AGGREGATION OF HETEROGENEOUS FIRMS

by

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Day (1963,1969) and a number of others (Miller, Lee, Spreen and Takayama, Marenco) have studied the aggregation problem within the linear programming context. Basically their results give the conditions under which a single representative firm model can be used to represent a number of firms.[1/] The conditions arrived at are restrictive requiring proportional objective functions and right hand sides along with identical technical coefficient matrices. Others have suggested following empirical approaches based on the most limiting resource (Sheehy and McAlexander, Frick and Andrews) or firm characteristics (Buckwell and Hazell). A characteristic of all of these approaches is the assumption that the representative firm model is of the same dimension in terms of rows and columns as each of the models it represents. This manuscript deals with the aggregation of heterogeneous firms where the firm models do not, in general, satisfy Day's conditions nor are they necessarily the same size. Herein we argue that when the firm models are of the same size but do not satisfy Day's conditions, then it is in general impossible to develop a satisfactory aggregate model. Arguments are also presented regarding the practical relationship between aggregate models and farm models. Subsequently, we draw together information from the literature including an aggregation approac proach based on McCarl and evidence pertaining to that approach from Hamilton, McCarl, and Adams which indicates the consequences of using variants of this scheme.

The Problem of Heterogeneity

First, let us address the impossibility of exact aggregation using a model of the same size as those to be aggregated when Day's conditions are not satisfied. This is best done by showing that it cannot be done with a simple case example.

Suppose we wish to aggregate the following two firm models

Maximize $P_1x_{11} + 2x_{21}$	Maximize $P_1x_{12} + 2x_{22}$
subject to $x_{11} + x_{21} \le 100$	subject to $x_{12} + x_{22} \le 300$
$4x_{11} + 2x_{21} \le 300$	$4x_{12} + 1.5x_{22} \le 900$

Note these models have proportional right hand sides and objective function coefficients but do not have identical technical coefficients in terms of the second constraint. Thus, Day's conditions are violated. Now, let us consider construction of an aggregate model. Suppose that x_{11} and x_{21} represent production of one commodity, whereas x_{12} and x_{22} represent production of a second commodity. Let us solve these models varying the price of the first commodity. The resultant solutions are

Price of Commodity 1	Production of				Total Production of Commodity	
	x_{11}	x_{21}	x_{12}	x_{22}	1	2
$-\infty \le P_1 \le 2.00$	0	100	0	300	0	400
$2.00 \le P_1 \le 4.00$	50	50	180	120	230	170
$4.00 \le P_1 \le 5.33$	75	0	180	120	255	120
$5.33 \le P_1 \le \infty$	75	0	225	0	300	0

A simple point may now be made. An aggregate model of the same dimensions as the two representative models - two variables and constraints - could not reproduce the above results. This statement is based on the properties of a linear programming basic solution. In this two-constraint model with two decision variables, the only possible way where the two decision variables can be nonzero is when they are both in the basis. But only one such basis matrix can be defined, thus two different solutions with them both nonzero cannot arise. However, in the above solution data, the second and third rows constitute two different solutions with non-zero levels for both commodities. Thus, it would be impossible to reproduce this solution set with one proper aggregate model and thus our point is shown. The question then becomes one of how proper aggregate models can be developed.

Size of Aggregate Models

Before addressing the question of heterogeneous firm aggregation, let us briefly discuss size relationships among "representative" and firm level models as they have arisen in practice.

Aggregation is required in many types of models. Suppose in this case we discuss agricultural sector models as reviewed in Heady and Srivastava, Norton and Schieffer, or McCarl and Spreen. Commonly, sector models depict a geographic region, such as the U.S., which is divided into a number of "homogeneous" production regions; e.g., there are 223 production regions in some of the models reviewed in Heady and Srivastava, 58 in Adams, Hamilton and McCarl, and 20 in Duloy and Norton. In each of these models there are relatively few technical constraints within a region and the depiction of each region amounts to a representative farm model. The Heady, Nicol, and Madsen model has constraints on four types of land and water at the production region, while the Adams, Hamilton and McCarl model constrains water, labor, and two types of land, and the Duloy and Norton model constrains monthly labor, water, and four types of land. In contrast, however, detailed farm level models such as that presented in McCarl *et al.* possess 100 or more constraints, constraining such things as land, operation sequencing, machinery availability, labor, and water on a seasonal basis. Furthermore, the McCarl *et al.* model is felt to be a valid farm representation for farms in a region that is represented by a much more aggregate model in both the Adams, Hamilton and McCarl and the Heady, Nicol, and Madsen studies. This shows that the farm level models can be much more detailed than the aggregate models. This is undoubtedly quite common as a fully developed farm level model is probably more detailed than a representative section in a much broader geographic scoped sector model can be. Thus, given that aggregate models cannot represent farm models which are of the same size, this point is likely exacerbated when the representative firm model is much smaller and simpler than the actual detailed firm models that it "represents."

Furthermore, when one considers the geographic sub-areas represented within sector models, one quickly discovers that the assumption of homogeneity of variables, constants, etc., is probably not entirely reasonable. The Adams, Hamilton and McCarl model contains a single representative farm model for Oregon, wherein there are many different soil types, rainfall conditions, microclimates, irrigation water patterns, etc. Thus, in general, the representative farm models are depicting heterogeneous firms which would be represented by different sized firm level models. However, within many sector models these are portrayed by small representative firm models which do not have provisions to avoid aggregation error. Serious errors can arise in this case.[2/]

Toward Aggregation of Non-Homogeneous Firms

Given acceptance of the concept that agricultural sector model usually contain aggregate representations of non-homogeneous firms which are much less complicated than appropriate firm level models, the question then becomes how this aggregation can properly be handled. One way this question can be addressed is to use a method based on Dantzig-Wolfe decomposition and the concepts in the paper on cropping activities in sector models by McCarl. Namely, the proper firm level representation in an aggregate sector model which neglects many firm level constraints is composed of extreme point solutions from the more detailed firm models.[3/] Here we provide an illustration of that.

Consider representation of the above example where the two non-homogeneous firms are to be represented as one. If the Dantzig-Wolfe decomposition procedure is followed for these firms as explained in McCarl, the final firm model is as given in figure 1. The λ's represent the proportion of of the acreage allocated to each of the 4 crop plans in the above solution. In turn, the technical coefficients under each λ are the summed solutions across the firms, and the important point is that the coefficients under each of the λ variables represent a feasible farm plan within the firm model constraints, which are ignored in this master problem. This illustrates a more comprehensive approach to developing appropriate aggregate models in the face of heterogeneity, which is that the activities within the aggregate model should be feasible farm plans within the constraints of the firms represented. One can also represent this model in terms of average production per acre as in figure 2. In general, when using the approach the firm solutions should reflect pricing conditions in the range expected for the shadow prices on the resources which are modeled in the aggregate sector model. Thus, if one considers a model with crop balance, land, labor and water constraints in the aggregate model, then one would need to develop solutions to the firm level models under alternative prices for the crops, imputed values of land, labor, and water. Such things as crop sequencing, detailed machinery considerations, etc., could be present in the submodel but ignored in the sector model as long as the solutions present in the sector models reflect feasibility within the domains of these constraints.

This procedure lends itself to two possible approaches to aggregation (McCarl). The first approach involves formal development of firm linear programming models and their use in a formal Dantzig-Wolfe type of procedure to construct the aggregate model. The second involves the use of historical feasible crop mixes in the regions being modeled as reflections of the feasible extreme point solutions from the non-homogeneous firms.

Figure 1. Aggregate Representative Problem for Decomposition

	Sell Commodity 1	Sell Commodity 2	Pattern 1	Pattern 1	Pattern 1	Pattern 1	Right Hand Side
Obj	P_1X_1	$+ 2X_2$					
Balance Commodity 1	X_1		$- 0 \lambda_1$	$-230\lambda_2$	$-255\lambda_3$	$-300\lambda_4$	≤ 0
Balance Commodity 2		X_2	$-400\lambda_1$	$-170\lambda_2$	$-120\lambda_3$	$- 0 \lambda_4$	≤ 0
			λ_1	$+ \lambda_2$	$+ \lambda_3$	$+ \lambda_4$	≤ 1

Figure 2 - Per Acre Representation of Problem for Decomposition

	Sell Commodity 1	Sell Commodity 2	Pattern 1	Pattern 1	Pattern 1	Pattern 1	Right Hand Side
Obj	P_1X_1	$+ 2X_2$					
Balance Commodity 1	X_1		$- 0 \lambda_1$	$-.575\lambda_2$	$-.6375\lambda_3$	$-.75\lambda_4$	≤ 0
Balance Commodity 2		X_2	$- 1 \lambda_1$	$-.425\lambda_2$	$-.30\lambda_3$	$- 0 \lambda_4$	≤ 0
			λ_1	$+ \lambda_2$	$+ \lambda_3$	$+ \lambda_4$	≤ 400

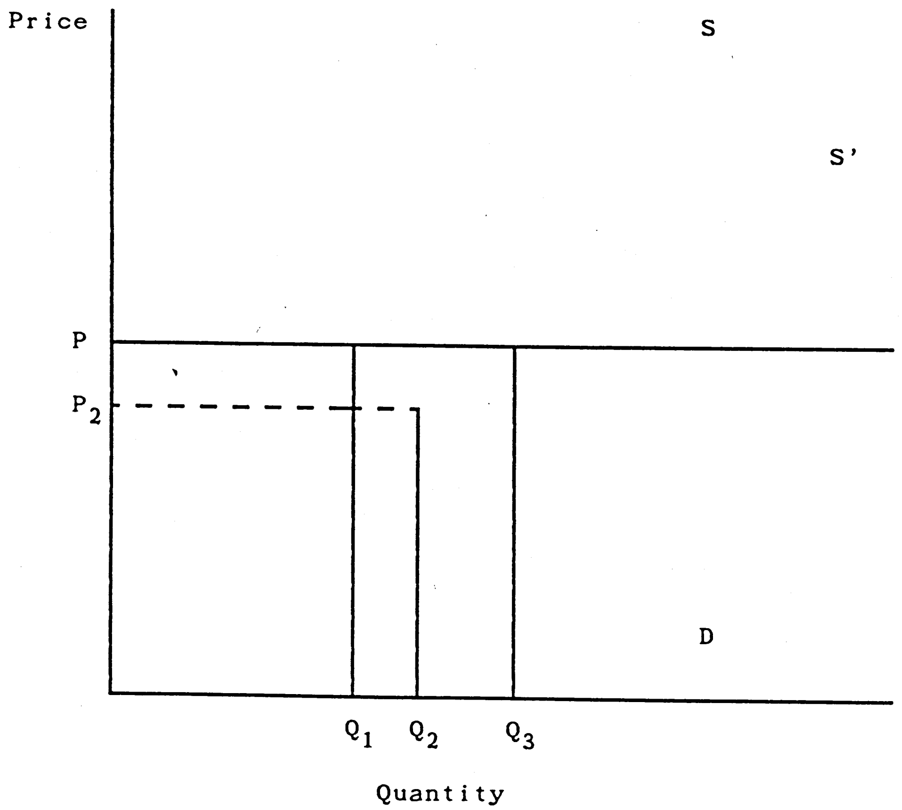
McCarl argues that the formal Dantzig-Wolfe decomposition procedure is too complex and computationally burdensome. Therefore, he argues that the second approach should be more commonly used. Both of these approaches have been implemented and tested by Hamilton, McCarl and Adams. The historical data approach has been implemented in Hamilton and McCarl.

Explorations with These Aggregation Procedures - Review of a Case Study

Hamilton, McCarl and Adams studied the effects of different aggregation schemes upon the results of reduced ozone consentrations in the Corn Belt. The aggregation schemes they tested, among others, consisted of the use of the aggregate farm model with the use of: (A) detailed firm level linear programming models; (B) a simple sector model representation with regional land and labor constraints; (C) the model in (B) with the addition of flexibility constraints; (D) a sector model formed upon the McCarl/Dantzig-Wolfe aggregation procedures discussed above from the firm level models; and (E) a sector model using historical data. The results of this analysis are rather extensively presented in Hamilton, McCarl and Adams. Here, we highlight their results regarding acreage adjustment parameters, but first we present a brief theoretical argument regarding acreage adjustment.

The basic economic problem, considering using a farm level linear programming model vs. the agricultural sector model is as illustrated in figure 3. Note that in Figure 3 we have supply curves S and S', which represent supply before (S) and after reducing ozone (S'). We also have two demand curves: 1) the fixed price curve (P) used in the firm level linear programming appraisal, and 2) the demand curve (D) that the sector model faces, which intersects the supply curve S at price P. This framework illustrates that the quantity adjustment in the fixed price farm level models (Q_3-Q_1) should be greater than the quantity adjustment in the sector models with downward sloping demand

Figure 3



curves (Q_2-Q_1), providing the supply curve is adequately represented in both, as the sector model should reflect a diminished price (P_2), which in turn should dampen adjustment.

Turning to the evidence selected from Hamilton, McCarl, and Adams (Table 1), note that the adjustments in the farm level models involve more than a 9% change in corn and soybean acres as an average, whereas the average change in the data has a standard deviation of 2.18. However, notice that in the sector model without flexibility constraints, that the average adjustment is approximately equal in terms of the acreages, but that the standard error is much greater than in the farm models. This is indicative of aggregation error when modeling features are not introduced to cause the model to be a proper aggregate model. The above theoretical development is violated as this model is confronting downward sloping demand, yet has larger adjustments than the firm level linear programming model. Thus, the supply curve representation must fundamentally be in error and more detail is needed in the aggregate models. The situation is also studied with flexibility constraints limiting the adjustment to no more than 50% and 20% of that in the base period. Here the adjustment percentages only become lower in the LP when the crop acreage is adjusted to be within ± 20% of the base period acreage. Finally, we turn to two models which use the aggregation procedures as discussed above. The first model is the sector model using the historical crop mixes. Here note the adjustment is much smaller than that in the LP model. Further, in the sector model which uses results from the LP submodel, the adjustment is even smaller. It appears then, from this basis, that the models based on the aggregation procedures discussed above do a better job of representing aggregate supply and conforming with theory.

Table 1 Ozone Induced Changes in Corn Belt Crop Acreage

Scenario		Percent Change[a] from Regional total in			Acreage[b] Deviation Across States & Crops
		Corn Acreage	Wheat Acreage	Soybean Acreage	
Firm Level LP[c]	(A)	-9.39	-0.08	9.52	2.186
Sector Model REPFARM No Flexibility	(B)	-9.132	0.305	8.827	8.293
Sector Model REPFARM Limited Flexibility (20%)	(C.1)	0.067	1.082	-1.669	1.695
Sector Model REPFARM Partial Flexibility (50%)	(C.2)	2.439	0.093	-2.532	2.669
Sector Model - LP Crop Mix	(D)	0.13	-1.011	0.998	0.403
Sector Model - Historical Crop Mix	(E)	1.325	-0.241	-1.085	0.625

a) This is the percent change in acreage between the base case and the improved air quality cases.

b) This is the square reoot of the sum of the squared acreage differences between cases divided by the number of states times crops.

c) These are the results from a set of firm level LP models which confront a fixed price demand curve.

Concluding Remarks

This manuscript examines the aggregation of non-homogeneous firms. An appropriate aggregation procedure in such a case is to use a model wherein the activities are crop mixes arising from the firm level models. The specific forms considered herein are those suggested by McCarl; i.e., using activities from firm LP models and using historical statistics on activities. Use of the LP models is potentially superior, although computationally and datawise very burdensome. The use of historical statistics, however, is somewhat easier and also reasonable from a theoretical standpoint. One might also go to different approaches than those presented here, and use econometrically estimated adjustments in firm level activities to generate linear programming activities where necessary. The basic approach which probably should be used is to develop historical statistic data for those regions which cannot be studied intensively and to use linear programming firm models for those regions where researchers have models available and time to do a more detailed study.

Based on the results of this study, a detailed crop mix-based agricultural sector model is being developed in which 64 regions in the U.S. are used based on historical data from either Ag. Statistics or the Statistical Reporting Service. We anticipate using this model in a number of technological appraisals. We feel the model structure avoids some common criticisms of linear programming models, that the models are far too responsive to technical change, as illustrated by the results of Hamilton, McCarl, and Adams.

NOTES

1. In a quite different approach, Paris and Rausser derive conditions which simply require the firm solutions add up to the aggregated firm model.

2. An examination of Baker and McCarl's results indicates dropping timing in homogeneous cases can lead to large errors.

3. As argued below, these may be explicitly derived as extreme points or assumed to be extreme points.

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