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THE EQUILIBRIUM EFFECTS OF IMPROVED CLIMATE FORECASTING

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I. Introduction

Firms operate in uncertain environments. Sources of uncertainty include output and input prices and the production process itself. The consequences of this uncertainty on the decisions of firms have been studied extensively. In general, past studies have examined the effects of either price uncertainty (e.g., Sandmo, Holthausen, Batra and Ullah) or production uncertainty (Ratti and Ullah, Pope and Kramer). Often, producers face both types of uncertainty, and, in agriculture especially, prices and production levels are not necessarily uncorrelated. Agricultural supply shocks are often widespread enough to affect output prices. The degree to which producers have control or knowledge about random production partly determines how their output price expectations are formed. The link between random production and prices is often ignored. This paper develops a model in which this link between the two is explicitly considered. It is assumed that supply is a function of production decisions and random weather.

Production decisions must often be made before the levels of weather variables are observed. Producers may be willing to pay for information about the distributions of future weather variables when the returns from their decisions depend on the realizations of weather.

In general, the method used to calculate the producer value of weather information involves the specification and estimation of a production model from which decision rules, contingent on future weather events, are derived. The optimal decision rules are chosen by maximizing the expected value of the appropriate objective function (usually a utility function defined on profits). The probability distribution function over which the expectation is taken is commonly derived by combining farmers' prior information about weather events with the information embodied in weather forecasts. (See Chapter 1 of Zellner and Bikadane for a detailed overview of the procedure.)

Examples of empirical studies which follow this framework are numerous. Byerlee and Anderson, Tice and Clouser, and Rosegrant and Roumasset examine how the productivity of nitrogen fertilizer on wheat and corn is affected by weather variables. All three studies use estimated production functions to derive optimal application rates which depend on the expectation of future weather events. Hashemi and Decker show how the number of irrigations can be reduced by the use of rainfall forecasts. Two examples of the value of frost forecasts to orchardists are Baquet, Halter and Conklin, for pears farmers, and Stewart, Katz and Murphy for apple farmers. Wilks and Murphy show how forecasts of growing season precipitation can be used to determine whether haying or pasturing is more profitable in Western Oregon. Morzuch and Willis use an estimated cranberry supply function to better match the amount of leased post-harvest storage bins with forthcoming supply.

There are two common elements in these studies. First, they all estimate yield responses to a weather variable and a decision variable. The decision variable can be either a damage control variable (Lichtenberg and Zilberman), such as frost protection, or a normal production input, such as nitrogen fertilizer. Second, all ignore or assume away possible price effects from the use of the weather information. In many circumstances making price exogenous is probably justified because the weather information is so localized that affected producers, in aggregate, do not contribute enough to total production to significantly change price. That is, the demand facing affected producers is perfectly elastic. However, this simplification may often result in a greatly overestimated value of information to producers. For example, the studies of frost protection for pears and apples ignore the price effects from the saving of their crops. If demand is quite inelastic, the losses to the industry as a whole due to lower prices may outweigh the aggregate benefits to individual growers.

Lave's analysis of the California raisin industry supports this point. He first calculates that the gain to growers from perfect information about rainfall during drying time is $90.95 per acre. But when he then calculates the partial equilibrium effects on output price, he concludes that even a modest increase in supply from rainfall protection would lower total industry profits. The detrimental price effects outweigh the individual gains. The raisin industry as a whole would be better off with less than perfect weather forecasts.

Lave stopped short, however, of examining how raisin producers would react to the knowledge that better weather forecasts lead to lower average prices. Rational farmers take these price effects into account. The work of Newberry and Stiglitz shows how price effects can be embodied into optimal decision rules describing how producers react to information, by assuming that they form their price expectations rationally. No studies have yet utilized this framework to show how producers will value information about random production inputs which, when used by many growers, can result in changes in output prices. The purpose of this paper is to begin to fill this void.

The rest of this paper is organized as follows. First, a general framework is developed in the next section which shows how producers react to imperfect information about random production inputs and how their reactions change as the quality of the information improves. The exogenous price assumption used in past theoretical and empirical studies of the firm facing production uncertainty is relaxed. Comparative statics results indicate that the gain to producers from better information is overstated when price is erroneously assumed to be exogenously determined. After that an empirical example for the special case of a quadratic production function and a linear demand curve is presented. This example gives an indication of the likely magnitudes of changes that can be expected as weather information becomes more accurate. The concluding section discusses some implications of the analysis.

2. Farmer's Use of Weather Information

Consider a competitive farmer with a crop yield which depends on the level of a controllable input, x, and an uncontrollable weather input, w. For simplicity assume that w can only take on two values, w_h and w_l. It is assumed for convenience that $w_h > w_l$ and that these two states occur with equal probability. Denote the per-acre production function facing the farmer as

$$y = f(x,w), \quad w = w_h \text{ or } w_l \qquad [1]$$

where y represents yield per acre. As written in [1], w is the only stochastic element of production. The results derived in this paper also hold if a mean zero additive disturbance, uncorrelated with w, is included. It will be assumed throughout the paper that x must be applied before the realization of w is observed and that the productivity of x is affected by w ($f_{xw} \neq 0$).

Suppose that the farmer receives a forecast about the future level of w. Let this forecast be a categorical forecast; that is, it consists of a pronouncement that a particular weather state will occur. Let the forecasts be correct $100 \cdot \rho$ percent of the time, where $.5 \leq \rho \leq 1$.

The parameter ρ measures forecast accuracy. It is assumed that each weather state is forecast with the same degree of reliability, that is ρ is the same for both forecasts and that the probability of receiving a particular forecast is equal to the long-run probability that the particular weather state will occur. When $\rho = .5$ the forecasts contain no information, since it is assumed that a farmer's own expectations about weather is that each weather state occurs with equal probability. When $\rho = 1$, the forecasts represent perfect information, since the probability that the weather state not forecast will occur is zero. Forecast accuracies of less than .5 do not need to be considered since this would imply that weather forecasts and weather realizations are negatively correlated. In this case, forecast users would simply redefine the state that is forecast to occur.

Assuming that both the forecaster and the farmer have the same information concerning forecast accuracy, the farmer's conditional density functions of w--which assign probabilities to each possible weather state given a categorical forecast--are given by

$$s_h(w) \begin{cases} \rho & \text{if } w = w_h \\ 1 - \rho & \text{if } w = w_l \end{cases}$$

$$s_l(w) \begin{cases} 1 - \rho & \text{if } w = w_h \\ \rho & \text{if } w = w_l \end{cases}$$

where the subscripts on the functions s denote the forecast. If weather forecasts give any information about future weather, and future weather helps determine supply, then forecasts and output price are correlated, unless demand is perfectly elastic. Some factors which determine the degree of correlation include: 1) the percentage of total supply coming from the region covered by the forecast, 2) the amount of uncontrollable variability in the region's supply not explained by weather, and 3) the correlation between one region's weather and the weather in other supply regions.

To highlight the relationship between one region's weather forecast and output price, it will be assumed that total supply comes from a single representative farmer. It is further assumed that this farmer operates with a production function in which weather is the only random element. In addition, this farmer is a price taker with rational expectations concerning random output price. The demand function is assumed to be nonstochastic once the level of the weather variable is observed. Such a model set-up follows the work of Newberry and Stiglitz. It allows production uncertainty and the implied price distribution to be examined jointly.

The assumption that the industry consists of one competitive producer with rational expectations is equivalent to assuming that there are a large number of identical producers, none of which are large enough to affect output. The assumption of rational expectations means that each producer knows how the rest of the producers in the market will respond to changes in information so that the resulting distribution of output price is that implied by the aggregate actions of all producers. No individual producer will diverge from what is optimal for all other producers, since the actions of any single producer does not affect output price.

The farmer chooses the level of a variable input to apply after receiving a forecast of future weather. The two objective functions and first-order conditions are

FORECAST OF w_h

$$\max_x E_h(\pi) = \rho P[f(x,w_h)]f(x,w_h) + (1-\rho)P[f(x,w_l)]f(x,w_l) - P_x x \qquad [2]$$

$$\text{FOC } \rho P[f(x,w_h)]f_x(x,w_h) + (1-\rho)P[f(x,w_l)]f_x(x,w_l) - P_x = 0 \qquad [3]$$

FORECAST OF w_l

$$\max_x E_l(\pi) = \rho P[f(x,w_l)]f(x,w_l) + (1-\rho)P[(x,w_h)]f(x,w_h) - P_x x \quad [4]$$

$$\text{FOC} \quad \rho P[f(x,w_l)]f_x(x,w_l) + (1-\rho)P[f(x,w_h)]f_x(x,w_h) - P_x = 0. \quad [5]$$

Denote the solutions to [3] and [5] as x_h and x_l. These solutions define maximums to [2] and [4] if $f_{xx} < 0$ and if interior solutions exist.

When choosing the level of x to apply, the competitive farmer does not take into account the change in output price implied by the level chosen. That is, competitive farmers cause no marginal revenue effects. However, because the farmer uses all available information, the price distribution used to choose x is exactly that distribution implied by the choice. Otherwise, the farmer's choice is suboptimal. For these reasons the derivative of output price with respect to input use does not appear in the first-order conditions, but the input level chosen must satisfy the first-order condition, including the resulting price distribution.

In general, little can be said about the effects of an increase in ρ without specifying a functional form for the demand curve. For example, displacing [3] with respect to x and ρ yields

$$\frac{\partial x_h}{\partial \rho} = \frac{P[f(x_h,w_h)]f_x(x_h,w_h) - P[f(x_h,w_l)]f_x(x_h,w_l)}{[+]} \quad [6]$$

where [+] denotes a positive expression. If x and w are complements, the sign of [6] becomes more positive (or less negative) as the price under weather year w_h approaches the price under weather year w_l ($w_h > w_l$). The two prices become closer the more elastic is demand. As the prices diverge, [6] becomes more negative (or less positive). This occurs when demand is more inelastic. To focus on the central role that demand elasticity plays in signing the effects of an increase in forecast accuracy, let the inverse demand function be given by

$$P(x,w) = [f(x,w)]^{-\alpha} \qquad [7]$$

where $\alpha > 0$. The demand elasticity is, therefore, $-\alpha^{-1}$.

One further restriction on functional forms is needed before any qualitative effects from an increase in forecast accuracy can be signed. Without separating the effects of the random input and the controllable input in some manner, no identification of the source of changes in price and output can be made. Therefore, let the production function be written

$$f(x,w) = g(x)T(w) \qquad w = w_h \text{ or } w_l. \qquad [8]$$

The analytical tractability of this function has not been purchased at zero cost. Specifying this production function restricts the cross partial derivatives of f(x,w) to be positive, if both x and w are to have positive marginal products. The comparative statics analysis of this section will be restricted to the case where the production function is given by [8] with x and w being complementary production inputs with positive marginal products ($T_w > 0$, $g_x > 0$) and f(x,w) is of the form given in [8]. There are two types of comparative statics which will be examined. These are ex post and ex ante.

Ex post comparative statics are defined here as the changes in input demand, expected supply, and expected profits after a particular forecast is received, but before the level of w is observed. Ex ante comparative statics are defined as the expected or average changes. They are the weighted average of the ex post changes, with weights given by the probability that a particular forecast will be made. Thus, ex post results show the effects of an increase in ρ for a particular forecast, while ex ante results give the average, or long-run, value of such improvements.

Changes in Ex Post Input Demands

Using the inverse demand function [7] and the production function [8], the displacement of the first-order conditions [3] and [5] with respect to x and ρ yield

$$\frac{\partial x_h}{\partial \rho} = \frac{g_{xx}}{g_{xx} - \alpha g^{-1} g_x^2} \cdot \frac{T(w_h)^{1-\alpha} - T(w_1)^{1-\alpha}}{\rho T(w_h)^{1-\alpha} + (1-\rho) T(w_h)^{1-\alpha}} \quad [9]$$

when the forecast is for w_h, and

$$\frac{\partial x_1}{\partial \rho} = \frac{g_{xx}}{g_{xx} - \alpha g^{-1} g_x^2} \cdot \frac{T(w_1)^{1-\alpha} - T(w_h)^{1-\alpha}}{\rho T(w_1)^{1-\alpha} + (1-\rho) T(w_h)^{1-\alpha}} \quad [10]$$

when the forecast is for w_1. Given that $g_{xx} < 0$, the signs of [9] and [10] are determined by the value of α. When $\alpha < 1$ (i.e., demand is elastic)

$$T(w_h)^{1-\alpha} - T(w_1)^{1-\alpha} > 0$$

which implies that

$$\frac{\partial x_h}{\partial \rho} > 0 \text{ and } \frac{\partial x_1}{\partial \rho} < 0.$$

The signs are reversed if demand is inelastic. The intuition behind these results is straightforward.

Suppose that w_h has been forecast with increased accuracy. The farmer knows that the expected value of marginal product of x increases due to the increased likelihood that the future weather state will be w_h, but that it also decreases due to the greater chance of a lower price. Which effect dominates depends on the demand elasticity. If demand is very elastic, output price tends to drop little with an increase in production so the positive productivity effect of good weather is more likely to dominate the negative price effect. If this dominance occurs, input use increases. The price effect tends to dominate if demand is very inelastic, so that input use would then tend to decline. With the constant elasticity inverse demand function and the multiplicative production function, the elasticity which causes the expected price and productivity effects to be exactly offsetting is minus one.

With a more accurate forecast of w_1 the opposite results hold. When demand is elastic, less weight is given to the state with a high marginal product, w_h, so the expected value of marginal product decreases, which requires a reduction in x. When demand is inelastic, more weight is given to the state with a high value of marginal product, w_1, so input use increases.

Changes in Ex Post Expected Supply

The two expressions which describe the changes in expected supply after the forecast is received are

$$\frac{\partial E_h(y)}{\partial \rho} = [T(w_h) - T(w_1)]g(x_h) + [\rho T(w_h) + (1-\rho)T(w_1)]g_x(x_h) \frac{\partial x_h}{\partial \rho} \qquad [11]$$

$$\frac{\partial E_1(y)}{\partial \rho} = [T(w_1) - T(w_h)]g(x_1) + [\rho T(w_1) + (1-\rho)T(w_h)]g_x(x_1) \frac{\partial x_1}{\partial \rho}. \qquad [12]$$

Again there are two effects, one due to the change in the distribution of weather and one due to the change in input use. The change in input use has an obvious effect. If input use increases, so too does expected supply, given that the weather distribution is constant. But, an increase in forecast accuracy also changes the farmer's perceived weather distribution, which casues an independent change in expected supply.

When w_h is forecast more accurately, expected supply tends to increase simply because more weight is given to the high output state w_h. If demand is elastic, this effect is reinforced by the increase in input use. Expected supply unambiguously increases. But if demand is inelastic the two effects work in opposite directions, since input use decreases. Thus, the change in expected supply cannot be unambiguously signed with an inelastic demand.

Similar reasoning can be applied to a more accurate forecast of w_1. If demand is elastic, input use decreases. This reinforces the decrease in expected supply from the lower likelihood that the future weather state will be w_h. If demand is inelastic, expected supply increases due to an increase in input use and decreases due to a lower probability of good weather. Again, the two effects work in opposite directions and, in general, the net effect cannot be determined.

Changes in Ex Post Expected Profits

There are three distinct effects a change in forecast accuracy has on expected profits. These are: 1) the change in the distribution of profits, holding input use constant, 2) the change in profits from the price change caused by a different input use, holding output and the distribution constant, and 3) the change in profits due to input use changing, holding price and the distribution constant. Only the first two effects need to be discussed since the envelope theorem (Varian) guarantees that the third effect equals zero.

The price effect and the distribution effect pull expected profits in opposite directions. When demand is elastic and w_h is forecast more accurately, the change in profits, holding price constant, tends to be positive, since the likelihood of future weather being w_h has increased. But, a more accurate forecast of w_h also results in more of x being used, which tends to lower price.

After some algebraic manipulations and the substitution for the partial derivatives of x_h and x_1 ([9] and [10]), the changes in expected profits for the two forecasts are

$$\frac{\partial E_h(\pi)}{\partial \rho} = \frac{g^{1-\alpha} g_{xx}}{g_{xx} - \alpha g^{-1} g_x^{\ 2}} \left[T(w_h)^{1-\alpha} - T(w_1)^{1-\alpha} \right] \qquad [13]$$

$$\frac{\partial E_1(\pi)}{\partial \rho} = \frac{g^{1-\alpha} g_{xx}}{g_{xx} - \alpha g^{-1} g_x^{\ 2}} \left[T(w_h)^{1-\alpha} - T(w_1)^{1-\alpha} \right] \qquad [14]$$

It is clear from [13] and [14] that the direct effects on revenue from increased confidence that a certain state will occur, dominate the indirect revenue effects from changes in prices. When demand is elastic and w_h becomes more probable, the increase in expected revenues from the now more favorable distribution of w are greater than the decrease in expected revenues from the lower expected price. And, when w_1 becomes more probable by a more accurate forecast, the decrease in expected revenues from the changed distribution are greater than the increased expected profits brought about by the price increase. The signs are reversed when demand is inelastic. Expected revenues fall when w_h is forecast more accurately and increase when w_1 becomes more probable.

The results from the preceding ex post analysis are summarized in Table 1. It is immediately apparent that the ex ante comparative statics cannot be signed by simply inspecting the signs of the ex post results since most of the signs under the two forecasts are different.

The signs of the ex ante comparative statics cannot be determined with the production function specification of [8]. For example, signing the average change in input use requires knowledge of the sums, products and quotients of $g(x)$, $g_x(x)$, and $g_{xx}(x)$ for both levels of x. These combinations make any general exploration into the signs of the ex ante comparative statics quite tedious. To give an indication of the average effects for a typical production function specification, g(x) will be specified as a Cobb-Douglas function. That is, let

$$g(x) = Ax^{\beta}. \qquad [15]$$

Changes in Ex Ante Input Demand

With g(x) given by [15], the analytic functions of x_h and x_1 are

$$x_h = K \left[\rho T(w_h)^{1-\alpha} + (1-\rho)T(w_1)^{1-\alpha} \right]^{\frac{1}{1-\beta(1-\alpha)}} \qquad [16]$$

$$x_1 = K \left[\rho T(w_1)^{1-\alpha} + (1-\rho)T(w_h)^{1-\alpha} \right]^{\frac{1}{1-\beta(1-\alpha)}} \qquad [17]$$

where

$$K = \left(\frac{\beta A^{1-\alpha}}{P_x} \right)^{\frac{1}{1-\beta(1-\alpha)}}.$$

Average input use, x, is $\frac{x_h + x_1}{2}$. Therefore, the change in average demand for x due to a change in ρ is found by dividing the sum of [16] and [17] by 2. This yields

$$\frac{\partial x}{\partial \rho} = K_1 \left[\frac{x_h}{\rho T(w_h)^{1-\alpha} + (1-\rho)T(w_1)^{1-\alpha}} - \frac{x_1}{\rho T(w_h)^{1-\alpha} + (1-\rho)T(w_h)^{1-\alpha}} \right] \qquad [18]$$

where

$$K_1 = \frac{1}{2} \left(\frac{K}{1-\beta(1-\alpha)} \right) \cdot [T(w_h)^{1-\alpha} - T(w_1)^{1-\alpha}]$$

It can be shown, with some algebra, that the bracketed term in [18] is always positive. Thus, the sign of $\partial x/\partial \rho$ equals the sign of K_1. When demand is elastic, $K_1 > 0$, and average input use increases. When demand is inelastic, $K_1 < 0$, and average input use decreases.

Changes in Ex Ante Expected Supply

Average supply is simply half the sum of expected supplies under the two forecasts evaluated at x_h and x_1. Therefore, the average change in expected supply from an increase in ρ with g(x) given by [15] can be written as

$$\frac{\partial E(y)}{\partial \rho} = \frac{1}{2} [g(x_h) - g(x_1)] \cdot [(T(w_h) - T(w_1)] +$$

$$\beta K_1 \left[g(x_h) \frac{\rho T(w_h) + (1-\rho)T(w_1)}{\rho T(w_h)^{1-\alpha} + (1-\rho)T(w_1)^{1-\alpha}} - g(x_1) \frac{\rho T(w_1) + (1-\rho)T(w_h)}{\rho T(w_1)^{1-\alpha} + (1-\rho)T(w_h)^{1-\alpha}} \right] \cdot \quad [19]$$

The sign of [19] is not immediately apparent. The first term is the effect on average supply due to a change in the distribution of w. The sign of this effect depends on the relative magnitudes of x_h and x_1, which, on inspection of [16] and [17], are determined by the relative magnitudes of the two bracketed terms, since $1-\beta(1-\alpha) > 0$. Some straightforward algebra proves that the bracketed term in [16] is greater (less) than its [17] counterpart when demand is elastic (inelastic). Thus, $x_h > x_1$ when demand is elastic, and $x_h < x_1$ when demand is inelastic. This enables us to sign the first term in [19]. Because the marginal product of both x and w are positive, an elastic (inelastic) demand makes this term positive (negative).

The second term in [19] measures the change in average supply from changing optimal input use under the two weather forecasts. Some tedious manipulations involving the substitution of [16] and [17] reveals that when demand is inelastic, the drop in expected supply from the distribution effect is reinforced by the drop in supply from decreased average use of x. But, it is not necessarily true that the average increase in input use when demand is elastic results in a reinforcement of increased average supply from the distribution effect. A sufficient condition for both effects to be positive is for $\beta(2-\alpha) > 1$. As β increases this condition is more likely to be met. That is, as the production function becomes less concave (i.e., as β increases), the decrease in supply from using less x when w_l is forecast is increasingly offset by the increase in supply from using more x when w_h is forecast.

Changes in Ex Ante Expected Profits

Ex ante expected profits are half the sum of the maximized expected profit functions [2] and [4]. The substitution of the appropriate derivatives of x_h and x_l with respect to ρ for the Cobb-Douglas specification for g(x) yields a rather simple expression for the change in ex ante expected profits.

$$\frac{\partial E(\pi)}{\partial \rho} = \frac{1-\beta}{2[1-\beta(1-\alpha)} \left[T(w_h)^{1-\alpha} - T(w_l)^{1-\alpha} \right] \cdot \left[g(x_h)^{1-\alpha} - g(x_l)^{1-\alpha} \right] \quad [20]$$

The sign of [20] depends on the relative magnitudes of x_h and x_l, and on the elasticity of demand. As discussed above, if demand is elastic, $(\alpha < 1)$, $x_h > x_l$, which implies that $g(x_h) > g(x_l)$, $g(x_h)^{1-\alpha} > g(x_l)^{1-\alpha}$, and $T(w_h)^{1-\alpha} >$

$T(w_l)^{1-\alpha}$. This means that with an elastic demand, expected profits increase with an increase in forecast accuracy. But, when demand is inelastic, average profits fall. In this case $T(w_h)^{1-\alpha} < T(w_l)^{1-\alpha}$, but because $x_h < x_l$, $g(x_h)^{1-\alpha}$ remains greater than $g_x(x_l)^{1-\alpha}$. The result that profits can fall as information improves is somewhat counter-intuitive. It would seem that if the use of better information makes a farmer worse off, then the information would be ignored. What must be remembered is that the ex ante change in profits is the average of the two ex post changes. When average profits decline, the expected loss from increased knowledge that w_l will occur more than offsets the expected gain when w_h has a greater chance of occurrance when it is forecast. The assumption that individual farmers are price takers makes this result possible. They do not take into account the effect that their input decisions have on output price. If a single farmer were to ignore the information, the market price would be unaffected, so the individual farmer would be acting suboptimally. Acting ignorantly would make any individual farmer worse off. Of course, if all farmers acted ignorantly, they would be made better off than if they all fully optimized, but if this were to happen, there would be substantial incentives for individuals to take the information into account when making their own production decisions. The ignorant equilibrium is unsupportable.

At first glance the result that agricultural producers can be made worse off from better information seems similar to the finding that farmers who ride the technological treadmill are made worse off due to the inelastic nature of food demand, or that agricultural extension efforts benefit consumers at the expense of farmers by lowering the price of food. But whereas both of these arguments rely on a supply expansion to lower profit, the findings here show that farmer profits can decrease even though average supply decreases.

3. An Empirical Application

The results of the previous section examine the circumstances under which input demand, expected supply, and expected profits increase or decrease as information concerning a random production input becomes more accurate. In this section the likely magnitudes of the changes in forecast accuracy are calculated empirically for a southern U. S. dryland cotton farm.

Cotton farmers in the south generally apply much of their nitrogen fertilizer at or just before planting time. The productivity of fertilizer may increase or decrease with subsequent rainfall. Too much rainfall can depress yields by leaching the nitrogen beyond the root zone or by denitrification of the available nitrates (Huber, et. al.). But moderate levels of soil moisture will increase the productivity of nitrogen (Pesek, Heady and Venezian).

For this application it is assumed that at planting time the only rainfall which significantly interacts with applied fertilizer, and the only rainfall about which farmers have information, is the precipitation which occurs one month after planting time.

To determine how nitrogen fertilizer and rainfall affect cotton yields requires an estimated production function. The data used for estimation were generated at the Mississippi Agricultural Experiment Station in the azoo-Mississippi Delta, and are reported in Grissom and Spurgeon. The planting date for cotton was, in most years, the first part of May, so the 40 years of corresponding May rainfall data for the area was used as the rainfall variable. The functional form, estimated parameters and their standard errors are reported in Table 2. All the parameters have expected signs and the point estimates give reasonable ranges of positive marginal productivity for both fertilizer and rainfall. Since α_5 is positive, fertilizer productivity increases with increases with rainfall.

Rainfall, of course, is a continuous random variable. To fit this application to the previously developed conceptual model, the rainfall distribution is made discrete. It is assumed that rainfall can take on only two values and that the weather forecast categorically states which value will occur. To continue the assumption that each weather state occurs half the time, the two states of nature must lie on either side of the median of the rainfall distrbution. Therefore, the mean of rainfall, given that rainfall is above the

median, is chosen for the high weather state, and the mean of rainfall, given that rainfall is below the median, is chosen for the low weather state.

The effects on an individual producer of increasingly accurate forecasts are reported in Table 3. As ρ increases from .5 to 1, fertilizer use increases by about seven pounds per acre when the high rainfall state is predicted, and declines by the same amount when the low rainfall state is predicted. The two fertilizer effects exactly offset each other since the derivative of the marginal product of x with respect to w is a constant. Expected profits increase by about $60. per acre when good weather is predicted, and decline by about $58. per acre when bad weather is predicted. Expected output in the two forecasts increases by about six percent with the good forecasts and decreases by not quite the same amount with the forecasts of bad weather.

The average of these ex post results show that input use stays constant (as expected since the production function is a quadratic function), expected supply marginally increases, and expected profits increase by $0.90 per acre when weather is forecast perfectly. This increase in expected profits is on the same order of magnitude found in previous studies of the value of weather information when fertilizer is the choice variable. Byerlee and Anderson calculate that the per-acre gain for Australian wheat farmers from weather information is between \$.03 and \$.36, while Tice and Clouser estimate that utilizing current weather information can result in increased profits of between \$1.48 and \$3.99 per acre. But the latter study included acreage allocated to soybeans as an additional decision variable so not all their calculated gain can be attributed to fertilizer.

The \$0.90 per-acre gain is the maximum gain from information. When the demand elasticity is less than infinite, this gain must decrease, and perhaps become negative. To investigate the likely changes in profits when demand is less elastic, a linear demand equation is included to reflect the endogenous price. That is, price is assumed to be a linear function of realized supply. Since supply is a quadratic function of x and w, the first-order conditions, which define optimal fertilizer levels for the two forecasts, are cubic equations. These cubic equations correspond to the general equations [21] and [23] in Section 3. The roots of these cubic equations were found using the IMSL subroutine ZRPOLY. Only one of the roots for each equation turned out to be real. A check of the second-order conditions showed that this real root defined maximum expected profits.

To facilitate the comparison of outcomes from simulations on demand elasticity, it is best to keep price and quantity somewhat constant. So instead of simply varying the slope of the demand curve as a new elasticity is considered, both the intercept and the slope are made functions of the chosen demand elasticity and a pivot point. This pivot point was chosen so that the average prices and quantities obtained from the optimal solutions were about the same. This point is 24 hundredweight of cottonseed and $39.75 per hundredweight which corresponds to average yields of California cotton and 1986 prices. Tables 4a-4e report the ex post and ex ante results corresponding to a range of demand elasticities from -.3 to -15.

One of the most surprising results is that fertilizer use increases with forecast accuracy when w_h is forecast, regardless of demand elasticity. Even when an individual farmer knows that a low price is coming, the benefits of increasing fertilizer use to take advantage of the higher rainfall outweigh the effects from a lower output price. The value of marginal product increases with forecast accuracy even when demand is very inelastic. This is not predicted in Section 2. The reason for this discrepancy is that the quadratic production function does not correspond to the general form of [8] used in that section. If the output price elasticity of input demand is very low relative to the weather elasticity of input demand, there will be little effect on input demand from a change in output price. The extreme example of this is the fixed proportion production function. If the limiting factor is rainfall, there will be only one fertilizer level for each forecast, regardless of output price, so long as the forthcoming market price results in enough revenue to cover variable costs. Knowledge of the price distribution can largely be ignored if the output price elasticity of demand is small.

The other results of the empirical analysis follow the developed theory much closer. With a very low demand elasticity, farmers are made worse off from more accurate information. But with a sufficiently high elasticity, the gains become positive, reaching a maximum of $.90 per acre.

Implications of the Analysis

To justify investment in weather forecasting skills on the basis of economic gain to the agricultural sector, it is necessary for policy makers to first identify the beneficiaries of weather forecasts and then quantify their likely gain (or loss). The results developed in this paper shed light on which producers are most likely to benefit from improved weather forecasting and what market and weather characteristics are likely to lead to the largest marginal benefits.

The basic result from this analysis is that more accurate weather forecasts tend to benefit those farmers who face an elastic demand for their output and tends to hurt farmers who face an inelastic demand. This generalizes the result of Lave who found that raisin growers facing an inelastic demand who knew when good weather was going to occur would be made worse off because of the large supply response.

It is shown in this paper that when demand is inelastic, a more accurate forecast of good weather makes competitive farmers worse off. But this result does not necessarily hold over all possible weather forecasts. When bad weather can also be forecast with increased accuracy, the expected benefits to farmers facing an inelastic demand tend to be, but are not always, negative. It is an empirical question if the benefits of more accurate forecasts of bad years are dominated by the decreased profits from more accurate forecasts of a good year. This is the case with the empirical example presented. An obvious implication from this is that those farmers facing an inelastic demand are better off, in aggregate, without weather forecasting.

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Table 1. Ex Post Effects of an Increase in Forecast Accuracy When Price is Endogenous

Comparative Statics	Forecast	Elastic Demand	Inelastic Demand
Input	w_h	+	-
Demand	w_l	-	+
Expected	w_h	+	?
Supply	w_l	-	?
Expected	w_h	+	-
Profits	w_l	-	+

Table 2. Estimated Cotton Production Function*

Functional Form

$$y = \alpha_0 + \alpha_1 F + \alpha_2 F^2 + \alpha_3 R + \alpha_4 R^2 + \alpha_5 RF + \epsilon$$

Parameter Estimates

	α_0	α_1	α_2	α_3	α_4	α_5
Estimate	8.32	0.134	-0.0005	1.21	-0.10	0.0045
Standard Error	1.34	0.021	0.00008	0.45	0.035	0.0027

R^2=0.37

*Cotton yield, y, measured in hundredweight of cottonseed per acre. Fertilizer, F, is pounds of nitrogen per acre, R is inches of May rainfall, and ϵ represents remaining stochastic production elements.

Table 3. Effects of Increasing Forecast Accuracy with No Price Effects On a U. S. Cotton Farmer*

Ex Post Effects

Forecast Accuracy	Forecast of w_h			Forecast of w_l		
	F	E(y)	E(π)	F	E(y)	E(π)
0.5	165.5	24.7	548.45	165.5	24.7	548.45
0.7	168.3	25.3	572.20	163.4	24.1	524.90
0.85	170.4	25.8	590.20	160.5	23.6	507.50
1.0	172.6	26.2	608.50	158.4	23.2	490.20

Ex Ante Effects

Forecast Accuracy	F	E(y)	E(π)
0.5	165.5	24.69	548.45
0.7	165.5	24.70	548.60
0.85	165.5	24.71	548.90
1.0	165.5	24.72	549.35

*F represents pounds of fertilizer per acre, y, hundredweight of cottonseed per acre, and π, profits per acre.

Table 4 a. Effects of Increasing Forecast Accuracy with an Endogenous Price for a U. S. Cotton Farmer*

Demand Elasticity = -0.3

Ex Post Effects

Forecast Accuracy	Forecast of w_h			Forecast of w_l		
	F	E(y)	E(π)	F	E(y)	E(π)
0.5	163.3	24.7	443.50	163.3	24.7	443.50
0.7	165.6	25.3	381.50	161.4	24.1	504.30
0.85	167.7	25.8	334.10	160.1	23.6	549.10
1.0	170.1	26.2	285.90	159.0	23.2	593.400

Ex Ante Effects

Forecast Accuracy	F	E(y)	E(π)
0.5	163.3	24.68	443.52
0.7	163.5	24.69	442.90
0.85	163.9	24.70	441.63
1.0	164.5	24.71	439.66

**F* represents pounds of fertilizer per acre, *y*, hundredweight of cottonseed per acre, and π, profits per acre.

Table 4 b. Effects of Increasing Forecast Accuracy with an Endogenous Price for a U. S. Cotton Farmer*

Demand Elasticity = -0.7

Ex Post Effects

Forecast Accuracy	Forecast of w_h			Forecast of w_l		
	F	E(y)	E(π)	F	E(y)	E(π)
0.5	164.6	24.7	502.98	164.6	24.7	502.98
0.7	167.3	25.3	489.84	162.1	24.1	515.83
0.85	169.4	25.8	479.81	160.3	23.6	532.39
1.0	171.7	26.2	469.65	158.7	23.2	534.49

Ex Ante Effects

Forecast Accuracy	F	E(y)	E(π)
0.5	164.6	24.69	502.98
0.7	164.7	24.70	502.83
0.85	164.9	24.70	502.53
1.0	165.2	24.72	502.07

*F represents pounds of fertilizer per acre, y, hundredweight of cotttonseed per acre, and π, profits per acre.

Table 4 c. Effects of Increasing Forecast Accuracy with an Endogenous Price for a U. S. Cotton Farmer*

Demand Elasticity = -1.0

Ex Post Effects

Forecast Accuracy	Forecast of w_h			Forecast of w_l		
	F	E(y)	E(π)	F	E(y)	E(π)
0.5	164.9	24.7	516.57	164.9	24.7	516.57
0.7	167.6	25.3	514.51	162.1	24.1	518.53
0.85	169.8	25.8	512.91	160.3	23.6	519.90
1.0	172.0	26.2	511.28	158.7	23.2	521.20

Ex Ante Effects

Forecast Accuracy	F	E(y)	E(π)
0.5	164.9	24.69	516.57
0.7	164.9	24.70	516.52
0.85	165.1	24.70	516.41
1.0	165.3	24.72	516.24

*F represents pounds of fertilizer per acre, y, hundredweight of cotttonseed per acre, and π, profits per acre.

Table 4 d. Effects of Increasing Forecast Accuracy with an Endogenous Price for a U. S. Cotton Farmer*

Demand Elasticity = -2.0

Ex Post Effects

Forecast Accuracy	Forecast of w_h			Forecast of w_l		
	F	E(y)	E(π)	F	E(y)	E(π)
0.5	165.2	24.7	532.50	165.2	24.7	532.50
0.7	168.0	25.3	543.39	162.5	24.1	521.72
0.85	170.1	25.8	551.63	160.5	23.6	513.68
1.0	172.3	26.2	559.94	158.5	23.2	505.68

Ex Ante Effects

Forecast Accuracy	F	E(y)	E(π)
0.5	165.19	24.69	532.50
0.7	165.22	24.70	532.55
0.85	165.29	24.70	532.65
1.0	165.40	24.72	532.81

**F* represents pounds of fertilizer per acre, *y*, hundredweight of cotttonseed per acre, and π, profits per acre.