AGRICULTURAL ECONOMICS

PAPER PRESENTED AT THE 1987 AAEA ANNUAL MEETING.

department of agricultural economics
virginia polytechnic institute and state university
blacksburg, virginia 24061

REVISED NOVEMBER, 1981

#1875

SP-87-7

Choice of Utility Functional Form: Its Effect on Classification of Risk Preferences and the Prediction of Farmer Decisions*

by

Segu M. M. Zuhair, Daniel B. Taylor, and

Randall A. Kramer**

*Paper presented at the American Agricultural Economics Association Annual Meeting in East Lansing, Michigan, August 2-5, 1987.

**The Authors are respectively: Lecturer in Agricultural Economics at the Faculty of Agriculture, University of Peradeniya, Peradeniya, Sri Lanka; Assistant Professor and Associate Professor of Agricultural Economics, Virginia Tech, Blacksburg, Virginia, USA, 24061.

ABSTRACT

Choice of Utility Functional Form: Its Effect on Classification of Risk Preferences and the Prediction of Farmer Decisions

Subjective utility values were elicited from Sri Lankan producers of minor export crops. Exponential, quadratic, and cubic utility functions were estimated. The choice of functional form was found to affect both the classification of risk attitudes and the prediction of harvesting strategy.

Choice of Utility Functional Form: Its Effect on Classification of Risk Preferences and the Prediction of Farmer Decisions

Introduction

One of the important steps in analyzing decision making under the expected utility hypothesis is specifying and then estimating a suitable utility function. For this purpose, several functional forms have been used in empirical studies [Lin and Chang; Halter and Dean; Lin, Dean, and Moore; Musser *et al.*]. Researchers generally agree that utility functions should possess some desirable properties such as continuity and decreasing absolute risk aversion. However, beyond that, there is little guidance for researchers to use in selecting functional forms. Using data collected from Sri Lankan farmers, this study first compares the risk attitudes given by quadratic, exponential, and cubic utility functions. Then an investigation is made of whether or not the ranking of prospects is independent of the utility functional form. If the functional form affects the ranking of prospects, choosing a functional form is fundamental to the methodology of expected utility analysis.

Most studies have arbitrarily chosen a particular functional form and then proceeded with the analysis. In one evaluation of functional forms, Lin and Chang criticized the forms usually employed because all of them require certain *a priori* assumptions on their specification. These authors suggested a Box-Cox transformation as a means of determining the form of the function rather than assuming it. Despite the appeal of this approach, Buccola (1982a) has demonstrated that the Box-Cox transformation is not consistent with Bernoullian decision theory.

As demonstrated by Musser *et al.*, the choice of functional form is critical because it can affect the classification of decision makers based on their risk attitudes. Musser *et al.*, classified twelve subjects (graduate students) in their study, using the second order derivative of the utility function as a measure of risk aversion. The utility functions used were quadratic ($U = a + bx - cx^2$), semi-log ($U = a + b \ln x$), and non-linear ($U = a + bx^c$). The quadratic function classified three subjects as risk preferring and the rest as risk indifferent. The semi-log function classified all subjects as risk averse, and the non-linear function classified all subjects as risk indifferent.

Regardless of the specific functional form chosen, certain desirable properties are sought in a functional form. Important ones include: the ease of estimating the parameters of the function, the ease with which the function can be mathematically manipulated to determine summary measures such as the mean and variance, and the behavior of the measures of the risk aversion. Ideally, a utility function should exhibit decreasing absolute risk aversion with respect to increasing wealth. The range of income used is another factor which needs to be considered, particularly for the semi-log forms, because the log of a negative number is not defined. Of the functional forms suggested in the literature, the quadratic and the exponential seem to be the most popular. Analyzing a California farmer's marketing problem, Buccola (1982b) reported that quadratic and exponential functions gave the same optimal portfolio under the assumption of a normal distribution.

Hanoch and Levy, in a theoretical comparison of the quadratic and the cubic functions, concluded that the cubic function has certain properties which are preferred to those of the quadratic function. These properties are: (1) expected utility depends on the third moment of the distribution, skewness, (2) this added parameter results in greater flexibility and better approximates the general utility function, (3) within certain restrictions on the coefficient, it is monotonically increasing, (4) it exhibits a decreasing degree of risk aversion at certain intermediate levels, and (5) it allows for risk preference (convexity) at certain intervals of high returns. The arguments for a cubic utility function, therefore, rest on the assumption of non-normal returns or non-zero skewness. Studies by Mendelbrot, and extensions by Fama, have shown that the skewness of the distribution of profit is negligible; therefore, for all practical purposes, an assumption of a normal distribution is valid.

Alternative Functional Forms for Utility

The quadratic utility function (QUF) can be represented as:[1]

$$U = a + bx - cx^2, \qquad [1]$$

where, a, b, and c are parameters. The second derivative of the function is [-2c < 0], implying diminishing marginal utility over the entire range of x, thus ruling out risk preferring behavior. The Arrow-Pratt absolute risk aversion coefficient R_a is given by:

$$\frac{2c}{b - 2cx} \qquad [2]$$

Function [2] will remain positive for x < (b/2c). Consequently, within this range of x, the quadratic function will exhibit increasing risk aversion, and for values of x > (b/2c) the function will exhibit decreasing risk aversion.

The exponential utility function (EUF) can be represented as:

$$U = K - \theta e^{-\lambda x} \text{ for } \quad K, \theta, \lambda, > 0, \qquad [3]$$

where, K and θ are parameters and e is the base of natural logarithms. The second derivative of the function is:

$$-\lambda^2 \theta e^{-\lambda x} < 0, \qquad [4]$$

implying diminishing marginal utility. The Arrow-Pratt absolute risk aversion coefficient, R_a is λ, which is positive and constant. The exponential utility function, therefore, exhibits constant risk aversion over all net returns.

The cubic utility function (CUF) can be expressed as:

$$U = a + bx + cx^2 + dx^3 \qquad [5]$$

---

[1] In all the utility functional forms discussed, U will refer to the utility index and x to the monetary measure. In this paper the monetary measure will be income in *Sri Lankan Rupees* (R_S).

where a, b, c, and d are parameters. The second derivative is given by: 2c + 6dx, the sign oq which depends on the sign and the magnitude of the parameters c, and d. Thus increasing and decreasing marginal utility are both possible. The Arrow-Pratt absolute risk aversion coefficient is:

$$R_a = -\left[\frac{2c + 6dx}{b + 2cx + 3dx^2}\right] \qquad [6]$$

The R_a thus can be positive or negative depending on the parameter values.

The Setting

The information presented in this paper was obtained as part of a comprehensive study of the decision making of Sri Lankan farmers who produce minor export crops.[2] The specific decision which the study examined was the harvesting behavior of these farmers. The farmers often harvest their crops prematurely due to fear of theft, damage by insects, cash needs, and other factors. The study developed methods to predict whether farmers would harvest maturely or prematurely. One of the methods used elicited utility functions to compare the income earned from premature harvesting with the income earned in a future period from mature harvesting.

The data were collected through farm surveys in the districts of Kandy and Matale in the central province of Sri Lanka. The surveys were conducted between December 1985 and February 1986. The sample size was 240, with an equal number from each district. The first survey was used to collect general information about farms in the two districts. Using the electoral voters' register as a sampling frame, the farm households were sampled randomly.

A second survey was conducted among 30 farmers selected at random from the larger sample of 240 farmers. The second survey included more detailed information on decision making activities concerning the farm, with the chief decision maker as the unit of inquiry. This survey included elicitation of subjective utility functions and probability distributions.

The two most widely used, direct elicitation methods for utility are the Ramsey method and the modified von Neumann-Morgenstern method. The Ramsey method elicits certainty equivalents for several risky alternatives. The modified von Neumann-Morgenstern method elicits

---

2 The minor export crops considered in this study were cocoa, coffee, pepper, cardamoms, and nutmeg. Many other crops, primarily spices and vegetables, also are referred to as minor export crops in Sri Lanka.

certainty equivalents for a series of lotteries. The modified von Neumann-Morgenstern method was deemed appropriate for this study in view of its simplicity [Anderson *et al.*] and the proven effectiveness of the method in Sri Lanka in a previous study [Herath *et al.*].

Results

Utility Function Estimation

The quadratic (QUF) and cubic functions (CUF) were estimated by applying the method of ordinary least squares while the exponential (EUF) functions were estimated by the method of maximum likelihood. For the exponential functions, a set of parameters (K, θ, and λ) was determined for each function which minimized the sum of squares of the error terms. These estimates were used as the starting values in the maximum likelihood method. Space limitations preclude presentation of the estimated functions. The reader is referred to Zuhair for more details.

For the EUF, estimates of Arrow-Pratt risk aversion coefficient (λ) were significant at least at the 0.05 level for all the farmers. For the QUF and CUF, the risk aversion coefficients are non-linear functions of the utility functions parameters; thus determination of the level of significance of the risk aversion coefficients for these functions was not possible.

For the QUF, the adjusted-R^2 ranged from 0.80 to 0.99, corrected to two decimal places. With the exception of one function, which was significant at 0.01 level, all the other functions were significant at the 0.001 level, based on an F-test. For the CUF, the adjusted-R^2 ranged from 0.86 to 0.99. The functions were significant at the 0.01 level for four farmers and at 0.001 for the rest of the farmers. For the EUF, the adjusted-R^2 ranged from 0.86 to 0.99. Comparing the R^2 across functions, the CUF gave the highest adjusted-R^2 for 14 farmers, the EUF for 3 farmers and the QUF for 2 farmers. For four farmers, all three functions gave the same adjusted-R^2. Four farmers had the same adjusted-R^2 for the CUF and EUF, which was higher than for their QUF. The QUF and the CUF were tied for highest adjusted-R^2 for two farmers, while the QUF and the EUF had the same higher adjusted-R^2 for one farmer.

Farmer Risk Attitudes

The Arrow-Pratt risk aversion coefficient, R_a, was computed for each farmer. The R_a, when computed with the EUF, is independent of the level of income, while for the QUF and the CUF the R_a is a function of income. To study the change with income, if any, in the risk attitude of farmers, the R_a was computed at the mean income and the annual income. The mean income for this computation was the mean of the income variable (certainty equivalents) used to estimate the individual utility functions. The annual income was the estimate of farm income obtained from the farmers. The risk aversion coefficients are given in Table 1 and the risk attitude classifications are given in Table 2.

The EUF classified all farmers as risk averse, $R_a > 0$. The R_a, as given by λ, ranged from 0.00001611 (farmer 39) to 0.0035684 (farmer 162). The QUF classified 27 farmers as risk averse and three farmers as risk preferring at the mean income. For the risk averse farmers, the R_a ranged from 0.0000055 (farmer number 39) to 0.0035835 (farmer 162). The risk averse farmers with the lowest and the highest R_a with the QUF at mean income also have the lowest and highest R_a with the EUF. For the risk preferring farmers, the R_a ranged from -0.0000505 (farmer 186) to -0.0015813 (farmer 72). At the annual income, the QUF classified 17 farmers as risk averse and 13 farmers as risk preferring. For the risk averse farmers, the R_a ranged from 0.000006 (farmer 39) to 0.022107 (farmer 28). For the 17 farmers classified as risk averse at both mean as well as annual income, two had decreasing risk aversion, the others had increasing risk aversion. For the risk preferring farmers, the R_a ranged from -0.000044 (farmer 186) to -0.010734 (farmer 204).

The CUF classified 15 farmers as risk averse and 15 farmers as risk preferring at the mean income. The R_a for risk averse farmers ranged from 0.0000006 (farmer 39) to 0.0008111 (farmer 195). Note that the same farmer had the lowest level of risk aversion with all three functions. For the risk preferring farmers, the R_a ranged from -0.0000078 (farmer 66) to -0.0022678 (farmer 72). The three farmers classified as risk preferring by the QUF at the mean income were also classified as risk preferring by the CUF at the annual income.

The CUF classified five farmers as risk averse and 25 farmers as risk preferring at the annual income. For the risk averse farmers, the R_a ranged from 0.000086 (farmer 26) to 0.038248 (farmer 95). For the risk preferring farmers, the R_a ranged from -0.000061 (farmer 62) to -0.018145 (farmer 162). All of the farmers who were risk preferring at the mean income were also risk preferring at the annual income. Four of the risk averse farmers were classified as exhibiting increasing risk

aversion, while one farmer exhibited decreasing risk aversion.[3] Of the risk preferring farmers two farms exhibited increasing risk aversion. The CUF therefore, classified 24 farmers as exhibiting decreasing risk aversion at the annual income.

The risk attitude of farmers, when determined with the QUF and CUF, is a function of the income at which the R_a is computed as well as the parameters of the utility function. The question then is what income level to use to compute the R_a. Musser *et al.* used the mid-point of the income scale to estimate the risk aversion coefficients. They were able to compare across the sample because all the subjects in their study had the same income level. When the income across the sample is not equal, the mean of the sample can lie above the income level of certain elements in the sample making the estimated coefficients unrealistic. Another alternative is to use the mid-point in the income scale of each individual in the sample. A third alternative is to use the mean of the elicited certainty equivalents. Because the estimated risk aversion coefficients can vary with the income level chosen to estimate them, it is reasonable to estimate the coefficients at the income level appropriate to the decision under consideration. For the premature versus mature harvesting decision considered below, the mean income's risk aversion coefficient appears to be most applicable.

Preferred Harvesting Strategies

Table 3 gives indices of expected utility for the three utility functions analyzed with respect to income which could be earned from the two harvesting strategies. The estimated utility functions were used to compute the expected utility of income under two alternatives, mature harvesting (M), and premature harvesting (P). The income from each alternative was computed using a simulation model which sampled prices and yields from elicited subjective probability distributions (Zuhair). These indices represent a measure of the expected utility derived from each strategy by each farmer.

For the EUF, the index ranged from 19.255 (farmer 95) to, 85.474 (farmer 113) for Strategy-M and from 14.693 (farmer 170) to 84.637 (farmer 69) for Strategy-P. Strategy-M,

[3] For farmers classified as risk averse at both mean and annual income, a higher (lower R_a at annual income is considered as increasing (decreasing) risk aversion. For farmers classified as risk preferring at both mean and annual income, a higher (lower) $-R_a$ at annual income is considered as increasing (decreasing) risk preference. Increased risk preference can be considered as decreased risk aversion. Also, risk aversion at a lower level of income and risk preference at a higher level of income is considered to be decreasing risk aversion.

therefore, has a narrower range for the utility index than Strategy-P. The utility index for Strategy-M was higher than that of Strategy-P for 29 farmers.

For the QUF, the utility indices ranged from 20.920 (farmer 72) to 91.595 (farmer 19) for Strategy-M and from 14.656 (farmer 170) to 91.055 (farmer 19) for Strategy-P. The range of the utility indices for Strategy-P was wider than that of Strategy-M. The QUF ranked Strategy-M higher for 26 farmers and lower for four farmers.

For the CUF, the utility indices ranged from 21.045 (farmer 72) to 192.910 (farmer 19) for Strategy-M and it ranged from 14.367 (farmer 170) to 190.120 (farmer 19) for Strategy-P. Strategy-P therefore has a wider range for the utility indices than Strategy-M. The CUF ranked Strategy-M higher for 21 farmers and lower for nine farmers. The CUF, thus, favored the premature harvesting strategy for the largest number of farmers.

These results demonstrate that different utility functional forms can have different preference ordering of the same set of prospects. Comparing the EUF and the QUF, five reversals in ranking of the prospects are observed. Between the EUF and CUF there are ten reversals in ranking, and between the CUF and QUF, seven reversals exist.

Conclusions

The present study has demonstrated that the choice of a utility function is an important aspect of the methodology of applying expected utility theory. The importance of choosing an appropriate utility function cannot be over emphasized. Depending on the functional form chosen by the researcher, farmers may be classified as risk averse or risk preferring. Similarly, one utility functional form can reverse the rankings of another functional form. Only one study prior to this one has compared the ranking of the exponential utility function and the quadratic utility function [Buccola (1982b)]. That study, with one farm, did not find any difference in the portfolio prescribed by the two utility functions. In the present study, with thirty farms, there were numerous preference reversals. This study examined the simple choice of either mature or premature harvesting. The large number of preference reversals suggests that if the analysis was extended to the more complicated case of portfolio selection, portfolios also would be sensitive to choice of functional form.

References

Anderson, J. R., J. L. Dillon, and J. B. Hardaker. *Agricultural Decision Analysis*. Ames, Iowa: Iowa State University Press, 1977.

Buccola, S.T. "Specification of Bernoullian Utility Function in Decision Analysis: Comment." *Ag. Econ. Res.* 34(1982a): 19-21.

Buccola, S.T. "Portfolio Selection under Exponential and Quadratic Utility." *West. J. Ag. Econ.* 7(1982b): 43-51.

Fama, E.F. "Mandelbrot and the Stable Paretian Hypothesis." *J. of Business*. 36(1963): 420-9.

Halter, A.N., and G.W. Dean. *Decision under Uncertainty with Research Applications*. Cincinnati: South-Western Publishing, 1971.

Hanoch, G., and H. Levy. "Efficient Portfolio Selection with Quadratic and Cubic Utility." *J. of Business*. 43(1970): 181-9.

Herath, H.M.G, J.B. Hardaker, and J.R. Anderson. "Choice of Varieties by Sri Lanka Rice Farmers: Comparing Alternative Decision Models." *Amer. J. Agr. Econ.* 64(1982): 87-93.

Lin, W., and H.S. Chang. "Specification of Bernoullian Utility Functions in Decision Analysis." *Ag. Econ. Res.* 30(1978): 30-6.

Lin, W., G.W. Dean, and C.V. Moore. "An Empirical Test of Utility vs Profit Maximization in Agricultural Production." *Amer. J. Agr. Econ.* 56(1974): 497-508.

Mendelbrot, B. "The Variation of Certain Speculative Prices." *J. of Business*. 36(1963): 394-419.

Musser, W.N., M.E. Wetzstein, S.Y. Reece, L.M. Musser, P.E. Varca, and C.C.J. Chou "Classification of Risk Preferences with Elicited Utility Data: Does the Functional Form Matter?" *West. J. Agr. Econ.* 9(1984): 322-28.

Von Neumann, J., and O. Morgenstern. *Theory of Games and Economic Behavior*. Princeton, New Jersey: Princeton University Press, 1953.

Zuhair, Segu M. M. "Harvesting Behavior of Perennial Cash Crops: A Decision Theoretic Study." Virginia Tech, Department of Agricultural Economics: Unpublished Ph.D. dissertation, 1986.

Table 1: Arrow-Pratt Absolute Risk Aversion Coefficients

Farmer number	*Exponential*	*Quadratic* Mean income	*Quadratic* Annual income	*Cubic* Mean income	*Cubic* Annual income
19	0.00005270	0.0000314	-0.000052	-0.0000273	-0.000065
24	0.00040450	0.0002923	-0.001559	-0.0001453	-0.000918
26	0.00019760	0.0001910	0.000284	0.0002057	0.000086
34	0.00033224	0.0003250	0.000608	0.0002270	0.002784
35	0.00036092	0.0003438	0.003458	0.0001235	-0.002010
36	0.00011053	0.0001079	0.000312	0.0000322	-0.001552
38	0.00029795	0.0002793	0.022107	0.0000282	-0.001204
39	0.00001611	0.0000055	0.000006	0.0000006	0.000276
40	0.00012613	-0.0005170	-0.000346	-0.0018647	-0.001169
42	0.00010392	0.0000448	0.000049	0.0001436	-0.000464
52	0.00009431	0.0000764	-0.000508	-0.0000104	-0.000254
62	0.00005919	0.0000306	-0.000046	-0.0000190	-0.000061
64	0.00047985	0.0003431	-0.000939	0.0000325	-0.000884
66	0.00001732	0.0000172	0.000068	-0.0000078	-0.000094
69	0.00011673	0.0001366	0.000207	-0.0000631	-0.001790
72	0.00022695	-0.0015813	-0.001091	-0.0022678	-0.001813
95	0.00005872	0.0006515	0.001423	0.0003576	0.038248
96	0.00027857	0.0001787	-0.000332	0.0000035	-0.000385
103	0.00062817	0.0005613	0.009114	-0.0000190	-0.002614
113	0.00008684	0.0000817	0.000219	-0.0000263	-0.000822
162	0.00356840	0.0035835	0.019934	-0.0007416	-0.018145
168	0.00046407	0.0002165	-0.000376	0.0004030	-0.000980
170	0.00007983	0.0000219	0.000023	0.0000293	0.000189
182	0.00007236	0.0000663	0.000234	0.0000154	-0.000630
186	0.00007145	-0.0000505	-0.000044	-0.0000493	-0.000158
195	0.00136620	0.0012160	0.013310	0.0008111	-0.011474
196	0.00015313	0.0001708	0.000515	-0.0000146	-0.001094
199	0.00033195	0.0001954	-0.000375	0.0001722	-0.000391
201	0.00017842	0.0001341	-0.000323	-0.0000283	-0.000322
204	0.00253540	0.0021626	-0.010734	-0.0007770	-0.006289

Table 2: **Classification of Risk Attitude Based on A-P Risk Aversion Coefficients**

Farmer number	*Exponential*	*Quadratic* Mean income	*Quadratic* Annual income	*Cubic* Mean income	*Cubic* Annual income
19	A	A	P	P	P
24	A	A	P	P	P
26	A	A	A	A	A
34	A	A	A	A	A
35	A	A	A	A	P
36	A	A	A	A	P
38	A	A	A	A	P
39	A	A	A	A	A
40	A	P	P	P	P
42	A	A	A	A	P
52	A	A	P	P	P
62	A	A	P	P	P
64	A	A	P	A	P
66	A	A	A	P	P
69	A	A	A	P	P
72	A	P	P	P	P
95	A	A	A	A	A
96	A	A	P	A	P
103	A	A	A	P	P
113	A	A	A	P	P
162	A	A	A	P	P
168	A	A	P	A	P
170	A	A	A	A	A
182	A	A	A	A	P
186	A	P	P	P	P
195	A	A	A	A	P
196	A	A	A	P	P
199	A	A	P	A	P
201	A	A	P	P	P
204	A	A	P	P	P

NOTE: A = risk averse ($R_a > 0$)
P = risk prefering ($R_a < 0$)

Table 3: Indices of Expected Utility and Preferred Harvesting Strategy

Farmer number	*Exponential* (EUF) M	P	*Quadratic* (QUF) M	P	*Cubic* (CUF) M	P
19	75.126*	74.744	91.595*	91.055	192.910*	190.120
24	80.136*	77.833	81.825*	81.451	99.079	108.206*
26	34.155*	29.237	34.071*	29.113	34.225*	29.349
34	64.754*	48.623	65.052*	48.796	66.088*	48.791
35	69.630*	68.904	71.521*	70.838	77.690*	76.899
36	40.656*	39.656	40.742*	39.703	38.706*	37.492
38	81.481*	81.094	81.114*	80.956	87.764	88.316*
39	70.688*	67.330	71.581*	67.824	73.282*	70.065
40	61.237*	53.015	64.753*	50.164	68.155*	44.951
42	59.622*	50.105	59.486*	49.524	59.320*	49.497
52	83.099*	82.923	81.704	81.782*	85.316	86.441*
62	65.696*	61.425	76.228*	69.191	104.218*	88.281
64	79.319*	78.757	85.545	85.619*	103.110	104.229*
66	73.316*	73.032	74.246*	73.994	96.512*	96.373
69	85.084*	84.637	84.147*	83.745	81.987	82.935*
72	31.480*	27.678	20.920*	16.965	21.045*	17.646
95	19.255*	18.928	27.358*	23.899	25.563*	22.100
96	72.562	72.735*	82.560*	80.274	102.004*	94.479
103	72.882*	61.441	74.379*	63.227	84.799*	70.311
113	85.474*	83.032	83.767*	82.090	74.113	79.730*
162	72.628*	68.116	73.922*	69.791	90.926*	86.075
168	80.711*	80.679	84.159	84.633*	79.004*	78.913
170	30.336*	14.693	29.688*	14.656	29.546*	14.367
182	83.878*	83.532	82.520*	82.400	77.720	78.900*
186	31.127*	20.518	29.055*	19.240	29.572*	19.894
195	41.715*	33.983	41.867*	33.859	41.731*	33.339
196	78.832*	76.421	78.536*	76.607	89.318	90.428*
199	82.163*	82.070	85.509	85.939*	86.710	87.239*
201	72.317*	66.342	81.286*	74.038	112.684*	96.195
204	55.197*	42.952	58.377*	43.827	71.340*	42.435

Note: M = Mature harvesting strategy
P = Premature harvesting strategy
* = Preferred strategy