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The Distribution of Futures Prices:

A Test of the Stable Paretian and Mixture of Normals Hypotheses

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Abstract

Two alternate hypotheses, the stable Paretian and mixture of normals, have been proposed to explain the observed thick-tailed distributions of futures price movements. The two hypotheses are tested by applying the stability-under-addition test of stable distribution parameters to twenty lengthy time series of changes in daily closing futures prices. Tests are conducted on both the original data series and randomized data. The results offer support for the mixture of normals hypothesis.

The Distribution of Futures Prices: A Test of the Stable Paretian and Mixture of Normals Hypotheses

I. Introduction

The distribution of daily commodity futures price movements is important in economic modeling. For example, portfolio models of asset allocation and option pricing models are typically derived assuming price changes are normally distributed with a constant variance.[1] Furthermore, statistical tests of the efficient market hypothesis which use the random walk model are based on the same assumption. If speculative price movements are not distributed normally, variance may not be an appropriate measure of dispersion and statistical tests based on finite variance are likely to give misleading results (Fama [12]). If distributions are normal, but variance is non-constant, then an adjustment for heteroskedasticity must be made before conducting statistical tests (Taylor [30]).

While the distribution of daily speculative price movements is often assumed to have a normal distribution, research on stock prices (Fama [11]; Officer [26]; Teichmoeller [31]; Barnea and Downs (2)) and futures prices (Mann and Hiefner [23]; Cornew, *et al*. [6]; Hudson, *et al* [17]; Gordon [9])[2] has found distributions that are leptokurtic relative to the normal distribution (i.e., having more values near the mean and in the extreme tails than a normal distribution). Two hypotheses have been proposed to explain the observed departures from normality. Under the first hypothesis, the stable Paretian, distributions conform to non-normal members of the stable Paretian family with infinite variance (Mandelbrot [22]). Under the second hypothesis, the mixture of normals, distributions are drawn from a population in which the variance changes (Fama [12]).

Recent research on the distribution of stock prices provides reasonably strong evidence in favor of the mixture of normals hypothesis, either through tests of subordinated stochastic process models of prices and trading volumes

(Epps and Epps [9]; Morgan [25]; Westerfield [32]; Harris [16]), tests based on sample variances for successively larger sample sizes (Perry [27]), or tests based on the stability-under-addition property of stable distributions (Fielitz and Rozelle [15]). Research on futures prices has not been as exhaustive or conclusive. Clark [4] and Tauchen and Pitts [29] applied subordinated stochastic process models to cotton and U.S. Treasury bill futures prices, respectively, and both reported results supportive of the mixture of normals hypothesis. Mann and Hiefner [23] conducted a stability-under-addition test of stable distribution parameters for nine commodity futures price series. However, their results are of questionable value due to computational errors.[3] So [28] recently estimated parameters of stable Paretian distributions for currency futures prices and tested whether non-stationarity of scale, attributable to a relationship between time-to-maturity and variability, could explain the observed distributions. He concluded that the nonnormality of foreign currency futures prices was not caused by nonstationarity of scale and that the stable Paretian distribution adequately described price changes for most currencies and most contracts.

The stable Paretian and mixture of normals hypotheses are tested in this paper by applying the stability under addition test of stable distribution parameters. This study uses daily futures price movements of 20 commodities. Financial, metal, and agricultural futures are included in the sample. Only nearby futures prices are used; thus, relative to past research, longer time series of data are available. The longer time series should be able to remove much of the effects of seasonality or differing maturities. The statistical procedure is applied to two different series. The first, which is the original series, is in chronological order. The second series uses the same data but the order is random. Randomly ordering the data ensures that adjacent price changes are independent and variance across time is not correlated. Estimates from the randomly ordered series are compared to the estimates from the original series. This randomization procedure has been applied by Fielitz and Rozelle to stock returns but has not been applied to futures prices.

II. The Alternative Hypotheses

The assumption of a normal distribution for price changes arises from the central limit theorem. The central limit theorem states that the sum and mean of a large number of independent random variables with finite variance approach a normal distribution as the sample size approaches infinity. Most researchers have found that daily price change movements are not distributed normally but are leptokurtic.[4] Mandelbrot suggested the stable Paretian approach to describing such distributions. A stable Paretian distribution is defined to be any distribution that is invariant under addition (Fama [11]). Stable Paretian distributions are the only possible limiting distributions for sums of independent, identically distributed variables (Fama [11]).

The logarithm of the characteristic function for the family of stable Paretian distributions is

1) $\log f(t) = i\delta t - \gamma|t|^{\alpha} (1 + i\beta(t/|t|) \tan (\pi\alpha/2))$,

where t is any real number and i is $\sqrt{-1}$. Stable Paretian distributions can be described by the four parameters, α, β, δ, and γ. The parameter γ, or c where $\gamma = c^{\alpha}$, is the scale parameter, δ is the location parameter, β is a measure of skewness, and α is the characteristic exponent. If α is greater than one, δ is equal to the mean of the distribution. The parameter β can have values in the interval $-1 \leq \beta \leq 1$. If $\beta < 0$ or if β is > 0 the distribution is skewed left or right, respectively. If $\beta = 0$ the distribution is symmetric. The characteristic exponent, α, determines the total probability in the extreme tails (Fama [11]).

The characteristic exponent, α, is the most important parameter for determining the type of distribution. Values of the characteristic exponent are limited to the interval $0 < \alpha \leq 2$. When $\alpha = 2$ the distribution is normal and the variance exists. When $0 < \alpha < 2$ there are more observations in the tails than under the normal distribution and the variance is not defined. The smaller the value of α the thicker the tails of the distribution (Fama [11]).

The mixture of normal distributions hypothesis is an alternative to the stable Paretian hypothesis (Fama [11]). The mixture of normal distributions hypothesis suggests commodity futures price movements are combinations of normal distributions with different variances. This could arise for example, if variance were proportional to the actual number of days rather than trading days (Fama and Roll [14]). Doukas and Rahman point out that the differing maturity of adjacent closing prices may cause differences in variability. But, Cornew argues that differences in maturity will be unimportant in a data set like the one used here. Results of Anderson and Kenyon, et al. suggest that seasonality may also have an influence in the grains. For example, Anderson found that the seasonality effect exceeded the maturity effect by a wide margin in explaining the variance of nine futures price series; in four of the nine studied, by a factor of at least ten-to-one.

Either of the mixture of normals or stable Paretian hypotheses can explain the observed leptokurtic distributions. Both hypotheses can be tested by estimating the characteristic exponent, α, for the entire sample, and for non-overlapping sums of observations from the sample (Fama and Roll [13]). The test arises from the stability under addition property of stable distributions. That is, as the stable Paretian distribution is invariant under addition, the distribution of sums of a stable Paretian distribution is stable Paretian with the same values of α and β (Fama [11]). Therefore, if the underlying distribution is non-normal stable the value of the estimated characteristic exponent, $\hat{\alpha}$, should tend to equality across the sums (Fama and Roll [13]). If the underlying distribution is a mixture of normals the values of $\hat{\alpha}$ for the sums should be closer to 2 than for the entire sample (Fama and Roll [13]).[5]

III. Procedure and Data

Following the procedure suggested by Fama and Roll [13], the two alternate hypotheses, the mixture of normal and the stable Paretian are tested by estimating α for the entire sample and for non-overlapping sums of observations from the sample. As indicated previously, the hypothesis of the mixture of normals is supported if the values of $\hat{\alpha}$ for the sums are closer to 2 than for the entire sample. If the alternate hypothesis of the stable Paretian distribution is appropriate, $\hat{\alpha}$ should tend to equality across the sums. This approach assumes symmetry and gives results similar to more complex methods which estimate all four parameters if asymmetry is not present. Leitch and Paulson compared estimating parameters with and without the assumption that β, the measure of skewness, equaled zero and found the effect of β on α was small and decreased as α approached 2. Since symmetry is assumed the distribution of c, and therefore α, is independent of the location parameter of the underlying variable (Fama and Roll [13]).

Logarithms of commodity futures price changes based on daily closing prices are used.[6] Log changes in closing prices of the futures contract nearest delivery are used until approximately two weeks before the delivery date. Then log changes in closing prices for the next nearest delivery month are used. The procedure is repeated as each "nearby" contract reaches its delivery month.[7] This gives a long time series and minimizes differences in maturity. The number of years of data and thus total number of observations vary by commodity (see Table 1).

The characteristic exponent is calculated for distributions based on all data available and for distributions including data for the six years, 1979-84. The 1979-84 period provides a sample of the same size for comparisons between commodities and allows for a comparison within commodities as the size of the sample is increased. Adjacent price movements are summed into groups of increasing size and the characteristic exponent estimated for each distribution. Due to the large amount of data the inclusion of larger sums is possible. Sum sizes are 2, 4, 10, 20, 30, and 60. The larger sums should be able to detect effects due to differing maturities. However, sample size decreases as the sum size increases and therefore, estimates for larger sums are subject to greater variance.

Daily futures price movements are also randomized, which makes adjacent price changes independent and removes any correlation in variance over time. Sums are taken again over the randomized series and the characteristic exponent estimated for each of these distributions. If adjacent price changes in the original series are independent and variance is not correlated over time, the estimated α's for the randomized series should follow the same pattern as those for the original series.

To estimate the characteristic exponent, α, $\hat{Z}_f$ is calculated. $\hat{Z}_f$ is an estimate of the f^{th} fractile of the standardized symmetric stable distribution with characteristic exponent, α. The estimate of α is from a table of standardized symmetric stable cumulative density functions whose f fractile matches $\hat{Z}_f$ (Fama and Roll [14]). $\hat{Z}_f$ is defined as:

2) $$\hat{Z}_f \equiv \frac{\hat{X}_f - \hat{X}_{1-f}}{2\hat{c}}, \text{ where}$$

3) $$\hat{c} \equiv (1/2(.827))\,[\hat{X}_{.72} - \hat{X}_{.28}].$$

$\hat{Z}_f$ then becomes:

4) $$\hat{Z}_f = (.827)\,\frac{\hat{X}_f - \hat{X}_{1-f}}{\hat{X}_{.72} - \hat{X}_{.28}}.$$

Fama and Roll conclude that estimates of α using $.95 \leq f \leq .97$ were robust and relatively free of bias with a slight downward bias for $\alpha > 1.7$ due to the truncation of the sampling distribution at $\hat{\alpha} = 2$ and for a sample size ≤ 99 (Fama and Roll [13]). For this study $f = .96$ is used.[8]

IV. Empirical Results

Estimates of α for the original data series (Tables 2 and 3) present a varied picture. Within the financials group, the two interest rate contracts, treasury bills and bonds, exhibit rising $\hat{\alpha}$'s across the sums for the full data set, but the $\hat{\alpha}$'s do not approach two. Treasury bills exhibit a similar pattern over the 1979-84 sample. Treasury bonds, however, have a pattern of rising $\hat{\alpha}$'s that approach two or reach two over the 1979-84 sample. The four currency contracts, for both the full data set and the 1979-84 sample, exhibit patterns of rising $\hat{\alpha}$'s that equal or approach two. In the metals group, gold shows a slight tendency for $\hat{\alpha}$ to increase across the sums but $\hat{\alpha}$ does not approach two for either data set. Copper exhibits rising $\hat{\alpha}$'s over the full data set and rising $\hat{\alpha}$'s that equal or approach two for the 1979-84 sample. Silver, which had long time periods of little or no price change and time periods with very volatile price changes, has the unique distinction of having all $\hat{\alpha}$'s less than one.

Within the agricultural group, one-half of the commodities, corn, soybeans, wheat, sugar, and cotton, show no evidence of rising $\hat{\alpha}$'s over the full data sample. A distinct change is observed, however, when the sample is restricted to the 1979-84 period. All five of the commodities exhibit a pattern of rising $\hat{\alpha}$'s that approach or are equal to two. This suggests that an extraordinarily large variance shift occurred prior to 1979, which is in fact what occurred for these commodities over the 1972-75 period. This was especially true for corn, soybeans, wheat, and cotton and is reflected in the low $\hat{\alpha}$'s for these commodities. The other five agricultural commodities, cocoa, pork bellies, live cattle, live hogs, and lumber, exhibit patterns of rising $\hat{\alpha}$'s that approach or equal two for both the full sample and the 1979-84 sample periods. However, lower $\hat{\alpha}$'s are shown for the full data set, suggesting that the same variance effect that affected the other agricultural commodities also affected these. Coffee, for which data is only available from 1979-84, shows no tendency for $\hat{\alpha}$ to rise across the sums.

The results for the original data series offers some support for the mixture of normal distributions hypothesis. Differences in maturity appear to be relatively unimportant in explaining the observed leptokurticity in futures prices. If non-stationarity in variance due to maturity is present the summing should remove most of these effects. Therefore, the larger sums would be expected to be much closer to 2 than smaller sums and this is not the case. The estimates of α for the randomized series (Tables 4 and 5) however are consistent with the mixture of normals hypothesis for all commodities except silver. The tendency for $\hat{\alpha}$ to approach two is much more dramatic for the randomized series. For example, 50 percent of the $\hat{\alpha}$'s for both the full randomized sample and the 1979-84 randomized sample are between 1.9 and 2.0.

These results support the hypothesis that commodity price movements are mixtures of normals with differing variances. This conclusion is generally only valid after the data is randomized. That the mixture of normals hypothesis is only clearly supported after the data is randomized suggest that adjacent price movements in the original series are not independent which leads to the observed leptokurtic distributions. It is well known that autocorrelation in futures price changes is small. Therefore, the probable cause of the dependence in the distribution is the presence of serially-correlated variances in the original series.

V. Concluding Comments

Two hypotheses, the stable Paretian and mixture of normals, have been proposed to explain the leptokurticity in observed distributions of futures price movements. The two hypotheses are tested by applying the stability-under-addition test of stable distribution parameters to twenty long series of the change in daily futures price closes. This paper improves upon past research by including a larger number of commodities, longer time series, and randomly ordering the data. For log changes in the original series the results were suggestive of the mixture of normals. But, even with sum sizes much larger than those in past research, the sums were still not distributed normally. The results with the randomized data were strikingly different. Sums of the randomized data did appear normally distributed. This suggests that the observed leptokurtic distribution can be explained by price changes not being independent. The most likely reason for rejecting independence is that variance is serially correlated.

The results have three important implications. First, classical statistical methods may be validly applied to most daily futures price series if an adjustment for heteroskedasticity is made to the data. Second, the process generating the non-constant variance is likely to be complex, responding to seasonal effects, structural shifts in demand and/or supply parameters, changes in government policies, etc., and represents a promising area for future study. Third, option pricing models that assume a constant or deterministically changing variance are likely to exhibit significant biases in predicting futures options premiums. Recently proposed models that allow variance to change stochastically (Johnson and Shanno [18]; Hull and White [18]) represent a promising improvement.

Footnotes

[1] More specifically, option pricing models are derived assuming logarithmic price relatives are normally distributed with a constant variance.

[2] The references cited are by no means an exhaustive list. Mandelbrot (1963) cited studies that observed the "fat-tailed" phenomena as early as 1915.

[3] Hudson, et al [16].

[4] See references in paragraph two, introduction.

[5] However, if the scale parameter, c, is changing $\hat{\alpha}$ has also been shown to increase as the number in the sum increases even though the underlying distribution may not necessarily be normal (Barnea and Downes [2]; Brenner [3]). But Cornew (1986, p. 680) argues that even when corrections are made for nonstationarity in the scale parameter, the corrected distribution is still decidedly non-normal.

[6] This is similar to the procedure used by Cornew, et al. [6].

[7] To avoid a discontinuity on the date of switch-over between contracts, the first price change of the new series is calculated as the logarithmic price relative of the closing price of the new contract on the day of the switch-over and the day previous to the switch-over.

[8] $\hat{Z}_{.96}$ monotonically decreases from 7.916 to 2.477 for the interval $1 < \alpha \leq 2$ (Fama and Roll [14]).

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Table 1. Commodities, Years of Data, and Total Number of Observations in Sample.

Commodity	(Years)	Total Number of Observations	Number of Observations in 6 Year Subset (1979-84)
Financials			
Treasury Bills	(1976-84)	2263	1514
Treasury Bonds	(1978-84)	1764	1513
Deutsche Mark	(1977-84)	2015	1514
Swiss Franc	(1977-84)	2015	1514
British Pound	(1977-84)	2012	1513
Japanese Yen	(1977-84)	2014	1513
Metals			
Copper	(1960-84)	6236	1510
Gold	(1975-84)	2511	1512
Silver	(1964-84)	5234	1509
Agricultural			
Corn	(1960-84)	6287	1511
Soybeans	(1960-84)	6282	1511
Wheat	(1960-84)	6288	1511
Sugar	(1962-84)	5720	1508
Cocoa	(1960-84)	6225	1509
Cotton	(1960-84)	6246	1510
Pork Bellies	(1965-84)	5028	1512
Live Cattle	(1966-84)	4778	1514
Live Hogs	(1970-84)	3773	1512
Lumber	(1974-84)	2769	1513
Coffee	(1979-84)	1507	1507

Table 2. Estimates of Characteristic Exponent For All Data.

Commodity	Sum Sizes 1	2	4	10	20	30	60
Financials							
Treasury Bills	1.21	1.25	1.33	1.42	1.43	1.26	1.20
Treasury Bonds	<1.0	<1.0	1.11	1.31	1.31	1.50	1.49
Deutsche Mark	1.51	1.66	1.69	1.74	1.72	1.92	2.0
Swiss Franc	1.67	1.71	1.64	2.0	2.0	1.78	1.81
British Pound	1.50	1.51	1.53	1.59	1.99	1.82	1.67
Japanese Yen	1.58	1.63	1.57	1.99	2.0	1.94	1.98
Metals							
Copper	1.49	1.52	1.51	1.59	1.63	1.84	1.66
Gold	1.45	1.50	1.49	1.47	1.39	1.73	1.54
Silver	<1.0	<1.0	<1.0	<1.0	<1.0	<1.0	<1.0
Agricultural							
Corn	1.36	1.40	1.38	1.39	1.49	1.43	1.50
Soybeans	1.26	1.23	1.26	1.21	1.21	1.23	<1.0
Wheat	1.38	1.41	1.39	1.49	1.55	1.45	1.53
Sugar	1.69	1.60	1.67	1.78	1.63	1.55	1.61
Cocoa	1.75	1.62	1.60	1.80	1.85	1.54	1.82
Cotton	1.15	1.21	1.27	1.35	1.32	1.19	1.40
Pork Bellies	2.0	1.74	1.82	1.89	1.82	1.55	1.71
Live Cattle	1.53	1.50	1.60	1.62	1.63	2.0	1.77
Live Hogs	1.65	1.68	1.71	2.0	2.0	1.95	2.0
Lumber	2.0	1.90	1.97	1.95	1.71	2.0	1.68

Note: Estimates were calculated following Fama and Roll (1971). Values are truncated at 2.0.

Table 3. Estimates of Characteristic Exponent for 1979-84 period.

	Sum Sizes						
Commodity	1	2	4	10	20	30	60
<u>Financials</u>							
Treasury Bills	1.33	1.43	1.35	1.49	1.53	1.38	1.68
Treasury Bonds	1.04	1.00	1.21	1.48	1.77	2.0	1.92
Deutsche Mark	1.55	1.69	1.65	2.0	1.45	1.37	2.0
Swiss Franc	1.67	1.70	1.88	2.0	1.71	1.16	1.70
British Pound	1.64	1.74	1.81	1.85	1.65	1.75	1.81
Japanese Yen	1.62	1.60	1.58	1.75	2.0	1.60	1.37
<u>Metals</u>							
Copper	1.61	1.61	1.78	1.85	2.0	1.67	2.0
Gold	1.46	1.36	1.45	1.49	1.54	1.35	1.55
Silver	<1.0	<1.0	<1.0	<1.0	<1.0	<1.0	<1.0
<u>Agricultural</u>							
Corn	1.65	1.73	1.61	1.94	1.58	1.49	2.0
Soybeans	1.58	1.76	1.74	1.49	1.50	2.0	1.43
Wheat	1.70	1.75	1.66	1.98	2.0	1.74	1.47
Sugar	1.95	1.65	1.72	1.92	1.67	2.0	2.0
Cocoa	1.83	1.82	1.76	2.0	2.0	1.80	2.0
Cotton	1.61	1.67	2.0	1.58	1.61	2.0	1.70
Pork Bellies	2.0	1.95	1.95	1.76	2.0	2.0	1.70
Live Cattle	1.74	1.79	1.88	2.0	2.0	1.97	2.0
Live Hogs	1.76	1.73	1.80	2.0	1.80	1.68	2.0
Lumber	2.0	2.0	2.0	1.94	2.0	2.0	1.32
Coffee	1.51	1.56	1.57	1.66	1.70	1.40	1.12

Note: Estimates were calculated following Fama and Roll (1971). Values are truncated at 2.0.

Table 4. Estimates of Characteristic Exponent for Randomized Series (All Data).

Commodity	Sum Sizes 1	2	4	10	20	30	60
Financials							
Treasury Bills	1.21	1.71	1.80	1.77	1.94	2.0	1.32
Treasury Bonds	<1.0	1.84	1.79	2.0	2.0	1.62	1.99
Deutsche Mark	1.51	2.0	1.92	2.0	1.74	1.68	1.80
Swiss Franc	1.67	2.0	2.0	2.0	2.0	1.86	2.0
British Pound	1.50	2.0	1.98	2.0	2.0	2.0	2.0
Japanese Yen	1.58	1.95	2.0	2.0	2.0	2.0	1.50
Metals							
Copper	1.49	1.44	1.40	1.55	1.58	1.36	1.71
Gold	1.45	1.66	1.90	1.50	1.49	2.0	1.39
Silver	<1.0	<1.0	<1.0	<1.0	<1.0	<1.0	<1.0
Agricultural							
Corn	1.36	1.97	1.86	2.0	2.0	1.80	1.69
Soybeans	1.26	2.0	2.0	2.0	2.0	2.0	1.55
Wheat	1.38	1.97	1.79	1.92	1.90	1.94	1.74
Sugar	1.69	2.0	2.0	2.0	2.0	2.0	1.58
Cocoa	1.75	1.79	1.91	1.77	1.88	1.87	1.45
Cotton	1.15	2.0	2.0	2.0	2.0	2.0	2.0
Pork Bellies	2.0	1.66	1.75	1.64	1.65	1.46	2.0
Live Cattle	1.53	1.89	1.93	1.94	1.98	1.68	2.0
Live Hogs	1.65	2.0	2.0	2.0	2.0	2.0	2.0
Lumber	2.0	1.71	1.80	2.0	2.0	2.0	1.87

Note: Estimates were calculated following Fama and Roll (1971). Values are truncated at 2.0.

Table 5. Estimates of Characteristic Exponent for Randomized Series for 1979-84 period.

Commodity	Sum Sizes 1	2	4	10	20	30	60
Financials							
Treasury Bills	1.33	1.78	1.78	1.86	2.0	1.72	2.0
Treasury Bonds	1.04	2.0	1.98	1.87	2.0	1.86	1.91
Deutsche Mark	1.55	2.0	2.0	2.0	2.0	1.95	1.72
Swiss Franc	1.67	2.0	2.0	2.0	2.0	1.44	2.0
British Pound	1.64	2.0	2.0	2.0	2.0	2.0	1.80
Japanese Yen	1.62	1.87	2.0	1.51	1.90	1.87	2.0
Metals							
Copper	1.61	2.0	1.98	1.90	2.0	2.0	1.12
Gold	1.46	1.82	1.51	1.93	2.0	1.52	1.60
Silver	<1.0	1.58	1.56	1.65	1.43	1.51	1.51
Agricultural							
Corn	1.65	2.0	2.0	1.94	2.0	2.0	2.0
Soybeans	1.58	2.0	1.98	2.0	1.74	2.0	1.69
Wheat	1.70	1.75	1.88	1.94	2.0	1.94	2.0
Sugar	1.95	2.0	2.0	2.0	1.90	2.0	1.77
Cocoa	1.83	1.73	1.77	2.0	2.0	1.60	2.0
Cotton	1.61	1.74	1.64	1.72	1.29	1.66	1.95
Pork Bellies	2.0	1.69	1.70	1.63	1.62	1.62	1.28
Live Cattle	1.74	2.0	1.98	1.81	1.89	2.0	2.0
Live Hogs	1.76	2.0	1.77	2.0	2.0	2.0	2.0
Lumber	2.0	2.0	2.0	2.0	1.69	2.0	1.83
Coffee	1.51	1.83	2.0	1.86	1.75	2.0	1.77

Note: Estimates were calculated following Fama and Roll (1971). Values are truncated at 2.0.