (the first two terms of the second
equality), consumption also depends on an extra wealth effect (or
"profit" effect in Strauss' parlance) which is a function of
stocks on hand, expected future output, and the mechanism by
which the household forms price expectation. It is therefore
possible that the overall response to a price increase might be
positive if this wealth effect is sufficiently large.

Equation (18) clarifies the differences between the model

which has been developed in this paper and the more traditional approach in terms of behavioral responses implied by each. It will be observed that if inventories are ignored ($S_t = 0$), the future is not discounted ($b = 1$), and price expectations are static in the sense that $E_t P_{t+j} = P_{it}$, then equation (18) is equivalent to equation (IA-8) in Strauss (1986) with the exception that expected output is substituted for current output.[4] Failure of one or more of these conditions to hold would lead to (possibly sizeable) empirical differences between the two approaches.

The author is currently estimating an expanded version of the model presented in this paper, using eight years of quarterly consumption, production, and inventory data collected from fifteen households in each of three South Indian villages. The first order condition governing inventory demand is being estimated using a Generalized Method of Moments procedure. Demand and supply systems are being estimated separately using duality results and flexible functional forms. Combining the results of the two exercises will allow computation of demand and marketed surplus elasticities. These results are forthcoming.

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[4] Alternatively, taking each period to be one growing season and assuming inventories are not held and the future is completely discounted ($b = 0$) renders the two results identical.

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