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IDENTIFYING A CSD EFFICIENT SET OF MIXTURES OF RISKY ALTERNATIVES FOR RISK PREFERRING DECISION MAKERS*

by

Francis McCamley**

Risk

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**The author is an Associate Professor of Agricultural Economics at the University of Missouri-Columbia.

"Identifying a CSD Efficient Set of Mixtures of Risky Alternatives for Risk Preferring Decision Makers." Francis McCamley (University of Missouri-Columbia)

For risk preferring decision makers, identifying the SDWRF efficient set of mixtures of risky alternatives can be difficult. It appears to be easier to identify the efficient set for a combination of the CSD and SDWRF criteria. A procedure is suggested and illustrated using data from Hazell.

IDENTIFYING A CSD EFFICIENT SET OF MIXTURES OF RISKY ALTERNATIVES FOR RISK PREFERRING DECISION MAKERS

Several methods are available for identifying risk efficient mixtures of risky alternatives. The oldest are mean-variance analysis and MOTAD. A few years ago, Target MOTAD was introduced by Tauer and Watts, Held and Helmers. Direct utility maximization techniques have also been proposed (Kroll, Levy and Markowitz; Lambert and McCarl; and Collender and Chalfant). These methods can be very useful for identifying appropriate mixtures for risk averse decision makers whose utility functions are relatively well known.

Although many decision makers are risk averse, there is evidence that others prefer risk. Wilson and Eidman reviewed earlier studies which suggested that decision makers' absolute risk aversion coefficients range from -.0002 to .0012. In their own survey of hog producers, 69 percent of the respondents had risk aversion coefficients in the -.0002 to .0003 interval; 22 percent of the respondents had negative risk aversion coefficients.

Imprecise knowledge of decision makers' utility functions has led to the use of stochastic dominance to rank mutually exclusive risky alternatives. Meyer's stochastic dominance with respect to a function (SDWRF) (also known as generalized stochastic dominance) is a criterion frequently used for this purpose by agricultural economists.

This and other stochastic dominance criteria are not as frequently employed for problems involving mixtures of risky alternatives. As Cochran has noted, stochastic dominance techniques are not well developed for this class of problems. There have been a few attempts to develop methods for problems involving mixtures. Knight et al. examined alternative indicators of diversification prospects. In a recent paper, Witt, Tew and Reid evaluated the effectiveness of stochastic dominance techniques in predicting the components of mean-variance efficient enterprise mixtures. Other work (McCamley and Kliebenstein) suggests that it may be possible to apply the SDWRF criterion to problems involving mixtures of risky alternatives when the class of decision makers consists only of risk neutral or risk averse individuals.

This paper considers the possibility of applying the SDWRF criterion to problems involving mixtures of risky alternatives when all members of the class of decision makers are risk seekers. It concludes that identification of all SDWRF efficient mixtures may be more difficult than identification of those which (also) satisfy the convex set stochastic dominance (CSD) criterion. A procedure for identifying the CSD refinement of the SDWRF efficient set for a relevant class of problems is presented and illustrated. Since this paper combines ideas which, by themselves, are all either very simple and/or well discussed elsewhere, no formal proofs are included. Interested persons are invited to contact the author directly for proofs and other details.

Preliminary Considerations

The basic ideas can be revealed by a simple example. Assume that there are two equally likely states of nature and two enterprises. Let x_1 and x_2 be (nonnegative) activity levels for the enterprises. Assume that the net return received if the first state of nature occurs is

(1) $y_1 = 100x_1 + 40x_2$.

If the second state of nature occurs, then

(2) $y_2 = 80x_1 + 120x_2$

is received. Finally, assume that the sum of x_1 plus x_2 is constrained to be no greater than one. This implies that the set of feasible combinations of y_1 and y_2 is represented by the triangle OAB in figure 1.

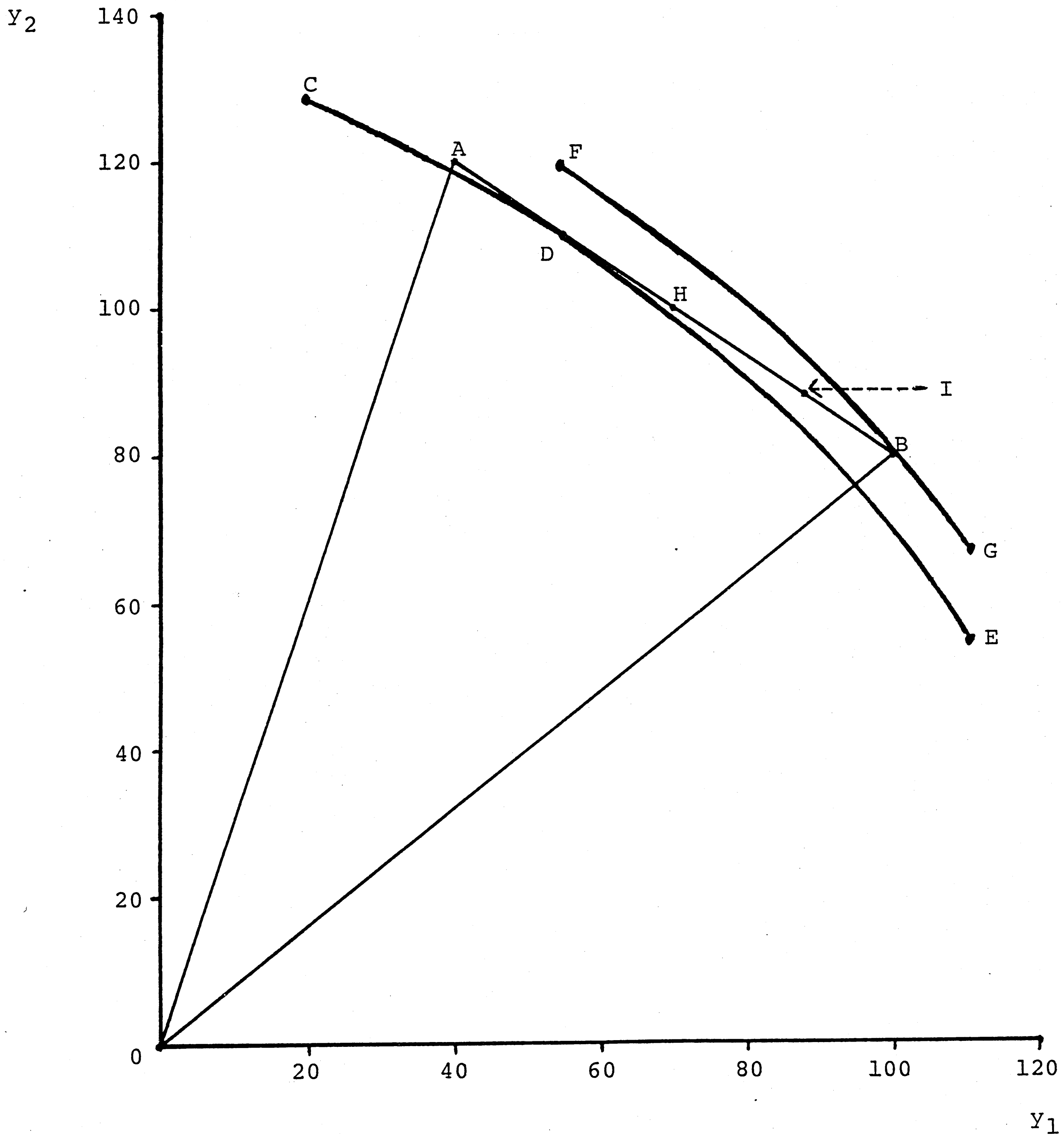

Figure 1. Feasible y vectors

Consider the problem of determining whether the mixture ($x_1 = \frac{1}{4}$, $x_2 = 3/4$) associated with D in figure 1 is SDWRF efficient. Earlier work exploited necessary (tangency) conditions for maximization of expected utility. It is relatively easy to show that these necessary conditions are satisfied for any interval which includes (1/55)ln(2/3) or approximately -.00737. The utility function

(3) $u = (2/3)^{(-m/55)}$

(where m is net returns) has an absolute risk aversion coefficient equal to this value. Given utility function (3) and the probability assumptions presented above, the tradeoff between y_1 and y_2 (at D) which is required to keep expected utility constant is the same as that implied by line segment AB. The curve CDE shows some of the combinations of y_1 and y_2 values for which the expected value of function (3) equals 1.875.

While the necessary conditions are satisfied it is clear that D does not maximize expected utility subject to the appropriate constraints. In fact, D <u>minimizes</u> (rather than maximizes) the expected value of function (3) subject to (1), (2) and the requirement that x_1 plus x_2 either be <u>at least</u> (rather than no greater than) one. The curve FBG shows some of the y_1 and y_2 combinations associated with a higher level of expected utility and suggests that B maximizes the expected value of (3).

Utility function (3) is only one of the many utility functions associated with any risk aversion coefficient interval which includes (1/55)ln(2/3). Nonetheless, it does exhibit a characteristic shared by all utility functions associated with risk aversion coefficient intervals which include only negative values. For each utility function of this sort, expected utility is a strictly convex function of the vector, y, of outcomes associated with the various states of nature. This means that no strictly convex combination of A and B can maximize expected utility for any utility function in this class.

This does not guarantee that no strictly convex combination of A and B can be SDWRF efficient. H and each strictly convex combination of H and I are dominated by a convex combination of I and B according to the first degree stochastic dominance criterion (FSD). Thus, they cannot be SDWRF efficient for any interval of risk aversion coefficients. However, given a sufficiently wide interval of risk aversion coefficients, many of the other combinations of A and B would be SDWRF efficient. Those near B have larger means than those lying closer to A (on AB) and thus would be undominated for any interval whose upper bound is sufficiently close to zero. Those near A have larger maximum outcomes than those lying closer to B and thus would be undominated when sufficiently large negative risk aversion coefficients are permitted.

The preceding two paragraphs suggest that it may be easier to identify the CSD efficient set for a simple problem such as the one discussed above than to identify the SDWRF efficient set.[1/] For any class of risk preferring decision makers, the only candidates for CSD efficiency are the extreme vectors A and B. The fact that no strictly convex combination of A and B can maximize any utility function which reflects a preference for risk means that none of these vectors can be CSD efficient. The vectors associated with the balance of the triangle OAB are obviously inefficient according to the FSD criterion and can be neither SDWRF or CSD efficient. Ordinary SDWRF algorithms can be used to rank A and B for any given risk aversion coefficient interval.[2/]

A More General Class of Problems

Much of the foregoing discussion is valid for more complicated problems. One class of such problems is considered in the balance of the paper. It assumes s states of nature. The probability of the ith state of nature is p_i. The s elements of the column vector, y, are the (total) net returns associated with the states of nature. The y vector is related to enterprise activity levels as follows:

(4) $y = Cx$

where x is a column vector of n activity levels and C is a matrix of per unit net returns associated with the activities and the states of nature. Specifically, C_{ij} is the net return per unit of activity j when the ith state of nature occurs. As usual, linear resource constraints of the form

(5) $Ax \leqq b$ and

(6) $x \geqq 0$

are assumed.

To simplify the exposition, it will be assumed that the feasible set of y vectors implied by (4), (5) and (6) is a convex polyhedron.[3/] The procedure to be outlined and illustrated in this paper is valid for many problems even when the set of feasible y vectors is not bounded. However, discussion of this situation is somewhat more difficult.

Three characteristics of the simple example are valid for the more general model. Expected utility is a strictly convex function of y whenever only risk preferring decision makers are considered. Only extreme vectors (corner points) can be CSD efficient. Ordinary SDWRF algorithms can be used to help rank alternative mixtures.

The (potentially) larger dimensions of the more general model lead to two complications. First, it may not be quite as easy to identify the relevant extreme vectors. This is significant because not all agricultural economists are familiar with techniques for identifying extreme vectors. Fortunately, this task is simplified somewhat by the fact that only extreme vectors which are also vector efficient need to be identified.

Second, application of the SDWRF criterion to extreme vectors will not, in general, identify the CSD efficient set. It is a useful screening device but the CSD criterion must be applied to guarantee identification of the CSD efficient set.[4/]

A Suggested Procedure

The following three step procedure is suggested:

1. Identify the set of vector efficient extreme y vectors.
2. Apply a standard SDWRF algorithm to these y vectors (and associated probability vector).
3. Apply the CSD criterion.

Methods such as those proposed by Murty (p. 468) and Steuer (pp. 233-244) can be adapted to identify the vector efficient extreme vectors.[5/] Almost any SDWRF algorithm could be used for step 2 when each state of nature is equally likely. This probability assumption is consistent with our tendency to use empirical distributions. Some algorithms would have to be modified when unequal probabilities are assumed.

An Example

Data from Hazell are used to illustrate the suggested procedure. Each state of nature (year) is assumed to be equally likely.

This problem involves three resource constraints and seven activities (including slack activities). Problems with these dimensions can have no more than 35 basic feasible solutions and will usually have fewer than that. The Hazell problem has fifteen basic feasible solutions. Five of these are associated with the same y vector. Thus, there are eleven extreme y vectors.

Enterprise mixtures associated with the eight vector efficient extreme vectors are presented in table 1.

One of the risk aversion coefficient subintervals, -.0002 to -.00005, used by Bosch, Eidman and Gill is selected for this paper. When the SDWRF criterion was applied, mixtures d, f, g and h were eliminated. For each of the remaining mixtures, it was possible to identify a utility function for which that mixture yields greater expected utility than each of the other three remaining mixtures. Thus, the CSD efficient set consists of mixtures a, b, c and e.

Concluding Remarks

A method for identifying a CSD efficient set for classes of risk preferring decision makers has been proposed and illustrated. Three comments are offered about this procedure.

In the past, stochastic dominance techniques have typically been applied to sets of mutually exclusive alternatives. The first step was usually the explicit identification of relevant alternatives. The set of alternatives was then reduced to an efficient set. Sometimes, several increasingly stringent criteria (e.g., FSD, SSD and TSD) were successively applied resulting in progressively smaller efficient sets. The procedure suggested in this paper is somewhat analogous. By contrast, the procedures which have been proposed for classes of risk averse decision makers tend to begin by identifying a subset of the efficient mixtures and then "search" for other efficient mixtures.

At least one major aspect of the application of the CSD/SDWRF criterion to mixtures of risky alternatives has apparently not been fully resolved. The author is aware of no procedure to identify the efficient set when the risk aversion coefficient interval includes both negative and positive values. It

Table 1. Enterprise Mixtures Associated with Vector Efficient y Vectors

Identifier	Enterprise Mixture Carrots x_1	Celery x_2	Cucumbers x_3	Peppers x_4	CSD Efficiency Status[a]
	---(acres)---				
a	--	27.45	100.00	72.55	X
b	100.00	23.53	--	76.47	X
c	--	100.00	100.00	--	X
d	--	--	123.33	76.67	
e	100.00	100.00	--	--	X
f	119.35	--	--	80.65	
g	--	--	200.00	--	
h	200.00	--	--	--	

[a]The risk aversion coefficient interval used was -.0002 to -.00005. X's indicate CSD efficient mixtures.

is certainly possible to divide such an interval into its negative and positive portions, and determine the efficient sets associated with each subinterval. The mixtures in these two efficient sets must belong to the efficient set for the overall interval but the union of the subinterval efficient sets may not include all of the mixtures belonging to the efficient set for the complete interval.

Finally, it should be obvious that linearity of the resource constraints and the relationship between y and the activity level vector x plays an important role in the procedure proposed here. If some of these relationships are nonlinear (but define a convex set of feasible y vectors) the problem of identifying the set of CSD efficient mixtures is more complex. This has perhaps more theoretical than practical significance given our tendency to approximate nonlinear relationships with linear or piecewise linear relationships.

Footnotes

1/ Cochran, Robison and Lodwick (p. 291) suggest that although convex set stochastic dominance provides a way of refining the efficient sets associated with any of several stochastic dominance criteria, it is perhaps most useful when combined with the SDWRF criterion. Therefore, in this paper the phrase, CSD efficient set, refers to the CSD refinement of the SDWRF efficient set.

2/ The CSD criterion is of no benefit when there are only two alternatives.

3/ Several definitions of the phrase, convex polyhedron, exist. The one adopted here assumes boundedness. In some texts, the less familiar term, polytope, is used to denote this sort of polyhedron.

4/ No claim is made in this paper that an extreme vector which is undominated (according to the SDWRF criterion) by other extreme vectors is SDWRF efficient. The possibility that such a vector could be dominated according to the SDWRF criterion by a nonextreme vector is not investigated since it is of little consequence when the CSD efficient set is desired. Each nonextreme vector is dominated, according to the CSD criterion, by the set of extreme vectors of which it (the nonextreme vector) is a convex combination. Transitivity of preferences ensures that any vector dominated by a nonextreme vector will not belong to the CSD efficient set.

5/ Murty's procedure is designed to identify the extreme points which are vector minima. With trivial adjustments, such as changing the sign of the elements in the C matrix, vector maxima can be identified.

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