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IMPACT OF IRRIGATION AND FERTILIZERS ON THE GROWTH OF OUTPUT IN ANDHRA PRADESH: A BAYESIAN APPROACH

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INTRODUCTION

The subject matter of this paper concerns estimation of "production functions" for regions (*e.g.*, a State) when time-series data on outputs and inputs are available for sub-regions (*e.g.*, districts). An approach is proposed and applied which is an alternative to working with aggregate regional data and which derives regional functions from estimates for sub-regions. Not only is the resulting relationship meaningful as a description of the regional input-output relationship but also it has certain attractive statistical properties based on optimal extraction of information from the sub-regional functions.

The methodology underlying this is empirical Bayesian and has been developed by Lindley and Smith,[1] and Novick, Jackson, Thayer and Cole.[2] A key feature of this methodology is a certain hypothesis, the hypothesis of exchangeability, about similarities between sub-regions. This enables us to 'pool' data for these regions in a way that not only leads to 'improved' statistical estimates for each region but also to an estimate of the regional relationship. Both the hypothesis of exchangeability and its econometric implications seem to be potentially useful in applied economic research and this paper illustrates how such a framework might be exploited.

Many writers including Fisher[3], Johansen[4] and Sato[5] have emphasized the distinction between the fundamental concepts of an *ex ante* micro technique relation and an *ex post* macro statistical relationship and have shown that the theoretical aggregate production function exists only under rather stringent conditions. Also when one's primary interest lies in measurement and comparison of marginal products of different inputs, as in many agricultural studies, micro economic investigations seem appropriate, data deficiencies notwithstanding. Nevertheless, in many practical situations the aggregate statistical production functions are needed and found to be useful.

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1. D. V. Lindley and A. F. M. Smith, "Bayes Estimates for the Linear Model", *Journal of the Royal Statistical Society*, Series A, Vol. 34, 1972, pp. 1-41.
2. M. R. Novick, P. M. Jackson, D. T. Thayer and N. S. Cole, "Estimating Multiple Regressions in m Groups: A Cross-Validation Study", *British Journal of Mathematical and Statistical Psychology*, Vol. 35, 1972, pp. 33-50.
3. F. M. Fisher, "The Existence of Aggregate Production Functions", *Econometrica*, Vol. 37, Nol 4, October 1969, pp. 553-577.
4. L. Johansen: Production Functions: An Integration of Micro and Macro, Short-run and Long-run Aspects, North Holland Publishing Company, Amsterdam, 1975.
5. K. Sato: Production Functions and Aggregation, North Holland Publishing Company, Amsterdam, 1975.

For example, if planning for the provision of public inputs is on a regional basis (as in India) then broad comparisons between marginal productivity of inputs at regional level become necessary. (At the same time, however, if wide dispersion in output response to inputs occurs at the level of sub-regions then this too is relevant information for planners.) Furthermore, even when micro economic investigations have been made, there would always be doubts about (*a*) their generality and (*b*) the appropriate method of combining the micro estimates to obtain macro estimates. Because of these doubts macro estimates would usually be obtained for purposes of comparison. A third reason for being interested in macro estimates is that they can form the basis of a broad summary of the contribution of various inputs to the growth of output.

Proceeding on the assumption that statistical macro production functions have some uses, we have estimated a Cobb-Douglas type specification for four major crops (rice, sugarcane, groundnuts and tobacco) of the State of Andhra Pradesh in India. The available data[6] have restricted us to only four inputs—irrigated and unirrigated land, fertilizers and rainfall. Of course, our choice of specification is very restrictive; elsewhere in the paper we give reasons for these restrictions. In the absence of these data difficulties and other restrictions, the usefulness of our approach would be shown to greater advantage.

SPECIFICATION AND DATA

The production function specification we use is of the Cobb-Douglas type, *viz.*,

$$\ln Y_{ijt} = \alpha_{ij} + \beta_{ij} \ln I_{ijt} + \gamma_{ij} \ln UI_{ijt} + \delta_{ij} \ln F_{it} + \lambda_{ij} \ln R_{it} + \varepsilon_{ijt} \qquad \ldots (1)$$

where i, j and t are respectively district, crop and time subscripts, Y denotes production in tonnes, I denotes irrigated acreage, UI denotes unirrigated acreage, F denotes the consumption of chemical fertilizers[7] in each district as a whole, *not* on the basis of individual crops, R denotes rainfall measured on district basis and ε is the stochastic disturbance term.

The obvious deficiencies are that output and inputs cover heterogeneous entities. In a sense this is a mismeasurement of variables entering the production function. Omission of relevant inputs including machinery, purchased labour, seeds and pesticides, timing and frequency of irrigation, bullock labour and so forth, necessarily implies a loss of precision in the measurement of marginal products. We find it extremely hard to speculate about the direction of the omitted variable bias. It must be said, however, that at the level of district or the State data deficiencies are hard to overcome and even when micro economic investigations are conducted the measurement of some of the inputs mentioned above is at least dubious in traditional agriculture.

We have data on twenty districts of Andhra Pradesh, namely, Srikakulam, Visakhapatnam, East Godavari, West Godavari, Krishna, Guntur, Nellore,

6. A complete description of data can be obtained from the authors. Data on other inputs are not available at the district level in Indian agriculture.

7. Data on chemical fertilizers were expressed as aggregate units of nitrogen, phosphorus and potash used. Some approximate method could be used to allocate total nutrients by crops, but we found that most of the approaches were ad hoc and did not produce any significantly different results of the Cobb-Douglas model.

Kurnool, Cudappah, Chittoor, Hyderabad, Nizamabad, Medak, Mehboobnagar, Nalgonda, Warangal, Khammam, Karimnagar, Adilabad and Anantapur. All grow rice, all except Anantapur grow groundnuts, all except Cudappah and Chittoor grow tobacco and all except Visakhapatnam, Nalgonda, Warangal, Khammam and Adilabad grow sugarcane. The time-series of observation spans from 14 to 16 annual observations depending upon the crop. The data period starts in 1959-60 and ends in 1974-75. Especially with respect to rice, this period includes a sub-period (towards the end of the sample) when farmers were conducting much experimentation with High-Yielding Varieties (HYVs).[8] This kind of structural change could impart systematic variations in production function estimates which we have studied elsewhere.[9] To analyse these data one could focus either on the time-series dimension or the cross-section dimension or pool the two. Our approach has been to pool either the time-series or the cross-sections but not both simultaneously. The reason for this is explained in the subsequent section.

ESTIMATION

For estimating specification (1) we employed the standard Ordinary Least Squares (OLS) procedure to each district time-series or to each year cross-section and a Bayesian procedure (BAY) based upon the hierarchical prior model analysed by Lindley and Smith[10] and Smith.[11] In what follows, we shall be interested primarily in the nature and motivation underlying the estimation procedure and refer the reader to the original sources for additional information. We begin with a brief exposition of hierarchical prior model.

Let m denote the number of linear regression equations, time-series or cross-section, which we wish to estimate. Then one version of a hierarchical prior model has the following representation:

$$Y_i/\beta_i \sim N\,(X_i\,\beta_i,\ \sigma_i^2 I_N) \qquad i=1,\ldots,m \qquad \ldots.(2)$$

$$\beta_i/\mu \sim N\,(\mu,\ C_1) \qquad i=1,\ldots,m \qquad \ldots.(3)$$

$$\mu/\theta \sim N\,(\theta,\ C_2) \qquad \ldots.(4)$$

In (2), (3) and (4), Y_i is $(N \times 1)$ vector, X_i is $N \times K$ matrix of linearly independent regressors, β_i, μ and θ are all $(K \times 1)$ vectors of unknown parameters, C_1 and C_2 are $(K \times K)$ non-singular variance matrices.

Equation (3) represents prior beliefs about β_i; in the present case these are that the coefficients for group i $(i=1, 2,\ldots,m)$, β_i, are *a priori* a random drawing from a normal (hyper) population with a common mean μ and variance matrix C_1. Equation (4) represents a second stage Bayesian prior on μ. Third stage or higher stage priors could be added. Where one stops depends on whether the higher stages represent informative or diffuse prior information.

8. See S. S. Gupta, A. K. Banerjee, P. C. Malhotra and M. Rajagopalan, "A Study on High Yielding Varieties of Rice in Andhra Pradesh", *Agricultural Situation in India*, Vol. XXVIII, No. 1, April 1973, pp.17-21.

9. See A. Parikh and P. Trivedi, "Estimation of Cross-Section Production Functions under Structural Change in Indian Agriculture", in D. A. Peel, D. Currie, and W. Peters (Eds.): Micro-economic Analysis, Croom Helm Ltd., 1981, pp. 413-453.

10. Lindley and Smith, *Journal of the Royal Statistical Society*, 1972, *op. cit.*

11. A. F. M. Smith, "A General Bayesian Linear Model", *Journal of the Royal Statistical Society*, Series B, Vol. 35, 1973, pp. 67-75.

In our own case prior information is informative only at the first stage, *i.e.*, about β_i, and it is vague about μ, *i.e.*, $C_2^{-1} \rightarrow 0$.

Equation (3) embodies the assumption of *exchangeability between regressions.* Essentially, it provides a link between the β's for any one group and those for any other group, through the hyperparameter μ. If, for example, C_1 and σ_i^2 were known and $C_2^{-1} \rightarrow 0$, then the posterior distribution of β_i would be normal with mean β_i^* where

$$\beta_i^* = M_1^i \hat{\beta}_i + M_2^i \mu^* \quad (i=1,\ldots,m) \qquad \ldots.(5)$$

where $\hat{\beta}_i$ and μ^* are respectively the least squares estimate of β_i and a pooled estimate of μ, M_1^i and M_2^i are both (K $\times$ K) matrices implicitly defined in (8) below. In other words, β_i^* is a matrix weighted average of two sets of estimates, one of which depends on group information about group i alone and the other, μ^*, depends on information about all groups. These are:

$$\hat{\beta}_i = (X_i' X_i)^{-1} X_i' Y_i \qquad \ldots.(6)$$

$$\mu^* = \sum_{i=1}^{m} W_i \hat{\beta}_i \qquad \ldots.(7)$$

$$W_i = \left[\sum_{j=1}^{m} (\sigma_j^{-2} X_j' X_j + C_1^{-1})^{-1} \sigma_j^{-2} X_j' X_j \right]^{-1} (\sigma_i^{-2} X_i' X_i + C_1^{-1})^{-1} . X_i' X_i \sigma_i^{-2} \qquad \ldots.(8)$$

The intuitive explanation underlying the structure of the estimator μ^* is simply that (2), (3) and (4) can be combined to yield an equivalent model

$$Y_i/\mu \sim N (X_i \mu, X_i' C_2 X_i + C_1) \qquad \ldots.(2a)$$

and

$$\mu/\theta \sim N (\theta, C_2) \qquad (4)$$

Further, given that the prior on μ is diffuse, it is straightforward to see that the posterior mean of μ is μ^*. Using standard results from the Bayesian analysis with conjugate normal priors it is easily shown that

$$B_i \sim N (\beta_i^*, D_i^*) \qquad \ldots. (9)$$

where

$$\beta_i^* = \left[\sigma_i^{-2} X_i' X_i + C_1^{-1} \right]^{-1} \left[\sigma_i^{-2} X_i' X_i \hat{\beta}_i + C_1^{-1} \mu^* \right] \qquad \ldots.(10)$$

$$D_i = \left[\sigma_i^{-2} X_i' X_i + C_1^{-1} \right]^{-1} \qquad \ldots.(11)$$

$$D_i^* = D_i + D_i C_1^{-1} V C_1^{-1} D_i \qquad \ldots.(12)$$

$$\text{and } V = \left[\sum_{j=1}^{m} \{ \sigma_j^2 (X_j' X_j)^{-1} + C_1 \}^{-1} \right]^{-1} \qquad \ldots.(13)$$

The matrix V is the variance matrix of μ^*. In summary, the procedure involves 'pooling' to estimate the hyperparameter μ, and shrinking towards μ^* in order to obtain Bayes point estimates of β_i.

What are the merits of this procedure ? Pooling can lead to efficiency gains in estimation if it is based on 'good' prior information. It is debatable whether the exchangeability assumption is a good one when dealing with m districtwise time-series or yearwise cross-sections. For example, specialisation of districts by crops could be an indication of differences rather than similarities between districts. In dealing with a time-series of cross-sections the regression coefficients may vary over time in which case the assumption of exchangeability between cross-section regressions may be unsatisfactory. It must be admitted that exchangeability assumption is a *hypothesis* which may in specific instances be counterfactual. We accepted it as a first approximation mainly because the districtwise time-series of observations is short and the data are 'weak' in the sense of Learner[12] so that no convincing demonstration of between district differences is possible, except for a few pairs of districts. We felt, however, that pooling of data would lead to some gain in the precision of estimates of the hyperparameters at least.[13] The motivation behind pooling of cross-sectional information was also similar. Except in the case of rice, we had no *a priori* reasons for believing that systematic variation had affected the relationship. On the other hand, cross-section estimates typically do show some temporal variation so that exchangeability is a reasonable starting point.

There are formal and algebraic similarities between the Bayesian hierarchical prior model and the random coefficient model. For example, if i in (2) is a group subscript, (3) can be regarded as a specification of stochastic linear restrictions $\beta_i = \mu + \varepsilon_i$ (i=1,...,m), $\varepsilon_i \sim N(0, C_1)$. We do not explore this connection here but refer the reader to Smith[14] and Parikh and Trivedi[15] for further details.

In this section we have dealt with the case where σ_i^2 and C_1 appearing in (2) and (3) are known. In estimation, of course, this constraint was relaxed by postulating further prior distributions on σ_i^2 and C_1 as in Lindley and Smith.[16] The computer programme we used has been described by Fearn.[17] The main limitation of this package from our viewpoint was that (*a*) we were restricted to *linear*[18] equations with not more than four slope coefficients in each equation and (*b*) the programme could not satisfactorily handle the cases where the prior distribution of σ_i^2 is different for each i.[19] The Bayesian estimates were then obtained after assuming $\sigma_i^2 = \sigma^2$ (all i) and postulating a (common) inverse chi-squared prior distribution on σ^2. The main practical consequence of this is likley to be that β_i^* may be pulled or shrunk too much towards μ^*. This is not always desirable. Increasing speed in computation and availability of better software may help to eventually overcome these deficiencies in future.

RESULTS

OLS and Bayesian procedures were applied to (*i*) districtwise time-series data and (*ii*) yearwise cross-section data. This yielded OLS estimates $\hat{\beta}_i$ and

12. E. Learner: Specification Searches: Ad Hoc Inference with Non-experimental Data, Wiley, New York, 1978.

13. For an elaboration of this idea, see P. Trivedi, "Small Samples and Collateral Information: An Application of the Hyperparameter Model", *Journal of Econometrics*, Vol. 12, 1980, pp. 301-318.

14. Smith, *Journal of the Royal Statistical Society*, 1973, *op. cit.*

15. A. Parikh and P. Trivedi, "Estimation of Returns to Inputs in Indian Agriculture", Australian National University Discussion Paper No. 008, Australian National University, Canberra, Australia, 1979.

16. Lindley and Smith, *Journal of the Royal Statistical Society*, 1972, *op. cit.*

17. T. Fearn, "Toward a Bayesian Package", *Compstat*, 1978, pp. 473-480.

18. In the context of the Bayesian estimation technique, the theory and computer programmes are only available for handling linear models.

19. The option of different σ_i^2 was tried but the estimates failed to converge.

Bayes estimates β_i^* and μ^*. As shown earlier, μ^* is simply the average of the β_i^* (i = 1,...,m). Subsequenly, we refer to it as posterior mean. In what follows, μ^* will be treated as an estimate of the State relationship and β_i^* as an estimate of the ith district (year) relationship. Note that there will be two sets of estimates for the State, first an average of district relationships and the other an average of cross-section relationships and in general these would be different. Which should one choose and, in any case, what is the relation between the two?

To answer the question first consider why time-series estimates of production function may differ from cross-section estimates. It is sometimes argued that estimates of parameters based on cross-section data are likely to reflect long run influences whereas the time-series would reflect a short run relationship. Recently, Stapleton[20] has traced the antecedents of this view to Meyer and Kuh[21] and also Tinbergen.[22] Both Kuh[23] and Houthakker[24] have argued that cross-section data which are averages of time-series data are more likely to yield long run estimates than the data based on a single year. The argument appears to be as follows. In any one year, the quantity of fixed or quasi-fixed factor may deviate from its equilibrium value but in different years the deviations around the equilibrium may be randomly distributed. When time averages are taken over a period which is sufficiently long relative to the speed of adjustment, deviations around the equilibrium will sum to zero and something analogous to a long run relationship can be estimated. Another source of difference between the cross-section and time-series estimates is that the range of variation of outputs and inputs is typically greater in the former than in the latter. Such arguments are suggestive rather than conclusive. We ourselves agree with Stapleton's conclusion that while cross-section estimates are likely to be closer to long run coefficients than time-series estimates, both are likely to be between the long and short run coefficients in many circumstances.

In what follows we shall emphasize broad characteristics and not go into great detail in presenting and analysing the results but refer the reader to two of our forthcoming papers for such detail.[25]

In Table I we have given a small selection of time-series results for the four crops. For each crop, we give the average district estimates as well as those for two other districts and the latter have been selected so as to indicate the range of variation of the elasticities. As one would expect, for wet season

20. D. C. Stapleton, "Inferring Long-term Substitution Possibilities from Cross-Section and Time-Series Data", in E. R. Berndt and B. C. Field (Eds.): Measuring and Modelling Natural Resource Substitution, M. I. T. Press, Cambridge, 1980.

21. J. R. Meyer and E. Kuh: The Investment Decision: An Empirical Study, Cambridge, Massachusetts, U.S.A., 1959.

22. J. Tinbergen: Econometrics, George Allen & Unwin Ltd., London, 1951.

23. E. Kuh, "The Validity of Cross-Sectionally Estimated Behavioral Equations in Time-Series Applications", *Econometrica*, Vol. 27, No. 2, April 1959, pp. 197-214.

24. H. S. Houthakker, "New Evidence on Demand Elasticities", *Econometrica*, Vol 33, No. 2, April 1965, pp. 277-288.

25. Parikh and Trivedi, "Estimation of Cross-Section Production Functions under Structural Change in Indian Agriculture", *op. cit.* and "Estimation of Marginal Productivity for the Groundnut Crop: A Cross-Section and Time-Series Analysis", *Sankhya*, Series 'D', Vol. 42, December 1980, pp. 68-100.

TABLE I—BAY POSTERIOR MEAN ESTIMATES, THEIR VARIANCES AND MEAN OF OLS ESTIMATES AND RANGE OF PARAMETER ESTIMATES OBTAINED FOR RICE, GROUNDNUTS, TOBACCO AND SUGARCANE

Name of the crop	Name of the district	Estimation method	Elasticity coefficients with respect to				Residual variance	Durbin-Watson ratio statistic
			Irrigated area	Unirrigated area	Chemical fertilizer	Rainfall		
Rice ..	BAY mean	BAY	1·01590		0·06540	0·16530	0·03700	
	BAY variance		(0·00398)		(0·00056)	(0·00979)		
	OLS mean	OLS	1·15630		0·04740	0·14170		
	West Godavari*	BAY	1·05540		0·09430	—0·06360		
		OLS	2·20680		0·13006	—0·37120	0·00540	1·4294
		S.E. of OLS	(0·59820)		(0·03464)	(0·14117)		
	Mehboobnagar*	BAY	0·95730		0·06890	0·24500		
		OLS	0·62250		0·03060	0·45590	0·01670	1·3748
		S.E. of OLS	(0·30900)		(0·0453)	(0·2774)		
Tobacco ..	BAY mean	BAY	0·11850	0·63265	0·17489	0·09769	0·23180	
	BAY variance		(0·00514)	(0·02368)	(0·00612)	(0·03482)		
	OLS mean	OLS	0·12720	0·65633	0·19300	0·19545		
	Hyderabad[3]	BAY	0·31617	1·23900	0·67646	0·79826		
		OLS	0·40738	1·77090	0·93926	1·59290	3·1573	2·2434
		S.E. of OLS	(0·59778)	(1·54620)	(0·60602)	(2·72624)		
	West Godavari[4]	BAY	0·15540	0·00549	0·08939	—0·20592		
		OLS	0·06753	0·00147	0·09230	—0·56010	0·1069	2·3581
		S.E. of OLS	(0·68108)	(0·05710)	(0·15061)	(0·44722)		
Groundnuts ..	BAY mean	BAY	0·12488	0·66358	0·07414	0·15549	0·0916	
	BAY variance		(0·00147)	(0·01758)	(0·00343)	(0·01355)		
	OLS mean	OLS	0·15890	0·65480	0·06420	0·14530		
	Nizamabad[3]	BAY	0·09058	1·00330	—0·02887	0·34854		
		OLS	0·03852	1·09050	—0·09652	0·48617	0·0629	1·9400
		S.E. of OLS	(0·04070)	(0·36590)	(0·10580)	(0·26160)		
	Nellore[4]	BAY	0·18314	0·03948	0·28890	0·18739		
		OLS	0·19803	0·04197	0·29289	0·22455	0·44560	1·67
		S.E. of OLS	(0·15800)	(0·08280)	(0·22490)	(0·55400)		

(Contd.)

TABLE I (*Concld.*)

Name of the crop	Name of the district	Estimation method	Elasticity coefficients with respect to				Residual variance	Durbin-Watson ratio statistic
			Irrigated area	Unirrigated area	Chemical fertilizer	Rainfall		
Sugarcane ..	.. BAY mean	BAY	0·98567		—0·04984	0·00880	0·01160	
	BAY variance		(0·00126)		(0·00057)	(0·00255)		
	OLS mean	OLS	1·00550		—0·05345	0·00132		
	Karimnagar[1]	BAY	1·06070		—0·19147	0·02652		
		OLS	1·10130		—0·24603	0·01078	0·0275	1·3504
		S.E. of OLS	(0·07040)		(0·07248)	(0·14640)		
	Medak[2]	BAY	0·90930		—0·09671	0·21162		
		OLS	0·83375		—0·07243	0·29382	0·0295	2·8449
		S.E. OF OLS	(0·18176)		(0·09851)	(0·17111)		

* These are chosen on the basis of maximum (West Godavari) and minimum (Mehboobnagar) average irrigated area under rice.

1. The selected district is corresponding to the maximum elasticity coefficient obtained for irrigated area using BAY estimation.
2. The selected district is corresponding to the minimum elasticity coefficient obtained for irrigated area using BAY estimation.
3. The selected district is corresponding to the maximum elasticity coefficient obtained for unirrigated area using BAY estimation.
4. The selected district is corresponding to the minimum elasticity coefficient obtained for unirrigated area using BAY estimation.
5. All the results can be obtained from the authors.

TABLE II—BAY POSTERIOR MEAN, ITS VARIANCE, OLS ESTIMATES FROM AGGREGATE DATA

Crop	Elasticity with respect to	Irrigated area		Unirrigated area		Fertilizers		Rainfall	
		Time-series	Cross-section	Time-series	Cross-section	Time-series	Cross-section	Time-series	Cross-section
Sugarcane ..	BAY	0·9856	0·9965	—	—	—0·0498	0·0586	0·0081	—0·0106
	Variance	(0·0012)	(0·00005)	—	—	(0·00057)	(0·0002)	(0·00255)	(0·0027)
	OLS (aggregated)	0·9320	0·9956	—	—	—0·0492	0·0661	—0·0486	—0·0235
Groundnuts ..	BAY	0·1248	0·0892	0·6636	0·7908	0·0741	0·0984	0·1554	0·0262
	Variance	(0·0014)	(0·0004)	(0·0176)	(0·0049)	(0·0039)	(0·0029)	(0·0135)	(0·0725)
	OLS (aggregated)	0·3461	0·0818	0·6004	0·8957	—0·0132	0·0895	0·4954	0·5056
Tobacco ..	BAY	0·1185	0·1932	0·6326	0·6450	0·1749	0·3630	0·0976	—0·2821
	Variance	(0·0051)	(0·0011)	(0·0236)	(0·0066)	(0·0061)	(0·0148)	(0·0348)	(0·0617)
	OLS (aggregated)	0·0900	0·2351	0·1130	0·6746	0·3386	0·3158	0·0980	—0·6718
Rice ..	BAY	1·0159	0·8504	—	—	0·0654	0·1858	0·1653	0·0382
	Variance	(0·0040)	(0·0041)	—	—	(0·00056)	(0·0024)	(0·0097)	(0·0125)
	OLS (aggregated)	1·6121	0·8637	—	—	0·0186	0·1682	—0·2472	—0·2129

crops, rice and sugarcane, irrigated area is the most important input with the average elasticity close to unity. For groundnuts and tobacco, the corresponding values are of the order of 0.1 only but the elasticity with respect to the unirrigated area is between 0.6 and 0.7. The Bayes point estimates do actually have the expected sign in more cases than the OLS estimates. This is partly the result of shrinkage towards the coefficients' mean vector. The OLS estimates have quite large associated standard errors. Almost all the presented results[26] have very high R^2 and although Durbin-Watson ratio statistic suggests the possibility of serial correlation in a few cases, quite often the value falls in the indecisive zone.

Table II collects the posterior means from the separate time-series and crosss-section analysis. The differences between them do not show any simple pattern. For rice and groundnuts the time-series elasticity estimates for irrigated area is larger than the cross-section estimates, but for tobacco the converse holds. For both groundnuts and tobacco the unirrigated area elasticity estimates from cross-section is higher than the time-series estimate. For all crops the cross-section fertilizer elasticity is estimated to be higher than the time-series one. However, recall that fertilizer data are not available by crop use. If cross-section estimates are taken as estimates of long run relationship and time-series estimates as those of the short run relationship the results of Table II do not give much support to a widespread (?) belief that the long run output response to irrigation is much greater than the short run response. But it seems justifiable to retain skepticism about identifying the cross-section and time-series with the long and short run relationships respectively.

In Table III we present a small subset of information on marginal productivity of various inputs.[27] Too much should not be claimed for these results. However, for what they are worth, they show that the marginal productivity of irrigated area compared with that of unirrigated area is about 1½ times higher in the case of groundnuts and tobacco. From cross-section estimates the factor is about the same for groundnuts but about 4½ for tobacco. More detailed examination confirms that even for crops where unirrigated area is the single most important input, the marginal productivity of irrigated land is higher by a substantial margin, which one should expect to be the case in water scarce agriculture. These results have some bearing on policy making, and it is not necessarily true that a quantitative analysis on cropwise returns to inputs is completely useless. In Indian agriculture, the choice of a particular crop-mix is location specific since soil, weather and other natural factors are not controlled by government policy makers. Regions growing rice show higher marginal productivity with respect to fertilizer input and

26. R^2 is not presented because it is above 0·9 in all cases.

27. Among the six districts for which marginal productivity computation presented in Parikh and Trivedi, "Estimation of Returns to Inputs in Indian Agriculture," *op. cit.*, West Godavari seems to have significantly higher than average marginal productivity of irrigated land used in rice and sugarcane production and higher than average marginal productivity for chemical fertilizers. West Godavari is one of the most advanced districts and this result is in agreement with one's expectations.

TABLE III—MARGINAL PHYSICAL PRODUCTS EVALUATED AT POSTERIOR MEAN ESTIMATES FROM: (A) DISTRICTWISE TIME-SERIES DATA AND (B) CROSS-SECTION DATA TOGETHER WITH (MIN, MAX) VALUES IN EACH CASE

Name of the crop		Irrigated area	Unirrigated area	Chemical fertilizers
Rice	(*a*)	1·411 (0·979, 1·784)	—	2·451 (0·931, 20·367)
	(*b*)	1·18 (0·908, 1·593)	—	6·96 (0·669, 16·998)
Groundnuts	(*a*)	0·97 (0·38, 59·65*)	0·60 (0·12, 1·65)	0·39 (negative, 3·91)
	(*b*)	0·69 (0·73, 4·97)	0·72 (0·31, 0·85)	0·52 (negative, 1·42)
Tobacco	(*a*)	2·31 (negative, 21·84*)	0·75 (0·0637, 1·09)	0·08 (negative, 0·25)
	(*b*)	3·77 (1·61*, 10·83*)	0·76 (0·37, 0·92)	0·17 (0·02, 0·31)
Sugarcane	(*a*)	7·837 (6·57, 9·10)	—	negative (negative, 0·15)
	(*b*)	7·92 (7·01, 8·85)	—	0·16 (0·09, 0·59)

* Indicates that value calculated using a very high output-input ratio.

greater allocation of fertilizers to rice growing regions will presumably be used for rice crop since rice cultivation is region specific due to soil and weather complex.[28]

We computed average districtwise growth rates and the contribution of the growth of various inputs to the growth in output. Broadly speaking, groundnuts, sugarcane and tobacco showed a high growth tendency while only five districts registered more than 3 per cent average growth in rice production and eleven registered decline. The contribution of the fertilizer input is consistently positive, and in the case of many districts it coincides with reduction in the irrigated acreage. In general, very little of the growth in output is explained by increased fertilizer use. For groundnuts and tobacco, the increase in the irrigated acreage is the dominant source of output growth. These conclusions are conditional on our somewhat unsatisfactory fertilizer data.

Finally, we mention that we carried out an extensive series of predictive tests to evaluate the Bayes point estimates vis-a-vis the OLS estimates. As the former are based on more information, in a correctly specified model they would be superior to the latter in a predictive mean square error sense. To carry out this exercise, the time-series were truncated by 4 to 6 observations

28. It is not entirely the case that aggregate production function for all crops together will yield a more useful estimate of returns to inputs than crop specific marginal productivities of inputs.

and Bayes and OLS estimates were obtained from the truncated sample data. Outside sample predictions based on Bayes point estimates β_i^*, the posterior mean estimate μ^* and OLS estimates $\hat{\beta}_i$ were generated and various mean square errors of prediction performance were compared. We found some evidence for prediction based on μ^* (which we could think of as smoothed estimates) to be more accurate than $\hat{\beta}_i$ in the mean square error sense.[29] But as the predictive comparison raises difficult issues which cannot be expounded briefly we refer the interested reader to Parikh and Trivedi.[30]

CONCLUSION AND QUALIFICATION

There is a sense in which the notion of an exchangeable prior distribution over sub-regions enables us to construct estimates of regional relationship. As we have shown by example, the Bayes procedure involves pooling of data for the sub-regions and it is an alternative to the research strategy of aggregating the data first and then estimating the aggregate function. Certain versions of random coefficient models (see for example, Swamy and Mehta[31] will yield results similar to our Bayesian method and in addition they could be free from limitations to which the Bayesian approach has suffered, namely, the constraint on the number of parameters, absence of easily implemented diagnostic checks on priors based on 'weak' beliefs and the restrictiveness of prior stochastic assumptions. But the empirical Bayesian approach also has some advantages (which we have not fully exploited because of lack of data). For example, the differences in β_i could be linearly related to a set of group specific variables, say Z_i. Thus (3) could be replaced by

$$\beta_i/\mu \sim N\ (Z_i\eta,\ C_1) \qquad \ldots.(3a)$$

and this would make a far richer model. The main difficulties with the hierarchical prior model at present seem to be computational rather than conceptual and these would probably be covercome in future.

The paper also argues against the generally prevailing view that cross-section and time-series relationships respectively yield long and short run relationships. Our conclusion on this issue is that cross-section estimates are likely to be closer to long run coefficients than the time-series estimates; both are likely to be between the long and short run coefficients in many circumstances and our study seems to confirm this.

29. The prediction comparison was made using OLS mean estimates and these also have smaller prediction mean square error like predictions based on μ^*. In the Bayesian framework smoothing and averaging is justified by the assumption of exchangeability between regressions, and therefore we might say that the use of this prior information is the cause of this improvement. OLS mean does not have any such theoretical justification.

30. Parikh and Trivedi, "Estimation of Returns to Inputs in Indian Agriculture", *op. cit.*

31. P. A. V. B. Swamy and J. S. Mehta, "Estimation of Linear Models with Time and Cross-Sectionally Varying Coefficients", *Journal of the American Statistical Association*, Vol 72, No. 360, December 1977, pp. 890-898.