Dynamic Oligopoly in Rice:

Estimation and Tests of Market Structure and Subgame Perfection

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A linear-quadratic cost-of-adjustment model is used to estimate market structure, firm rationality, and adjustment costs. Market structure is measured by an index that is analogous to a conjectural variation. We consider both open-loop and subgame perfect equilibria. The model is used to study the rice export market.

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Dynamic Oligopoly in Rice: Estimation and Tests of Market Structure and Subgame Perfection

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In recent years, dramatic shifts in the international rice export market have occurred. In particular, the share of the largest exporter (Thailand) has grown substantially, while the share of the second largest (the U. S.) has fallen. A key question in determining policy in each of these countries is the degree of competitiveness in the market. All of the major exporters except the U. S. (Thailand, China, Burma, and Pakistan) have a single marketing board that handles exports, so that their behavior can reasonably be described as resembling that of a single firm. Thus, we introduce a model of dynamic oligopoly behavior to describe behavior in these markets.

The first part of the paper presents a brief summary of our theoretical model. We consider both open-loop and feed-back models. Our formulation is flexible enough to allow for price-taking behavior, oligopoly, or monopoly. The second part of the paper presents simulation results to show the properties of our model. Finally, our empirical results on the rice market are presented.

A Model of Dynamic Oligopoly

It is not possible to instantaneously adjust output at zero cost. Each country's (firm's) quadratic cost of adjustment is:

$$\left(\delta_{0,i} + \frac{\delta u_{i,t}}{2}\right) u_{i,t}\,\varepsilon, \quad \text{where } u_{i,t}\,\varepsilon = q_{i,t} - q_{i,t-\varepsilon}$$

The firm's quadratic cost of production is:

$$\left(\theta_{0,i} + \frac{\theta q_{i,t}}{2}\right) q_{i,t}\,\varepsilon,$$

The industry faces a linear demand curve:

$$p = a - bq, \quad q = \sum q_i,$$

so each firm has revenues in period t of:

$$p_{i,t}q_{i,t} = [a - b(q_{i,t} + \sum_{j \neq i} q_{j,t})]\, q_{i,t}\, \varepsilon$$

[Note: we can allow each firm i to have a unique intercept, a_i, and slope b_i, at some cost in complexity]

Each firm's objective is to maximize its discounted stream of profits (given interst rate r):

$$\sum_{t=1}^{\infty} e^{-r(t-1)\varepsilon} \left[\left(p_{i,t} - \frac{\theta}{2}\, q_{i,t}\right)\, q_{i,t} - \frac{\delta u_{i,t}}{2}\, u_{i,t}\right]\, \varepsilon \tag{1'}$$

where, for simplicity, we set $\theta_{0,i} = \delta_{o,i} = 0$.

In matrix form, the ith firm's objective is:

$$\sum_{t=1}^{\infty} e^{-r(t-1)\varepsilon}[ae_i'(q_{t-\varepsilon}-u_t\varepsilon)- \frac{1}{2}(q_{t-\varepsilon}+u_{t-\varepsilon}\varepsilon)'K_i(q_{t-\varepsilon}+u_t\varepsilon)-\frac{1}{2}u_t'S_iu_t]\varepsilon$$

where e_i is the ith unit vector, e is a column vector of 1's, $S_i = e_ie_i'\delta$, $K_i = b(ee_i' + e_ie') + \theta e_ie_i'$ [K_i is a matrix with b's on the ith row and column except for the (i,i) element which is $2b + \theta$; all other elements are 0].

As $\varepsilon \to 0$, this expression approaches:

$$\int_0^{\infty} e^{-rt}\, [ae_i'q_t - \frac{1}{2}\, q_t'K_iq_t - \frac{1}{2}u_t'S_iu_t]dt \tag{1b}$$

Since $q_{i,t}$ is unconstrained so that negative sales are possible. Negative prices can be interpreted as very low prices. When prices fall below a certain level, firms would prefer to be buyers rather than sellers; they must bear the adjustment cost to make the transition.

Two Families of Equilibria

We consider two families of equilibria: open-loop and feed-back. Members of each family are indexed by a parameter v ($= \partial u_{j,t}/\partial u_{i,t}$ for $i \neq j$), which describes the behavioral assumption that determines the outcome. In a static model, v could be interpreted as a constant conjectural variation. Since the open-loop game is equivalent to a static problem, the same interpretation can be used, but that interpretation does not apply to the feed-back model. We use the neutral description of v as a player's behavioral assumption. This assumption is taken as primitive and not explained by strategic considerations (except for the three leading cases of collusion, Nash-in-quantities, and price-taking):

A. Collusion: $v = 1$ (where all firms are identical): match changes in output -- maintain market share.

B. Nash-in-quantity: $v = 0$: can't change output within a period.

C. Price-taking: $v = -1/n$ (where there are n+1 firms): total output is unchanged.

In the open-loop equilibrium, each player chooses a sequence of changes in output, $u_{i,t}$, using a particular behavioral assumption, v. The equilibrium levels can be expressed in feed-back form; in which case, strategies are open-loop with revision. This revision is unanticipated. When players choose their current levels, they act as if they were also making unconditional choices regarding future levels. A control approach can be used to obtain the open-loop equilibrium for arbitrary v.

In the feed-back equilibrium, players recognize that their future choices will be conditioned on the future state, so players choose control rules rather than levels. The feed-back equilibrium is obtained by the simultaneous solution of the n+1 dynamic programming equations:

$$J_i(q_{t-\varepsilon}) = \max_{u_{i,t}} [(ae_i'q_t - \frac{1}{2} q_t'K_iq_t - \frac{1}{2}u_t'S_iu_t)\varepsilon + e^{-r\varepsilon}J_i(q_t)] \qquad (2)$$

where $q_t = q_{t-\varepsilon} + u_t\varepsilon$. When v =0, equation (2) gives the feed-back Nash-Cournot game. When v = -1/n or 1, we obtain the price-taking or collusive outcome (i.e., the feed-back and the open-loop solutions are the same).

Consistent conjectures

We call v a "behavioral assumption." If, however, one viewed v as a conjectural variation, it is natural to ask if it could be made endogenous, as has been done in static games, by imposing the consistency requirement of Laitner, Breshnahan, Kamien and Schwartz, and Perry. This approach can be used in the open-loop games because they are essentially static. A different interpretation is used in the feedback game. At the beginning of a period of length ε, players anticipate an equilibrium in the current period that depends on lagged quantities. They expect that any deviation from this equilibrium will be met by an instantaneous response from their rivals. If the conjectural response is optimal (to a first-order approximation), then the conjectures are said to be consistent.

In the discrete time feed-back game, the consistent conjecture depends on ε. As $\varepsilon \to 0$, the consistent conjecture goes to 0. As $\varepsilon \to \infty$, the game becomes static and (with $\theta = 0$) you get Breshnahan's consistent conjecture of $-1/n$.

Intuition (based on equation (2)): A change of Δu_j from equilibrium changes q_j by $\Delta u_j \varepsilon$. Firm i's loss from a failure to respond to a change in u_j has two components: the reduction of its profits in the current period and the present value of the loss of having a suboptimal q_i in the subsequent period. Both these components depend on $\Delta u_j \varepsilon$ and ε directly, since the current period's profits are a flow of profits times ε and next period's value function is discounted by $\exp(-r\varepsilon)$.

When ε is large, for give Δu_j, the first component dominates -- firm i can suffer a substantial loss from not responding to a change in u_j, so the slope of i's reaction function (and hence the value of j's consistent conjecture) should be large in absolute value. When ε si small, a given Δu_j has negligible effect on i's payoff, so the consistent conjecture is small.

Effect of an increase in the number of firms, n+1

If $\theta = 0$, we can normalize by setting $\delta = (n+1)c$, where $c > 0$ is constant. This normalization implies that the open-loop price-taker and collusive equilibria are invariant to n. As $n \to \infty$, the Nash-Cournot feed-back equilibrium converges to the Nash-Cournot open-loop equilibrium. Reynolds showed that, as $\delta \to 0$, the steady-state open-loop and feed-back Nash-Cournot equilibria do not converge. Our result shows that as both δ and $n \to \infty$, so that $\delta/(n+1)$ remains constant, the two steady states do converge. Apparently, for intermediate values of v, the open-loop and feed-back equilibria still differ, but we have not formally proven this result.

Simulation results

To illustrate the properties of these models, we first use simulations. In the following simulations, the basic parameters are: two firms (n = 1), demand intercept (a) = 255, demand slope (b) = 10, adjustment cost parameters ($\delta_{0,i}$, δ) = 5, discount parameter (β) = .95, initial industry output = 20, the length of a time period (ε), and constant marginal cost (θ) = 0. Given these parameters, the collusive, Nash and price-taker static equilibria industry output levels are: 12.75, 17.5, and 25.5. The corresponding steady-state values are 12.738 (collusive), 16.983 (Nash open-loop), 17.483 (Nash feed-back), and 25.476 (price-taker). Thus,

Observation 1: The Nash feed-back steady-state industry output is greater than the Nash open-loop output.

Observation 2 : As the number of firms increase, the collusive and price-taking steady-state industry output levels are, of course, not affected, but the industry output rises under either of the Nash models, approaching the price-taker level. At least for this example, the Nash feed-back output increases more rapidly than does the Nash open-loop output as the number of firms increases:

Table 1
The Effect of Increasing the Number of Firms on Industry Output

Number of Firms	Nash Open-Loop	Nash Feed-Back
2	16.983	17.483
3	19.106	19.915
4	20.380	21.348
5	21.229	22.270
10	23.159	24.162
15	23.883	24.743

Parameters: $n = 1$, $a = 255$, $b = 10$, $\delta_{0,i} = 5$, $\beta = .95$, $\delta = 2.5(n+1)$

The collusive steady-state output = 12.7375, the price-taker output = 25.475.

Observation 3: There is no distinction between the feed-back and open-loop steady-state outputs where firms act collusively ($v = 1$) or are price-takers ($v = -1/n$). Under other behavioral rules, the feed-back output is greater than the open-loop output. The greatest difference does not occur at the Nash v (= 0).

See Figure 1. In this example, the greatest difference occurs at approximately $v = -0.7$.

One implication of this result is that using a standard open-loop model does not cause problems when firms are collusive or are price-takers. Unfortunately, where firms use other behavioral assumptions, the feed-back and open-loop models produce different results (both in terms of steady-state values and adjustment paths). In our example, the feed-back steady-state output was as much as 5 percent higher than the open-loop output.

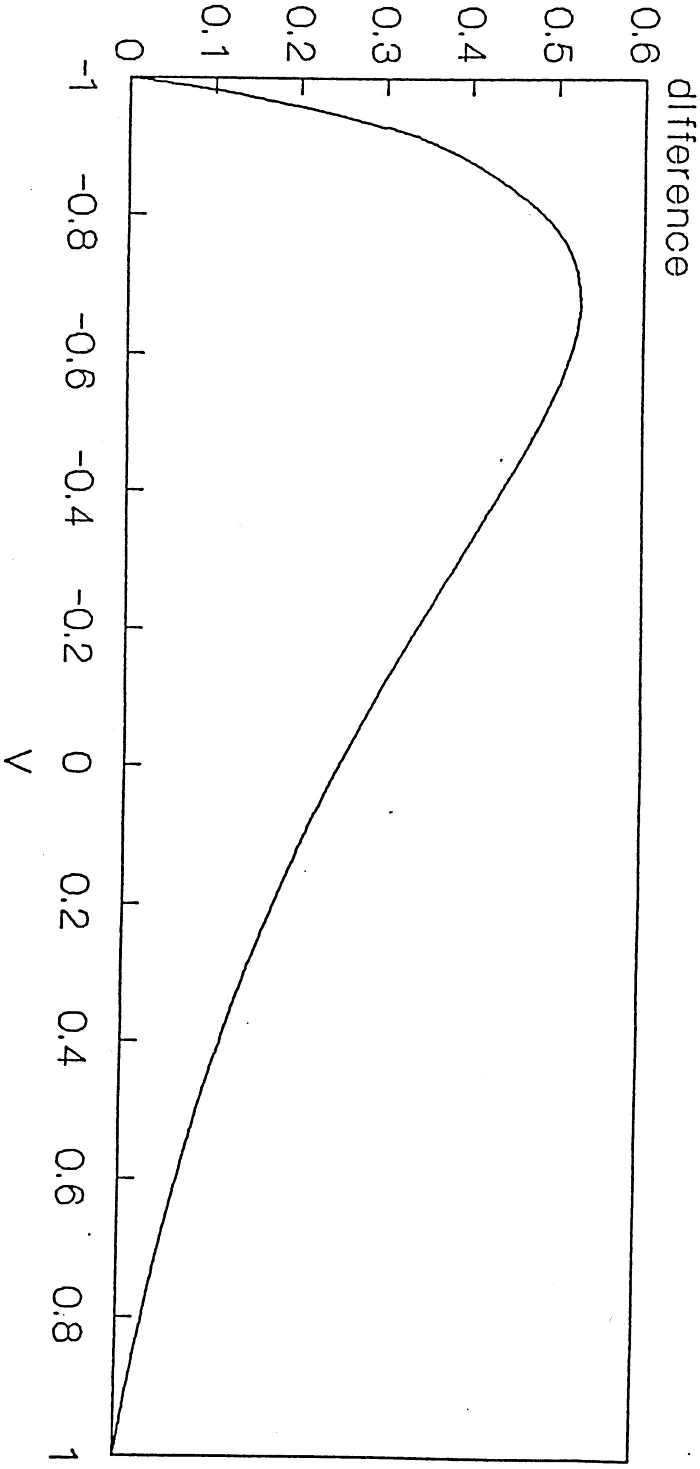


Figure 1
Difference between
Feedback and Open-Loop Output

The Adjustment Equations and Restrictions

In order to estimate this system we need the basic control rules. The feed-back control rule is:

$$q_{i,t} = \phi + (1 + \frac{\rho_0 + nv\rho_1}{\delta})q_{i,t-1} + \frac{\rho_1 + nvz}{\delta} \sum_{j \neq i} q_{j,t-1}$$

The Control rule equivalent to the open-loop is:

$$q_{i,t} = \phi + (1 + \frac{\rho_0}{\delta})q_{i,t-1} + \frac{\rho_1}{\delta} \sum_{j \neq i} q_{j,t-1}$$

The restrictions are:

$$0 = r\delta\rho_0 + \delta[(2 + nv)b + \theta] - \rho_0^2 - n\,\rho_1^2 \qquad \text{(A2a)}$$

$$0 = r\delta\rho_1 + \delta b - 2\rho_0\rho_1 - (n - 1)\rho_1^2 \qquad \text{(A2b)}$$

$$h = \frac{a\delta}{(r\delta - \rho_0 - n\rho_1)} \qquad \text{(A3)}$$

Rice

Our empirical work is still in the early stages. We first estimate the linear demand curve for rice using three-stage least squares (asymptotic standard errors in the parentheses):

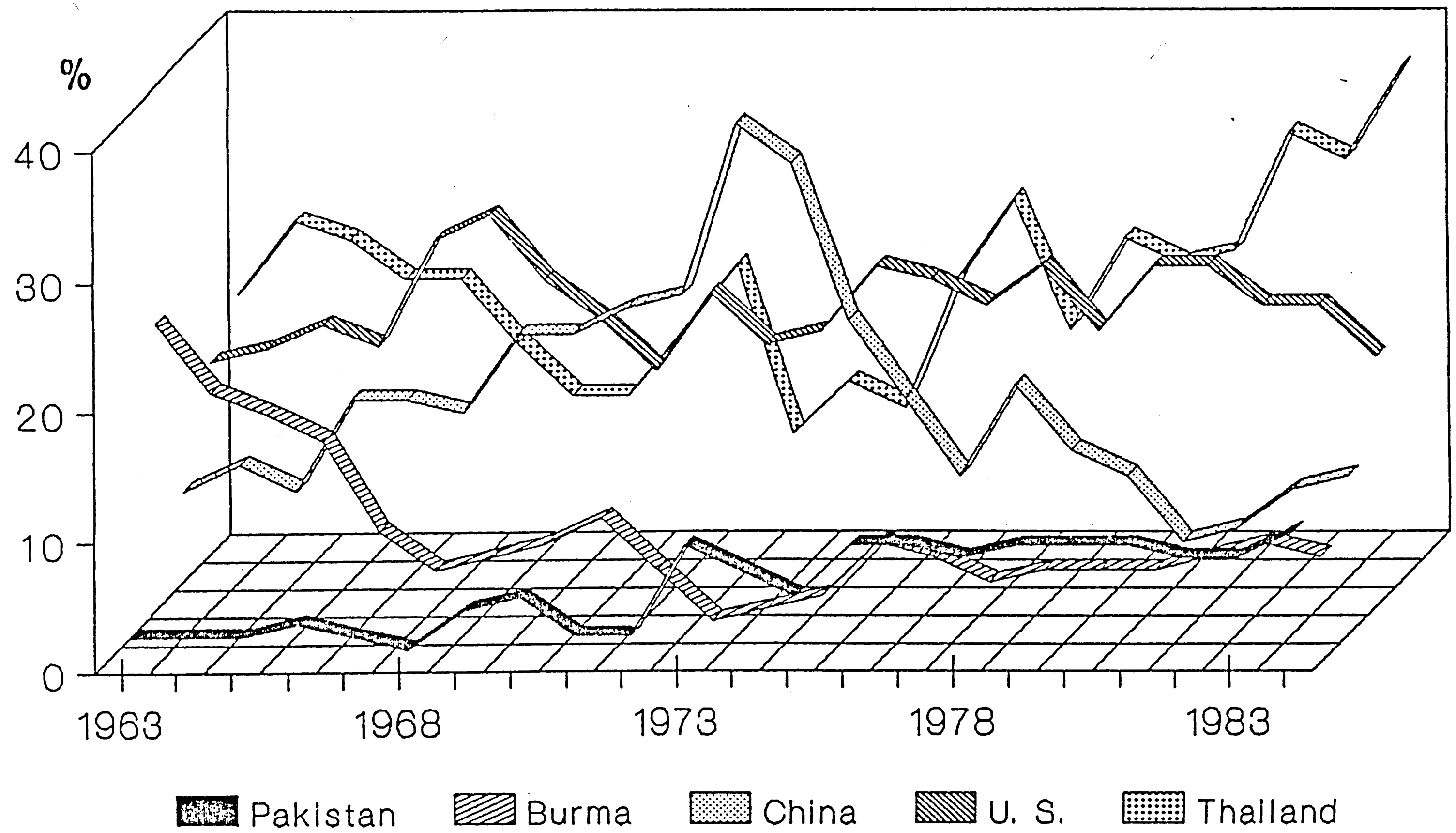
$$\begin{array}{l} p_r = 98.026 - 0.6202\, q + 0.2279\, p_w - 0.5690\, I \\ \qquad (5.127) \quad (0.107) \quad (0.046) \quad (0.081) \end{array}$$

where p_r = real price of rice, q = total rice exports, I = income, p_w = real price of wheat.

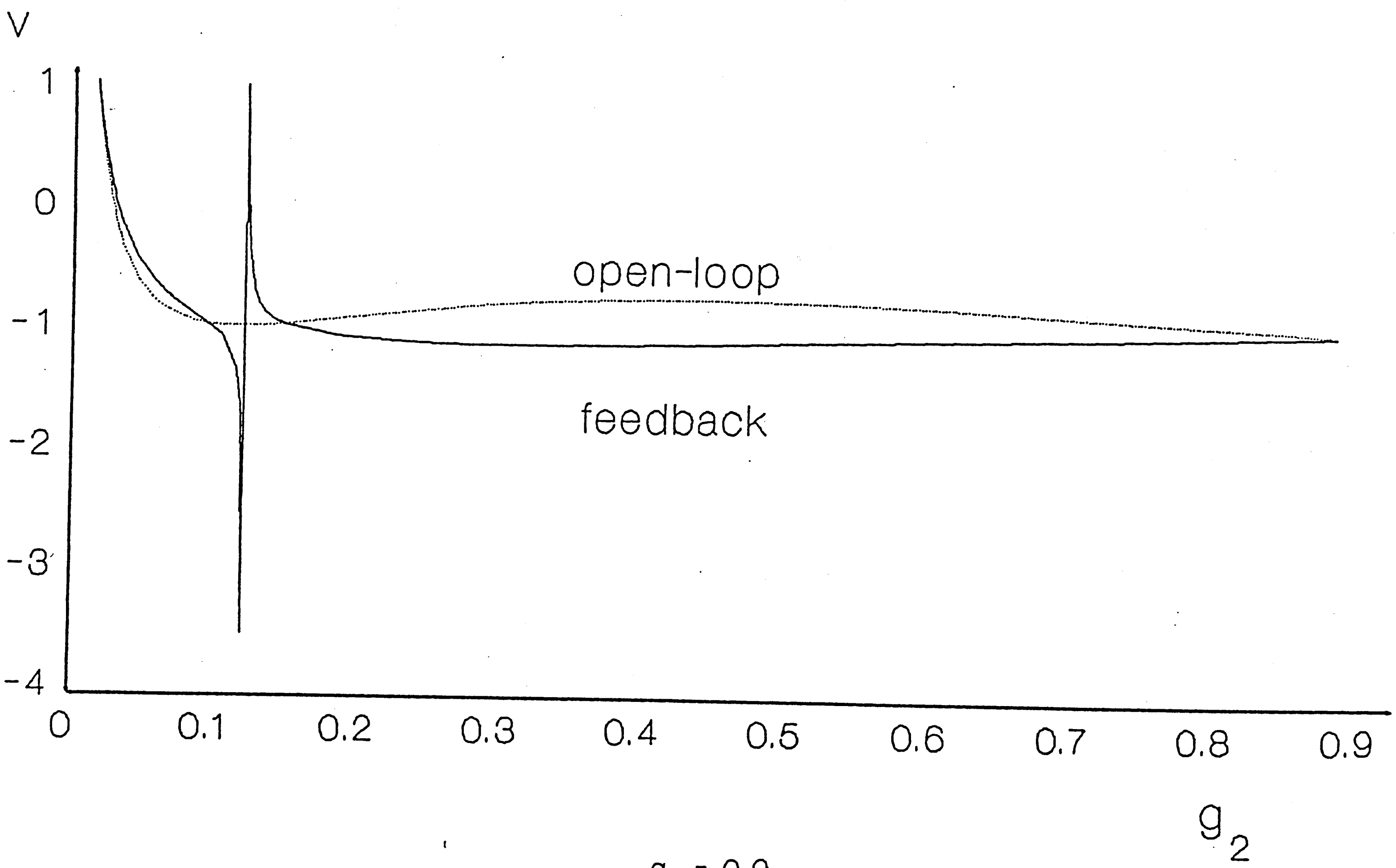
Next we estimate the adjustment equation parameters g_1 (diagonal elements of the adjustment matrix) and g_2 (off-diagonal elements) based on the assumption of symmetry. The parameters $g_1 = 0.57728$ (t-statistic = 7.13) and $g_2 = -0.054478$ (1.56). The test that these parameters are zero across the equations are $F(5,85) = 6.62$ for g_1 and $F(20,85) = 2.99$ for g_2, so we can reject the hypotheses that these parameters are zero.

Based on these estimates of b, g_1 and g_2, the open-loop parameters are $\delta_o = 3.2954$ and $v_o = .05958$ (standard error = .4033). The feed-back parameters are $d_f = 3.3736$ and $v_f = .1253$ (.4126). Thus, the point estimates indicate the market is closer to Nash-Cournot ($v = 0$) than competitive ($v = -0.25$), but we cannot reject competition at the 0.05 level.

Share of Exports
by the Five Largest Exporters



Market Structure, v
Feedback and Open-Loop Models



$g_1 = 0.9$

Two identical firms