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Testing for unit roots in three-dimensional heterogeneous panels in the presence of cross-sectional dependence*

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Abstract

This paper extends the cross-sectionally augmented IPS (CIPS) test of Pesaran (2006) to a three-dimensional (3D) panel. This 3D-CIPS test is correctly sized in the presence of cross-sectional dependency. Comparing its power performance to that of a bootstrapped IPS (BIPS) test, we find that the BIPS test invariably dominates, although for high levels of cross-sectional dependency the 3D-CIPS test can out-perform the BIPS test.

JEL Classification: C12; C15; C22; C23

Keywords: Heterogeneous dynamic panels; Monte Carlo; unit roots; cross-sectional dependence.

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1 Introduction

The problem of testing the presence of unit roots in panels of data has received a great deal of attention over the past decade; see for instance Quah (1994), Maddala and Wu (1999) and more recently Im, Pesaran, and Shin (2003) (IPS). These panel unit root tests combine information from the time-series dimension with that from the cross-section dimension, such that fewer time observations are required for these tests to have power. A critical assumption underlying these tests is that of cross sectional independence. Failing to account for potential cross-sectional dependence leads to over-rejection of the unit root test statistics, the extent of which is related to the degree of this dependence.

In this paper we examine the time series properties of a panel of data that consists of three dimensions, which were first looked at by Ghosh (1976) and subsequently by Baltagi (1987). Empirical examples of three dimensional panels are still relatively rare, but are starting to appear in the literature, see for example, Goldberg and Verboven (2005), who assembled a large three dimensional panel (using make of car, countries and time) to investigate market integration and convergence in the European car market, and Davies and Lahiri (1995) who analyse forecasts of inflation available for different individuals, across varying time horizons over the period from 1977 through to 1992. Within the context of a three-dimensional (3D) panel, N might denote countries or industries and M might be regions or firms within that country or industry. With such panels there is the possibility of an error covariance matrix in which there is different correlation across the N units compared to across the M units.

This paper uses Monte Carlo simulation to investigate the small sample properties of the IPS test, for a 3D panel of data that allows for cross-sectional correlation over both the N and M dimensions. The simulation results show a severe size distortion of the IPS test, and we consider two alternative procedures to account for potential cross-sectional dependency, namely, an extended version of the cross-sectionally augmented IPS (CIPS) test statistic put forward by Pesaran (2006), and a procedure based on a bootstrap of the residuals of the IPS test. In this paper we tabulate a new set of critical values that are required to apply the Pesaran (2006) test in a 3D setup.

The plan of the paper is as follows. Section 2 briefly reviews the IPS approach to unit root testing in panels with two cross-sectional dimensions. Section 3 presents the design of the Monte Carlo simulations. Section 4 discusses the main results.

2 IPS unit root test and basic framework

IPS presented a method to test for the presence of unit roots in dynamic heterogeneous panels, which is based on averaging individual ADF unit root test statistics. Extending the IPS unit root testing procedure to the context of a 3D panel, we assume that the stochastic process $y_{ij,t}$ is generated by a first-order autoregressive process:

$$\Delta y_{ij,t} = a_{ij} + b_{ij} y_{ij,t-1} + \sum_{r=1}^{p} c_{ijr} \triangle y_{ij,t-r} + \varepsilon_{ij,t}, \tag{1}$$

where $i = 1, \ldots, N,\ \ j = 1, ..., M,\ \ t = 1, \ldots, T$. In this setting the null hypothesis to test the presence of a unit root becomes $H_0 : b_{ij} = 1$ for all i, j, against the alternative that at least one of the individual series in the panel is stationary, that is $H_1 : b_{ij} < 1$ for at least one i, j.

The IPS test averages the ADF statistics obtained in equation (1) across the NM cross-sectional units of the panel, denoted as:

$$\widetilde{tbar}_{NMT} = (NM)^{-1} \sum_{i=1}^{N} \sum_{j=1}^{M} \tilde{t}_{ij,T},$$

where $\tilde{t}_{ij,T}$ is the ADF test for the ij^{th} cross-sectional unit. IPS show that a suitable standardisation of the $\widetilde{tbar}_{NMT}$ statistic, denoted as $Z_{\widetilde{tbar}}$, follows a standard normal distribution.

3 Monte Carlo simulation design

We assume that the stochastic process $y_{ij,t}$ is generated by a first-order autoregressive process:

$$y_{ij,t} = \left(1 - \phi_{ij}\right) \mu_{ij} + \phi_{ij} y_{ij,t-1} + \varepsilon_{ij,t},$$

where $i = 1, \ldots, N,\ \ j = 1, ..., M,\ \ t = 1, \ldots, T$, $\mu_{ij} \sim N(0, 1)$, $\varepsilon_{ij,t} \sim N\left(0, \sigma_{ij}^2\right)$, $\sigma_{ij}^2 \sim U[0.5, 1.5]$ and U stands for a uniform distribution. Under the null hypothesis $\phi_{ij} = 1$ for all i, j, while $\phi_{ij} = 0.9$ for all i, j under the alternative hypothesis. All of the parameter values such as μ_{ij} or σ_{ij}^2 are generated independently of ε_{ijt} once, and then fixed throughout replications. Simulations are carried out for $N = 5, 10, 20, 25,\ M = 2, 5, 10$ and $T = 10, 20, 25, 40$. The number of replications is set to 2,000 and the first 50 observations are discarded. We allow the cross-sectional dependence to differ across the i, j cross-sectional units, of the form:

$$E(\varepsilon_{ikt}, \varepsilon_{ilt}) = \begin{cases} \sigma^2 & k = l \\ \omega_{kl} & k \neq l \end{cases}, E(\varepsilon_{pjt}, \varepsilon_{qjt}) = \begin{cases} \sigma^2 & p = q \\ \omega_{pq} & p \neq q \end{cases},$$

where, following O'Connell (1998), we set $\omega_{kl} = 0.3, 0.5, 0.7, 0.9$, and restrict the cross-sectional correlation over i to be a constant proportion of that over j, that is, $\omega_{pq} = \theta\omega_{kl}$, where $\theta = 0.0, 0.25, 0.5, 0.75, 1.0$.

Recently, Pesaran (2006) suggested modifying the IPS test by including auxiliary terms into equation (1) In particular, he augments equation (1) with the cross section averages of lagged level and lagged first-differences of the individual series in the panel. In a standard two-dimensional (2D) panel the test of the unit root hypothesis would be based on the following p^{th} order cross-sectionally augmented ADF regressions:

$$\Delta y_{it} = a_i + b_i y_{i,t-1} + \sum_{r=1}^{p} c_{ir} \triangle y_{it-r} + d_i \bar{y}_{t-1} + \sum_{r=0}^{p} f_{ir} \Delta \bar{y}_{t-r} + \varepsilon_{it}, \tag{2}$$

where $\bar{y}_t$ is the cross section mean of y_{it}, defined as $\bar{y}_t = (N)^{-1} \sum_{i=1}^{N} y_{it}$. The corresponding cross-sectionally augmented version of the IPS (CIPS) test statistic is:

$$\overline{CADF} = (N)^{-1} \sum_{i=1}^{N} t_i,$$

where t_i is the cross-sectionally ADF statistic for the i^{th} unit. The critical values of the $\overline{CADF}$ statistic are tabulated by Pesaran (2006) for models without intercepts or trends (Case I), with intercepts only (Case II), and with intercepts and trends (Case III).

Pesaran (2006) (p.27) suggests the CIPS test could be generalised "... for a richer pattern of cross dependence". For a 3D panel the CIPS test is be based on the following p^{th} order cross-sectionally augmented ADF regressions:

$$\Delta y_{ij,t} = a_{ij} + b_{ij} y_{ij,t-1} + \sum_{r=1}^{p} c_{ijr} \triangle y_{ij,t-r} + d_{ij} \bar{y}_{t-1} + \sum_{r=0}^{p} f_{ijr} \Delta \bar{y}_{t-r} + g_{ij} \bar{y}_{it-1} + \sum_{r=0}^{p} h_{ijr} \Delta \bar{y}_{it-r} + \varepsilon_{ij,t}, \quad (3)$$

where $\bar{y}_t$ is the cross-sectional mean of $y_{ij,t}$, defined as $\bar{y}_t = (NM)^{-1} \sum_{i=1}^{N} \sum_{j=1}^{M} y_{ij,t}$, and $\bar{y}_{i,t}$ is the cross-sectional mean formed over the M individuals of group i, that is $\bar{y}_{i,t} = (M)^{-1} \sum_{j=1}^{M} y_{ij,t}$. The corresponding 3D-CIPS test, would then be:

$$\overline{CADF} = (NM)^{-1} \sum_{i=1}^{N} \sum_{j=1}^{M} t_{ij},$$

where t_{ij} is the cross-sectionally ADF statistic for the ij^{th} cross-sectional unit.

The critical values of the 3D-CIPS test are tabulated via a Monte Carlo simulation (based on 20,000 replications) in Table 1 for different values of N, M and T, for models without intercepts or trends (Case I), with intercepts only (Case II), and with intercepts and trends (Case III). It should be noticed that these critical values are larger in absolute terms than the corresponding values tabulated by Pesaran (2006).

As an alternative procedure to test the presence of unit roots in panels that exhibit cross-sectional dependency, Maddala and Wu (1999) and more recently Chang (2004) have considered bootstrapping unit root tests. In order to implement this procedure, we start off by resampling the restricted residuals $\Delta y_{ij,t} = y_{ij,t} - y_{ij,t-1} = \varepsilon_{ij,t}$ after centring, since $y_{ij,t}$ has a unit root under the null hypothesis; this is what Li and Maddala (1996) refer to as the sampling scheme $S_{3,}$ which is appropriate in the unit root case. To preserve the cross-correlation structure of the error term within each cross section i, j, and following Maddala and Wu (1999), we resample the restricted residuals with the cross-section index fixed. Also, in order to ensure that initialisation of $\varepsilon^*_{ij,t}$, i.e. the bootstrap samples of $\varepsilon_{ij,t}$, becomes unimportant, we follow Chang (2004) who advocates generating a large number of $\varepsilon^*_{ij,t}$, say $T + Q$ values and discard the first Q values of $\varepsilon^*_{ij,t}$ (in our simulations we choose Q equal to 50). Lastly, the bootstrap samples of y^*_{ijt} are calculated by taking partial sums of $\varepsilon^*_{ij,t}$, that is $y^*_{ij,t} = y^*_{ij,0} + \sum_{k=1}^{t} \varepsilon^*_{ij,k}$, where $y_{ij,0}^*$ is set equal to zero. These Monte Carlo simulation results are based on 2,000 replications each of which uses 100 bootstrap repetitions. This bootstrapped IPS test will be denoted as BIPS.

4 Main results

The results for the empirical size of the 3D-CIPS and BIPS tests (while not reported, are available from the authors upon request) show that both tests are approximately correctly sized, although the empirical size of the BIPS test tends to be too low when ω is small and N and/or M are large.

Table 2 reports the power probabilities for both the 3D-CIPS and the BIPS tests for $\theta = 0.0$, 0.25, 0.75, 1.00. The power of both the BIPS and the 3D-CIPS tests increase with increases in either the number of cross-sectional units, N or M, or with increases in T. However, while the power of the BIPS test falls as the degree of cross-sectional correlation (in either ω_{kl} or θ) increases, the power of the 3D-CIPS test is largely invariant to the degree of cross-sectional correlation.

In comparing the power of the two tests for $\theta = 0.0$ (a case in which there is cross-correlation over $j = 1, \ldots, M$, but none over $i = 1, \ldots, N$), the BIPS test unambiguously dominates the 3D-CIPS test for all values of ω_{kl}; for example for $N = 20$, $M = 10$, $T = 20$ and $\omega_{kl} = 0.3$, the BIPS test has power of 95.85% compared to just 16.75% for the 3D-CIPS test. For the case when $\theta = 0.0$, aggregating the data over the $j = 1, \ldots M$ units to produce a 2D panel removes the cross-sectional dependency, although at the cost of reducing the number of cross-sectional units from NM to N. In this case, the standard IPS test applied over the 2D aggregated data is correctly sized, and its power is better than that of the 3D-CIPS test, but remains inferior to that of the BIPS test. Note, however, that even for relatively small values of θ aggregation over the $j = 1, \ldots M$ units yields incorrectly sized tests as correlation remains over the $i = 1, \ldots, N$ units and is therefore inappropriate.

In general, the BIPS test has greater power compared to the 3D-CIPS test for all values of θ considered, when $\omega_{kl} < 0.5$. For $\omega_{kl} \geq 0.5$ the power of the 3D-CIPS test is occasionally greater than that of the BIPS test, with the dominance of the 3D-CIPS over the BIPS test more likely when θ is bigger and N, M or T are larger; for example for $\theta = 0.75$ ($\theta = 1.00$), $N = 25$, $M = 10$ and $T = 40$ the power of the 3D-CIPS test is 76.2% (75.2%) compared to 30.1% (16.0%) for the BIPS test.

It should be noted that when $\theta = 1.00$, as the correlation over i and j is identical, the CIPS test as outlined in equation (2) can be applied. In these cases the empirical size results for the CIPS test is correct and the power of this test is greater than that of the 3D-CIPS test; for example, in the case of $\theta = 1.00$ reported above, the CIPS test has power of 82.0%.

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Table 1. Critical values of the 3D-CIPS test

N	M	$T=10$			$T=20$			$T=25$			$T=40$			$T=80$			$T=100$		
		1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
		Case I: No intercept, no trend																	
5	2	-3.00	-2.36	-2.08	-2.57	-2.18	-1.99	-2.52	-2.18	-2.00	-2.50	-2.16	-1.99	-2.48	-2.18	-2.01	-2.49	-2.17	-2.01
5	5	-2.14	-1.88	-1.74	-2.01	-1.82	-1.71	-2.02	-1.83	-1.73	-2.03	-1.86	-1.76	-2.03	-1.86	-1.77	-2.04	-1.86	-1.77
5	10	-1.93	-1.74	-1.64	-1.87	-1.73	-1.65	-1.89	-1.74	-1.66	-1.90	-1.76	-1.69	-1.91	-1.78	-1.71	-1.92	-1.78	-1.71
10	2	-2.44	-2.06	-1.87	-2.22	-1.97	-1.84	-2.21	-1.97	-1.84	-2.20	-1.97	-1.84	-2.21	-1.99	-1.86	-2.20	-1.98	-1.86
10	5	-1.92	-1.73	-1.63	-1.87	-1.72	-1.65	-1.87	-1.73	-1.65	-1.89	-1.75	-1.68	-1.91	-1.77	-1.70	-1.91	-1.78	-1.70
10	10	-1.80	-1.65	-1.57	-1.78	-1.66	-1.60	-1.79	-1.68	-1.62	-1.81	-1.70	-1.64	-1.84	-1.73	-1.67	-1.84	-1.74	-1.67
20	2	-2.12	-1.86	-1.73	-2.01	-1.81	-1.71	-2.00	-1.81	-1.71	-2.02	-1.84	-1.74	-2.01	-1.84	-1.75	-2.03	-1.85	-1.76
20	5	-1.78	-1.65	-1.57	-1.78	-1.66	-1.60	-1.79	-1.68	-1.61	-1.80	-1.70	-1.64	-1.83	-1.73	-1.67	-1.84	-1.73	-1.67
20	10	-1.71	-1.59	-1.53	-1.71	-1.63	-1.58	-1.73	-1.65	-1.60	-1.75	-1.67	-1.62	-1.79	-1.70	-1.64	-1.80	-1.71	-1.65
25	2	-2.07	-1.81	-1.69	-1.94	-1.77	-1.68	-1.94	-1.79	-1.70	-1.98	-1.81	-1.71	-1.97	-1.82	-1.73	-1.98	-1.82	-1.73
25	5	-1.76	-1.62	-1.55	-1.75	-1.65	-1.59	-1.77	-1.66	-1.60	-1.79	-1.69	-1.63	-1.82	-1.72	-1.66	-1.83	-1.72	-1.66
25	10	-1.69	-1.58	-1.52	-1.70	-1.62	-1.57	-1.72	-1.64	-1.59	-1.75	-1.67	-1.62	-1.77	-1.70	-1.64	-1.79	-1.70	-1.65
		Case II: Intercept only																	
5	2	-4.02	-3.14	-2.76	-3.12	-2.76	-2.56	-3.08	-2.74	-2.56	-3.03	-2.71	-2.56	-3.02	-2.73	-2.57	-3.03	-2.73	-2.57
5	5	-2.90	-2.53	-2.35	-2.57	-2.37	-2.28	-2.56	-2.38	-2.28	-2.57	-2.40	-2.30	-2.56	-2.41	-2.32	-2.56	-2.41	-2.33
5	10	-2.63	-2.37	-2.24	-2.41	-2.27	-2.19	-2.41	-2.28	-2.21	-2.42	-2.30	-2.23	-2.45	-2.33	-2.26	-2.44	-2.32	-2.26
10	2	-3.32	-2.76	-2.51	-2.78	-2.51	-2.39	-2.77	-2.53	-2.39	-2.74	-2.53	-2.40	-2.74	-2.54	-2.42	-2.74	-2.54	-2.43
10	5	-2.62	-2.35	-2.23	-2.41	-2.27	-2.19	-2.42	-2.28	-2.21	-2.42	-2.30	-2.23	-2.44	-2.32	-2.26	-2.45	-2.33	-2.26
10	10	-2.45	-2.26	-2.16	-2.31	-2.20	-2.15	-2.32	-2.22	-2.16	-2.35	-2.25	-2.19	-2.36	-2.27	-2.22	-2.36	-2.27	-2.22
20	2	-2.91	-2.53	-2.36	-2.56	-2.37	-2.27	-2.55	-2.37	-2.28	-2.56	-2.39	-2.30	-2.56	-2.40	-2.32	-2.57	-2.41	-2.33
20	5	-2.43	-2.25	-2.15	-2.32	-2.21	-2.15	-2.33	-2.22	-2.17	-2.35	-2.25	-2.19	-2.37	-2.27	-2.22	-2.37	-2.27	-2.22
20	10	-2.35	-2.19	-2.11	-2.26	-2.17	-2.12	-2.27	-2.19	-2.14	-2.29	-2.21	-2.17	-2.32	-2.24	-2.19	-2.32	-2.24	-2.20
25	2	-2.80	-2.46	-2.31	-2.51	-2.33	-2.24	-2.50	-2.34	-2.25	-2.51	-2.36	-2.27	-2.52	-2.38	-2.30	-2.51	-2.38	-2.30
25	5	-2.41	-2.23	-2.14	-2.29	-2.19	-2.13	-2.30	-2.20	-2.15	-2.33	-2.24	-2.19	-2.35	-2.26	-2.21	-2.35	-2.26	-2.21
25	10	-2.33	-2.17	-2.10	-2.24	-2.16	-2.11	-2.26	-2.18	-2.13	-2.29	-2.21	-2.16	-2.31	-2.23	-2.19	-2.31	-2.24	-2.19
		Case III: Intercept and trend																	
5	2	-5.93	-4.15	-3.55	-3.65	-3.22	-3.02	-3.54	-3.21	-3.02	-3.50	-3.19	-3.03	-3.48	-3.19	-3.04	-3.47	-3.19	-3.04
5	5	-3.98	-3.30	-3.03	-3.03	-2.83	-2.73	-3.02	-2.84	-2.75	-3.03	-2.86	-2.77	-3.02	-2.87	-2.79	-3.03	-2.87	-2.80
5	10	-3.48	-3.09	-2.89	-2.87	-2.73	-2.65	-2.87	-2.74	-2.67	-2.87	-2.76	-2.70	-2.89	-2.78	-2.72	-2.88	-2.78	-2.72
10	2	-4.65	-3.63	-3.25	-3.26	-2.98	-2.85	-3.24	-2.99	-2.86	-3.20	-2.98	-2.87	-3.18	-3.00	-2.89	-3.20	-3.00	-2.89
10	5	-3.49	-3.07	-2.87	-2.87	-2.73	-2.65	-2.87	-2.74	-2.67	-2.89	-2.76	-2.70	-2.90	-2.78	-2.73	-2.89	-2.79	-2.73
10	10	-3.23	-2.92	-2.77	-2.77	-2.66	-2.60	-2.77	-2.68	-2.62	-2.79	-2.70	-2.65	-2.81	-2.72	-2.67	-2.81	-2.73	-2.68
20	2	-3.98	-3.32	-3.04	-3.03	-2.84	-2.73	-3.03	-2.83	-2.74	-3.02	-2.86	-2.77	-3.01	-2.87	-2.79	-3.02	-2.87	-2.80
20	5	-3.20	-2.91	-2.77	-2.78	-2.66	-2.60	-2.78	-2.68	-2.62	-2.79	-2.70	-2.65	-2.81	-2.72	-2.68	-2.82	-2.73	-2.69
20	10	-3.05	-2.83	-2.71	-2.71	-2.62	-2.57	-2.72	-2.64	-2.59	-2.74	-2.67	-2.62	-2.76	-2.69	-2.65	-2.77	-2.69	-2.66
25	2	-3.81	-3.21	-2.96	-2.97	-2.80	-2.70	-2.96	-2.80	-2.72	-2.96	-2.82	-2.74	-2.97	-2.84	-2.77	-2.97	-2.84	-2.77
25	5	-3.16	-2.88	-2.74	-2.75	-2.64	-2.59	-2.77	-2.66	-2.61	-2.77	-2.69	-2.64	-2.80	-2.71	-2.67	-2.79	-2.72	-2.67
25	10	-3.02	-2.80	-2.69	-2.71	-2.61	-2.56	-2.71	-2.63	-2.58	-2.73	-2.66	-2.62	-2.75	-2.68	-2.65	-2.75	-2.69	-2.65

Table 2. Empirical power of the 3D-CIPS and bootstrapped IPS (BIPS) tests

N	M	T	3D-CIPS ($\theta = 0.0$)				BIPS ($\theta = 0.0$)				3D-CIPS ($\theta = 0.25$)				BIPS ($\theta = 0.25$)			
			ω_{kl}				ω_{kl}				ω_{kl}				ω_{kl}			
			0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9
5	2	10	5.9	5.6	5.2	4.7	10.6	10.5	10.1	9.6	5.9	6.1	5.7	5.0	10.8	10.0	9.5	9.0
5	5	10	5.3	5.3	5.3	5.5	14.1	12.3	10.9	9.3	5.0	5.5	5.8	5.6	14.2	12.9	11.0	9.4
5	10	10	6.5	6.3	6.4	6.6	15.8	13.8	12.5	11.4	7.2	6.6	6.8	6.4	15.9	13.1	11.4	10.2
10	2	10	7.1	6.7	5.7	5.3	13.7	12.9	11.9	10.8	6.9	7.2	6.3	5.7	13.2	12.1	10.6	10.2
10	5	10	6.1	6.5	6.5	6.2	17.8	15.0	13.8	11.9	7.1	7.3	6.1	6.3	17.5	13.9	12.7	10.9
10	10	10	6.9	7.4	7.2	7.1	21.3	18.4	15.4	12.7	8.0	7.7	7.2	7.8	20.8	16.9	13.4	11.6
20	2	10	5.1	4.6	4.7	6.0	16.6	15.9	16.1	15.2	6.0	6.1	6.2	5.8	16.8	15.5	14.1	12.9
20	5	10	6.5	7.3	7.2	6.6	22.8	20.2	17.8	15.4	6.4	7.4	7.4	7.2	21.0	18.0	14.6	13.2
20	10	10	8.4	7.9	7.5	8.4	29.5	23.8	18.9	14.9	8.1	8.0	7.5	7.5	26.2	20.2	16.3	13.7
25	2	10	6.2	6.3	6.2	5.8	19.5	18.7	17.1	16.1	5.7	5.8	6.1	6.3	18.0	16.4	15.1	12.5
25	5	10	7.8	7.4	8.1	7.9	24.9	21.7	18.7	16.3	7.4	7.4	8.1	8.0	22.1	17.3	14.9	12.3
25	10	10	8.4	8.3	8.5	8.2	33.5	25.9	19.2	15.9	7.7	7.4	7.3	7.5	28.1	20.0	15.6	12.9
5	2	20	6.4	6.2	6.6	7.0	18.6	17.6	16.3	15.1	5.9	5.9	6.0	6.7	18.2	17.6	15.5	13.2
5	5	20	10.0	10.0	9.8	9.7	31.9	26.4	21.1	16.9	10.2	9.8	9.6	9.8	30.8	25.9	20.3	15.8
5	10	20	11.3	11.7	11.9	12.1	44.2	33.6	24.8	17.1	11.4	11.3	12.2	11.8	42.5	32.0	23.1	16.2
10	2	20	8.7	8.4	8.8	8.6	30.4	30.0	26.8	23.0	8.6	8.9	8.7	8.7	29.2	27.6	23.7	19.7
10	5	20	13.1	12.5	13.1	12.8	49.4	40.0	33.1	24.5	12.5	12.5	12.8	12.8	46.5	36.0	29.1	20.7
10	10	20	15.5	16.0	15.4	16.4	63.3	47.8	34.6	24.0	15.1	15.2	15.5	15.5	59.7	43.3	30.2	21.1
20	2	20	9.8	10.2	10.0	10.6	46.8	44.8	41.7	36.4	9.9	10.1	10.0	10.4	43.1	38.1	31.7	26.3
20	5	20	15.5	14.8	15.0	14.8	69.5	57.2	45.3	35.7	14.9	14.5	14.4	14.7	63.5	49.2	37.3	27.4
20	10	20	16.8	16.6	16.8	17.4	85.9	70.9	55.2	40.1	17.7	17.9	18.1	17.6	76.9	58.2	43.3	32.9
25	2	20	10.8	10.3	9.8	10.3	53.6	51.5	45.9	39.5	11.0	10.9	10.7	11.0	50.9	43.8	36.2	29.8
25	5	20	14.7	15.3	15.3	16.0	77.3	66.1	53.9	41.7	15.5	15.4	15.6	16.0	69.5	54.0	40.0	30.2
25	10	20	18.2	18.6	18.2	17.8	91.1	77.0	60.1	46.8	17.4	18.1	17.9	18.5	82.1	61.9	45.6	34.3
5	2	25	9.1	8.6	8.7	8.8	27.6	24.8	21.6	19.2	9.1	8.8	8.2	8.4	27.3	23.9	21.2	18.4
5	5	25	14.0	14.6	13.3	14.0	46.1	36.5	28.2	22.2	13.7	13.9	12.9	13.8	44.3	34.7	27.5	20.4
5	10	25	16.9	17.9	17.0	17.0	61.5	47.0	35.0	25.7	17.2	18.0	17.8	16.5	59.0	44.5	32.6	23.1
10	2	25	10.9	10.5	10.2	10.0	43.8	41.3	36.6	31.5	10.6	10.0	10.3	9.9	43.0	38.7	32.8	26.4
10	5	25	17.4	16.9	16.9	18.0	66.9	56.7	45.8	34.8	17.0	16.9	17.1	17.8	64.9	52.7	40.6	30.2
10	10	25	24.2	23.6	22.6	23.3	84.0	68.0	49.8	35.4	23.1	22.7	22.5	21.9	79.4	61.2	43.5	31.3
20	2	25	14.3	14.5	14.6	15.2	68.7	64.7	60.9	52.6	14.2	15.0	15.2	14.9	63.6	55.6	46.5	37.9
20	5	25	20.8	21.1	21.6	21.2	89.2	80.7	66.8	52.9	21.9	22.0	21.6	21.3	83.4	68.8	54.5	42.4
20	10	25	27.0	27.2	27.0	26.5	97.4	88.0	72.1	53.5	27.3	26.9	26.7	25.9	92.1	76.7	58.5	42.3
25	2	25	14.5	14.1	14.2	15.0	74.9	72.1	67.7	59.8	14.2	14.7	13.9	14.9	70.7	61.4	52.3	42.5
25	5	25	24.5	24.7	24.2	25.0	94.1	86.0	74.1	59.7	24.5	23.7	23.7	24.8	87.2	72.3	57.2	44.2
25	10	25	28.3	27.5	27.2	28.0	98.9	93.6	81.7	63.4	27.0	28.5	28.1	28.0	94.5	81.3	64.7	48.1
5	2	40	11.7	12.7	12.5	13.0	55.2	51.8	46.3	41.2	11.9	12.3	12.5	13.4	55.1	49.8	44.2	37.5
5	5	40	28.5	28.5	27.5	28.7	86.5	76.0	61.4	45.1	28.3	28.5	27.9	28.0	84.3	71.8	55.5	40.5
5	10	40	45.7	46.3	46.0	46.9	95.3	83.6	64.5	44.1	45.1	44.8	46.1	46.7	93.9	80.1	60.1	40.6
10	2	40	17.9	18.5	19.7	19.8	82.5	79.3	73.9	65.2	18.1	19.0	19.6	19.1	81.2	73.6	65.0	52.1
10	5	40	42.8	42.7	43.3	44.2	98.2	92.5	82.0	67.3	43.0	43.9	44.1	44.8	96.1	87.7	72.7	57.7
10	10	40	60.6	60.6	58.8	59.1	99.8	97.3	87.7	70.3	59.7	59.5	59.9	60.1	99.2	93.4	79.7	60.6
20	2	40	28.5	28.3	28.2	28.6	98.5	97.5	94.9	89.8	27.6	28.3	27.9	28.9	96.7	90.5	81.7	71.5
20	5	40	59.9	59.8	60.3	59.7	100	99.8	97.7	91.0	59.0	59.8	60.1	59.0	99.5	96.1	87.5	75.4
20	10	40	74.7	74.3	74.1	74.1	100	100	99.0	92.7	74.2	74.6	73.8	73.0	99.8	98.3	91.1	77.1
25	2	40	35.0	34.4	35.2	35.1	99.4	99.0	97.7	94.9	34.2	33.5	34.0	34.5	97.6	94.3	86.4	76.9
25	5	40	63.7	63.4	62.8	62.4	100	99.9	98.7	94.9	62.9	63.0	63.4	64.5	99.7	97.4	90.3	79.4
25	10	40	75.8	75.5	75.8	76.0	100	100	99.9	96.9	75.5	76.3	76.0	74.7	99.9	98.4	93.4	82.5

Table 2 (cont'd). Empirical power of the 3D-CIPS and bootstrapped IPS (BIPS) tests

N	M	T	3D-CIPS ($\theta = 0.75$)				BIPS ($\theta = 0.75$)				3D-CIPS ($\theta = 1.0$)				BIPS ($\theta = 1.0$)			
			ω_{kl}				ω_{kl}				ω_{kl}				ω_{kl}			
			0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9	0.3	0.5	0.7	0.9
5	2	10	5.3	5.6	5.7	5.9	10.2	8.7	7.6	7.3	5.2	5.7	5.3	5.4	9.4	8.3	8.2	7.5
5	5	10	5.2	6.0	6.0	5.9	12.4	10.8	9.2	7.8	5.8	5.7	6.1	6.2	11.3	9.2	7.6	7.0
5	10	10	6.6	7.3	6.9	6.9	13.7	11.4	10.0	8.0	6.2	6.4	6.0	6.6	11.9	8.7	8.4	7.2
10	2	10	6.7	5.8	5.8	4.9	11.8	9.9	9.1	7.7	6.5	5.9	5.7	5.8	11.1	9.5	8.0	7.6
10	5	10	6.5	6.4	7.0	6.4	13.6	10.6	9.1	8.1	7.1	6.8	7.5	7.6	11.9	8.7	8.4	7.2
10	10	10	8.0	8.1	7.1	6.7	15.6	11.3	9.2	8.1	7.7	7.2	7.2	7.0	12.3	8.7	7.6	7.3
20	2	10	6.6	6.5	6.6	6.6	12.1	10.3	9.2	7.5	6.8	6.4	6.3	6.4	11.1	9.1	8.3	7.7
20	5	10	7.0	7.5	7.6	7.2	15.8	10.7	8.3	7.8	7.3	6.7	6.6	7.2	12.3	8.7	7.6	7.3
20	10	10	8.4	8.9	8.8	8.4	16.3	12.1	10.3	9.0	8.8	9.1	9.1	8.1	14.5	10.6	9.5	7.5
25	2	10	5.8	6.4	6.9	6.3	13.7	10.6	9.0	7.3	6.0	6.4	6.1	6.1	11.9	8.7	8.4	7.2
25	5	10	7.5	7.2	6.4	7.2	14.6	10.4	8.8	7.2	7.9	7.4	7.1	7.3	13.0	10.0	8.4	7.0
25	10	10	7.5	6.8	6.7	7.0	16.7	11.7	8.9	8.2	7.8	7.0	7.6	8.0	12.1	9.0	6.7	6.7
5	2	20	5.9	5.9	5.9	6.3	16.8	15.0	13.2	10.8	5.7	5.4	5.6	6.1	16.3	13.3	11.4	8.4
5	5	20	10.2	10.0	9.0	9.8	26.9	20.0	14.4	10.9	9.7	9.5	9.2	9.9	24.0	16.7	12.2	8.9
5	10	20	11.2	10.6	11.1	11.6	34.2	23.6	15.2	11.7	11.5	11.6	11.1	11.8	28.8	18.4	13.4	10.1
10	2	20	9.0	9.1	9.2	8.8	24.6	19.0	14.7	11.8	9.2	9.3	9.9	9.5	21.6	15.1	11.2	9.0
10	5	20	12.7	13.1	12.3	11.8	35.0	23.8	17.0	13.1	12.9	13.4	13.5	13.2	28.8	18.4	13.4	10.1
10	10	20	14.4	14.3	14.4	14.6	41.3	25.1	17.5	13.6	14.9	15.0	14.6	15.2	32.9	20.5	13.9	9.7
20	2	20	9.9	10.1	10.1	10.3	31.2	22.1	15.5	12.3	10.1	10.3	10.8	11.1	26.4	17.2	12.1	9.4
20	5	20	14.5	14.1	13.8	14.0	41.9	25.8	18.1	13.9	14.3	14.0	14.5	14.4	32.9	20.5	13.9	9.7
20	10	20	18.3	18.7	18.3	17.6	47.4	28.1	19.6	13.4	18.6	18.3	18.4	17.9	35.8	20.2	13.6	10.7
25	2	20	11.1	11.3	11.3	11.0	35.7	24.4	17.0	13.9	11.4	10.8	10.8	11.1	28.8	18.4	13.4	10.1
25	5	20	14.5	15.1	14.8	16.1	42.3	26.0	17.1	12.6	14.4	14.9	15.2	15.5	32.5	19.0	13.0	9.3
25	10	20	17.8	17.5	17.9	17.8	49.6	28.2	19.7	15.4	18.4	17.7	17.9	18.4	38.3	20.7	13.7	10.1
5	2	25	9.1	8.9	8.5	8.4	25.4	20.8	17.0	13.8	9.3	8.8	8.3	8.4	23.6	18.3	14.2	10.9
5	5	25	13.1	12.7	12.6	12.7	36.6	27.1	19.1	13.9	12.7	12.7	13.0	12.9	33.2	22.1	14.3	10.0
5	10	25	17.4	17.1	17.3	18.0	48.2	31.1	22.3	16.2	17.4	17.4	17.0	17.3	41.0	24.3	16.6	11.4
10	2	25	10.6	10.6	10.7	9.2	34.0	25.4	19.1	13.4	11.2	10.6	9.6	9.4	30.1	20.5	14.6	10.7
10	5	25	17.4	17.7	17.9	18.1	48.7	32.8	22.0	14.9	17.3	17.5	18.0	17.5	41.0	24.3	16.6	11.4
10	10	25	24.2	23.2	22.6	21.8	57.4	35.1	22.6	15.9	24.5	23.4	22.6	22.7	47.0	25.8	15.6	9.7
20	2	25	14.3	14.3	14.7	14.5	45.5	30.3	20.9	14.7	14.1	14.3	13.9	13.8	38.4	21.8	14.0	9.7
20	5	25	21.9	21.6	21.2	21.2	57.4	36.1	23.8	16.3	21.9	21.6	20.9	21.1	47.0	25.8	15.6	9.7
20	10	25	26.7	27.5	27.7	26.3	62.7	37.3	23.6	15.5	27.0	27.4	27.1	26.4	47.8	25.9	15.1	9.9
25	2	25	14.1	13.5	13.8	14.8	50.3	33.8	22.5	17.3	14.1	14.3	14.1	14.3	41.0	24.3	16.6	11.4
25	5	25	24.8	23.8	24.0	24.4	58.3	35.9	24.9	17.6	24.3	23.8	22.8	23.3	46.9	26.8	17.2	11.3
25	10	25	27.4	27.8	28.4	27.1	66.2	41.1	25.9	17.6	27.8	27.0	27.3	26.6	52.0	28.1	18.1	10.4
5	2	40	12.3	12.3	12.6	12.5	49.6	39.7	31.7	24.0	12.2	12.2	12.4	11.9	45.3	34.3	25.0	17.2
5	5	40	28.1	28.2	28.5	28.1	72.0	52.2	35.7	23.4	28.1	28.5	28.2	28.2	64.5	42.2	25.3	16.2
5	10	40	45.5	46.0	44.9	46.0	81.3	58.1	37.2	23.6	44.7	46.6	45.9	45.5	73.4	44.1	25.9	14.7
10	2	40	18.2	18.9	19.1	19.1	66.2	49.4	34.5	24.7	19.1	18.8	18.7	18.6	58.4	37.7	24.1	15.4
10	5	40	43.7	43.5	43.9	42.9	82.7	58.5	40.2	26.3	44.4	44.5	44.4	44.2	73.4	44.1	25.9	14.7
10	10	40	59.4	58.9	57.8	58.4	87.1	62.4	40.8	26.6	59.2	57.8	57.2	57.6	76.3	44.8	26.9	15.2
20	2	40	27.5	26.8	28.0	28.2	79.6	58.2	40.2	27.6	28.0	27.3	27.7	27.6	69.6	43.4	25.7	15.8
20	5	40	60.0	58.6	59.1	58.1	87.4	61.8	40.1	27.8	59.7	58.8	58.5	58.5	76.3	44.8	26.9	15.2
20	10	40	72.9	73.1	72.6	72.1	91.6	67.8	45.4	29.2	72.9	72.3	71.9	72.6	81.5	49.6	28.4	16.1
25	2	40	34.8	35.0	35.2	34.0	83.5	60.7	40.3	25.9	35.3	35.6	34.9	35.9	73.4	44.1	25.9	14.7
25	5	40	63.3	63.8	63.7	64.6	89.4	64.8	44.6	31.0	63.9	63.6	64.1	63.8	78.3	48.9	30.0	16.6
25	10	40	77.5	76.7	76.7	76.2	91.8	67.9	45.9	30.1	77.2	77.1	76.1	75.2	80.2	50.6	29.6	16.0