Does Tax Competition Really Promote Growth?

**No 810**

# WARWICK ECONOMIC RESEARCH PAPERS

**DEPARTMENT OF ECONOMICS**

THE UNIVERSITY OF
WARWICK

# Does Tax Competition Really Promote Growth?

Marko Koethenbuerger*and Ben Lockwood†

**Abstract**

This paper considers the relationship between tax competition and growth in an endogenous growth model where there are stochastic shocks to productivity, and capital taxes fund a public good which may be for final consumption or an infrastructure input. Absent stochastic shocks, decentralized tax setting (two or more jurisdictions) maximizes the rate of growth, as the constant returns to scale present with endogenous growth implies "extreme" tax competition. Stochastic shocks imply that households face a portfolio choice problem, which may dampen down tax competition and may raise taxes above the centralized level. Growth can be lower with decentralization. Our results also predict a negative relationship between output volatility and growth, consistent with the empirical evidence.

Keywords: tax competition, uncertainty, stochastic growth.

JEL Classification: H77, E62, F43

---

*Mailing Address: CES, University of Munich, Schackstr. 4, 80539 Munich, Germany. E-mail: koethenbuerger@lmu.de.

†Mailing address: Department of Economics, University of Warwick, Coventry CV4 7AL, England. Email: B.Lockwood@warwick.ac.uk

# 1. Introduction

The link between fiscal decentralization and economic growth is increasingly attracting the attention of economists. In particular, a growing body of empirical research is investigating the links between measures of fiscal decentralization and growth, both at the country and sub-national level. Overall, the evidence is mixed. In particular, cross-country studies, which generally use similar measures of fiscal decentralization, can find positive or negative effects, depending on precise measure of decentralization, sample, estimation method, etc. (Davoodi and Zhu (1998), Wooler and Phillips (1998), Zhang and Zou (1998), Iimi (2005), Thornton (2007)). More recently, two studies on US data have found more robust evidence that fiscal decentralization increases growth (Akai and Sakata (2002), Stansel (2005)). For example, Stansel (2005), in a study of growth over 30 years in 314 US metropolitan areas, has found that the degree of fractionalization (the number of county governments per million population in a metropolitan area) significantly increases growth.

On the theoretical side, explanation of the mechanisms linking fiscal decentralization and growth are thin on the ground. Two mechanisms have been studied. First, as shown by Lejour and Verbon (1997), and Hatfield (2006), tax competition can lower the tax on capital, thus increasing the return to savings, and thus growth, in an endogenous growth model. Second, Brueckner (2006) shows that centralization, if it imposes uniform public good provision across regions, can lower the rate of savings and thus growth, although this mechanism appears to require difference in the mix of young and old across fiscal jurisdictions.

This paper makes a contribution to understanding of the tax competition mechanism. In formalizing this mechanism, there is a basic dilemma[1]. To understand the effect of fiscal policy on growth, a simple endogenous growth model is required. But, simple endogenous growth models have constant returns to scale. As a consequence, in (for example) an AK-type growth model, the firm's demand for capital is perfectly elastic at the tax-inclusive price of capital. This in turn, under the standard assumption of perfect mobility of capital across regions (i.e. that households can move their capital costlessly across regions) implies "extreme" or Bertrand tax competition: each jurisdiction can undercut the others by a fraction and capture all the capital in the economy. The result is that under decentralization, the tax is driven down to zero, and the growth rate is maximized.

More recently, Hatfield (2006) has shown that a similar conclusion holds very generally[2]

---

[1] See Lejour and Verbon (1997), who implicitly make this point.

[2] This holds even when labour supply is endogenous and is taxable, when there are consumption and

2

in Barro's (1990) model of infrastructure growth: that is, there is a tax-inclusive price of capital at which firms are willing to employ any level of the capital input. So, the allocation of capital is determined by households, who move capital to the region with the highest price. Thus, again there is implies extreme or Bertrand tax competition: jurisdictions compete to set taxes to achieve the highest pre-tax price of capital. As there is an infrastructure public good funded by the capital tax, this price-maximising tax is not zero, but strictly positive. Nevertheless, the conclusion is the same as in the AK model: under decentralization, the rate of return on savings, and thus the growth rate is maximized.

So, if capital mobility is assumed perfect (costless) between jurisdictions, we have a rather limited theory of the link between tax competition and growth, which cannot easily accommodate the findings of some empirical studies that this relationship is negative. In this paper, we suggest a way out of this dilemma, which does not rely on any ad hoc assumptions about imperfect mobility of capital.

We start with a standard AK model with a consumption public good financed out of a tax on capital. We assume only that there are independent stochastic shocks to production in each of $n$ regions. This generates stochastic returns to capital invested by a household in each region. Thus, a standard portfolio argument implies that - if taxes are not too different - the household located in one region will want to invest some of its accumulated capital in all regions. This in turn generates a negative fiscal externality: an increase in the tax in any "foreign" region reduces the return to savings in the "home" region, and also thus the capital employed in the "home" region.

The key point is that this negative externality offsets the positive externality arising from mobile capital (i.e. that an increase in the foreign region's tax leads to a capital outflow from the foreign region to the home region). This implies that when the second externality dominates, taxation under decentralization will be higher, and growth lower, than with centralization. Analytical results show that this occurs when (i) the number of regions is small; (ii) when the variance of the shock is sufficiently high.

We then modify the AK model to the Barro (1990) model of infrastructure growth in which we allow the production technology to be stochastic. This has the consequence that the pre-tax rate of return to capital becomes stochastic, an effect which magnifies as the amount of infrastructure goods increases. Since each region also hosts capital from non-residents, this specification introduces a third type of externality of decentralized

infrastructure goods, and where agents are different within regions (with respect to their endowments of capital) and decisions are made by majority voting.

government policy. A higher tax increases the riskiness of investment and thus the risk-bearing of non-residents. We find that when the variance is zero, centralization yields a tax rate which is too high to be growth maximizing, while decentralization yields a tax rate which is growth maximizing (replicating the result in Hatfield, 2006). By a continuity argument, growth is higher under decentralized government when the variance of the shock is small. But, as the variance of the shock increases, centralization may generate *higher* growth than decentralization, as in the consumption public good case.

One of the interesting predictions of our model concerns the relationship between the variance of stochastic shocks and growth. With a public consumption good, we show analytically that growth is (at least weakly) decreasing in the variance of the output shock. With a public infrastructure good, and fiscal decentralization, simulation results indicate a negative relationship between growth and the variance of output shock. This is consistent with the macroeconomic evidence (see Ramey and Ramey, 1995), although of course there are other mechanisms linking output shocks and growth (Jones and Manuelli, 2005).

The most closely related paper[3] to ours is Lejour and Verbon (1997). They were the first to point out that if the household has a diversified (across tax jurisdictions) portfolio of investments across regions, a negative externality via savings could arise in equilibrium. But, the modelled this in an ad hoc way. They just *assume* that there is some convex mobility cost of moving capital between regions. The increasing marginal cost ensures that capital moves smoothly between regions in response to price (and thus tax) differential. Moreover, to generate higher taxes under decentralization, Lejour and Verbon (1997) have to introduce *another* ad hoc element into the mobility cost function: that is, it is assumed that when returns to capital in two regions are the same, households have a strict preference for investing some of their capital in the foreign region i.e. a preference for diversity[4].

The paper is also related to an existing literature on public finance in models of stochastic growth, where fiscal policy rules are taken to be fixed (Turnovsky, 2000 and Kenc,

---

[3]Another related paper is by Lee (2004), who studies the impact of output shocks on tax competition in a static model. However, in his model, as the number of regions is assumed large, investors can be sure of a certain return on capital ,and only face uncertain wage income. Thus, the negative externality arising though portfolio choice which is studied here does not arise in his model.

[4]Formally, if $s$ is the share of capital invested outside the home region, the mobility cost is $c(s) = -vs + \mu s^2/2$. Given this specification, it is possible to establish that for $v$ large enough relative to $\mu$, the equilibrium tax on capital can be higher under decentralization, and thus growth lower. Unfortunately, the $v$ and $\mu$ parameters are almost impossible to interpret empirically.

2004). By contrast, in this paper, taxes are optimized by governments. So, this paper is the only one, to our knowledge, that studies endogenous tax policy in a stochastic growth model. For endogenous tax policy in a deterministic growth models, see Philippopoulos (2003) and Philippopoulos and Park (2003). But these two papers do not deal with the issue of capital mobility and its impact on tax policy.

The rest of the paper is organized as follows. Section 2 introduces the model, solves for equilibrium conditional on fixed government policy, and identifies the fiscal externalities at work in the model. Section 3 contains the main results. Section 4 modifies the model to include infrastructure public goods. Section 5 concludes.

## 2. The Set-Up

### 2.1. The Model

Our model is a dynamic stochastic version of the Zodrow-Mieszkowski (1986) model, where regional government use source-based capital taxes to finance the provision of a public good. The economy evolves in continuous time: $t \in [0, \infty)$. There are $n$ regions, $i = 1, ..n$. There is one firm in each region, which produces output from capital according to the constant returns production function. Expressed in differential form, changes in output in region $i$ over the interval $(t, dt)$ is :

$$dy_i = k_i(dt + dz_i)$$

where $k_i$ is the capital stocks at $t$, and the stochastic variable[5] $dz_i$ is temporally independent with variance $\sigma_i^2 dt$ over the period $(t, dt)$.

Firms in region $i$ are willing to operate at any scale iff, over any interval $(t, dt)$, the change in output per unit of capital $dt + dz_i$, is equal to the cost of capital, $r_i dt + \tau_i dt$, where $r_i$ is the rental price of capital at $t$, and $\tau_i$ is the tax on the use of capital. Thus, the rental price of capital in region $i$ is determined by:

$$r_i dt = dt + dz_i - \tau_i dt = (1 - \tau_i)dt + dz_i \qquad (2.1)$$

The actual allocation of capital across regions will then be determined by households, as described below.

---

[5]The assumption that random shocks take the form of Brownian motion is standard in stochastic growth models. It describes a situation where productivity is subject to frequent small changes; see Turnovsky (2000) for more discussion.

Each region is populated by a number of identical infinitely-lived households, and the population in each region is normalized to 1. In region $g_i$, each household has a flow of utility from private consumption and a public good:

$$u(c_i, g_i) = \ln c_i + \beta \ln g_i,$$

where each of the two utility functions has the same fixed degree of relative risk-aversion equal to unity. At each $t$, the household in region $i$ has an endowment of the private good (wealth) $w_i$, which it can rent to any firm. Let $s_{ij}$ be the share of $w_i$ rented to the foreign firm. The household chooses $c_i, s_{ij}$ to maximize the expected present value of utility - this problem is considered in Section 2.2 below.

Finally, in region $i$, the public good is wholly financed by a source-based tax on capital i.e. $g_i = \tau_i k_i$. This is without loss of generality, as there is no wage income in the model, so our analysis would go through if $\tau_i$ were re-specified as an output tax.

We can now see more formally why in the absence of uncertainty ($\sigma_i \equiv 0$), taxes and public good provision would be zero under decentralization. Without uncertainty, the households in any region will simply allocate its capital to the region where the return on savings is highest. Given the negative relationship between $r$ and $\tau$ in (2.1), it is then clear that all capital will flow into region(s) where $\tau_i$ is lowest. This in turn implies that without mobility costs, there will be a "race to the bottom" in capital taxes, with the only possible equilibrium tax being zero[6].

## 2.2. Solving The Household Problem

Households solve a portfolio allocation/savings problem under uncertainty. First, wealth evolves according to

$$dw_i = \sum_{j=1}^{n} s_{ij} r_j w_i dt - c_i dt \tag{2.2}$$

where $\sum_j s_{ij} r_j dt$ is the overall return on wealth over the interval $(t, dt)$. Combining (2.2) and (2.1), we get:

$$dw_i = [\mu_i w_i - c_i] dt + w_i \sum_{j=1}^{n} s_{ij} dz_j \tag{2.3}$$

---

[6]We cannot aviod this conclusion by assuming decreasing returns i.e. $y = f(k)$, with $f''(k) < 0$, as then we are back to a Solow-type growth model, where taxes do not affect growth (in the long run).

where $\mu_i = \sum_j s_{ij}(1 - \tau_j)$ is the deterministic part (average) rate of return on wealth. Thus, household in region $i$ chooses $c_i,\ s_{ij}$ to maximize

$$E\left[\int_0^\infty e^{-\rho t} \ln c_i dt\right] \tag{2.4}$$

subject to (2.3) and $\sum_{j=1}^n s_{ij} = 1$. Finally, we assume that the taxes (and thus the mean return to investing in any given region) are time-invariant (this assumption will be verified below). The solution to this problem is well-known (e.g. Jones and Manuelli, 2005) and easily stated. First, the optimal consumption rule is simply

$$c_i = \rho w_i \tag{2.5}$$

Second, the portfolio shares $s_{ij},\ j \neq i$ are determined by the $n-1$ first-order conditions

$$\tau_i - \tau_j - \left[\left(\sum_{j\neq i} s_{ij} - 1\right)\sigma_i^2 + s_{ij}\sigma_j^2\right] = 0, \quad j \neq i. \tag{2.6}$$

To get intuition, consider the two-region case. Then, (2.6) solve for regions $1, 2$ to give

$$s_{12} = \frac{\tau_1 - \tau_2 + \sigma_1^2}{\sigma_1^2 + \sigma_2^2}, \quad s_{21} = \frac{\tau_2 - \tau_1 + \sigma_2^2}{\sigma_2^2 + \sigma_1^2}. \tag{2.7}$$

The portfolio rule is simple and intuitive; invest more "abroad" i.e. outside the home region $i$; (i) the higher the difference in average returns as measured by $\tau_i - \tau_j$; (ii) the lower the relative uncertainty of investing abroad i.e. $\sigma_j^2/\sigma_i^2$.

## 2.3. Fiscal Externalities

In this section, we identify the fiscal externalities at work in the model. We focus on the two-region case, thinking of region 1 as the home region. We trace the effects of a change in $\tau_2$ on home welfare. To simplify further, assume that $\sigma_2 = \sigma_1 = \sigma$. Then the portfolio allocation rule for home and foreign households is

$$s_{12} = \frac{1}{2} + \frac{\tau_1 - \tau_2}{2\sigma^2}, \quad s_{21} = \frac{1}{2} + \frac{\tau_2 - \tau_1}{2\sigma^2}. \tag{2.8}$$

Moreover the capital employed in the home region is

$$k_1 = (1 - s_{12})w + s_{21}w_2. \tag{2.9}$$

Finally, in the home region, the household's optimal accumulation of capital follows the rule (2.3), given also (2.5);

$$dw_1 = w_1\left(\mu_1 - \rho\right)dt + (1 - s_{12})dz + s_{12}dz^*.$$

By inspection of (2.8),(2.9), (2.3), we can identify two fiscal externalities in the model.

First, $\tau_2$ up implies $s_{21}$ up and $s_{12}$ down from optimal portfolio choice, implying $k_1$ up. This is the well-known *positive capital mobility externality:* an increase in the foreign tax causes a capital outflow, benefitting the home region. Note that it is measured (inversely) by $\sigma$; the higher $\sigma$, the weaker is this externality, as the higher $\sigma$, the stronger the incentive for the household to maintain a balanced portfolio.

Second, $\tau_2$ up has two related effects. First, it directly affects home welfare by lowering $\mu_1 = (1 - s_{12})\tau_1 - s_{12}\tau_2$, the deterministic part of the return on $w_1$. Second, it lowers $\mu_2$, and thus $w_2$ and thus $k_1$. We call this the *negative rate of return externality.* Note that the size of this externality is measured by $s_{12}, s_{21}$ : specifically, this externality only operates in equilibrium if $s_{12}, s_{21} > 0$. But, the portfolio allocation rule ensures that at symmetric equilibrium, $s_{12} = s_{21} = 0.5$.

## 3. Equilibrium Taxes

In this section, we further simplify by assuming $\sigma_i = \sigma$, $i = 1,..n$. Our approach here is the following. It is difficult to solve for government $i's$ problem given arbitrary taxes $\tau_j$, $j \neq i$. So, we *assume* that all regions $j \neq i$ set the same (time-invariant) tax $\tau^*$, and solve for region $i's$ best response to $\tau^*$. We show that this is also time-invariant i.e. some $\tau$. Then, imposing $\tau = \tau^*$ gives a condition for the Nash equilibrium tax.

We proceed as follows. First, let $s_{ij} = s$, $j \neq i$, be the share of wealth invested by the households in region $i$ in any region $j \neq i$. Note that from (2.6), and $\tau_j = \tau^*$, $j \neq i$, these share must all be the same. Indeed, evaluating (2.6) at $\sigma_i = \sigma$, $s_{ij} = s$, and $\tau_i = \tau$, $\tau_j = \tau^*$, and solving, we get

$$s = \frac{\tau - \tau^*}{n\sigma^2} + \frac{1}{n} \tag{3.1}$$

Next, let $s^* = s_{ji}$, $j \neq i$, $\hat{s} = s_{jk}$, $k \neq j \neq i$ be the shares of wealth invested by the households in region $j \neq i$ in region $i$ and the $n - 2$ regions $k \neq j \neq i$ respectively. Again from (2.6), $s^*, \hat{s}$ solve the two simultaneous equations

$$\tau^* - \tau - \sigma^2[-1 + 2s^* + (n - 2)\hat{s}] = 0 \tag{3.2}$$
$$-\sigma^2[-1 + (n - 1)\hat{s} + s^*] = 0. \tag{3.3}$$

We are only interested in $s^*$. Solving (3.2), (3.3), we get

$$s^* = \frac{(n - 1)(\tau^* - \tau)}{n\sigma^2} + \frac{1}{n}. \tag{3.4}$$

8

Now let $w$ be the wealth of region $i$, and $w^*$ be the average wealth of regions $j \neq i$. Then, by definition,

$$k = (1 - (n-1)s)w + (n-1)s^*w^*. \tag{3.5}$$

Then, using the government budget constraint $g = \tau k$, and the consumption rule $c = \rho w$, the government of $i$'s instantaneous payoff can be written

$$\ln c + \beta \ln g = \kappa + \ln w + \beta \ln k + \beta \ln \tau. \tag{3.6}$$

So, it is clear from (3.6) and (3.5) that there are *two* state variables in the problem, $w$ and $w^*$. These follow the following processes. First, from (2.3), given also (2.5), $\sigma_i = \sigma$, and the definition of $s$;

$$dw = w\left(1 - (1-(n-1)s)\tau - (n-1)s\tau^* - \rho\right)dt + (1-(n-1)s)dz_i + (n-1)sdz^*, \tag{3.7}$$

where $dz^* = \frac{1}{n-1}\sum_{j\neq i}dz_j$. Second, the process for some $w_j$ is

$$dw_j = w\left(1 - (1-s^*)\tau^* - s^*\tau - \rho\right)dt + s^*dz_i + (1 - s^* - (n-2)\hat{s})dz_j + \hat{s}\sum_{k\neq j\neq i}dz_k.$$

So, as $w^* = \frac{1}{n-1}\sum_{j\neq i}w_j$, tedious but straightforward calculation gives

$$dw^* = w^*\left(1 - (1-s^*)\tau^* - s^*\tau - \rho\right)dt + s^*dz_i + (1-s^*)dz^*. \tag{3.8}$$

So, the problem for the government of region $i$ is to choose $\tau$ to maximize

$$E\left[\int_0^\infty \left(\ln w + \beta \ln \left((1-(n-1)s)w + (n-1)s^*w^*\right) + \beta \ln \tau\right)e^{-\rho t}dt\right]$$

subject to (3.7),(3.8), and the portfolio allocation rules (3.1) and (3.4).

Our approach to this problem follows Turnovsky (2000). We first write down the Bellman equation defining $V(w, w^*)$, and thus characterizing the optimal choice of $\tau$, given $\tau^*$. We then evaluate this Bellman equation at $\tau = \tau^*$. To get a closed-form solution for the equilibrium $\tau$, we must guess the correct form of the value function $V(w, w^*)$, which we are able to do, using the fact, from (3.7),(3.8), that symmetric equilibrium $(s = s^* = \frac{1}{n})$, $w = w^*$. Specifically, we guess that $V(w, w) \equiv A + \psi \ln w$. All these steps are dealt with in detail in the Appendix, and the end result is:

**Proposition 1.** *The equilibrium tax rate with $n$ regions is*

$$\tau_n = \frac{\rho\beta}{\frac{\rho\beta(n-1)}{\sigma^2} + \frac{1+\beta}{n}}. \tag{3.9}$$

*The tax rate is increasing in the preference for the public good, $\beta$, the rate of discount, $\rho$, and the size of the output shock, $\sigma$.*

A number of comments are in order here. This formula and the comparative statics are intuitive. First, the higher the marginal valuation of the public good, the higher the tax. Second, the less the household values future growth i.e. the higher $\rho$, the more it is willing to increase the tax rate to fund the public good now, at the expense of future growth in the tax base. Third, the higher $\sigma$, the weaker the response of the capital stock in any region to a change in the tax rate in that region (when $n > 1$), and so the smaller the mobile tax base externality, thus increasing the equilibrium tax. Intuitively, the smaller the variance, the more willing are investors to move their wealth between regions in response to tax differences, thus increasing the mobility of the tax base.

More formally note from (3.5), in symmetric equilibrium,

$$\begin{aligned}
\frac{\partial k}{\partial \tau} &= -(n-1)w\frac{\partial s}{\partial \tau} + (n-1)w^*\frac{\partial s^*}{\partial \tau} \\
&= -(n-1)w\frac{1}{n\sigma^2} - (n-1)^2 w^*\frac{1}{n\sigma^2} \\
&= -k(n-1)\frac{1}{\sigma^2}, \tag{3.10}
\end{aligned}$$

where in the last line we used $w = w^* = k$. So, the semi-elasticity $\frac{1}{k}\frac{\partial k}{\partial \tau} = (n-1)/\sigma^2$ is clearly decreasing in $\sigma$. Note also that the more regions, the bigger this elasticity. This is a similar result to those for the standard static model of tax competition. Note finally this effect is operative only when $n > 1$; with full centralization ($n = 1$), the size of stochastic shocks make no difference to $\tau$.

Last, we turn to comparative statics in $n$. This is a key question because $n$ measures the degree of fiscal decentralization. Inspection of (3.9) shows that $n$ affects the denominator of $\tau$ in two places, corresponding to the two different externalities identified above. First, a higher $n$ increases the tax base elasticity, as already remarked; this is measured by the term $\frac{\rho\beta(n-1)}{\sigma^2}$ which is simply $\rho\beta$ times the semi-elasticity of the tax base with respect to $\tau$.

Second, an increase in $n$ increases the rate of return externality, corresponding to the term $\frac{1}{n}(1 + \beta)$. Intuitively, any resident of region $i$ only invests $\frac{1}{n}$ of his wealth at home

in equilibrium. So, the government of region $i$ ignores the negative effect of the tax in region $i$ on the rate of return to investors in all the other regions, measured by $\frac{n-1}{n}(1+\beta)$.

As the externalities have opposite effects on $\tau$, we expect that the effect of $n$ on $\tau$ is not monotonic, and this is confirmed by the following result:

**Proposition 2.** *There is a critical value of $n$, $\hat{n} = \sqrt{\frac{(1+\beta)\sigma^2}{\rho\beta}}$, such that $\tau_n$ is increasing in $n$ for $n < \hat{n}$, and decreasing in $n$ for $n > \hat{n}$. Finally, $\tau_n \longrightarrow 0$ as $n \longrightarrow \infty$, so that if there are sufficiently many regions, the decentralized tax is lower than the centralized tax.*

**Proof.** The denominator of (3.9), $\frac{\rho\beta(n-1)}{\sigma^2} + \frac{1+\beta}{n}$, is a convex function of $n$, with a minimum at $n = \sqrt{\frac{(1+\beta)\sigma^2}{\rho\beta}} = \hat{n}$. Thus, $\tau_n$ must be a quasi-concave function of $n$, with a maximum at the same value. Finally, by inspection, $\tau_n \longrightarrow 0$ as $n \longrightarrow \infty$. $\square$

The most interesting result here is that the effect of $n$ on $\tau_n$ is generally ambiguous. This is of course, because an increase in $n$ increases both the mobile tax base and rate of return externalities, which have opposite signs. So, Proposition 2 says that for $n$ small, the rate of return externality dominates, whereas for $n$ large, the mobile tax externality dominates. Figure 3.1 illustrates the capital tax rate as a function of the number of regions (assuming $\beta = 1$ and $\rho = 0.1$). For instance, for $\sigma^2 = 0.5$ the tax rate is maximal at $n \approx 2$. The rate of return externality dominates the mobile tax externality for $n \in \{2, 3, 4\}$.

The final step in our analysis is to relate taxes and growth. First, at any instant, output must be divided between private and public consumption, i.. $y = c + g$. Next, from the government budget constraint and (2.5), we have in symmetric equilibrium that $y = \rho w + \tau k = (\rho + \tau)w$. So, the growth of output is just proportional to that of wealth. Finally, from (3.8) and the definition of $dz^*$, in symmetric equilibrium:

$$dw = (1 - \tau - \rho)w + \frac{1}{n}\sum_i dz_i.$$

Growth has a stochastic and deterministic component, and only the latter is affected by taxes, being decreasing in the tax. In what follows, we refer to $1 - \tau - \rho$ as the average growth rate.

So, Figure 3.1 indicates that full centralization ($n = 1$) yields higher growth than moderate decentralization ($n < 5$). It is only when $n \geqq 5$ that the mobile tax base externality dominates and thus growth under decentralization is higher.
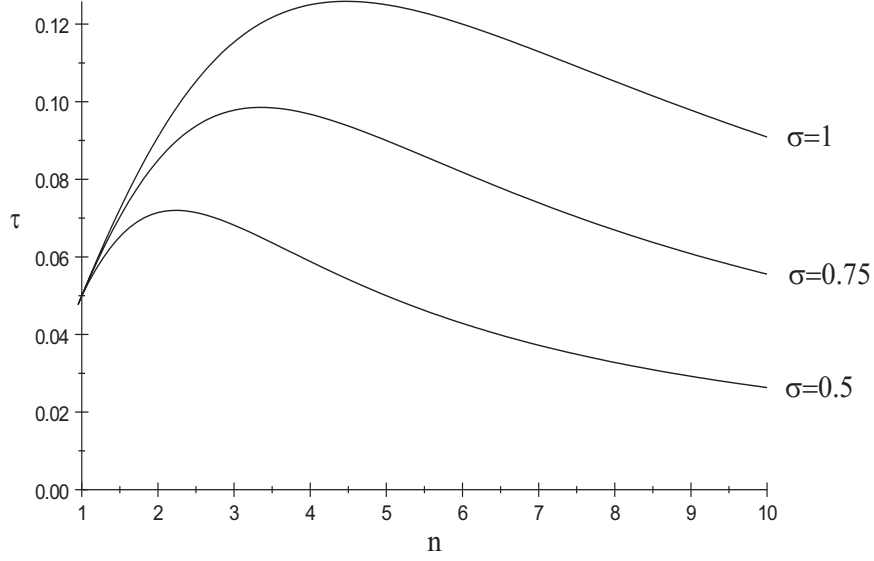
11

Figure 3.1: $\tau$ as a function of $n$ for $\beta = 1$ and $\rho = 0.1$.

## 4. Infrastructure Public Goods

### 4.1. The Model

We now modify the baseline set-up by allowing the government to spend on a public infrastructure good rather than consumption good. For tractability, we assume two regions, unstarred (home) and starred (foreign). We will focus on home region. Output follows the process

$$dy = g^{1-\alpha}k^{\alpha}(dt + dz).$$

Following Turnovsky (2000) and Kenc (2004), we assume that the pretax wage, $a$, over the period $(t, t + dt)$ is determined at the start of the period and is equal to expected marginal product

$$
\begin{aligned}
adt &= (1-\alpha)g^{1-\alpha}k^{\alpha}dt \\
&= (1-\alpha)\tau^{1-\alpha}kdt = \theta(\tau)kdt
\end{aligned}
\tag{4.1}
$$

using the budget constraint $g = \tau k$. So, the wage is non-random. The rate of return (pre-tax) over the period $(t, t + dt)$ is thus determined residually:

$$
\begin{aligned}
\frac{dy - adt}{k} &= \alpha g^{1-\alpha}k^{\alpha-1}dt + g^{1-\alpha}k^{\alpha-1}dz \\
&= \alpha\tau^{1-\alpha}dt + \tau^{1-\alpha}dz
\end{aligned}
\tag{4.2}
$$

12

using the budget constraint $g = \tau k$. So, the post-tax rate of return thus follows the process

$$dR = (\alpha \tau^{1-\alpha} - \tau)dt + \tau^{1-\alpha}dz = r(\tau)dt + \tau^{1-\alpha}dz. \tag{4.3}$$

where $r(\tau) = \alpha \tau^{1-\alpha} - \tau$ plays an important role below.

## 4.2. Solving the Household Problem

For simplicity, consumer preferences are logarithmic. The consumer maximizes

$$E\left[\int_0^\infty e^{-\rho t} \ln c\, dt\right] \tag{4.4}$$

subject to the stochastic wealth equation

$$
\begin{aligned}
dw &= (1-s)wdR + swdR^* + adt - cdt \tag{4.5}\\
&= [(1-s)r(\tau) + sr(\tau^*)]wdt + \theta(\tau)kdt - cdt + (1-s)w\tau^{1-\alpha}dz + sw(\tau^*)^{1-\alpha}dz^*.
\end{aligned}
$$

Unlike consumption good case, this problem is non-standard, and so for completeness, we provide a solution in the Appendix. Moreover, in deriving the solution, we suppose that the household believes[7] that $k \equiv w$, a belief which is true in equilibrium. The consumption and portfolio allocation rule for the home region are

$$c = \rho w \quad \text{and} \quad s = \frac{(r^* - r)/\sigma^2 + \tau^{2(1-\alpha)}}{(\tau^{2(1-\alpha)} + (\tau^*)^{2(1-\alpha)})}. \tag{4.6}$$

We can now compare (4.6) to (2.8). The difference is only in the portfolio allocation rules; for comparison, the consumption good rule can be written $s = \frac{1}{2} + \frac{r^*-r}{2\sigma^2}$ in the notation of this Section. The difference is that in (4.6), $\tau, \tau^*$ affect the rule directly, and not just via their effects on $r$. This is because a higher tax rate in the home region increases the pre-tax rate of return on capital in the home region and thereby the riskiness of the investment from (4.3).

## 4.3. Fiscal Externalities

Besides the tax base externality and the rate-of-return externality identified in the AK-model, the infrastructure model exhibits a third type of externality. Following (4.2)

---

[7]This assumption is slightly different than rational expectations, as when $\tau \neq \tau^*$, $k \neq w$ in general. However, we cannot solve the household problem in closed form under fully rational expectations when $\tau \neq \tau^*$.

the pre-tax rate of return is stochastic and a higher tax rate magnifies the exposure of investors to risk. Since each region also hosts capital from non-residents, this specification introduces a negative externality, i.e. a higher tax increases the riskiness of investment and thus the risk-bearing of non-residents. We call this externality the *risk-exposure externality*. We will now characterize the equilibrium tax policy under (de)centralization and relate it to the externalities.

## 4.4. Centralization

Under centralization, using (4.6), the government maximizes

$$E\left[\int_0^\infty e^{-\rho t}\ln c\, dt\right] = E\left[\int_0^\infty e^{-\rho t}\ln w\, dt\right] + const$$

subject to $\tau = \tau^*$. It also understands that

$$k = w, \ \ s = s^* = 0.5.$$

So, there is a single state variable $w$:

$$dw = [r(\tau) - \rho]w\, dt + \theta(\tau)k\, dt + w\tau^{1-\alpha}[0.5dz + 0.5dz^*].$$

The Bellman equation is

$$\rho V(w, w^*) = \max_\tau \left\{ \ln w + [r(\tau) + \theta(\tau) - \rho]V_w w + \frac{V_{ww}w^2}{2}\sigma_w^2 \right\},$$

where $\sigma_w^2 = 0.5\tau^{2(1-\alpha)}\sigma^2$. The FOC is

$$V_w w[r' + \theta'] + \frac{V_{ww}w^2 2\sigma^2(1-\alpha)\tau^{1-2\alpha}}{2} = 0.$$

Now, we guess $V(w, w) = A + B\ln w$. Then, the FOC becomes, cancelling $B$,

$$r' + \theta' - \frac{\sigma^2(1-\alpha)\tau^{1-2\alpha}}{4} = 0. \tag{4.7}$$

To interpret this condition, it is helpful to note that (following exactly the argument in the consumption case), the average growth rate in output is $r(\tau) - \rho$, where $r(\tau) = \alpha\tau^{1-\alpha} - \tau$ is a strictly concave function with a unique maximum at $\hat\tau = (\alpha(1-\alpha))^{1/\alpha} > 0$.

Although (4.7) cannot be solved explicitly for the tax rate, we can see that in the absence of uncertainty, $r' + \theta' = 0$. As $\theta' = (1 - \alpha)^2\tau^{-\alpha} > 0$ i.e. the tax has a positive effect on the wage, $r' < 0$. Thus, the tax rate is too high to be growth-maximising. This reproduces the finding of Alesina and Rodrik (1994). Note that uncertainty implies

14

$r' + \theta' > 0$, i.e. it tends to lower $\tau$.[8] This is because an increase in $\tau$ raises the pre-tax return on capital (4.2), and thus the riskiness of investments. So, in principle, $\tau_C$ could maximize growth when there are stochastic shocks to production.

## 4.5. Decentralization

Using using (4.6), the government in the home region maximizes

$$E\left[\int_0^\infty e^{-\rho t}\ln c\, dt\right] = \left[\int_0^\infty e^{-\rho t}\ln w\, dt\right] + const.$$

It also understands that

$$k = (1-s)w + s^*w^*, \ \ k^* = (1-s^*)w^* + sw \tag{4.8}$$

and

$$s = \frac{(r^*(\tau^*) - r(\tau))/\sigma^2 + \tau^{2(1-\alpha)}}{\left(\tau^{2(1-\alpha)} + (\tau^*)^{2(1-\alpha)}\right)}, \ \ s^* = \frac{(r(\tau) - r^*(\tau^*))/\sigma^2 + (\tau^*)^{2(1-\alpha)}}{\left(\tau^{2(1-\alpha)} + (\tau^*)^{2(1-\alpha)}\right)}. \tag{4.9}$$

As in the AK model, there are two state variables, $w$ and $w^*$. Their equations are:

$$dw = [(1-s)r(\tau) + sr(\tau^*) - \rho]w\,dt + \theta(\tau)k\,dt + (1-s)w\tau^{1-\alpha}dz + sw(\tau^*)^{1-\alpha}dz^*$$
$$dw^* = [(1-s^*)r(\tau^*) + s^*r(\tau) - \rho]w^*dt + \theta(\tau^*)k^*dt + (1-s^*)w^*(\tau^*)^{1-\alpha}dz^* + s^*w^*\tau^{1-\alpha}dz.$$

Again, we set up the Bellman equation and guess the functional form of the value function in order to derive the FOC. All these steps are dealt with in detail in the Appendix, and the resulting FOC, evaluated at symmetric equilibrium, is:

$$0.5r' + \theta' + \theta\frac{k_\tau}{k} - \frac{\sigma^2(1-\alpha)\tau^{1-2\alpha}}{4} = 0. \tag{4.10}$$

Comparing to the centralized case,

$$r' + \theta' - \frac{\sigma^2(1-\alpha)\tau^{1-2\alpha}}{2} = 0,$$

one observes three differences.

- First, due to the mobile tax base externality, we have the term $\theta\frac{k_\tau}{k}$ in the FOC. $\frac{k_\tau}{k}$ measures the percentage capital outflow due to a 1 percentage point increase in the tax.

---

[8]In fact, invoking the implicit function theorem we unambiguously find $d\tau/d\sigma < 0$ if the share of output accruing to capital $\alpha \le 0.5$.

- Second, due to the rate of return externality, $r'$ enters with a weight of only 0.5. The rationale is that half of the total return to capital goes to foreigners and the effect of tax policy on it is external to the government.

- Third, the impact of capital taxation on risk exposure of foreigners is not recognized (risk exposure externality), explaining the weight 0.5 in the last term.

Thus, in general the average growth rate of the economy, which is $r(\tau) - \rho$, may be higher or lower under decentralization relative to centralization. To begin the comparison, we can obtain an analytical result confirming (Hatfield, 2006) when there is no uncertainty. Clearly, (4.7) reduces to $r' = -\theta'$ when $\sigma = 0$, whereas (4.10) reduces to $r' = 0$.[9] Thus, in the absence of productivity shocks the government sets the tax rate so as to maximize the average growth rate $r(\tau) - \rho$ with decentralization, but with centralization, the tax is too high to be growth-maximising:

**Proposition 3.** *If $\sigma = 0$, $\tau_D = \hat{\tau} < \tau_C$, so or all $n$, so the average growth rate is always higher under decentralization.*

What happens with stochastic shocks? Although we cannot solve explicitly for the equilibrium tax rates $\tau_C$ and $\tau_D$, appealing to Proposition 3 and a continuity argument we know that we know that $\tau_C > \tau_D$ also holds for a small enough variance of the production shock. This allows us to conclude that decentralization yields higher growth when the variance is small.

The latter finding does however not extend to any size of the variance. First, consider how taxes change as $\sigma$ increases - see figure 4.1. It might be surprising that the decentralized tax rate decreases with the variance of the shock. At first sight, it appears that a higher variance should increase the benefit of portfolio diversification, thereby making the investment decision less sensitive to the tax rate differential (recall Proposition 1). As a first observation, a higher variance increases the pre-tax return on capital (4.2). In response the government lowers the tax rate - last term in (4.10). A second effect operates through the mobility of capital. Noting (4.8) and (4.9)

$$k_\tau = -s_\tau w + s_\tau^* w^* \text{ and } s_\tau = -s_\tau^* = -\frac{r'}{2\sigma^2 \tau^{2(1-\alpha)}} + \frac{(1-\alpha)}{2\tau}.$$

Thus, in symmetric equilibrium

$$\frac{k_\tau}{k} = \frac{r'}{\sigma^2 \tau^{2(1-\alpha)}} - \frac{(1-\alpha)}{\tau}. \tag{4.11}$$

---

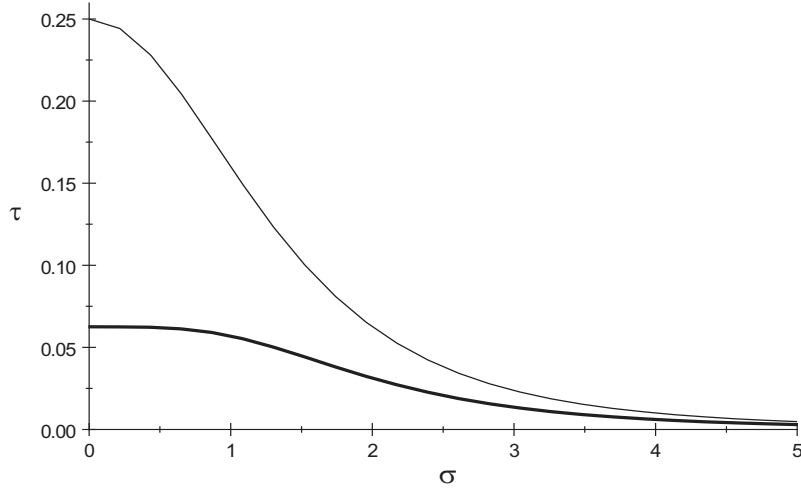[9]More firmly, following (4.1), (4.8) and (4.9) $\theta \frac{k_\tau}{w}$ reduces to $-\theta'$ when $\sigma \to 0$.

Figure 4.1: $\tau_C$ (thin line) and $\tau_D$ (solid line) as a function of $\sigma$ for $\alpha = 0.5$.

Under decentralization and $\sigma > 0$, we know that $r' > 0$ for $\alpha = 0.5$. So, when $\sigma$ rises, assuming $k_\tau < 0$, from (4.11), the semi-elasticity of capital with respect to the tax increases in absolute value. This in turn tends to decrease the equilibrium tax from (4.10). This is in contrast to the case with a public consumption good where a higher variance makes the tax base less elastic - see (3.10).

Note finally that as $k_\tau$ is independent of $\rho$ from (4.11), and (4.7),(4.10) are otherwise independent of $\rho$, then $\tau_D, \tau_C$ are also independent of $\rho$. This is in contrast to the consumption public good case. This is because taxation and infrastructure provision have countervailing effects on growth which are equated at the margin.

Now we turn to the relationship between decentralization and growth, recalling that the average growth rate is $r(\tau) - \rho$. Figure 4.2 shows how $r_C = r(\tau_C)$ and $r_D = r(\tau_D)$ vary as $\sigma$ increases.

In the absence of uncertainty a decentralized government engages in Bertrand tax competition with the consequence of maximizing growth. The tax rate decreases as $\sigma$ increases and, since the growth rate is hump-shaped in the tax rate, growth is decreasing in $\sigma$. A centralized government sets a too high tax rate to be growth maximizing in the absence of uncertainty and lowers it as $\sigma$ rises. For sufficiently small values of $\sigma$ growth is rising. In fact, for $\sigma \approx 1.7$ the growth rate equals that under decentralization and for $\sigma \approx 2$ the centralized tax rate is growth maximizing. For larger values both a decentralized and centralized government operate on the upward sloping part of the growth-curve with centralization yielding higher growth. This is in contrast to the results of Hatfield (2006),
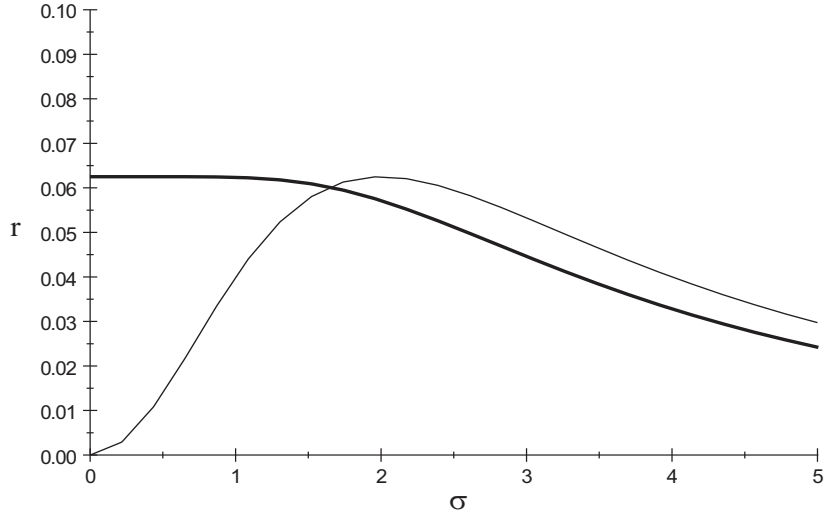
Figure 4.2: $r_C$ (thin line) and $r_D$ (solid line) as a function of $\sigma$ for $\alpha = 0.5$.

who assumes a deterministic growth model.

## 5. Conclusions

This paper has considered the relationship between tax competition and growth in an endogenous growth model where there are stochastic shocks to productivity, and capital taxes fund a public good which may be for final consumption or an infrastructure input. Absent stochastic shocks, decentralized tax setting (two or more jurisdictions) maximizes the rate of growth, as the constant returns to scale present with endogenous growth implies Bertrand tax competition. Stochastic shocks imply that households face a portfolio choice problem. Shocks dampen down tax competition and may raise taxes above the centralized level when the government provides a public consumption good. Growth can be lower with decentralization. In the public infrastructure model shocks may increase the tax base elasticity and the equilibrium decentralized tax rate may be too low to yield higher growth with decentralization. Our results also predict a negative relationship between output volatility and growth, consistent with the empirical evidence.

One might ask robust our results are. Two of our important simplifying assumptions are logarithmic utility of private consumption, and in the case of the infrastructure model, no taxes on labour. If logarithmic utility of private consumption is relaxed to iso-elastic utility, we can still solve the household savings and portfolio choice problem in closed form, but we cannot get a closed-form solution for the equilibrium tax, even in the case

18

of a consumption public good. But the key externalities at work remain the same, and as a result, it is possible to get higher taxes and lower growth with decentralization in the public consumption good case[10]. In the infrastructure model, we conjecture that our main conclusions would be unaffected, as long as the demand for the public good is high enough so that it is optimal to use a capital tax at the margin.

# References

Akai, N. and M. Sakata (2002), "Fiscal Decentralization Contributes to Economic Growth: Evidenc from State-Level Cross Section Data from the US", *Journal of Urban Economics* 52, 93-108.

Alesina, A., and D. Rodrik (1994), "Distributive Politics and Economic Growth", *Quarterly Journal of Economics* 109, 465-490.

Barro, R.J. (1990),"Government Spending in a Simple Model of Endogenous Growth", *Journal of Political Economy* 98, S103-S125.

Brueckner, J. (2006), "Fiscal Federalism and Economic Growth", *Journal of Public Economics* 90, 2107-2120.

Davoodi, H. and H. Zou (1998), "Fiscal Decentralization and Economic Growth: A Cross-Country Study", *Journal of Urban Economics* 43, 244-257.

Hatfield, J. (2006), "Federalism, Taxation, and Economic Growth", unpublished paper, Graduate School of Business, Stanford University.

Iimi, A. (2005), "Fiscal Decentralization and Economic Growth Revisited: An Empirical Note", *Journal of Urban Economics* 57, 449-461.

Jones, L.E. and R.E. Manuelli (2005), "Neoclassical Models of Endogenous Growth: The Effects of Fiscal Policy, Innovation and Fluctuations". In: P. Aghion and S.N. Durlauf, Editor(s), *Handbook of Economic Growth*, Volume 1, Part 1, 13-65, Elsevier.

Kenc, T. (2004), "Taxation, risk-taking and growth: a continuous-time stochastic general equilibrium analysis with labor-leisure choice", *Journal of Economic Dynamics and Control* 28, 1511-1539.

Lee, K., (2004), "Taxation of Mobile Factors as Insurance Under Uncertainty", *Scandinavian Journal of Economics* 106, 253-271.

Lejour, A. and H. Verbon (1997), "Tax Competition and Redistribution in a Two-Country Endogenous Growth Model", *International Tax and Public Finance* 4, 485-497.

---

[10]Details of these calculations are available on request.

Philippopoulos, A. (2003), "Are Nash Tax Rates too Low or Too High? The Role of Endogenous Growth in Models with Public Goods", *Review of Economic Dynamics* 6, 37-53.

Philippopoulos, A. and H. Park (2003), "On the dynamics of growth and fiscal policy with redistributive transfers", *Journal of Public Economics*, 515-538.

Ramey, G. and V. Ramey (1995), "Cross-Country Evidence on the Link Between Volatility and Growth", *American Economic Review* 85, 1138-1151.

Stansel, D. (2005), "Local Decentralization and Economic Growth: A Cross-Sectional Examination of US Metropolitan Areas" , *Journal of Urban Economics* 57, 55-72.

Thornton, J. (2007), "Fiscal Decentralization and Economic Growth Reconsidered", *Journal of Urban Economics* 61, 64-70.

Turnovsky, S. (2000), "Government Policy in a Stochastic Growth Model with Elastic Labour Supply", *Journal of Public Economic Theory* 2, 389-433.

Wooler and Phillips (1998), "Fiscal Decentralization and LDC Economic Growth: An Empirical Investigation", *Journal of Development Studies* 34, 139–148.

Zhang, T. and H. Zou (1998), "Fiscal Decentralization, Public Spending, and Economic Growth in China", *Journal of Public Economics* 67, 221-240.

Zodrow, G.R. and P. Mieszkowski (1986), "Pigou, Tiebout, Property Taxation, and the Underprovision of Local Public Goods", *Journal of Urban Economics* 19, 356 - 370.

# A. Proofs of Propositions and Other Results

**Derivation of the Solution to the Household Problem in the Public Infrastructure Good Case**

Using $k = w$ in (4.5), the problem is to maximize (4.4) subject to

$$dw = [(1 - s)r(\tau) + sr(\tau^*)]wdt + \theta wdt - cdt + (1 - s)w\tau^{1-\alpha}dz + sw(\tau^*)^{1-\alpha}dz^*. \quad (A.1)$$

Assume the value function for this problem takes the form $V(w) = A + B \ln w$. In this case the Bellman equation is

$$\rho (A + B \ln w) = \max_{c,s} \left\{ \ln c + (w[(1 - s)r + sr^* + \theta] - c)\frac{B}{w} - \frac{B}{2}\sigma_w^2 \right\}$$

where

$$\sigma_w^2 = [(1 - s)^2 \tau^{2(1-\alpha)} + s^2 (\tau^*)^{2(1-\alpha)}]\sigma^2.$$

is the variance of wealth. The first-order conditions for $c$ and $s$ are

$$\frac{1}{c} - \frac{B}{w} = 0 \tag{A.2}$$

$$B\left(r^* - r - \sigma^2[s(\tau^{2(1-\alpha)} + (\tau^*)^{2(1-\alpha)}) - \tau^{2(1-\alpha)}]\right) = 0 \tag{A.3}$$

and the Bellman equation becomes

$$\rho\left(A + B\ln w\right) = \max_{c,s}\left\{\ln w - \ln B + [(1-s)r + sr^* + \theta]B - 1 - \frac{B}{2}\sigma_w^2\right\}.$$

So, $B = 1/\rho$; using this in (A.2), (A.3) gives (4.6). $\square$

**Proof of Proposition 1.** First rewrite the stochastic terms in the state equations as

$$d\tilde{w} = (1 - (n-1)s)dz_i + (n-1)sdz^* \tag{A.4}$$

$$d\tilde{w}^* = s^*dz_i + (1 - s^*)dz^* \tag{A.5}$$

Noting that

$$E[(dz_i)^2] = \sigma^2 dt, \ \ E[(dz^*)^2] = \frac{\sigma^2}{n-1}dt, \ \ E[dz_i dz^*] = 0$$

one can compute from (A.4),(A.5) that

$$E[(d\tilde{w})^2] = [(1-(n-1)s)^2 + (n-1)s^2]\sigma^2 dt = \sigma_w^2 dt$$

$$E[(d\tilde{w}^*)^2] = [(1-s^*)^2/(n-1) + (s^*)^2]\sigma^2 dt = \sigma_{w^*}^2 dt$$

$$E[d\tilde{w}.d\tilde{w}^*] = [(1-(n-1)s)s^* + (1-s^*)s]\sigma^2 dt = \xi dt$$

So, assuming a value function $V(w, w^*)$, and the Bellman Equation for the government problem can be written

$$\rho V(w, w^*) = \max_{\{\tau\}}\{\ln w + \beta\ln k + \beta\ln\tau + V_w w\left[1 - (1-(n-1)s)\tau - (n-1)s\tau^* - \rho\right] \tag{A.6}$$

$$+V_{w^*}w^*\left[1 - (1-s^*)\tau^* - s^*\tau - \rho\right] + \frac{V_{ww}w^2}{2}\sigma_w^2 + \frac{V_{w^*w^*}(w^*)^2}{2}\sigma_{w^*}^2 + V_{ww^*}ww^*\xi\}$$

where $k = ((1-(n-1)s)w + (n-1)s^*w^*)$, and also taking into account the effect of $\tau$ on $s, s^*$, and thus on $\sigma_w^2, \sigma_{w^*}^2, \xi$ . Specifically,

$$s_\tau = \frac{1}{n\sigma^2}, \ s_\tau^* = -\frac{n-1}{n\sigma^2} \tag{A.7}$$

$$\frac{d\sigma_w^2}{d\tau} = \frac{2(n-1)}{n}[sn-1], \ \ \frac{d\sigma_{w^*}^2}{d\tau} = \frac{2}{n}[1-ns^*] \tag{A.8}$$

$$\frac{d\xi}{d\tau} = [(1-ns^*) - (n-1)(1-ns)]\frac{1}{n} \tag{A.9}$$

21

So, using (A.7), the FOC for the tax is

$$\frac{\beta}{\tau} + \frac{\beta}{k}\frac{\partial k}{\partial \tau} - V_w w(1 - (n-1)s) - V_{w^*}w^* s^* + \tag{A.10}$$

$$(V_w w + V_{w^*}w^*)(n-1)(\tau - \tau^*)\frac{1}{n\sigma^2} + \frac{1}{2}V_{ww}w^2\frac{d\sigma_w^2}{d\tau} + \frac{V_{w^*w^*}(w^*)^2}{2}\frac{d\sigma_{w^*}^2}{d\tau} + V_{ww^*}ww\frac{d\xi}{d\tau} = 0$$

where

$$\frac{\partial k}{\partial \tau} = -(n-1)ws_\tau + (n-1)w^* s_\tau^* \tag{A.11}$$

$$= -(n-1)w\frac{1}{n\sigma^2} + (n-1)^2 w^*\frac{1}{n\sigma^2}$$

At symmetric equilibrium, where $\tau^* = \tau$ and so $w^* = w = k$, $s = s^* = \frac{1}{n}$, using (A.11), and also noting from (A.8),(A.9) that as $s = s^* = \frac{1}{n}$, $\frac{d\sigma_w^2}{d\tau} = \frac{d\sigma_{w^*}^2}{d\tau} = \frac{d\xi}{d\tau} = 0$, we can rewrite (A.10) as:

$$\frac{\beta}{\tau} - \frac{\beta(n-1)}{\sigma^2} - \frac{1}{n}(V_w w + V_{w^*}w^*) = 0 \tag{A.12}$$

Next, note that in symmetric equilibrium, $w = w^*$, and assume $V(w,w) \equiv A + \psi \ln w$. Then, at symmetric equilibrium,

$$V_w w + V_{w^*}w^* = \psi$$

$$V_{ww}w^2 + V_{w^*w^*}(w^*)^2 + 2V_{ww^*}ww^* = -\psi$$

and consequently, (A.12) can be rewritten as

$$\frac{\beta}{\tau} - \frac{\beta(n-1)}{\sigma^2} - \frac{1}{n}\psi = 0 \tag{A.13}$$

It just remains to solve for $\psi$. Note also at symmetric equilibrium that $\sigma_w^2 = \sigma_{w^*}^2 = \xi = \frac{\sigma^2}{n}$. Using all these facts, the Bellman equation (A.6) reduces to

$$\rho(A + \psi \ln w) = \{(1+\beta)\ln w + \beta \ln \tau + \psi[1 - \tau - \rho] - \frac{1}{2n}\sigma^2\psi \tag{A.14}$$

So, by inspection, $\psi = (1+\beta)/\rho$. So, from (A.13) and $\psi = (1+\beta)/\rho$, we obtain the expression for $\tau_n$ in Proposition 1. $\square$

**Derivation of the FOC (4.10).** At a symmetric equilibrium, $w = w^*$. So, the Bellman is

$$\rho V(w, w^*) = \max_\tau \left\{ \begin{array}{c} \ln w + V_w[[(1-s)r(\tau) + sr(\tau^*) - \rho]w + \theta(\tau)k] \\ +V_{w^*}[[(1-s^*)r(\tau^*) + s^*r(\tau) - \rho]w^* dt + \theta(\tau^*)k^*] \\ + \frac{V_{ww}w^2}{2}\sigma_w^2 + \frac{V_{w^*w^*}(w^*)^2}{2}\sigma_{w^*}^2 + V_{ww^*}ww^*\xi \end{array} \right\}$$

$$\sigma_w^2 = [(1-s)^2\tau^{2(1-\alpha)} + s^2(\tau^*)^{2(1-\alpha)}]\sigma^2$$

$$\sigma_{w^*}^2 = [(1-s^*)^2(\tau^*)^{2(1-\alpha)} + (s^*)^2\tau^{2(1-\alpha)}]\sigma^2$$

$$\xi = [(1-s)s^*\tau^{2(1-\alpha)} + s(1-s^*)(\tau^*)^{2(1-\alpha)}]\sigma^2$$

22

Evaluate at $\tau = \tau^*$, the Bellman equation is

$$V_w[0.5r'w + \theta'k + \theta k_\tau] + V_{w^*}[0.5r'w^* + \theta k_\tau^*]$$
$$+ \left( \frac{V_{ww}w^2}{2} + \frac{V_{w^*w^*}(w^*)^2}{2} + V_{ww^*}ww^* \right) \frac{\sigma^2(1-\alpha)\tau^{1-2\alpha}}{2}$$
$$= 0,$$

where we already used the fact that

$$\frac{\partial \sigma_w^2}{\partial \tau} = \frac{\partial \sigma_{w^*}^2}{\partial \tau} = \frac{\partial \xi}{\partial \tau} = \frac{\sigma^2(1-\alpha)\tau^{1-2\alpha}}{2}.$$

Now, guess $V(w, w^*) = A + B \ln w$, i.e. the value function is independent of $w^*$. Then,

$$V_w w = B, \ V_{w^*} = 0$$
$$\left( \frac{V_{ww}w^2}{2} + \frac{V_{w^*w^*}(w^*)^2}{2} + V_{ww^*}ww^* \right) = -\frac{B}{2}.$$

Thus, the FOC becomes, cancelling $B$ and using $k = w$ in symmetric equilibrium:

$$0.5r' + \theta' + \theta\frac{k_\tau}{k} - \frac{\sigma^2(1-\alpha)\tau^{1-2\alpha}}{4} = 0.$$

This completes the proof. $\square$