THE EFFECTS OF SUPPLY SHIFTS ON PRODUCERS SURPLUS

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ABSTRACT. Changes in the supply curve caused by technological improvements in production or changes in national policy may or may not benefit producers as a whole, depending on the type of shift in the supply curve. Different changes in the supply curve that model this situation are studied and the change in producers surplus is examined depending on quantitative aspects of the supply and demand curves. It is shown that for supply and demand curves that are linear or power functions, a small downward pivot of the supply curve will increase producers surplus only if the equilibrium point is far enough into the elastic region of the demand curve, and any downward pivot of the supply curve will decrease producers surplus if the equilibrium point is in the inelastic region of the demand curve. Similar results are obtained for the more general case of convex supply and demand curves.

Shifts in the supply curve cause changes in producers surplus, consumers surplus, and net social benefits. These social welfare measures are very sensitive to the actual manner in which the shift occurs, i.e. whether it is pivotal, parallel, convergent, divergent, or some combination thereof.[1] This fact makes projected changes in the supply curve very important. Many economists have observed this. For example, Duncan and Tisdell showed the impact on producers surplus assuming polar elasticities of demand[2] with different types of shifts in the supply curve. Also, Lindner and Jarrett (1978, 1980) demonstrated the sensitivity of societal benefits to assumptions about the nature of the supply curve shift due to technological improvements in the production system.

In many agricultural industries, technological progress is likely to result in increased consumers surplus and net societal benefits, but decreased producers surplus because the demand curve is highly price inelastic. Moreover, with excess capacity in U.S. agriculture in the 1980's, the distribution of gains and losses from commodity policies are again a national issue. Amosson, et al., were concerned with this in evaluating alternative bovine brucellosis eradication programs. Their approach was to construct an economic model for this disease which incorporated cost-benefit analysis; they assumed pivotal supply curve shifts associated with different eradication programs and found a negative effect on producers surplus. Also, Hushak found that production and price constraints on corn from 1961 to 1966, which shifted the supply curve to the left, resulted in decreased consumers surplus and negative net societal benefits, but increased producers surplus because of highly price inelastic demand for corn. LaFrance and de Gorter estimate producers and consumers surplus changes from the dairy program in a market with technological change; the nature of the supply and demand curve shifts are assumed, not empirically estimated. Norton and Davis review some of the literature concerning shifts in the supply curve due to agricultural research and the effects that this has on consumers and producers surplus.

The results and examples that follow will show that often the specific assumptions made about supply curve shifts such as in LaFrance and de Gorter determine apriori the effect on producers surplus. Whether the supply curve is long-run or not, and whether producers surplus is a good measure of producer welfare or not, these results demonstrate the importance of assumptions made about supply curve shifts, especially if results are to be used for policy decisions that would affect producers as a whole.

First, the effect on producers surplus of a pivotal shift is described in the case of linear supply and demand. Then a combined pivot and parallel shift is discussed, again in the case of linear supply and demand. The result of a pivot is considered next when the supply and demand curves are first-order power functions. Finally, the general case of pivots of convex supply functions is examined. Possible implications of the results are discussed.

Linear Supply Curves with a Pivotal Shift

Two common examples of shifts in the supply curve are the pivot, where the intercept value remains fixed, and the outward parallel shift, where the curve's shape is retained but the supply curve drops uniformly to lower price levels. Consider the situation where supply and demand are linear. · · Figure 1, the supply curve S_1 represents the current supply while S_2 represents a pivotal shift after technological progress has been made.

In Figure 1, the supply and demand curves are given respectively by $P = S_1(Q) = aQ + c$, $P = S_2(Q) = raQ + c$, and $P = D(Q) = -bQ + d$, where $a,b,d > 0$, $c < d$ and $0 < r < 1$. As can be readily seen, consumers surplus always increases when shifting supply from S_1 to S_2 by the area of the trapezoid $P_1E_1E_2P_2$. Likewise, societal benefits increase by the area of the triangle cE_1E_2. But the effect on producers surplus is not clearly positive or negative. When supply shifts from S_1 to S_2, the change in producers surplus is given by the area of the triangle P_2E_2c minus the area of the triangle P_1E_1c (see Figure 1).

The area A representing producers surplus can be written as a function

$$A = \frac{a(d - c)^2}{2(a + b)^2} \tag{1}$$

Thus, $\frac{\partial A}{\partial a} > 0, < 0$, or $= 0$ as $b > a$, $b < a$, or $b = a$ respectively. Hence, if $b > a$ and a decreases, then A(a) decreases too. By the same reasoning, A(a) increases as a decreases if a is kept larger than b in the process. These observations give the following basic result:

Result 1. <u>With linear supply and demand curves, producers surplus is decreasing during a continuous downward linear pivot in supply if and only if the pivoting is through equilibrium points where the slope of the supply curve is smaller than the absolute value of the slope of the demand curve</u>.

Notice that with discrete shifts in supply as contrasted with the infinitesimal analysis implied by continuous shifts, it is possible that with fixed demand slope $-b$, the curve S can pivot downward from $a > b$ to $a < b$ while producers surplus increases or decreases. But if changes in a are very small, then producers surplus decreases exactly when $a < b$.[3]

The condition on slopes of Result 1 is not the same condition as whether the equilibrium point $E^* = (Q^*,P^*)$ is in the elastic or inelastic portion of the demand curve. Specifically, the point elasticity of demand e_D can be computed in terms of a,b,c, and d and E^* is in the inelastic portion of the demand if and only if $a < b - \frac{2bc}{d}$.

Therefore, if the equilibrium point E^* is in the inelastic portion of demand and $c \geq 0$, then $a < b$. That is, when equilibrium is in the inelastic portion of demand, then a downward pivot in supply will decrease producers surplus, by Result 1. For values of a between b and $b - \frac{2bc}{d}$, E^* is in the elastic portion of demand and Result 1 shows that a downward pivot will decrease producers surplus. For values of a in this range, the downward pivot will decrease producers surplus while at the same time increasing revenues to producers in the industry. For $c \geq 0$, the following holds:

Result 2. <u>With linear supply and demand curves</u>, <u>a downward linear pivot of the supply curve can continuously decrease producers surplus while increasing revenues to the producers</u>, <u>but it can never continuously decrease revenues to the producers while continuously increasing producers surplus</u>.[4]

As is shown later, the first part of Result 2 is common for more general supply curves, but for some non-linear supply curves the second part of Result 2 does not necessarily hold.

<u>Linear Supply Curves with a Combined Parallel and Pivotal Shift</u>

If the supply curve shifted outward in a parallel manner (assuming demand is not perfectly inelastic) the effect on both producers and consumers surplus would be positive. This type of shift has been studied to determine the actual change in societal benefits because it closely models changes in production costs resulting from the removal of a tariff or tax that affects producers independent of their efficiency. For linear supply and demand this corresponds to fixing all parameters except c, and then decreasing c .

If the supply curve movement is some combination of pivoting and shifting, i.e. changing a and c simultaneously, then the joint effect of these two competing factors is less clear. For example, a technological change which causes a downward pivot in the supply curve may be accompanied by a change in taxes which causes a parallel shift in the supply curve. The overall effect is termed a divergent shift in supply. It is shown here that this may cause an overall decrease in producers surplus instead of the increase expected from the parallel shift alone, depending on the value of the slope and intercept of the supply and demand curves.

Precisely, consider A as a function of both a and c. Assume demand is not perfectly inelastic, and that b and d are fixed. The differential for A(a,c) is

$$\partial A = \frac{(d - c)^2(b - a)}{2(a + b)^3}\,\partial a + \frac{-a(d - c)}{(a + b)^2}\,\partial c \quad . \tag{2}$$

Let E(a,c) denote the equilibrium point for fixed values of a and c. If equilibrium is changed from $E(a_0,c_0)$ to a nearby point $E(a_1,c_1)$, then the first order approximation to $\Delta A = A(a_1,c_1) - A(a_0,c_0)$ is equal to

$$\frac{(d - c_0)^2(b - a_0)}{2(a_0 + b)^3}\,(a_1 - a_0) + \frac{-a_0(d - c_0)}{(a_0 + b)^2}\,(c_1 - c_0) \quad . \tag{3}$$

The previous discussions were based either on $c_1 - c_0 = 0$ for the purely pivotal case, or on $a_1 - a_0 = 0$ for the parallel shift.

Now if both c and a decrease by a small amount, then producers surplus will still decrease, under the assumption that $b > a$ as in Result 1, if

$$\frac{\Delta a}{\Delta c} > \frac{2a_0(a_0 + b)}{(b - a_0)(d - c_0)} \quad . \tag{4}$$

So if the equilibrium point E_1 occurs when $b > a$, then a small combined linear pivot and parallel shift will decrease producers surplus if and only if $\frac{\Delta a}{\Delta c}$ is sufficiently large in the sense above. A similar analysis can be made for equilibrium points where $a > b$. This analysis shows that any small pivot and parallel shift that separately lower the supply curve will always increase producers surplus when the slope of the supply curve is greater than the absolute value of the slope of the demand curve.

General Supply Curves and Pivotal Shifts

It is worthwhile analyzing what effect pivotal shifts have on producers surplus, without making the restrictive assumption that supply and demand are linear. The pivotal shift discussed here models the effect that technological improvements, perhaps from newly implemented production techniques or improved conditions in the production process, can have in altering the supply curve and producers surplus. Let the new supply curve be $S_2(Q)$ and the old supply curve be $S_1(Q)$. Assume that $S_1(0) = S_2(0) = P_0$ and assume $\frac{\partial S_2}{\partial Q} < \frac{\partial S_1}{\partial Q}$ for $Q > 0$. That is, given a small increase in price, producers will be willing to respond with a larger increase in production after the technological improvements are made. Equivalently, $S_1(Q) - S_2(Q)$ is increasing with increasing Q. When this situation occurs, we say that $S_2(Q)$ is gotten from $S_1(Q)$ by a downward pivotal shift.

When the demand curve is perfectly inelastic or when the demand curve D just has very large negative slope everywhere relative to the slopes of S_1 and S_2, then producers surplus decreases in pivoting from S_1 to S_2. But if D is perfectly elastic or D has very small negative slope relative to the slopes of S_1 and S_2, then the producers surplus increases in pivoting from S_1 to S_2. The results to follow show that depending on the slopes of the supply and demand curves at equilibrium, the producers surplus will either increase or decrease in pivoting from S_1 to S_2 analogously to the above polar extremes. It is assumed in the rest of this section that the demand curve is negatively sloped. When solving $P = S_i(Q)$ for Q, the notation $Q = S_i^{-1}(P)$ will be used. That is, S_i^{-1} is the function inverse to S_i . Assume that $Q = S_i^{-1}(P)$ is strictly increasing with P and that $S_i^{-1}(P_0) = 0$.

There is always a function f relating S_2^{-1} to S_1^{-1} . This function is called a change of variables function because it is assumed to satisfy $f(P_0) = P_0$ and $S_2^{-1}(P) = S_1^{-1}(f(P))$ for all $P \geq 0$ (see Figure 2). Also, since $\frac{\partial S_2}{\partial Q} < \frac{\partial S_1}{\partial Q}$, it is easy to show that $\frac{\partial f}{\partial P} > 1$ for all $P > P_0$. If $\frac{\partial f}{\partial P}$ is a constant, i.e. f is linear, then S_2 is called a linear pivot of S_1. That is, S_2 is a linear pivot of S_1 if for some $r < 1$, $S_2(Q) = r(S_1(Q) - P_0) + P_0$ for all Q. Defining $S_2(Q)$ by this formula allows one to pass through a parameterized family of downward linear pivots of S_1 as r decreases.

The general results for non-linear supply and demand curves are easiest to derive and state when S_2 is a linear pivot of S_1. For example, suppose that S_1 and D are power functions of the form $S_1(Q) = CQ^s + P_0$ and $D(Q) = KQ^{-d}$ where C,K,s, and d are positive constants. Note that S_1 will not have constant elasticity if $P_0 \neq 0$ and that Result 3 following depends on P_0 in an essential way. Now if S_2 is a linear pivot of S_1, then S_2 is the power function $S_2(Q) = rCQ^s + P_0$. Since P_0 is not assumed to be 0, one cannot solve for producers surplus, PS, explicitly in terms of the constants above as was done in the linear case. But expressing PS in terms of (Q^*,P^*) and in terms of the constants above gives

$$PS = \frac{s}{s+1} r C Q^{*s+1} \quad . \tag{5}$$

Differentiating with respect to r gives

$$\frac{\partial PS}{\partial r} = \frac{s}{s+1} C Q^{*s+1} + s r C Q^{*s} \frac{\partial Q^*}{\partial r} . \tag{6}$$

If one differentiates the equilibrium equation, $r\,C\,Q^{*s} + P_0 = KQ^{*-d}$, with respect to r, then one gets

$$\frac{\partial Q^*}{\partial r} = \frac{-\,C\,Q^{*s}}{r\,C\,s\,Q^{*s-1} + d\,K\,Q^{*-d-1}} \quad . \tag{7}$$

Substituting (7) into (6) gives

$$\frac{\partial PS}{\partial r} = \frac{s}{s+1}\,C\,Q^{*s}\,\frac{d\,K\,Q^{*-d} - r\,C\,Q^{*s}}{r\,C\,s\,Q^{*s-1} + d\,K\,Q^{*-d-1}} \quad . \tag{8}$$

Hence, $\frac{\partial PS}{\partial r} > 0$ if and only if $d\,K\,Q^{*-d} - r\,C\,Q^{*s} > 0$. Since $d\,K\,Q^{*-d} - r\,C\,Q^{*s} = P_0 + (d-1)\,P^*$, this gives the following result for supply and demand functions that are power functions as above:

Result 3. A) <u>If</u> $d = 1$ (<u>demand is of unitary elasticity</u>), <u>then producers surplus is independent of</u> r (<u>the degree of pivot</u>) <u>when</u> $P_0 = 0$, <u>and producers surplus always decreases as</u> r <u>decreases if</u> $P_0 > 0$.

B) <u>If</u> $d > 1$ (<u>demand is inelastic</u>), <u>then producers surplus decreases as</u> r <u>decreases.</u>

C) <u>If</u> $d < 1$ (<u>demand is elastic</u>), <u>then producers surplus decreases as</u> r <u>decreases if the equilibrium value of price</u> P^* <u>satisfies</u> $(1-d)P^* < P_0$; <u>and producers surplus increases as</u> r <u>decreases if</u> $(1-d)P^* > P_0$.

D) <u>Except for the case</u> $d < 1$, <u>the above obtain regardless of the elasticity of supply</u>.

This shows that a downward pivot that decreases producers surplus can at the same time increase revenues to producers. Also, a continuous downward linear pivot which decreases revenues to producers must decrease producers surplus. These results for power functions are analogous to the linear supply and demand case. One can make the same computations when the supply curve is a power function and the demand curve is linear or has the form $D = K(Q + Q_0)^{-d}$ so that D does not become infinite at $Q = 0$. In all these cases, one finds that a small downward linear pivot of supply will increase producers surplus only if equilibrium is far enough into the elastic region of the demand curve (cf. Result 3, $d < 1$), but any downward linear pivot of supply will decrease producers surplus if equilibrium is in the inelastic region of the demand curve.[5]

Taking S_1 and D to be general convex curves, the same technique which gives Result 3 can be carried out for general curves and linear pivots. Take $S_2(Q) = r(S_1(Q) - P_0) + P_0$ so that PS depends on r. One cannot solve explicitly for PS, but

$$PS = \int_{P_0}^{P^*} S_1^{-1}\left(\frac{1}{r}(P - P_0) + P_0\right) dP . \tag{9}$$

Then differentiating (9) with respect to r, one can compute implicitly that

$$\frac{\partial PS}{\partial r} = \frac{\partial P^*}{\partial r} Q^* - \frac{1}{r} Q^* (P^* - P_0) + \frac{1}{r} PS . \tag{10}$$

Now differentiating the equilibrium equations, $P^* = S_2(Q^*) = D(Q^*)$, with respect to r and using this result to solve for $\frac{\partial Q^*}{\partial r}$ gives

$$\frac{\partial P^*}{\partial r} = \frac{\partial D}{\partial Q}(Q^*) \frac{\partial Q^*}{\partial r} = \frac{\partial D}{\partial Q}(Q^*) \frac{P_0 - S_1(Q^*)}{r\frac{\partial S_1}{\partial Q}(Q^*) - \frac{\partial D}{\partial Q}(Q^*)} . \tag{11}$$

Substituting (11) into (10) shows that $\frac{\partial PS}{\partial r} > 0$ at $r = 1$ if and only if

$$\frac{Q^*(P^* - P_0)}{PS} < 1 - \frac{\frac{\partial D}{\partial Q}(Q^*)}{\frac{\partial S_1}{\partial Q}(Q^*)} . \tag{12}$$

Hence, if the supply curve is convex, then $PS \geq \frac{1}{2} Q^*(P^* - P_0)$ and therefore a sufficient condition for $\frac{\partial PS}{\partial r} > 0$ is that $\frac{\partial S_1}{\partial Q}(Q^*) < -\frac{\partial D}{\partial Q}(Q^*)$. This gives the following result.

Result 4. <u>Given a convex supply curve</u>, <u>a small downward linear pivot of the supply curve</u> S_1 <u>will decrease producers surplus if at equilibrium</u> $E^* = (Q^*,P^*)$

$$\frac{\partial S_1}{\partial Q} < -\frac{\partial D}{\partial Q}. \tag{13}$$

This result dramatically extends part of Result 1 which was for linear supply and demand to the case of convex supply and arbitrary demand curves. Equation (12) also shows that a small downward linear pivot of the supply curve can increase or decrease producers surplus while revenues are increasing or decreasing, and all four possible combinations of increase/decrease are possible. This differs from the case of linear or power functions as supply and demand curves.

If the change of variables function f is non-linear, then similar results to Result 4 can be obtained. The conditions for a pivot to decrease producers surplus in this generality will include the variation in $\frac{\partial f}{\partial P}$ over the range of P values involved in pivoting from S_1 to S_2. See the appendix for a description of these results.

Finally, for any supply and demand curves, if S_2 is a convergent shift of S_1 (i.e. $S_2(Q) < S_1(Q)$ and $\frac{\partial S_2}{\partial Q} > \frac{\partial S_1}{\partial Q}$ for all Q), then producers surplus always increases in changing from the supply curve S_1 to the supply curve S_2.

Changes in Producers Surplus and Welfare Analysis

Many authors have treated the question of the change in consumer and/or producer welfare due to shifts in supply. Norton and Davis survey some of this literature especially of interest to the agricultural economist (see also Lindner and Jarrett, Rose, Wise and Fell, and Duncan and Tisdell). In these papers, both pivotal and parallel shifts are considered for linear and non-linear supply and demand curves. The focus of this paper has also been as in these references on the effect on producers surplus given a downward movement of supply.

One important feature of this study was to show that typically when equilibrium lies in part of the elastic or any of the inelastic region of the demand curve, a downward pivot of the supply curve will decrease producers surplus. This was shown to be true for any supply and demand functions that are linear or are power functions. Since demand in agricultural markets is generally inelastic, this shows that a welfare analysis concerning producers surplus, estimated with these types of supply and demand functions, which hypothesizes a pivot of the supply curve will predetermine the nature of the change in producers surplus independent of the empirical data. Consequently, care must be taken in the type of supply curve shifts that are presupposed.

Duncan and Tisdell have studied the perfectly elastic and inelastic demand extremes, and observed what the effect on producers and consumers would be in these cases. The substance of part of this analysis can be thought of as clarifying their results where demand elasticity is between the polar extremes. For pivoting by itself or for a combination of pivot and parallel shifts, these results exactly quantify their qualitative discussion. The results above also avoid the pitfalls in the analysis given by Lindner and Jarrett, as pointed out by Rose.

In addition, some conclusions are presented here that seem to be overlooked in many of the references cited in Norton and Davis. An important result of this analysis which has not been previously addressed is that elasticities are not the major determinant of changes in producers surplus. Rather the major determinants are the relative slopes of the supply and demand curves. Sampath, in evaluating returns to irrigation development, used the power function model of Result 3 with $P_0 = 0$ (that is, the supply curve passes through the origin). His findings of increasing producers surplus when demand is elastic and decreasing producers surplus when demand is inelastic as a result of irrigation development (i.e. outward pivot of the supply curve) corresponds to Result 3, B and the second half of C. His finding that elasticity was the major determinant of changes in producers surplus was a direct result of using a power function model passing through the origin.

An additional important result obtained here is that for general convex supply curves, producers surplus can decrease or increase while pivoting through an elastic or inelastic region of the demand curve. So revenues can increase while producers surplus is decreasing, and vice versa. This might seem somewhat contradictory. However, increasing revenues due to a pivot that are accompanied by increasing cost can certainly lead to a negative change in surplus profits or rent. Hence, a decrease in producers surplus together with an increase in revenues is an entirely feasible possibility, and for this reason the results are not contradictory.

Appendix

In the previous, it was assumed that the change of variable relating S_1 to S_2 was linear. But it is possible that a more general type of pivot would be relevant. To state the precise results, one needs to control the variation $\frac{\partial f}{\partial P}$ over the values of P involved in pivoting from S_1 to S_2. There are always numbers M_1 and M_2 which are respectively the maximum and minimum positive numbers such that $1 + M_1 \leq \frac{\partial f}{\partial P} \leq 1 + M_2$ for all P such that $P_0 \leq P \leq P_2$ (see Figure 2). Define the variation of f to be the ratio $V = \frac{M_2}{M_1}$. If f is linear, then the variation V = 1. Otherwise, V > 1. This result can be proved:

Result 5. For any supply and demand curves, a small downward pivot of the supply curve will increase producers surplus if

$$\frac{Q^*(P^* - P_0)}{PS} > V \left[1 - \frac{\frac{\partial D}{\partial Q}(Q^*)}{\frac{\partial S_1}{\partial Q}(Q^*)} \right] \tag{14}$$

But a small downward pivot of the supply curve will decrease producers surplus if

$$\frac{Q^*(P^* - P_0)}{PS} < \frac{1}{V} \left[1 - \frac{\frac{\partial D}{\partial Q}(Q^*)}{\frac{\partial S_1}{\partial Q}(Q^*)} \right] \tag{15}$$

This result generalizes the condition given by equation (12) that leads to Result 4. The proof of this result is quite technical and is omitted; it is available from the authors upon request.

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Footnotes

1. These technical terms are defined in the sections of this paper and illustrated in the figures.
2. This term, used by Duncan and Tisdell, means that the demand curve is either perfectly inelastic or perfectly elastic.
3. Also, the criterion $a < b$ on the slopes in Result 1 says that the pivoting is through equilibrium points which are unstable according to the cobweb model, i.e. the cobweb graph spirals outward away from equilibrium. Since price expectations are not currently modeled in this manner, the criterion $a < b$ is not referred to here as market instability.
4. For $c > 0$, any discrete pivot downward in supply whose overall effect is to decrease revenues must also decrease producers surplus. Indeed, suppose the pivot is from S_1 to S_2 as in Figure 1. If revenues satisfy $P_2Q_2 < P_1Q_1$, then since $c > 0$ and $Q_2 > Q_1$, it follows that $P_2Q_2 - cQ_2 < P_1Q_1 - cQ_1$. This shows that producers surplus has decreased. If $c < 0$, then the second half of Result 2 and the comment above no longer hold generally. However, Result 1 holds whether $c > 0$ or not.
5. If $S_1(Q) = CQ^s + P_0$ and $D(Q) = K(Q + Q_0)^{-d}$, then PS decreases with a downward linear pivot if and only if $P^*\dfrac{(1-d)Q^* + Q_0}{Q^* + Q_0} < P_0$. Also revenues R decrease with a downward linear pivot if and only if $\dfrac{Q^* + Q_0}{Q^*} < d$. If $S_1(Q) = CQ^s + P_0$ and $D(Q) = -bQ + d$, then PS decreases with a downward linear pivot if and only if $P^* < \dfrac{d + P_0}{2}$, and R decreases with a downward linear pivot if and only if $P^* < \dfrac{d}{2}$. It follows that if R is decreasing with a downward linear pivot, then PS is decreasing too in both of these cases.

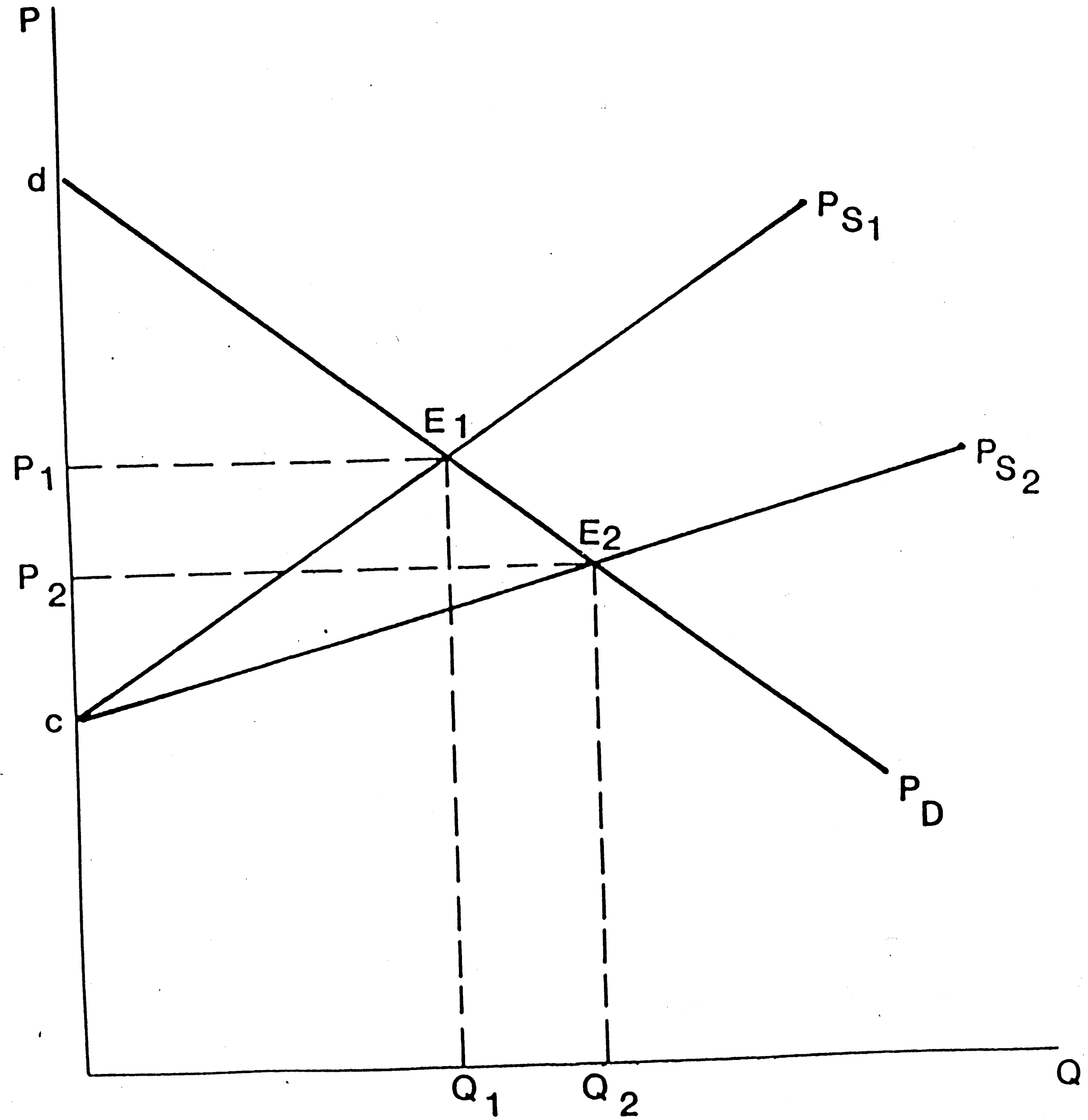

Figure 1.

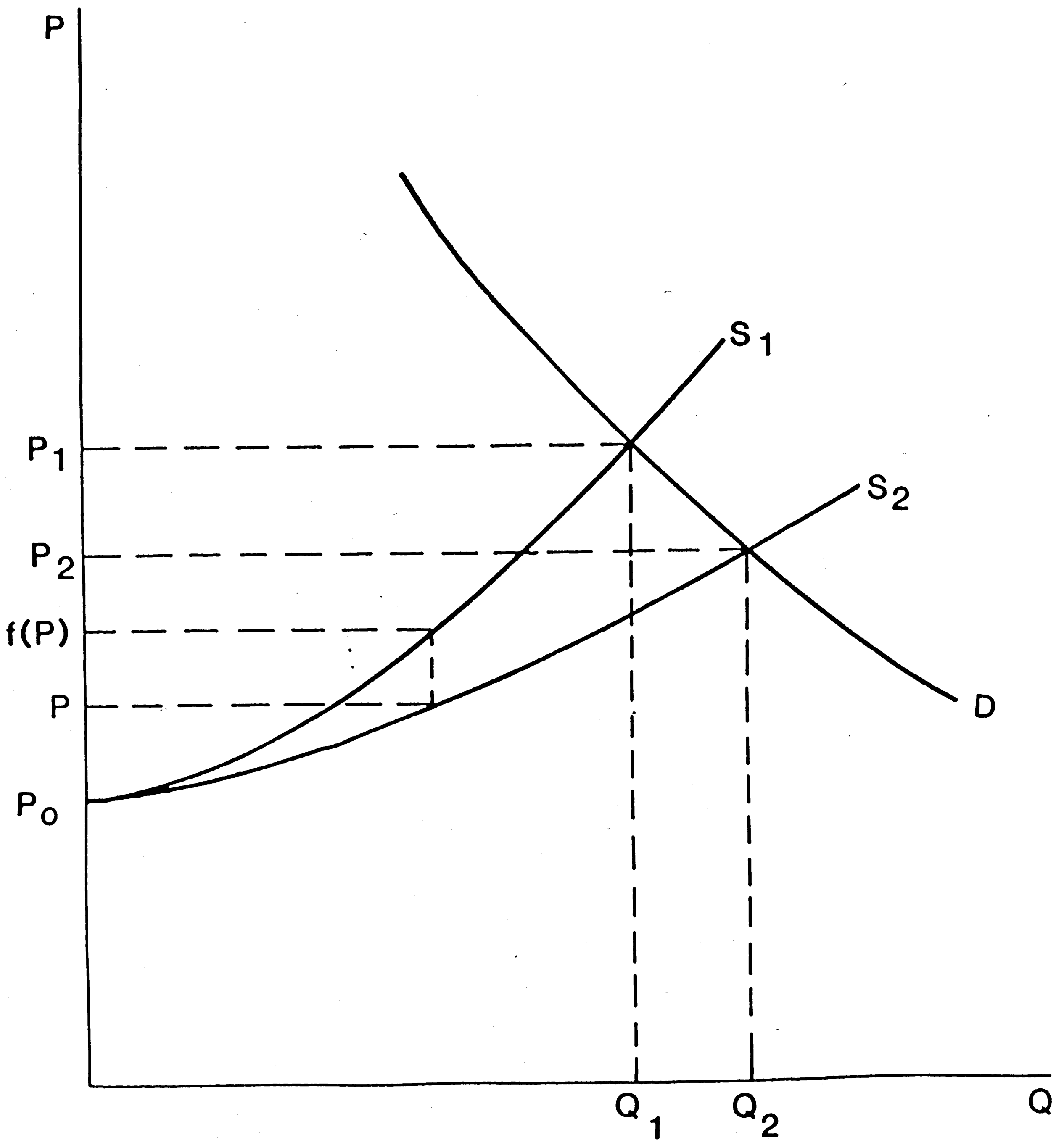

Figure 2.