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MISSPECIFICATION OF TIME SERIES MODELS IN U.S. AGRICULTURAL RESPONSE ANALYSIS

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Supply and demand -- Mathematical models

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Misspecification of Time Series Models in U.S. Agricultural Supply Response Analysis

Analysis of supply response over time for major agricultural commodities is of critical importance to policymakers who must decide the future direction of U. S. agricultural policy. Decisions based on this analysis could determine the structure of American agriculture for years to come. Because supply response analysis is a critical policymaking tool (Shumway) it is useful to consider the methodology employed in constructing the economic and statistical models which are used to generate forecasts of future economic activity. The purpose of this paper is to investigate one particular methodological practice, the specification of time as an explanatory variable in regression models, and to access the impact of this practice on supply response analysis.

The paper begins with a brief review of the supply response literature, emphasizing models using crop yield and acres planted as the dependent variable in the supply response equation. Specification of time as an independent regressor in these equations, the statistical environment of time series analysis, and different methods for modeling these data (trend stationary or difference stationary models) are then discussed. The results of this investigation suggest a lack of adequate diagnostic analysis of time series data employed in studies of supply response. Trend stationarity, as a maintained hypothesis, is shown to be tenuous, producing statistical results which may be seriously in error.

Supply Response Analysis

Models analyzing supply response frequently contain a linear trend term as an independent regressor. Specification of a functional dependence on time implies an assumption by the investigator of an underlying data generation process that is stationary in trend. In many instances the explicit justification for the trend variable is " to capture the effects of omitted variables that may have exerted systematic effects over time " (Morzuch et. al.). Omitted variables often include " technology ", suggesting smooth deterministic changes in technology, as opposed to abrupt and random changes. Studies of supply response considered in this analysis fall into two basic categories; those which analyze changes in crop yield or output per acre, and those analyzing changes in acreage devoted to a particular crop. Examples of the former methodology are, Menz and Pardey, Houck and Gallagher, Reed and Riggins, Butell and Naive, LaFrance and Burt, and, Lin and Davenport. These studies predominately use models which specify output as a deterministic function of time, with the exception of Reed and Riggins who employ a difference specification after finding trend to be explaining the major part of the variation in corn yields. Menz and Pardey specify a corn yield response equation where average corn yields (bu./acre) are assumed to be a linear function of the natural log of nitrogen (lbs./acre), weather, and a linear trend representing non-nitrogen technologies. Houck and Gallagher propose a similar yield response equation where trend represents all technological change. LaFrance and Burt suggest a specification including both differences and trend terms as regressors in a partial adjustment model of U.S. agricultural supply.

Examples from the literature of studies analyzing changes in acres planted are, Houck et.al., Gardner, Houck and Ryan, Morsuck et. al., Houck and Subotnik, and, Ryan and Able. The Houck et.al study uses several specifications including trend and difference specifications for several crops including corn, wheat, and soybeans. Houck and Ryan, and, Ryan and Able also estimate both specifications, but the major emphasis of the results are placed on the trend specification. Houck and Subotnik specify a differenced model as a means of capturing farmers expectations of future prices. The remaining studies all utilized models specifying trend as an explanatory variable with the implicit assumption of smooth technological change.

Statistical Background

Analysis of long-run supply response requires the evaluation of economic activity over time. Frequently, data for supply response analysis consists of historical time series with some level of aggregation, i.e. local, state, regional, national. Because it is often difficult to obtain more then one realization of a random variable over time, time series analysts must utilize a rigid set of assumptions about the underlying data generation process. Foremost is the assumption of linear stationarity, meaning that the process is both linear and stationary. Let X_t denote a sequence of observations of a random variable for which there is only a single realization available. The process generating X_t is linear if present values of X_t are generated as a linear combination of past values of X_t. The process generating X_t is said to be stationary if

$$E(X_t) = \mu, \quad Var(X_t) = \sigma^2 < \infty, \quad \text{and } Cov(X_t, X_{t-s}) = \tau_{t-s},$$

so that $\sigma^2 = \tau_0$. Thus the data generation process is stationary if it has a constant mean, finite variance, and a covariance structure not dependent on time itself, but only on the distance between any two observations (Granger and Newbold). This is the definition of weak stationarity which is sufficient if the process is assumed to be Gaussian. An autoregressive (AR) process is a way of expressing the current value of a discrete linear stochastic process in terms of the current periods disturbance and past observations. A moving average (MA) model expresses the current value of the process in terms of current and past disturbances. A mixed autoregressive moving average process (ARMA) is one containing both AR and MA terms. A MA process having a stationary AR representation is said to be invertible, thus an ARMA process is stationary and invertible if the AR component is stationary and the MA component can be represented as a stationary AR process (Nelson).

Some economic time series such as the rate of unemployment have been found to exhibit stationary behavior (Nelson and Plosser). Other economic time series are not stationary in that they move away from any constant level, and the measure of dispersion increases with time. These nonstationary time series fall into one of two categories depending on the characteristics of this divergent behavior. Series which deviate in one direction characterize a series which is nonstationary in trend. Trend is often modeled as a deterministic function of time with the residuals from the detrended series considered stationary. Series consistent with the hypothesis proposed by Box and Jenkins are characterized as an accumulation of first or higher order changes through time. Residuals from a differenced series are assumed to constitute a stationary series. Following Nelson and Plosser these two fundamentally different processes will be referred to as trend stationary (TS), and difference stationary (DS) processes, respectively. The importance of this distinction rests on the specification of the time series model. If the underlying data generation process is TS but the data are differenced in the analytical model, then there is the problem of specification error. The same is true if data generated by a DS process are inappropriately detrended.

Nelson and Plosser discuss the specification of the TS and DS processes as a set of alternative hypothesis. Without loss of generality the linear TS process and its DS counterpart in first differences will be considered. A linear TS model can be written

$$
\begin{aligned}
(1)\quad & \Phi(L)Y_t = \alpha + \beta t + \Phi(L)e_t, \\
& \Phi(L)e_t = \Theta(L)u_t \quad u_t \quad \text{iid } (0,\sigma^2),
\end{aligned}
$$

where α and β are parameters and $\Phi(L)$ and $\Theta(L)$ are autoregressive (AR) and moving average (MA) polynomials which satisfy the conditions of stationarity and invertibility.

Similarly a first order DS process takes the form

$$(2)\quad \tau(L)(1-L)Y_t = \beta + \tau(L)d_t,$$
$$\tau(L)d_t = \delta(L)u_t \quad u_t \quad \text{iid } (0,\sigma^2),$$

where $(1-L)$ is the difference operator and $\tau(L)$ and $\delta(L)$ are, again, AR and MA polynomials satisfying the stationarity and invertibility conditions. To illustrate the fundamental difference between these two specifications rewrite (2) as,

$$(3)\quad Y_t = Y_{t-1} + \beta + d_t,$$
$$Y_{t-1} = Y_{t-2} + \beta + d_{t-1},$$
$$Y_{t-2} = Y_{t-3} + \beta + d_{t-2},$$
$$\vdots$$

and substitute back to some point, Y_0, in time yielding,

$$(4)\quad Y_t = Y_0 + \beta t + \sum_{i=1}^{t} d_i .$$

Equation (4), the result of writing (2) as a linear function of time, reveals the source of specification error. Unlike (1), (4) is not stationary because the variance of d_t, $V(d_t) = t\sigma^2$, is unbounded as t increases. This implies that writing a DS process as a linear function of time violates the stationarity conditions.

Another way of showing the differences between the TS and DS processes is by examining the roots of the AR and MA polynomials in each specification. Taking first differences of the TS process (1) results in

$$(5)\quad \Phi(L)[(1-L)Y_t] = \beta + (1-L)\Phi(L)e_t,$$
$$\Phi(L)[(1-L)Y_t] = \beta + (1-L)\Theta(L)u_t .$$

Nelson and Plosser show that this differencing produces a unit root in the MA component of the ARMA process, implying that the process is not invertible. Similarly, representing the DS process in (2) in terms of the absolute levels of Y_t results in a process containing a unit root in the AR polynomial of the ARMA process, implying that the process has no convenient MA representation (Nelson and Plosser pp. 143). Therefore a series generated by a TS process would fail to reject the hypothesis of a unit root in the MA polynomial of the ARMA model in first differences, and a series generated by a DS process would fail to reject the hypothesis of a unit root in the AR polynomial of the ARMA process.

Consequences associated with the inappropriate specification of a time series are developed by Nelson and Kang. Using Monte Carlo techniques the properties of R^2, SSE, SST, and t-statistics for individual parameters are evaluated. The procedure is to generate 1000 samples of size N=100 from a random walk process, the simplest DS process, and to fit the data to a linear trend model. From (4) it is established that a DS process can be represented as a linear function of time, but the residuals from this model will not be a stationary series. The result of inappropriate detrending of the DS generated data is to remove approximately 86 percent of the variation in the data (Nelson and Kang pp. 76). It is observed that in models containing a drift term R^2 tended to one in the limit regardless of the true rate of drift. Conventional t-statistics are also found to be an unreliable indicator of significant trend in the series. Hypothesis tests conducted on the significance of the trend variable indicated a rejection of the null hypothesis of no time dependence in 87 percent of the cases. This finding suggests the existence of spurious regression phenomenon in which t tests predict significant relations between variables when none in fact exists. In their conclusion, Nelson and Kang suggest that investigators " regard stationarity around a function of time as a tentative rather than a maintained hypothesis ". Furthermore analysts should strongly consider formal test procedures to aid in determining the appropriate model specification.

Formal procedures for testing time series specifications are developed by Dickey and Fuller (1979, 1981). Each specification, TS and DS, is treated as one side of a mutually exclusive hypothesis, and combined into a single model. The test is developed under the assumption that if there is a unit root in the ARMA polynomials it will be in the AR polynomial corresponding to model (4) in which DS generated data were inappropriately represented in levels. The D-F test is derived from the model having the general form (Nelson and Plosser)

$$(6) \quad Y_t = \alpha + \beta t + e_t/(1-\Phi L),$$

multiplying both sides by $(1-\Phi L)$ yields

$$(7) \quad (1-\Phi L)Y_t = \alpha(1-\Phi) + \Phi\beta + \beta(1-\Phi)t + e_t .$$

If the TS hypothesis is correct, $|\ \Phi\ | < 1$, if the DS hypothesis is correct $\Phi = 1$, and (7) becomes

$$(8) \quad Y_t = Y_{t-1} + \beta + e_t .$$

Rewriting (7) in the compact form

$$(9) \quad Y_t = \alpha + \beta t + \phi Y_{t-1} + e_t ,$$

provides a simple model for testing the TS vs. DS hypothesis. Testing the null hypothesis, $\phi = 1, \beta = 0$, is equivalent to testing for a unit root in the AR polynomial. Failure to reject the null hypothesis indicates an underlying DS process, while rejection of the null hypothesis implies, $\phi = 1, \beta = 0$, and an underlying TS process (Nelson and Plosser). Dickey and Fuller (1979) represent the limiting distribution of ϕ, and derive a test statistic $t(\phi)$ for testing this hypothesis. Critical values are tabulated, and presented in Fuller for the one parameter test, and in Dickey and Fuller (1981) for the likelihood ratio test on the entire parameter space where the null hypothesis is $(\alpha,\beta,\phi) = (\alpha,0,1)$.

Results and Conclusions

The D-F test of the TS vs DS hypothesis is applied to aggregate USDA data for total crop yield, and acres planted, for the major crops, corn, soybeans, and wheat. Data are annual and observations are continuous for the period 1930 - 1986. Although this data is not the exact time period analyzed in any particular study, it can be considered representative of the data available. Results of the D-F test for both yields and acres planted are presented in table 1. For the yield data, the null hypothesis H0: $\phi = 1$ was rejected at the nominal .01 level in all cases. This implies an underlying TS generating process and suggests that the appropriate specification is one which involves a deterministic function of time. Results for acreage planted were quite the opposite, being unable to reject the null hypothesis in any case at the nominal .05 level[1]. The disagreement in these results requires further scrutiny.

With respect to the model specification in the supply response literature, it appears that those studies which chose to model yields as a deterministic function of time made the correct a priori assumption, and that those which chose to model acreage planted as a deterministic function of time did not. However these results can be explained in terms of the underlying assumptions of the TS and DS specifications. The critical assumption involves the nature of the technological change that is postulated to be captured by the dynamic model. If technology does in fact change in a relatively smooth way, than it is

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[1] In this case being unable to reject the null hypothesis at the .05 level is a stronger condition then not being able to reject at the .01 level because the larger the significance level, the smaller the value of $t(\phi)$ must be to maintain the null hypothesis.

Table 1. Results From Testing for Autoregressive Unit Roots.

	parameter estimate	standard error	Dickey-Fuller test statistic
Crop Yield			
Corn	0.350	0.129	-5.04
Soybeans	0.015	0.138	-7.14
Wheat	0.528	0.113	-4.18
Acres Planted			
Corn	0.783	0.086	-2.52
Soybeans	0.860	0.066	-2.12
Wheat	0.807	0.081	-2.38

reasonable to assume a TS process. Technological change in agriculture can by characterized in this way because of active and independent innovation related to output enhancing inputs. For example, a major breakthrough in seed technology may be followed by improvements in fertilizer, which in turn is followed by an advance in herbicides etc. Therefore it could be argued that aggregate yields for corn, soybeans, and wheat have increased along a deterministic trend. Acreage planted, on the other hand is more a function of policy changes from one farm bill to the next, and of prices and price expectations. These effects are likely to be random in nature, thus, data for acres planted would be expected to follow a DS specification. This observation is particularly disturbing in light of the discussion of spurious regression phenomena provided by Nelson and Kang. Following their argument leads to the conclusion that results from an inappropriately detrended series can provide seriously misleading information.

A review of the methodological approach for analyzing time series data in agricultural supply analysis reveals an insufficient appreciation of the time series literature and possible specification error. The model proposed by LaFrance and Burt containing both trend and lagged output as independent regressors is a multivariate example of a DS process written as a linear function of time. It is shown above that this specification is not stationary, and that the source of the specification error is a variance increasing over time without bound. Those studies using acreage response data and modeling that variable as a deterministic function of time are also faced with the problem of specification error and possible spurious results. Given the importance placed on supply response analysis and the results presented in this paper, a more thorough examination of the data generation process is required.

In conclusion, the purpose of this paper is to investigate alternative specifications of time series models commonly utilized in agricultural supply analysis. Reference is made to the statistical environment of time series, and the alternative specifications of trend stationarity and difference stationarity are introduced. A test developed by Dickey and Fuller is presented and a model which imbedded the alternative TS vs. DS hypothesis into a single equation is discussed. Modeling efforts from the supply response literature are briefly reviewed and discussed in the context of the statistical environment. The D-F test for unit roots is then applied to a set of six time series similar to those used in major supply response studies. The results indicate the possibility for model misspecification when hypothesis on the nature of the underlying data generation process are maintained a priori.

The procedure discussed in this paper is not without problems of its own. Although the D-F test statistics are " uniformly more powerful " (Dickey and Fuller, 1979) when compared to other test statistics, the power of the test is low, especially when values of ϕ are close to one. Because of this, it is recommended that researchers use their critical judgment when modeling and not rely on any one test when determining model specification. It is important, however not to maintain hypotheses a priori when the cost of doing so is very great.

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