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QUALITATIVE COMPOSITE FORECASTING*

by

Peter M. Feather

and

Michael S. Kaylen**

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**Peter M. Feather is a graduate student and Michael S. Kaylen is an assistant professor in the Department of Agricultural Economics, University of Missouri-Columbia.

Abstract

QUALITATIVE COMPOSITE FORECASTING

Two methods are proposed for aiding the decision maker in resolving conflicting qualitative forecasts: a logit model and a method termed "vector probability". An empirical application using directional forecasts of hog prices from an ARIMA model, an econometric model, and an expert suggest that both methods have merit.

QUALITATIVE COMPOSITE FORECASTING

Introduction

There has been considerable interest in the qualitative evaluation of forecasts. To speculators, the direction of price changes may be of primary concern; the magnitude of the changes may be of secondary importance. Farmers may benefit from meteorological forecasts phrased in terms of favorable or unfavorable for production of a particular crop. This type of forecast would be much simpler to interpret than a numerical forecast of a vector of meteorological variables (e.g., temperature, rainfall, solar radiation). Other types of qualitative forecasts may be useful to decision makers. For example, several recent studies have evaluated forecasting models in terms of their ability to predict turning points (Naik and Leuthold; Kaylen; Harris and Leuthold; Brandt and Bessler).

Methods of composite forecasting have also received considerable attention. The motivation for forming composite forecasts is to use all of the information contained in each individual forecast. Instead of seeking the best forecast of an event (when many forecasts are available) and possibly losing information contained in the discarded set of forecasts, a composite forecast can be formed. One of the most common approaches is to form a linear combination of the individual forecasts, e.g.,

$$F_c = \sum_{i=1}^{n} W_i F_i \qquad (1)$$

where F_c denotes the composite forecast, F_i denotes the i-th individual forecast, and W_i denotes the weight given to the i-th individual forecast.

Past studies have evaluated cardinal composites on the basis of their ability to make qualitative forecasts (Brandt and Bessler; Harris and Leuthold). The problem with this evaluation is that the cardinal composites are formed using methods which seek to optimize a cardinal criterion (e.g., Bates and Granger derive formulas for the weights in (1) assuming the decision maker wishes to minimize the mean squared forecast error). The purpose of this paper is to present methods of forming qualitative composites which optimize qualitative criteria.

The paper proceeds as follows. Two qualitative composite forecasting methods are discussed: a logit model and a method termed "vector probability". The latter approach was motivated by the outperformance method proposed by Bunn (1975). Consequently, the paper begins with a brief review of this procedure. The two proposed qualitative methods are discussed, followed by an empirical application using U.S. hog prices. The final section of this paper consists of a brief summary and concluding remarks.

The Outperformance Method

Bunn suggested treating the probability of a forecast outperforming all other forecasts as a random variable. An outperformance probability exists for each forecast. The vector of these random variables is assumed to be distributed as a Dirichlet probability density function. A composite forecast is then formed by taking a linear combination of the individual forecasts. Each forecast is weighted by the expected value of the probability that it will outperform all of the other forecasts. Although the concept of outperformance is qualitative, Bunn developed his approach using the cardinal measure of absolute error; specifically, one forecast outperforms another if it has lower absolute error.

For the two forecast case, with forecasting models F_1 and F_2, the outperformance method assumes there exists some probability θ that F_1 will outperform F_2. This outperformance probability is assumed to have a beta distribution with parameters a_1 and a_2. Each forecast realization produces a new datum δ which equals one if F_1 has outperformed F_2 and zero otherwise. The updating variable δ can be considered a Bernoulli variable. Since the beta distribution is a conjugate prior for the Bernoulli, the posterior distribution of θ will be a beta distribution with parameters

$$a_1 + \delta \quad \text{and} \quad a_2 + \delta. \tag{2}$$

Bunn has suggested that the weight for F_1 should be the expected value of θ taken from the posterior beta distribution, hence

$$k = \frac{a_1 + \delta}{a_1 + a_2 + 1} \tag{3}$$

with the weight for forecast two being 1-k.

Bunn (1975, 1977, 1980) has extended his use of outperformance to the n forecast case using the Dirichlet probability density function, a multivariate analogue of the beta distribution. Given n forecasts, θ_1 through θ_n are assumed to be distributed as a Dirichlet probability density function with parameters a_1 through a_n, with θ_i denoting the probability that forecast method i will outperform all others. Updating is similar to the two forecast case, that is, if forecast i outperforms all others, a_i increases by one. The weight for forecast i is the expected value of θ_i:

$$k_i = \frac{a_i}{a_1 + a_2 + \ldots + a_n} \tag{4}$$

where k_i denotes the weight given to forecast i.

It can be shown that the use of the qualitative outperformance criterion is inappropriate if the decision maker is interested in a cardinal measure of performance. For example, suppose the decision maker is interested in minimizing the absolute forecast error and two forecasting mechanisms (F_1 and F_2) are available. Further suppose that F_1 outperforms F_2 50 percent of the time. The outperformance method would simply average the two forecasts.

Consider the consequences if F_2 always has a "small" absolute error while F_1 has an "extremely large" absolute error when it is outperformed. In this situation, F_2 has a smaller average absolute error than F_1. Weights of zero and one for F_1 and F_2, respectively, would result in a smaller expected absolute error than if the outperformance weights were used.

Qualitative Approaches to Composite Forecasting

The formation of qualitative composite forecasts that optimize qualitative criteria has received little or no attention. The method of weighting individual forecasts, as in the cardinal case, is not appropriate when the forecasts are qualitative.

One approach is to recognize that the probability of a particular outcome may be conditional on the qualitative set of individual forecasts. To illustrate, imagine a commodities broker who provides forecasts as a service to his clients. He may perceive that clients are generally more interested in taking long positions than short positions. Since he works for a commission, he may (perhaps unintentionally) have a tendency to forecast higher prices. In particular, his bias may be so strong that he only forecasts falling prices in the face of overwhelming evidence that they will indeed fall. Consequently, although he rarely forecasts falling prices, when he does he is almost always correct. Suppose another forecasting mechanism exists which has no apparent bias. If the client is interested only in the qualitative up or down forecasts, anytime the broker forecasts prices going down, the client should use this as his "composite" forecast. Many other situations involving a group of forecasting mechanisms can be imagined. For example, when all mechanisms forecast the same outcome, that outcome may typically occur. Another example may be when all but one mechanism forecast the same outcome, the one that forecasts a different outcome may have access to information the others lack. All of these situations represent valuable information; they suggest it may be desirable to account for the success or failure of each mechanism conditional on the vector of forecast values.

The above considerations lead to the proposed Bayesian method termed "vector probability". Given n forecasting mechanisms and m mutually exclusive and exhaustive outcome categories (such as price rises or price falls), there are m^n unique combination vectors of forecasts. Let $\theta_{i/G}$ denote the probability that the actual value will be in outcome category i, given the particular forecast combination vector G. Following Bunn, treat the vector of these conditional probabilities as having a Dirichlet probability density function with parameters $a_{1/G}$ through $a_{m/G}$. Given a particular forecast combination vector, the "composite" forecast is then chosen as that outcome with the greatest expected conditional probability.

The conditional expectation of $\Theta_{i/G}$ (the probability that the actual value will be in outcome category i) is

$$E(\Theta_{i/G}) = \frac{a_{i/G}}{\sum_{j=1}^{m} a_{j/G}}. \tag{5}$$

Given a forecast realization, and treating the conditional Dirichlet probability density function as a prior distribution, the posterior distribution for the outcome probabilities may be derived. In particular, define

$$\delta_i = \begin{cases} 1 \text{ if the actual is in category i} \\ 0 \text{ otherwise} \end{cases}$$

The vector $(\delta_1,\ldots,\delta_m)$ has a multinomial distribution. Since the Dirichlet distribution is a conjugate prior for the multinomial, the posterior distribution of $(\Theta_{1/G},\ldots,\Theta_{m/G})$ is a Dirichlet with parameters $(a_{1/G} + \delta_1,\ldots, a_{m/G} + \delta_m)$.

An alternate composite method is a model of qualitative choice such as a logit model. When combining qualitative forecasts containing only two outcomes, the outcomes can be expressed in binary form. The use of qualitative independent variables in regressions models, especially binary variables, is a common and easily interpreted process. However, the use of a qualitative dependent variable that can take on only a limited range of values requires careful interpretation. Instead of simply regressing a binary dependent variable on a set of binary independent variables, a more appropriate technique would be to regress the probability of the dependent variable being either a zero or a one on the set of binary independent variables. One way to accomplish this is with the logit model, which is based on the cumulative logistic distribution function. The model is

$$P_j = [1 + \exp(-\alpha - \sum_{i=1}^{n} \beta_i X_i)]^{-1} \quad (6)$$

where P_j denotes the probability that the dependent variable (the composite forecast) will be a one, α denotes a constant term, β_i denotes the i-th estimated parameter and X_i denotes the i-th individual forecast. The logit model has the desirable property of transforming the problem of predicting probabilities with a (0,1) interval to the problem of predicting the odds of an events occurring within the range of the entire real line (see Fomby, et al., for a detailed discussion).

An Empirical Application

This section presents results of an application using quarterly hog price forecasts. Two qualitative categories are considered: price increases (a one) and price decreases (a zero). For illustrative purposes, three forecasting mechanisms were considered. The individual forecasting mechanisms consisted of an expert, an autoregressive integrated moving average (ARIMA) model, and an econometric model. The expert forecasts were supplied by Glenn Grimes, University of Missouri Extension Livestock Specialist. The ARIMA and econometric models were proposed by Brandt. The ARIMA model specifies the first difference of hog prices as having a fifth order moving average term. The econometric model consists of a single reduced form equation containing several demand and supply shifters. The econometric and ARIMA models were initially estimated from first quarter 1960 through fourth quarter 1975 and subsequently re-estimated each quarter. The initial estimates are reported in Table 1.

Table 2 shows the specification of the two qualitative composite forecasting techniques. The logit model requires an out-of-sample set of qualitative forecasts for initial estimation, so it was estimated from first quarter 1976 through fourth quarter 1979. The logit model was then used to combine qualitative forecasts and subsequently re-estimated each quarter to incorporate new data. The "vector probability" method was initially assigned uninformative prior Dirichlet parameters (all ones). Given a particular combination vector of forecasts G, the first conditional Dirichlet parameter ($a_{1/G}$) is updated (increased by one) if the actual is a one (price increases) otherwise, the second conditional Dirichlet parameter ($a_{2/G}$) is updated. When a particular combination vector occurs, the expected probability that the actual value will be a one is simply the first conditional Dirichlet parameter divided by the sum of both conditional Dirichlet parameters. If this expected value is greater (less) than 0.5, the "composite" forecast is that price increases (decreases); expected values equal to 0.5 are treated as uninformative forecasts. The initial and final parameters are displayed in Table 2. One problem with using uninformative priors is that uninformative composites (equal probabilities of both outcomes) are likely to result, especially early in the updating process.

Table 3 displays the results of the three individual and two composite techniques. All techniques except the logit method forecasted from first quarter 1976 through fourth quarter 1986. The logit model was used to forecast quarterly prices from first quarter 1980 through fourth quarter 1986; the first quarter 1976 through fourth quarter 1979 was used to obtain initial estimates for the logit model. Both time periods are displayed in Table 3. Twelve ties occurred in the "vector probability" method; since a tie is neither a correct or incorrect forecast, the performance results associated with the "vector probability" method include the fraction of correct forecasts counting and not counting ties. Ignoring times, the composites performed well over the 1980-86 prediction period, being outperformed only by the expert.

Summary and Conclusions

This paper has suggested the formation and use of qualitative composite forecasts. Two qualitative composite methods have been proposed: a logit model and a method termed "vector probability". The theoretical development was followed by an empirical application using quarterly hog prices. Three individual forecasting mechanisms were used: an expert, an ARIMA model, and an econometric model. Over the 1980 through 1986 forecast evaluation period, both composite methods performed at least as well as the ARIMA and econometric forecasts (ignoring the ties associated with the "vector probability" method). The best individual forecasting mechanism, the expert, performed only slightly better than either of the composites.

The performance of the composites is encouraging. For decision makers who prefer not to make marketing or investment strategies on the basis of a single forecast procedure, these composite alternatives warrant consideration. A problem associated with this study is the relatively small sample size utilized; a larger sample size may increase the accuracy of the composite methods.

Due to the initial uninformative conditional prior Dirichlet parameters used in the "vector probability" method, ties frequently occurred early in the forecasting period. As more forecast realizations occur, the conditional probabilities tend to stabilize. This may explain the better performance of the "vector probability" method in the 1980 through 1986 forecasting period. It may be desirable to use informative prior Dirichlet parameters which could lead to stabilization of the conditional outcome probabilities. Research along these lines is likely to be of increasing value as the number of forecasting mechanisms and/or the number of outcome categories increase. A further area of research could address forming composite cardinal forecasts using weights conditional on qualitative properties.

TABLE 1 THREE HOG PRICE FORECASTING MECHANISMS[a]

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(1) Econometric

$$\begin{array}{llllll} PH_t = & -191.4 & + 49.8IN_{t-1} & - 6.4SF_{t-2} & - 5.0SF_{t-3} \\ & (-14.4)^b & (15.7) & (-8.6) & (-7.1) \\ & + .54HC_{t-1} & - 3.4CS_{t-1} & - 67.8HATCH_{t-1} & \\ & (4.5) & (-4.0) & (-8.4) & \end{array}$$

$R^2 = .90$ D.W. = 2.1

(2) ARIMA

$$\begin{array}{lll} PH_t = PH_{t-1} & - .39U_{t-5} \\ & (-2.68) \end{array}$$

$Q^*(23) = 21.61$

(3) Expert Opinion

$$PH_t = f(\text{Quantitative and Qualitative market factors})$$

Note: The initial quarterly econometric and ARIMA models are based on a 1960 through 1975 estimation period. The models were re-estimated quarterly from 1976 through 1986 to incorporate new data.

[a]PH is the price of barrows and gilts at the seven terminal markets ($/cwt);

PH is a forecast of PH; IN is the natural logarithm of total disposable income ($ billion); SF is sows farrowing in the U.S. (million head); HC is the hog to corn ratio, U.S. basis; CS is U.S. commercial cattle slaughter (billion lbs); HATCH is broiler type eggs hatched in the U.S. (billion eggs); U is the white noise disturbance term.

[b]t-statistics appear in parentheses below estimated parameters.

TABLE 2 COMPOSITE APPROACHES FOR FORECASTING DIRECTION OF HOG PRICE MOVEMENTS[a]

(1) Logit[b]

$$P(PH=1) = [1+\exp(-z)]^{-1}$$

$$z = -1.18 + 1.52EX + 2.18AR + 0.47ECON$$

$$\quad (.95)^{c} \quad (1.5) \quad (1.5) \quad (1.4)$$

(2) Vector Probability[d]

Possible Vectors F_1 F_2 F_3	Final Parameters (a_1,a_2)
(0 , 0 , 0)	(2,6)
(0 , 0 , 1)	(3,9)
(0 , 1 , 0)	(3,2)
(1 , 0 , 0)	(3,2)
(1 , 1 , 0)	(3,1)
(1 , 0 , 1)	(3,2)
(0 , 1 , 1)	(4,3)
(1 , 1 , 1)	(11,3)

[a]Price increases are denoted by a one and price decreases by a zero.

[b]Initial binary logit model based on a fit period of 1976 through 1979. The model was re-estimated quarterly as new data became available. P(PH=1) denotes the probability of hog prices being a one (price increases). z is the estimated parameter where EX, AR and ECON denote binary expert opinion, ARIMA and econometric forecasts, respectively.

[c]Standard errors appear in the parentheses below estimated coefficients.

[d]Given a present combination vector, let (Θ_1,Θ_2) denote the probabilities of a one or zero occurring. (Θ_1,Θ_2) has a Dirichlet probability density function with parameters (a_1,a_2). The final parameters column shows the values of (a_1,a_2) for each of the possible forecast combinations after updating the prior (1,1) values using 44 forecast realizations.

TABLE 3 PERFORMANCE OF QUALITATIVE FORECASTS

	1976-1986		1980-1986	
	Fraction Correct (%)		Fraction Correct (%)	
Econometric	24/44	(54.5)	14/28	(50.0)
ARIMA	32/44	(72.7)	20/28	(71.4)
Expert Opinion	32/44	(72.7)	22/28	(78.6)
Logit			20/28	(71.4)
Vector[a]	22/44[b]	(50.0)	18/28[b]	(64.3)
Probability	22/32[c]	(68.8)	18/24[c]	(75.0)

Note: Since all forecasting models except the logit forecasted from 1976 through 1986, both time periods are shown. Fraction correct refers to the number of times out of total that each model correctly predicted direction.

[a]Due to the uninformative initial prior assigned to each vector, uninformative composites are likely to occur (equal probabilities of both outcomes).

[b]Number correct counting ties as incorrect forecasts.

[c]Number correct not counting ties (twelve ties were observed in 1976 to 1986 period; four ties were observed in 1980 to 1986 period).

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