1987

[redacted]

AAEA paper 1987

Production economics -- Mathematical models

DEPARTMENT OF AGRICULTURAL AND RESOURCE ECONOMICS
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Exact Welfare Measurement for Producers Under Uncertainty

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Working Paper No. 87-07

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July 1987

*Thanks are due Richard Just, Paul Flacco, Ted McConnell, and John Antle for helpful comments on this and earlier versions. This work is a result of research sponsored by the Alaska Sea Grant College Program, under grant No. NA82AA-D-00044, project R/14-08, and the University of Alaska with funds appropriated by the state. (Scientific Article No. A-4554, Contribution No. 7548, Maryland Agricultural Experiment Station)

Exact Welfare Measurement for Producers

Under Uncertainty

Abstract

Ex ante compensating and equivalent variations for producers under price or production uncertainty can frequently be obtained from properly specified ordinary supply and input demand functions, without assumptions about risk aversion. Exact welfare measures of single parameter changes are derived for several widely-used empirical specifications.

Exact Welfare Measurement for Producers Under Uncertainty

Many useful results have been derived for different characterizations of uncertainty in the theory of the firm (for example, Sandmo; Batra and Ullah; Pope and Kramer; and Blair). However, the limited literature available on producer welfare measurement under uncertainty (see Pope, Chavas, and Just; Pope and Chavas; and Just, Hueth, and Schmitz) is pessimistic about the possibilities for deriving exact welfare measures from ex ante supply functions. The only exception is Pope and Chavas, who derive error bounds for the use of producer's surplus as an approximation for true welfare change that are analogous to Willig's bounds for consumer's surplus as an approximation of compensating or equivalent variation. However, these results are presumably subject to the same shortcomings noted by Hausman (and amended by Haveman, Gabay, and Andreoni) for Willig's error bounds; even though compensating or equivalent variation is approximated well, a poor approximation of deadweight loss may result.

Pope, Chavas and Just show that the ex ante supply function derived from an uncertainty model is generally not sufficient for determining true welfare change except in the special case of absolute risk aversion. Because decisions depend on the firm's wealth, for non-neutral risk preferences a parameter (price) change induces a wealth effect in addition to an ordinary output effect. Thus, for exact welfare measurement under price uncertainty, the unobservable compensated (constant-expected utility) supply function is needed. Newbery and Stiglitz, among others, have shown that the area above an expected supply function is generally not a valid welfare measure.

This paper shows that exact ex ante welfare measurement for producers is not so forbidding a task as the foregoing discussion suggests. Using the approach of recent literature on consumer choice, especially Hausman and

Vartia, compensated _ex ante_ supply functions corresponding to maintained hypotheses about the form of the ordinary supply function under price uncertainty are derived in a straightforward manner. Under production uncertainty, compensated _ex ante_ input demand functions are likewise derived. Once the compensated supply or input demand functions are obtained, the exact _ex ante_ welfare measures are easily obtained in principle, and are presented for several common functional specifications.

The Expected Utility Model

The firm is assumed to maximize the expected utility of wealth, $E[U(W_0 + \pi)]$, where $E[\cdot]$ is the expectations operator, and $U(\cdot)$ is a von Neumann-Morgenstern utility function defined on wealth which has $U' > 0$ and $U'' < 0$ for a risk-averse firm. Wealth is the sum of initial wealth W_0 and profits, $\pi = pq - rx$, where p and q are corresponding m-dimensional output price and quantity vectors, and r and x are corresponding n-dimensional input price and quantity vectors, respectively. Both price vectors are random while the elements of q are concave and twice differentiable neoclassical production functions $q_i = q_i(x)$, with $q_i(0) = 0$ and non-negative marginal products (with at least one strictly positive).[1] That is, the technology is non-joint in inputs. This formulation, which follows Just, Hueth, and Schmitz, includes as special cases the models of Sandmo and Batra and Ullah, who consider only output price uncertainty, and Blair, who considers only input price uncertainty.

The firm's decision can thus be represented as

$$\max_{x} E[U(W_0 + pq - rx)] \tag{1}$$

for which the necessary and sufficient conditions are

$$E[U'(p \frac{\partial q}{\partial x_j} - r_j)] = 0, \; j = 1, \ldots, n \tag{2}$$

$$(3) \qquad \left\{ E\left[U''\left(p\frac{\partial q}{\partial x_i} - r_i\right)\left(p\frac{\partial q}{\partial x_j} - r_j\right) + U'\left(p\frac{\partial^2 q}{\partial x_i \partial x_j}\right)\right]\right\} < 0$$

where $\{Z_{ij}\} < 0$ implies negative definiteness of a matrix Z with elements Z_{ij}.

Under condition (3), the Jacobian of (2) is non-vanishing and the implicit function theorem permits, in principle, solving for the optimal input demands

$$(4) \qquad x_j = x_j(\mu, \gamma, \phi, \psi, W_0)$$

and optimal output supplies

$$q_i = q_i[x(\mu, \gamma, \phi, \psi, W_0)]$$

$$(5) \qquad = q_i(\mu, \gamma, \phi, \psi, W_0)$$

where μ and γ are means and ϕ and ψ denote higher moments of the distributions of p and r. Substituting (4) and (5) into (1) obtains the indirect expected utility function

$$V(\mu, \gamma, \phi, \psi, W_0) \equiv E[U(W_0 + pq - rx)]$$

which has the properties

$$\frac{\partial V}{\partial \mu_i} = E(U')q_i \quad \text{and} \quad \frac{\partial V}{\partial \gamma_j} = -E(U')x_j .$$

Also, since $\partial V/\partial W = E(U')$,

$$q_i = \frac{\partial V/\partial \mu_i}{\partial V/\partial W} \quad \text{and} \quad -x_j = \frac{\partial V/\partial \gamma_j}{\partial V/\partial W} .$$

Pope (1982a,b) shows that profit function results from the dual approach to production under certainty do not follow in the uncertainty case (for example, Hotelling's Lemma and homogeneity of factor demands and output supplies do not hold) but that symmetry and negative definiteness of the Slutsky matrix of input demands holds, and compensated output is positive.

Procedures for Exact Welfare Measurement

In the expected utility model, the compensating variation, c, of a change in any economically relevant parameter, θ, can be defined as

$$E[U(\theta_1, W_0) = E[U(\theta_2, W_0 - c)].^2$$

Just, Hueth, and Schmitz show for a single mean output price change from μ_i^o to μ_i^1 the compensating variation, when considered as an *ex ante* nonstochastic payment that keeps the individual's utility at the initial level, can be written

$$(6) \qquad c(\mu_i^o, \mu_i^1) = \int_{\mu_i^o}^{\mu_i^1} \left(\frac{\partial c}{\partial \mu_i}\right) d\mu_i = \int_{\mu_i^o}^{\mu_i^1} q_i^*\left(\mu, \gamma, \phi, \psi, V_0\right) d\mu_i$$

where V_0 denotes the initial level of indirect expected utility, and q_i^* is the compensated supply for output i; this contrasts with the ordinary supply function for output i in (5). The *ex ante* compensating variation of a mean input price is

$$(7) \qquad c(\gamma_j^o, \gamma_j^1) = -\int_{\gamma_j^o}^{\gamma_j^1} x_j^*(\mu, \gamma, \phi, \psi, V_0) d\gamma_j$$

where x_j^* denotes the compensated input demand function for input j; this contrasts with the ordinary demand for input j in (4).

Pope, Chavas, and Just derive Slutsky-Hicks equations that relate the slopes of ordinary and compensated demands

$$(8) \qquad \frac{\partial x_j^*}{\partial \gamma_j} = \frac{\partial x_j}{\partial \gamma_j} + x_j \frac{\partial x_j}{\partial W_0}$$

and the slopes of ordinary and compensated supplies

$$(9) \qquad \frac{\partial q_i^*}{\partial \mu_i} = \frac{\partial q_i}{\partial \mu_i} - q_i \frac{\partial q_i}{\partial W_0} .$$

These results show that q_i and q_i^* (x_j and x_j^*) are different functions with different mean price slopes except under constant absolute risk aversion (cara), where $\frac{\partial x_j}{\partial W_0} = \frac{\partial q_i}{\partial W_0} = 0$. The importance of (8) and (9) in this paper is that they provide a basis for employing the spirit of Hausman's consumer welfare approach in calculating the producer welfare measures (6) and (7) from ordinary empirical supplies and demands under uncertainty.

Hausman used duality to show that a consumer's c of a single price change could be computed from the parameters of the ordinary consumer demand functions for several widely used specifications. Roy's Identity is used to specify an ordinary differential equation in income and price. For some demand specifications, this differential equation can be solved for the expenditure function. Changes in the expenditure function resulting from parameter changes can then be evaluated to obtain c. Hausman considered the case of a single price change only.

Hausman's reasoning is helpful in the present context, even though his methods are not applicable since duality results have not been extended to the expected utility model under uncertainty.[3] However, when the Slutsky equation can be integrated directly, the compensated demands can be obtained from parameters of the ordinary demand function. Thus, the exact welfare measures can be obtained by integrating the resulting compensated demands. Furthermore, this approach extends directly to multiple price change cases.[4]

Consider some initial point (q_i^o, μ_i^o) on the ordinary supply function for q_i in price-quantity space for output i. Equation (9) gives the slope of the compensated supply functions which intersect the ordinary supply function $q_i(\mu, \gamma, \phi, \psi, W_0)$ as one moves along the ordinary supply curve at (q_i^o, μ_i^o). However, rewriting (9) as

$$(10) \qquad \frac{\partial q_i^*}{\partial \mu_i} = \frac{\partial q_i}{\partial \mu_i} - q_i^* \frac{\partial q_i}{\partial W_0}$$

gives an expression for the slope of the compensated supply function as one moves along the <u>compensated supply</u> at (q_i^o, μ_i^o) in terms of the parameters of the observed ordinary supply functions which intersect it. Noting that indirect expected utility and all other arguments of q_i^* are held constant along the compensated supply, (10) can be written as an ordinary differential equation in q_i^* and μ_i:

$$(11) \qquad \frac{dq_i^*}{d\mu_i} = \frac{\partial q_i}{\partial \mu_i} - q_i^* \frac{\partial q_i}{\partial W_0}.$$

The solution to (11), if it exists and can be found, is the compensated supply function $q_i^*(\mu, \gamma, \phi, \psi, V_0)$ which facilitates calculation of compensating variation following (6). A similar argument holds for the derivation of compensated input demands from ordinary input demands.

<u>Exact Welfare Measures of Price Change Implied By Common Supply Specifications</u>

This section derives exact welfare measures for several empirical supply specifications under price uncertainty.[5]

The Linear Case

Suppose equation (5) is parameterized as

$$(12) \qquad q = \alpha + \beta\mu + \delta W_0 + f(\phi, \psi)$$

where α and δ are m-dimensional column vectors and β is an m-dimensional square matrix of parameters. The linear specification, because of its simplicity, has the widest empirical precedent in the risk literature; for example, Behrman (1968) and Just (1974) used linear mean and variance parameters to estimate supply response functions.

Under the linear maintained hypothesis in (12), the Slutsky-Hicks equation (as modified by the analysis of the previous section) is

$$(13) \qquad dq_i^*/d\mu_i = \beta_{ii} - \delta_i q_i^*,$$

which is an ordinary differential equation with general solution

$$(14) \qquad q_i^* = \frac{\beta_{ii}}{\delta_i} + ke^{-\delta_i \mu_i}.$$

To find a particular solution to (14), observe that by definition $(q_i^{*o}, \mu_i^o) = (q_i^o, \mu_i^o)$, so

$$q_i^o = \frac{\beta_{ii}}{\delta_i} + ke^{-\delta_i \mu_i^o}.$$

which requires that

$$k = \frac{q_i^o - \beta_{ii}/\delta_i}{\exp(-\delta_i \mu_i^o)} .$$

Thus, the equation for the compensated supply function corresponding to the linear supply function (12) and passing through (q_i^o, p_i^o) is

$$(15) \qquad q_i^* = \frac{\beta_{ii}}{\delta_i} + \left(q_i^o - \frac{\beta_{ii}}{\delta_i}\right) e^{-\delta_i(\mu_i - \mu_i^o)}$$

Equation (15) is easily integrated to obtain the <u>ex ante</u> c of a change in the mean price of output i from μ_i^o to μ_i, <u>ceteris paribus</u>:

$$c(\mu_i^o, \mu_i) = \int_{\mu_i^o}^{\mu_i} q_i^* d\mu_i$$

$$= \frac{\beta_{ii}}{\delta_i}(\mu_i - \mu_i^o) - \frac{1}{\delta_i}\left(q_i^o - \frac{\beta_{ii}}{\delta_i}\right)\left[e^{-\delta_i(\mu_i - \mu_i^o)} - 1\right].$$

For multiple price changes, this procedure can be applied sequentially to successive price changes from μ_i^o to μ_i^1 given changes from μ_j^o to μ_j^1, $j < i$. Note that the total welfare effect is captured without examining cross price-quantity effects according to the derivation of Section II.

The parameter restrictions which assure a positive compensated output response to an increase in mean output price are $\beta_{ii} > \delta_i q_i^o$, from (13). Since δ_i is typically non-negative, the ordinary output response is generally positive.[6] However, cases could arise where $\delta_i < 0$ because of increasing absolute risk aversion (as with a quadratic utility function) or because of risk-loving behavior. In these cases a negative ordinary supply response can occur.

The Semilog Case

If equation (5) is specified as

$$q_i = \exp[\alpha + \beta\mu + \delta W_0 + f(\phi,\psi)]$$

the Slutsky-Hicks equation (11) is

$$dq_i^*/d\mu_i = \beta_{ii} q_i^* - \delta_i q_i^{*2}.$$

This equation has general solution

$$q_i^* = \frac{k\beta_{ii} \exp(\beta_{ii}\mu_i)}{1 + k\delta_i \exp(\beta_{ii}\mu_i)}$$

where k is some constant of integration. The compensated supply function through (q_i^o, μ_i^o) is

$$(16) \qquad q_i^* = q_i^o \left[\frac{\beta_{ii} \exp[\beta_{ii}(\mu_i - \mu_i^o)]}{\beta_{ii} - \delta_i q_i^o [1 - \exp(\beta_{ii}(\mu_i - \mu_i^o))]}\right]$$

for which the parameter restrictions necessary for $dq_i^*/d\mu_i > 0$ are again $\beta_{ii} > \delta_i q_i^o$. The compensating variation is obtained through integration of (16):

$$c(\mu_i^o, \mu_i) = \frac{1}{\delta_i} \left\{ \ln\left[\beta_{ii} - \delta_i q_i^o (1 - e^{\beta_{ii}(\mu_i - \mu_i^0)})\right] - \ln \beta_{ii} \right\}.$$

The Cobb Douglas Case

If equation (5) were specified to be linear in logs,

$$\text{(17)} \qquad \ln q = \alpha + \beta \ln \mu + \delta \ln W_0 + f(\phi, \psi)$$

the familiar Cobb-Douglas form results from taking the exponential of both sides. The Slutsky-Hicks equation corresponding to (17) is

$$\text{(18)} \qquad \frac{dq_i^*}{d\mu_i} = \beta_{ii}\frac{q_i^*}{\mu_i} - \delta_i \frac{q_i^{*2}}{W_0}.$$

The general solution for (18) is

$$q_i^* = k_1 \mu_i^{\beta_{ii}} (\mu_i^{\beta_{ii}+1} + k_2)^{\frac{\delta_i}{1-\delta_i}}$$

and using the initial conditions for the location and slope of q_i^* to obtain k_1 and k_2, the compensated supply function passing through (q_i^0, p_i^0) is

$$q_i^* = q_i^0 \ \mu_i^{\beta_{ii}}/\mu_i^{0\,\beta_{ii}+1}\left[\frac{z}{\mu_i^{\beta_{ii}} - \mu_i^{0\,\beta_{ii}+1} + z}\right]^{\frac{\delta_i}{\delta_i - 1}}, \quad \delta_i \neq 1$$

where $z = W_o/(\delta_i - 1)\ q_i^o$ and parameter restrictions sufficient for $dq_i^*/d\mu_i > 0$ are $\beta_{ii} > \delta_i \mu_i^o q_i^o / W_0$. The compensating variation for a change in μ_i is given by

$$c(\mu_i^o, \mu_i) = W_o \mu_i^{o\,-(\beta_{ii}+1)}/(\beta_{ii}+1)\left\{1 - \left[\frac{z}{\mu_i^{\beta_{ii}+1} - \mu_i^{o\,\beta_{ii}+1} + z}\right]^{\frac{1}{\delta_i - 1}}\right\}, \ \delta_i \neq 1.$$

When $\delta_i = 1$, the expression for compensated supply simplifies to

$$q_i^* = q_i^o \left(\frac{\mu_i}{\mu_i^o}\right)^{\beta_{ii}} \exp\left((\mu_i^{1+\beta_{ii}} - \mu_i^{o\,1+\beta_{ii}})/(1 + \beta_{ii})A\right),$$

where $A = -W_0 \mu_i^{o\,\beta_{ii}}/q_i^o$ is a constant determined by the initial point through which the compensated supply passess. The compensating variation for this case is also easily calculated but will not be given here.

Welfare Measurement for Changes in Price Distributions

Welfare measurement for a variety of changes in the price distribution parameters are also easily done using the general approach of specifying the behavioral (supply or demand) relationships directly. For example, all of the variables in (12) affect the slope and location of the compensated supply through shifts in q_i in (15). Thus, it is feasible to derive an ex ante measure of the c or e for, say, a change in the skewness of the output price distribution by considering areas to the left of the appropriate compensated supply function before and after the shift.[7] Similarly, evaluation of welfare effects for changing any individual moment or for linear shifts in the price distribution (i.e., of the form $\hat{p} = a + bp$) are possible. In the latter case, the change in price distribution alters not only the mean price but also the higher moments; when this happens, the output response could be negative.

To evaluate the effects of linear shifts in output price distributions, assume for illustration that only the first two moments of any price distribution affect firm decisionmaking; normality of the price distribution would be an example. Using (12), write the (linear) ordinary supply functions in terms of the moments of the reference price distribution for each output and linear shifts r and T:

$$q = \alpha + \beta(r + T\mu^o) + \rho T'TV^o + \delta W_0 \quad .$$

where r, μ^o, and V^o are m-dimensional column vectors representing additive shifts to, and reference means and variances of, the output price distributions; ρ is an m-dimensional square matrix of parameters: and $T = \{t_{ij}\}$ is an m-dimensional diagonal matrix of multiplicative shifters. When r is the zero vector and T is the identity matrix, $q = q^o$ corresponding to the reference. price distributions. This is the starting point for the analysis.

Consider a sequence of multiplicative shifts in the price distributions for the first $k \le m$ outputs: r is the zero vector and $t_{jj} = 0$, $j > k$.[8] The Slutsky-Hicks equation for output i is

$$(19) \qquad dq_i^*/dt_i = \beta_{ii}\mu_i^o + 2t_{ii}\rho_{ii}V_i^o - \delta_i q_i^*$$

for which

$$q_i^* = \frac{\beta_{ii}\mu_i^o\delta_i - 2\rho_{ii}V_i^o}{\delta_i^2} + \frac{2\rho_{ii}V_i^o}{\delta_i} t_{ii} + q_i^o e^{-\delta_i(t_{ii}-1)}$$

is the solution. The compensating variation associated with the i^{th} multiplicative shift is

$$(20) \qquad c(1,t_{ii}) = \frac{\beta_{ii}\mu_i^0\delta_i - 2\rho_{ii}V_i^o}{\delta_i^2}(1 - t_{ii}) + \frac{\rho_{ii}V_i^o}{\delta_i}(1 - t_{ii}^2) + \frac{q_i^o}{\delta_i}\left(1 - e^{\delta_i(1-t_{ii})}\right).$$

Because of path independence, the compensating variation of k multiplicative shifts can be determined by sequentially evaluating an expression like (20) for each output, successively allowing the system to adjust (through the q_i^o) to proceeding shifts. Also, it is immediately apparent from (19) that in a mean-variance model, if $\rho_{ii} < -\beta_{ii}\mu_i^o/2t_{ii}V_i^o$, the ordinary supply response to a multiplicative increase in the output i price distribution will be negative. An example of this behavior for risky multioutput production subject to capacity constraints is given by Just and Zilberman.

Welfare Measurement Under Production Uncertainty

The development in previous sections was restricted to cases of price uncertainty. When production is also uncertain, the derivation of welfare measures becomes somewhat more complex, but is still possible. As noted earlier, when production is uncertain, measurement of welfare based on supply, or expected supply, functions is not justified because output is a random variable. However, when the input decisions are made prior to realization of actual output, inputs are nonstochastic from the decisionmaker's point of view. Thus, results relating to welfare calculations based on derived demands continue to hold.

Where the firm's decisions are made subject to possible production and price uncertainty, profits can be defined as the difference between random total revenues (the sum of m random revenues variables, one for each output) and the sum of products of input quantities and their respective random prices. The firm's decision is to choose x to maximize $E[U(W_0 + Re - rx)]$, where R is an m-dimensional random revenues vector and e is the m-dimensional unit vector. The resulting input demands are of the form $x_j = x_j(\mu, \gamma, \rho, \phi, \psi, \tau, W_0)$, where ρ and τ denote output means and higher moments, respectively.

If the parameter changes to be evaluated are input mean prices, the total welfare change is obtained by successively integrating the demands for inputs whose prices change, following the analysis in Section III. More generally, if the parameters that change are means output prices or quantities, or other risk parameters, the welfare change can be found by integrating the demand for a necessary input.[9] If input j is a necessary input with mean price γ_j^o and shut down price $\hat{\gamma}_j^o$ before a change in some or all parameters (denoted by a vector θ), and with mean price γ_j^1 and shut down price $\hat{\gamma}_j^1$ after the parameter change, the compensating variation is

$$c(\theta^0, \theta^1) = \int_{\gamma_j^1}^{\hat{\gamma}_j^1} x_j^*(\rho^1, \gamma_j, \overline{\gamma}^1, \tau^1, V^o)\, d\gamma_j - \int_{\gamma_j^o}^{\hat{\gamma}_j^o} x_j^*(\rho^o, \gamma_j, \overline{\gamma}^o, \tau^o, V^o)\, d\gamma_j,$$

where $\overline{\gamma}$ denotes the (n - 1)-vector of mean input prices besides γ_j and x_j^* is the compensated demand function for input j.

Conclusions

For general price or production uncertainty, estimation of several commonly used functional forms for properly specified output supplies and factor demands permits derivation of the compensated *ex ante* supply and demand functions, and calculation of the true *ex ante* welfare effects of a parameter change. When linear, semilogarithmic, or log-linear (Cobb-Douglas) specifications are used, there is no need to use the Willig-type bounds calculated by Pope and Chavas, nor any need to assume the utility function exhibits constant absolute risk aversion in order to derive true welfare change measures.

Though the focus of this paper is on development of conceptual measures of welfare change, several interesting and challenging empirical issues for future research are suggested. Determining the moments which fully characterize the distributions of the random variables confronting the decision maker is complex, though methods exist for eliciting or estimating subjective probability beliefs. Accurate measurement of the initial wealth variable is crucial if the firm maximizes expected utility of wealth, since the income effect is the basis for divergence of compensated demands (supplies) from ordinary demands (supplies).

Footnotes

1. While the main results of the paper are developed under price uncertainty, the results also extend to production uncertainty, which is covered in a later section.

2. Equivalent variation is not considered here in the interest of brevity, since all results obtained for compensating variation extend to equivalent variation in a straightforward manner.

3. An alternative theory of choice under risk, which reverses the roles of payments and probabilities, has been proposed by Yaari, who terms this alternative theory a "dual" theory.

4. Since there is no problem of path dependence for the measures in (6) and (7), a multiple price change can be easily evaluated sequentially as a sum of integrals, where the integrals are allowed to adjust to preceding levels of compensation.

5. The analysis of the welfare effects of changes in mean input prices through integration of input demands proceeds in a completely analogous way but for the sake of space is omitted here.

6. Pope, Chavas, and Just note that dq_i/dW_0 (δ in the linear specification) > (=) 0 as absolute risk aversion is decreasing (constant). For a given price distribution, a firm with a higher initial wealth has a smaller Pratt risk premium and will choose a greater output.

7. Downside risk aversion (Menezes, Geiss, and Tressler) suggests that an increase in the skewness of price distribution would increase the risk averse firm's optimal output.

8. Additive shifts will not be treated explicitly here, since they were the subject of the previous section. Also, path independence assures that combinations of additive and multiplicative price shifts can easily be evaluated as a sequence of integrals, allowing the demand system to adjust after each partial compensation is obtained through integration.

9. I.e., an input x_j for which there is a mean price $\hat{\gamma}_j$ low enough to cause the firm to cease operating.

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