Hedge Your Costs: Exchange Rate Risk and Endogenous Currency Invoicing

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Hedge Your Costs: Exchange Rate Risk and Endogenous Currency Invoicing*

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Abstract

The choice of invoicing currency for trade is crucial for the international transmission of macroeconomic policy. This paper develops a three-country model that endogenizes the choice of invoicing currency and that allows for a share of firms' costs to be denominated in foreign currency, consistent with the empirical evidence on the high degree of pass-through to import prices. Invoicing decisions are driven by firms' desire to hedge costs but also by exchange rate volatility and currency comovements. The model is tested empirically with a data set that spans ten currencies and 24 reporting countries, confirming the importance of currency comovements for the decision to invoice in vehicle currency. The findings also imply that if the U.S. share of world output continues to fall, other currencies will increasingly replace the U.S. dollar as an international vehicle currency.

JEL classification: F3, F4
Keywords: Invoicing Currency, Exchange Rate Risk, Hedging

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1 Introduction

The choice of invoicing currency in international trade is crucial for the international transmission of macroeconomic policy. For example, Betts and Devereux (2000) demonstrate that the effects of monetary and fiscal policy on consumption and welfare can be radically different, depending on which currency is used for invoicing. But how do firms choose an invoicing currency for international trade? Clearly, an exogenous assumption about the invoicing currency is not satisfactory.

This paper endogenizes the choice of invoicing currency in a three-country model of monopolistic competition in which firms preset prices under exchange rate risk. They can either invoice in producer currency, in local currency or in a third vehicle currency and endogenously choose the currency that maximizes their expected profits. A key feature of the model is that firms may face a share of their production costs denominated in foreign currency, a feature which is motivated by empirical evidence showing that the pass-through of nominal exchange rates is considerably higher to import and wholesale prices than to consumer prices. One can think of oil as an anecdotal example. Since oil is traditionally priced in U.S. dollars, from the perspective of non-U.S. firms it counts as an input denominated in foreign currency. The model gives rise to a forthright hedging intuition in that firms have an incentive to invoice in a particular currency if they face a large share of their costs in that currency.

But apart from the hedging intuition, the optimal choice of invoicing currency is also driven by exchange rate properties. In particular, if a certain currency is relatively volatile, firms tend to invoice in other, more stable currencies in order to circumvent unnecessary exchange rate risk. Exchange rate correlations also play an important role. If a third currency is highly correlated with the vehicle currency, firms have an increased incentive to invoice in vehicle currency because the high correlation adds to the momentum of the vehicle currency. The model is partial equilibrium but the same invoicing decisions would emerge in a general equilibrium framework because as monopolistic competitors, firms take aggregate variables as given when making their invoicing decisions.

The essential building block that firms' costs are partially denominated in foreign currency offers an explanation for the special role that the U.S. plays whenever it is involved in international trade. The majority of trade involving the U.S. either as an exporter or importer is heavily priced in U.S. dollars to a degree that is unparalleled by other countries and their respective currencies. The model provides an intuitive explanation in that pricing in U.S. dollars is optimal both for exporters from and importers to the U.S. because it allows the firms involved to hedge their costs.

In addition, the model's predictions are tested empirically with a comprehensive data set that includes 24 reporting countries and ten invoicing currencies. A theoretical criterion based on the model is used to distinguish vehicle currency pricing from local currency

pricing and the econometric specifications are closely intertwined with the model. The findings confirm the importance of currency correlations for the decision to invoice in vehicle currency. They also imply that if the U.S. share of world output continues to fall, other currencies will increasingly replace the U.S. dollar as an international vehicle currency.

Empirical data on currency invoicing are still hard to find. Goldberg and Tille (2005) give an excellent overview of data availability. In addition, the European Central Bank (2005) has recently collected a number of invoicing observations on the euro which are analyzed by Kamps (2005). A number of authors explore other country-specific invoicing data. Donnenfeld and Haug (2003) examine Canadian invoicing. Oi, Otani and Shirota (2004) examine Japanese invoicing and Goldberg (2005) analyzes the invoicing of Eastern European EU accession countries. Wilander (2005) uses a multinomial logit model to explain the choice of invoicing currency by Swedish exporters. Goldberg and Tille (2005) focus on industry-specific characteristics such as demand elasticities and exporters' market shares but their sample is considerably smaller.

The theoretical invoicing literature is surveyed by Oi, Otani and Shirota (2004) who provide a detailed discussion of models that endogenize the choice of invoicing currency. Another review of the literature is presented by the European Central Bank (2005). A recent theoretical contribution has been made by Devereux, Engel and Storgaard (2004). Their general equilibrium framework predicts that exporters wish to invoice in the currency of the country with the more stable monetary policy. This prediction is related to the intuition about exchange rate volatility that arises in the present paper. But as monetary policy is equally stable in most industrial countries, this result in isolation might be more suitable for comparing firms' invoicing behavior across poor and rich countries. Furthermore, their two-country model does not allow for the possibility of vehicle currency pricing and therefore, no statement can be made about the role of exchange rate correlations.

Friberg (1998) develops a three-country partial equilibrium model in which a monopolist faces costs in domestic currency. The exchange rates, however, are assumed to be uncorrelated. Goldberg and Tille (2005) also present a three-country model with the possibility of vehicle currency pricing. Their hedging mechanism arises through the assumption of decreasing returns to scale in production and fluctuating marginal costs. By allowing some of firms' costs to be denominated in foreign currency and by explicitly incorporating currency comovements, the present model develops a richer hedging intuition that can also account for the prevalence of U.S. dollar invoicing.

Both Bacchetta and van Wincoop (2005) and Goldberg and Tille (2005) suggest models where industry-specific features matter. But industry-specific invoicing data are hardly available and hence, their models are difficult to test. The predictions of the present model,

however, are independent of industry-specific characteristics and therefore particularly suitable for testing.

The paper is organized as follows. Section 2 develops the theoretical model with the key feature that some costs are denominated in foreign currency, giving rise to the volatility and hedging intuitions. Section 3 proceeds to test this theory empirically, making use of a comprehensive data set and analyzing both vehicle currency pricing and local currency pricing. Section 4 discusses the special role of the U.S. dollar and highlights questions for future research. Section 5 concludes.

2 A Model of Endogenous Currency Invoicing

The continuum $[0, 1]$ is the range of all tradable goods in the world, each produced by one individual firm. There are three countries in the model denoted by k, l and m. Country k produces the tradable goods range and comprises the firm range $[0, n_k]$, country l is in the range $[n_k, n_l]$ and country m in $[n_l, n_m]$ with $n_m = 1$.

2.1 Consumers

Each country-j consumer maximizes a standard Dixit-Stiglitz consumption index defined over all tradable goods as

$$C_j^T \equiv \left(\int_0^1 (c_{ij})^{\frac{\rho-1}{\rho}} \, \mathrm{d}\, i \right)^{\frac{\rho}{\rho-1}} \tag{1}$$

where c_{ij} denotes the consumption of good i for a country-j consumer and T indicates tradable goods. The parameter $\rho > 1$ is the elasticity of substitution and it is assumed to be the same across countries. The price index, defined as the minimum expenditure for one unit of C_j^T, can be derived from (1) as

$$P_j^T = \left(\int_0^1 (p_{ij})^{1-\rho} \, \mathrm{d}\, i \right)^{\frac{1}{1-\rho}}$$

where p_{ij} denotes the price of the good c_{ij}. The demand function for good c_{ij} follows as

$$c_{ij} = \left(\frac{p_{ij}}{P_j^T} \right)^{-\rho} C_j^T \tag{2}$$

2.2 Firms

A key element of the model is the assumption that firms face production costs that are not solely denominated in domestic currency but partly in foreign currency. The literature so far has assumed that inputs are only denominated in domestic currency, for instance Devereux, Engel and Storgaard (2004) and Bacchetta and van Wincoop (2005).

As an anecdotal example one can think of oil, which is a crucial input factor for many industries and which is traditionally priced in U.S. dollars and thus in foreign currency from the perspective of non-U.S. firms.[1] Similarly, a vast range of raw materials and other standardized commodities such as certain chemical products are usually priced in U.S. dollars. Goldberg and Tille (2005) adopt the distinction devised by James Rauch (1999) of reference priced goods and goods traded on an organized exchange versus differentiated goods. They find that the former types of goods are priced considerably less in domestic (non-U.S. dollar) currencies than differentiated goods.[2]

Furthermore, the assumption that firms face a part of their costs in foreign currency is motivated by comprehensive empirical evidence showing that the degree of pass-through to import and wholesale prices is considerably higher than the degree of pass-through to consumer prices. This phenomenon is documented, for instance, by McCarthy (2000) and Campa and Goldberg (2005).[3]

2.2.1 Production

All firms within one country are assumed to be symmetric and the firm-specific subscript i will therefore be dropped. A country-j firm uses the Cobb-Douglas production technology

$$Y_j^T = N_{j,k}^{\alpha_{j,k}} N_{j,l}^{\alpha_{j,l}} N_{j,m}^{\alpha_{j,m}} \quad \text{for} \quad j = k, l, m \tag{3}$$

where Y_j^T is tradable output produced by a country-j firm. $N_{j,k}$, $N_{j,l}$ and $N_{j,m}$ denote input factors that originate from countries k, l and m, respectively, with $\alpha_{j,k}$, $\alpha_{j,l}$ and $\alpha_{j,m}$ being their weights in the production process. Thus, $N_{j,j}$ represents domestic input factors. The technology is assumed to exhibit constant returns to scale so that $\alpha_{j,k}+\alpha_{j,l}+\alpha_{j,m} = 1$.

It is furthermore assumed that all inputs are denominated in the currency of the country of origin. Let R_h denote the h-currency price of the input factors from country h for all $h = k, l, m$. Define the nominal exchange rate $e_{j,h}$ as the j-currency price of h-currency and $e_{h,j}$ as its inverse with $e_{j,j} = 1$ for all $j = k, l, m$ and $h = k, l, m$. Given this notation the cost function that is associated with production function (3) and that is denominated in j-currency can be written as

$$costs_j = e_{j,k} R_k N_{j,k} + e_{j,l} R_l N_{j,l} + e_{j,m} R_m N_{j,m} \tag{4}$$

The technical appendix shows that when firms minimize costs, cost function (4) can be expressed as

$$costs_j = B_j \left(e_{j,k} R_k\right)^{\alpha_{j,k}} \left(e_{j,l} R_l\right)^{\alpha_{j,l}} \left(e_{j,m} R_m\right)^{\alpha_{j,m}} Y_j^T \tag{5}$$

[1] For details about oil invoicing see European Central Bank (December 2005, Box 4).

[2] Goldberg and Tille (2005) consider industries in Australia, Japan and the UK.

[3] Bacchetta and van Wincoop (2003) as well as Corsetti and Dedola (2005) offer theoretical explanations.

with $B_j = \alpha_{j,k}^{-\alpha_{j,k}} \alpha_{j,l}^{-\alpha_{j,l}} \alpha_{j,m}^{-\alpha_{j,m}}$. The optimal cost function (5) is linear in output and marginal costs therefore do not depend on the amount of output produced.

2.2.2 Optimal Prices

Since marginal costs are constant, a country-j firm can maximize profits with respect to each individual consumer separately without taking into account the amount sold to other consumers. Using demand function (2) and cost function (5) one can express expected profits generically for any combination of $h, i, j = k, l, m$ as

$$\mathrm{E}\left[\pi_{j,i}^{h}\right] = \mathrm{E}\left[\left(e_{j,h}p_{j,i}^{h} - B_j \left(e_{j,k}R_k\right)^{\alpha_{j,k}} \left(e_{j,l}R_l\right)^{\alpha_{j,l}} \left(e_{j,m}R_m\right)^{\alpha_{j,m}}\right) \left(\frac{e_{i,h}p_{j,i}^{h}}{P_i^T}\right)^{-\rho} C_i^T\right] \quad (6)$$

where $\pi_{j,i}^{h}$ denotes the nominal profits denominated in j-currency that a country-j firm earns by selling its good to an individual country-i consumer for the price $p_{j,i}^{h}$. The superscript h in $\pi_{j,i}^{h}$ and $p_{j,i}^{h}$ indicates invoicing in h-currency. Through multiplying the price $p_{j,i}^{h}$ by the exchange rate $e_{j,h}$ the country-j firm converts its revenue into domestic j-currency. Through multiplying the price $p_{j,i}^{h}$ by the exchange rate $e_{i,h}$ the country-i consumer converts the price $p_{j,i}^{h}$ into country-i currency.

Firms preset prices $p_{j,i}^{h}$ before the exchange rates are known. When maximizing expected profits, they take the exchange rate risk into account, but as monopolistic competitors they take the input prices R_i, composite consumption C_i^T and the price index P_i^T as given for all $i = k, l, m$. Maximizing expected profits (6) and solving the first-order condition yields the optimal price

$$p_{j,i}^{h} = \frac{\rho}{\rho - 1} B_j R_k^{\alpha_{j,k}} R_l^{\alpha_{j,l}} R_m^{\alpha_{j,m}} \frac{\mathrm{E}\left[e_{j,k}^{\alpha_{j,k}} e_{j,l}^{\alpha_{j,l}} e_{j,m}^{\alpha_{j,m}} e_{i,h}^{-\rho}\right]}{\mathrm{E}\left[e_{j,h} e_{i,h}^{-\rho}\right]} \quad (7)$$

Firms are assumed to always invoice in domestic currency when selling to domestic consumers, i.e. they set the price $p_{j,j}^{j}$

$$p_{j,j}^{j} = \frac{\rho}{\rho - 1} B_j R_k^{\alpha_{j,k}} R_l^{\alpha_{j,l}} R_m^{\alpha_{j,m}} \mathrm{E}\left[e_{j,k}^{\alpha_{j,k}} e_{j,l}^{\alpha_{j,l}} e_{j,m}^{\alpha_{j,m}}\right]$$

which is a special case of the generic optimal price (7). But foreign consumers are by assumption not able to arbitrage away international price differences and firms can therefore price-discriminate across countries. Depending on which invoicing currency maximizes their expected profits, firms from country j have the option when selling in country i for $j \neq i$ of either producer currency pricing (PCP) by setting the price $p_{j,i}^{j}$, local currency pricing (LCP) by setting the price $p_{j,i}^{i}$ or vehicle currency pricing (VCP) by setting the price $p_{j,i}^{h}$ if h is the country of the vehicle currency.

For the discussion that follows, let k be the country of the vehicle currency and l and m the countries with non-vehicle currencies. As Figure 1 illustrates, three qualitatively different pricing relations arise. The first relation is pricing from the vehicle country to non-vehicle country l. From the perspective of the vehicle country there is no difference between PCP and VCP, of course, such that country-k firms face the choice between $p^k_{k,l}$ (PCP=VCP) and $p^l_{k,l}$ (LCP) when selling to country-l consumers. Similarly, as countries l and m are symmetric, country-k firms face the choice between $p^k_{k,m}$ and $p^m_{k,m}$ when selling to country-m consumers.

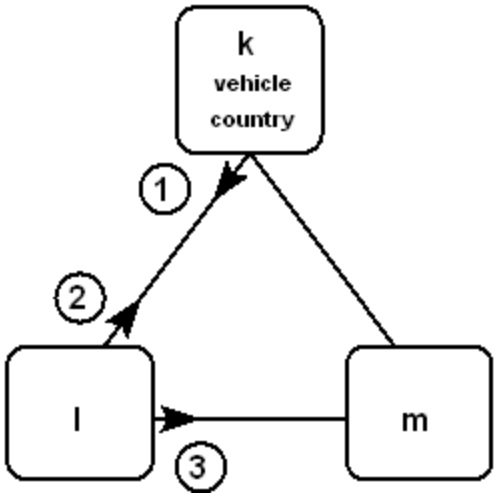

Figure 1: There are three qualitatively different pricing relationships. k is the vehicle country, l and m are non-vehicle countries.

The second relation in Figure 1 is pricing from a non-vehicle country to the vehicle country. Country-l firms can charge country-k consumers the price $p^l_{l,k}$ (PCP) or the price $p^k_{l,k}$ (LCP=VCP). The third relation is pricing between the two non-vehicle countries. A country-l firm faces three options of invoicing country-m consumers. It can set the price $p^l_{l,m}$ (PCP), $p^m_{l,m}$ (LCP) or $p^k_{l,m}$ (VCP).

2.2.3 The Stochastic Properties of the Exchange Rates

As one can see from the optimal price (7), various exchange rates appear multiplicatively in the expectations operator and thus, it is important to specify their stochastic properties. In order to circumvent Siegel's paradox, it is assumed that $e_{k,l}$ and $e_{k,m}$ are joint lognormally distributed with

$$\ln \begin{pmatrix} e_{k,l} \\ e_{k,m} \end{pmatrix} \sim N \left(\begin{bmatrix} \mu_{k,l} \\ \mu_{k,m} \end{bmatrix}, \begin{bmatrix} \sigma^2_{k,l} & \sigma_{l,m} \\ \sigma_{l,m} & \sigma^2_{k,m} \end{bmatrix} \right)$$

For simplicity let $\mu_{k,l} = \mu_{k,m} = 0$. Of course, the variances are always positive ($\sigma^2_{k,l} > 0$ and $\sigma^2_{k,m} > 0$) whereas the covariance can be negative ($\sigma_{l,m} \gtreqless 0$). As a result of triangular arbitrage the relationship $e_{l,m} = e_{k,m}/e_{k,l}$ holds.

2.3 Endogenous Choice of Invoicing Currency

Firms plug the optimal prices based on (7) into expected profits (6) and then compare which invoicing currency maximizes their expected profits. As it will be shown, firms' optimal invoicing decisions are generally driven by two factors - the currency denomination of costs (represented by the α's) as well as the comovement and volatility of exchange rates (represented by the σ's). These factors will now be discussed in the light of the three pricing relations depicted in Figure 1.

In general, note that all invoicing criteria that are explained in the following are independent of general equilibrium effects. Since monopolistic firms take aggregate variables including input prices as given, the optimal price (7) and subsequently the invoicing criteria would be the same in general equilibrium. A partial equilibrium set-up is therefore sufficient in this context to model the endogenous choice of invoicing currency.

2.3.1 Invoicing from the Vehicle Country to a Non-Vehicle Country

Vehicle country firms can set either price $p^k_{k,l}$ (PCP=VCP) or price $p^l_{k,l}$ (LCP) when selling to country-l consumers. If expected profits $\mathrm{E}\left[\pi^k_{k,l}\right]$ are higher than expected profits $\mathrm{E}\left[\pi^l_{k,l}\right]$, country-$k$ firms will choose PCP=VCP over LCP, and vice versa. If the expected profits are equal, firms will be indifferent. As it is shown in the technical appendix, this procedure leads to a necessary and sufficient condition for PCP=VCP to be chosen over LCP

$$\left(\frac{1}{2} - \alpha_{k,l}\right) > (1 - \alpha_{k,k} - \alpha_{k,l})\frac{\sigma_{l,m}}{\sigma^2_{k,l}} \tag{8}$$

The invoicing decision of country-k firms depends on the currency denomination of their costs ($\alpha_{k,k}$ and $\alpha_{k,l}$) and on exchange rate properties ($\sigma_{l,m}$ and $\sigma^2_{k,l}$).

Initially suppose $\sigma_{l,m} > 0$ and $\alpha_{k,l} < 1/2$. All else being equal the more inputs are denominated in domestic currency (i.e. the bigger $\alpha_{k,k}$), the more likely inequality (8) holds and the more likely country-k firms price in domestic currency (PCP=VCP). Intuitively, as a basic hedging argument firms prefer to invoice in domestic currency when a large share of their costs is denominated in domestic currency. Conversely, given $\sigma_{l,m} > 0$ country-k firms invoice in l-currency (LCP) if $\alpha_{k,l} > 1/2$, i.e. when most costs are denominated in the currency of the destination country. Figure 2 illustrates the relationship between the invoicing decision and the shares $\alpha_{k,k}$ and $\alpha_{k,l}$ for the numerical example of $\sigma_{l,m} = 1/2$ and $\sigma^2_{k,l} = \sigma^2_{k,m} = 1$.

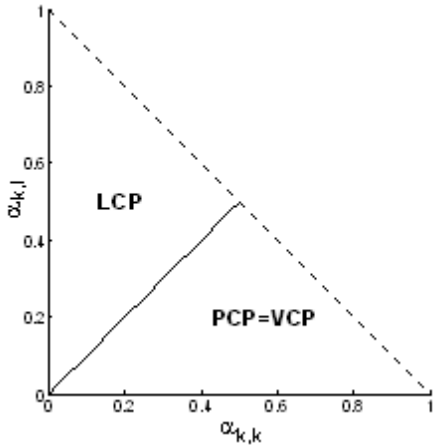

Figure 2: Invoicing from k to l for the numerical example of $\sigma_{l,m} = 1/2$ and $\sigma^2_{k,l} = \sigma^2_{k,m} = 1$.

Apart from the currency denomination of costs, exchange rate properties also play a decisive role in determining the choice of invoicing currency. Again suppose $\sigma_{l,m} > 0$ and $\alpha_{k,l} < 1/2$. A more volatile exchange rate between k and l (i.e. bigger $\sigma^2_{k,l}$) makes PCP=VCP more likely. Intuitively, bigger exchange rate volatility $\sigma^2_{k,l}$ means a less stable $e_{k,l}$ exchange rate and by invoicing in domestic currency firms decrease their exposure to exchange rate volatility. More formally, under PCP=VCP profits $\pi^k_{k,l}$ are a convex function of the exchange rate $e_{k,l}$ due to elastic demand ($\rho > 1$) such that firms are better off by invoicing in domestic currency, an explanation that goes back to Giovannini (1988). Conversely, under LCP profits $\pi^l_{k,l}$ are concave in the exchange rate $e_{k,l}$. Note that apart from the elasticity requirement $\rho > 1$ for consumption index (1), the invoicing criterion (8) and in fact all other invoicing criteria do not depend on any particular value of ρ because ρ is the same across countries.

The role of the covariance $\sigma_{l,m}$ is easier to understand when rewriting condition (8) as

$$\left(\frac{1}{2} - \alpha_{k,l}\right) > \alpha_{k,m}\frac{\sigma_{l,m}}{\sigma^2_{k,l}}$$

If the exchange rates $e_{k,l}$ and $e_{k,m}$ are positively correlated (implying $\sigma_{l,m} > 0$), then the currencies of countries l and m become rather similar from the country-k perspective. Given $\alpha_{k,l} < 1/2$, if the share $\alpha_{k,m}$ denominated in m-currency is sufficiently high, country-k firms are better off invoicing in l-currency, i.e. pricing in local currency. This is again a simple hedging intuition because firms will invoice in l-currency, which is similar to m-currency, if they face a sufficiently big share of their costs in m-currency.

Conversely, given $\alpha_{k,l} < 1/2$ when the exchange rates $e_{k,l}$ and $e_{k,m}$ are negatively

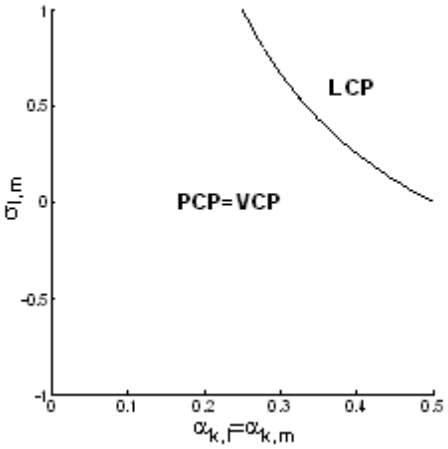

Figure 3: Invoicing from k to l for the numerical example of $\sigma^2_{k,l} = \sigma^2_{k,m} = 1$ and $\alpha_{k,l} = \alpha_{k,m}$.

correlated (implying $\sigma_{l,m} < 0$), PCP is always superior to LCP. Intuitively, when k-currency depreciates against l-currency, costs associated with inputs denominated in l-currency are higher. But as a result of the negative correlation, at the same time k-currency appreciates against m-currency, leading to lower costs associated with inputs denominated in m-currency. In total, the two changes tend to offset each other and the above argument about the convexity of expected profits under PCP=VCP applies. Figure 3 illustrates the effect of the covariance $\sigma_{l,m}$ and the share $\alpha_{k,m}$ on the choice of invoicing currency for the numerical example of $\sigma^2_{k,l} = \sigma^2_{k,m} = 1$ and $\alpha_{k,l} = \alpha_{k,m}$.

2.3.2 Invoicing from a Non-Vehicle Country to the Vehicle Country

The second invoicing relationship illustrated in Figure 1 is pricing from non-vehicle country l to vehicle country k. Country-l firms can charge country-k consumers either the price $p^k_{l,k}$ (LCP=VCP) or the price $p^l_{l,k}$ (PCP). Comparing expected profits (6) conditional on these two prices leads to the following necessary and sufficient condition for the choice of LCP=VCP over PCP[4]

$$\left(\frac{1}{2} - \alpha_{l,l}\right) > (1 - \alpha_{l,k} - \alpha_{l,l})\frac{\sigma_{l,m}}{\sigma^2_{k,l}} \tag{9}$$

Figures 4 and 5 illustrate the choice of invoicing currency depending on the variables in (9), again for the numerical example of $\sigma^2_{k,l} = \sigma^2_{k,m} = 1$.

As with condition (8), a basic hedging argument provides an intuitive interpretation. Suppose $\sigma_{l,m} > 0$ and $\alpha_{l,l} < 1/2$. The bigger the vehicle currency denominated share $\alpha_{l,k}$ of costs is, the more likely condition (9) holds and the more likely LCP=VCP is chosen. If

[4]See the technical appendix.

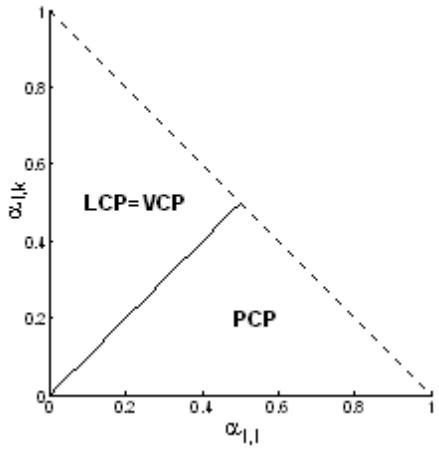

Figure 4: Invoicing from l to k for the numerical example of $\sigma_{l,m} = 1/2$ and $\sigma^2_{k,l} = \sigma^2_{k,m} = 1$.

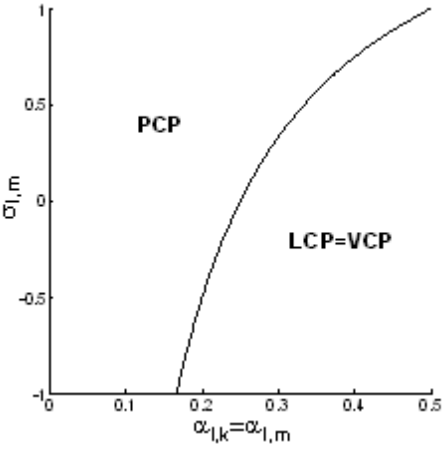

Figure 5: Invoicing from l to k for the numerical example of $\sigma^2_{k,l} = \sigma^2_{k,m} = 1$ and $\alpha_{l,k} = \alpha_{l,m}$.

the domestic currency denominated share $\alpha_{l,l}$ of costs exceeds 1/2 and is thus bigger than the other shares combined, then country-l firms choose to invoice in domestic currency (PCP). In addition, a bigger $\sigma^2_{k,l}$ makes LCP=VCP more likely. When the $e_{k,l}$ exchange rate is volatile relative to $e_{k,m}$ and thus $\sigma^2_{k,l}$ tends to be big relative to $\sigma^2_{k,m}$, then due to triangular arbitrage exchange rate volatility between countries l and m also tends to be big. Domestic l-currency is therefore more volatile than k-currency and country-l firms find it more attractive to invoice in k-currency.[5] Intuitively, firms try to avoid invoicing in currencies that are unstable because it unnecessarily exposes them to exchange rate risk.

If $\alpha_{l,l} < 1/2$ but $\sigma_{l,m} < 0$, then LCP=VCP is chosen. Intuitively, negative covariation between $e_{k,l}$ and $e_{k,m}$ means that $e_{l,k}$ and $e_{l,m}$ are positively correlated due to triangular arbitrage, i.e. from the perspective of country-l firms, the currencies of countries k and m tend to move in the same direction.[6] Given that the cost share $\alpha_{l,l}$ denominated in domestic currency constitutes less than half of total costs, country-l firms are therefore better off pricing in k-currency in order to hedge exchange rate risk (see Figure 5 for a numerical example).

As will be explained in Section 3, empirical data are available to test condition (9). Its testable implications can therefore be summarized as follows.

Proposition 1 *Suppose firms from non-vehicle country l invoice customers from vehicle country k. If the share $\alpha_{l,l}$ of costs denominated in the currency of country l is below 1/2, then invoicing in vehicle currency becomes more likely for (a) a bigger share $\alpha_{l,k}$ of costs denominated in vehicle currency and for (b) a smaller ratio $\sigma_{l,m}/\sigma^2_{k,l}$ of the exchange rate comovement with the currency of country m and the variance of the exchange rate between countries k and l. If in addition the ratio $\sigma_{l,m}/\sigma^2_{k,l}$ is below 1, then invoicing in vehicle currency becomes more likely for (c) a smaller share $\alpha_{l,l}$ of costs denominated in the currency of country l.*

2.3.3 Invoicing between Non-Vehicle Countries

The range of possible invoicing choices is biggest between non-vehicle countries. A country-l firm faces the three options of $p^k_{l,m}$ (VCP), $p^m_{l,m}$ (LCP) or $p^l_{l,m}$ (PCP) when invoicing country-m consumers. Again, examining expected profits conditional on these three prices leads to the following pairwise comparisons. A necessary condition for VCP to be chosen

[5] Given the lognormal distribution in Section 2.2.3, it can be shown that $Var(e_{k,l}) = Var(e_{l,k}) = \exp(\sigma^2_{k,l})\left(\exp(\sigma^2_{k,l})-1\right)$, $Var(e_{k,m}) = Var(e_{m,k}) = \exp(\sigma^2_{k,m})\left(\exp(\sigma^2_{k,m})-1\right)$ and $Var(e_{l,m}) = Var(e_{m,l}) = \exp(\sigma^2_{k,l}-2\sigma_{l,m}+\sigma^2_{k,m})\left(\exp(\sigma^2_{k,l}-2\sigma_{l,m}+\sigma^2_{k,m})-1\right)$. An increase in $\sigma^2_{k,l}$ therefore increases $Var(e_{l,k})$ and $Var(e_{l,m})$, making l-currency relatively more unstable.

[6] $e_{l,m} = e_{k,m}/e_{k,l}$ holds due to triangular arbitrage. It follows $e_{k,l} = e_{k,m}/e_{l,m}$ and $e_{l,k} = e_{l,m}/e_{k,m}$. If $e_{k,m}$ goes up, then $e_{l,k}$ tends to go up as well because of $\sigma_{l,m} < 0$ and thus $e_{l,m}$ tends to go up. Conversely, if $e_{k,m}$ goes down, then $e_{l,k}$ tends to go down and thus $e_{l,m}$ tends to go down. Hence, $e_{l,k}$ and $e_{l,m}$ tend to move in the same direction.

over PCP is[7]

$$\left(\frac{1}{2}-\alpha_{l,l}\right) > (1-\alpha_{l,k}-\alpha_{l,l})\frac{\sigma_{l,m}}{\sigma^2_{k,l}} \tag{10}$$

which is the same as condition (9). For VCP to be chosen over LCP it is necessary that

$$\left(\frac{1}{2}-\alpha_{l,m}\right) > (1-\alpha_{l,k}-\alpha_{l,m})\frac{\sigma_{l,m}}{\sigma^2_{k,m}} \tag{11}$$

Note that condition (11) is similar to condition (10) but with $\alpha_{l,m}$ taking the place of $\alpha_{l,l}$. In addition, the volatility $\sigma^2_{k,m}$ of the exchange rate between vehicle country k and the destination country m matters now.

Finally, a necessary condition for LCP to be chosen over PCP is

$$\left(\frac{1}{2}-\alpha_{l,l}\right)\frac{\sigma^2_{k,l}}{\sigma^2_{k,m}}+\alpha_{l,l}\frac{\sigma_{l,m}}{\sigma^2_{k,m}} > \left(\frac{1}{2}-\alpha_{l,m}\right)+\alpha_{l,m}\frac{\sigma_{l,m}}{\sigma^2_{k,m}} \tag{12}$$

If $\alpha_{l,l} < 1/2$ and $\alpha_{l,m} < 1/2$ and if $\sigma_{l,m}$ is sufficiently close to zero, then condition (12) is more likely to hold in favor of LCP in case of a big foreign currency denominated share $\alpha_{l,m}$ of costs, whereas it is more likely to hold in favor of PCP for a big domestic currency denominated share $\alpha_{l,l}$.

Perhaps the σ-variables in (12) can best be understood when considering the variance of the exchange rate between countries l and m. It is given by $Var(e_{l,m}) = Var(e_{m,l}) = \exp(\sigma^2_{k,l}-2\sigma_{l,m}+\sigma^2_{k,m})\left(\exp(\sigma^2_{k,l}-2\sigma_{l,m}+\sigma^2_{k,m})-1\right)$. In contrast, the variance $Var(e_{k,l}) = Var(e_{l,k})$ of the exchange rate between countries l and k is a function of $\sigma^2_{k,l}$ only and likewise, the variance $Var(e_{k,m}) = Var(e_{m,k})$ is a function of $\sigma^2_{k,m}$ only.[8] If $\sigma^2_{k,l}$ goes up, this increases $Var(e_{l,m})$ and $Var(e_{l,k})$ and thus makes l-currency more volatile relative to m-currency. Firms therefore try to avoid pricing in l-currency and LCP (i.e. pricing in m-currency) becomes more likely. If $\sigma^2_{k,m}$ goes up, this increases $Var(e_{l,m})$ and $Var(e_{k,m})$ and thus makes m-currency more volatile relative to l-currency. If $\alpha_{l,l}$ in (12) is sufficiently high compared to $\alpha_{l,m}$, then firms will try to avoid m-currency and PCP (i.e. pricing in l-currency) becomes more likely.

But if $\sigma_{l,m}$ goes up, this decreases $Var(e_{l,m})$ only and the volatility of l-currency relative to m-currency is not affected. A change in $\sigma_{l,m}$ therefore does not matter with respect to volatility but rather with respect to hedging. If $\sigma_{l,m}$ is negative, this implies that from the country-l perspective the currencies of countries k and m tend to move in the same direction such that LCP (i.e. pricing in m-currency) is optimal for sufficiently

[7] As opposed to condition (9), condition (10) is no longer sufficient for choosing VCP because LCP is now a distinct third alternative. In the context of condition (9) LCP is the same as VCP.

[8] $Var(e_{k,l}) = Var(e_{l,k}) = \exp(\sigma^2_{k,l})\left(\exp(\sigma^2_{k,l})-1\right)$ and $Var(e_{k,m}) = Var(e_{m,k}) = \exp(\sigma^2_{k,m})\left(\exp(\sigma^2_{k,m})-1\right)$. Also see footnote 5.

high $\alpha_{l,m}$.[9] If $\sigma_{l,m}$ is positive, this implies that the currencies of countries l and m tend to move in the same direction such that PCP (i.e. pricing in l-currency) is optimal for sufficiently high $\alpha_{l,m}$. An increase in $\sigma_{l,m}$ thus makes PCP more likely for sufficiently high $\alpha_{l,m}$ (if $\alpha_{l,m} > \alpha_{l,l}$ in condition (12)). Conversely, if $\sigma_{l,m}$ is negative, PCP is optimal for sufficiently high $\alpha_{l,l}$ and thus, an increase in $\sigma_{l,m}$ makes LCP more likely if $\alpha_{l,l} > \alpha_{l,m}$ in (12).

As will be explained in Section 3, empirical data are available to test condition (12). Its testable implications can therefore be summarized as follows.

Proposition 2 *Suppose firms from non-vehicle country l invoice customers from non-vehicle country m. If the share $\alpha_{l,l}$ of costs denominated in the currency of country l is below $1/2$ and if the share $\alpha_{l,m}$ of costs denominated in the currency of country m is below $1/2$, then invoicing in local currency (i.e. in the currency of country m) becomes more likely for (a) a bigger ratio $\sigma^2_{k,l}/\sigma^2_{k,m}$ of exchange rate variances. If in addition $\sigma_{l,m}/\sigma^2_{k,m} < 1$, the invoicing in local currency becomes more likely for (b) a bigger share $\alpha_{l,m}$ of costs; if $\sigma_{l,m}/\sigma^2_{k,m} < \sigma^2_{k,l}/\sigma^2_{k,m}$, then invoicing in local currency becomes more likely for (c) a smaller share $\alpha_{l,l}$ of costs; if $\alpha_{l,l} > \alpha_{l,m}$, then invoicing in local currency becomes more likely for (d) a bigger ratio $\sigma_{l,m}/\sigma^2_{k,m}$ of the exchange rate comovement and the variance of the exchange rate between vehicle country k and importing country m.*

Furthermore, the pairwise comparisons (10)-(12) can be combined to yield sufficient conditions for each type of invoicing. For example, a particularly simple case arises if one assumes positive correlation between $e_{k,l}$ and $e_{k,m}$ as well as relatively big variances ($\sigma^2_{k,l} > \sigma_{l,m} > 0$ and $\sigma^2_{k,m} > \sigma_{l,m} > 0$). It is shown in the technical appendix that in this case $\alpha_{l,k} > 1/2$ is a sufficient condition for VCP, $\alpha_{l,m} > 1/2$ is a sufficient condition for LCP and $\alpha_{l,l} > 1/2$ is a sufficient condition for PCP. But as exchange rates are often negatively correlated, invoicing decisions will in practice depend on parameter values.

Figures 6-8 illustrate some cases of invoicing between the non-vehicle countries l and m by combining conditions (10)-(12) graphically. $\sigma^2_{k,l} = \sigma^2_{k,m} = 1$ is again picked as a numerical example. Figures 6 and 7 show that in line with the hedging argument PCP occurs when the domestic currency denominated share $\alpha_{l,l}$ of costs is sufficiently high, whereas LCP occurs with a sufficiently high destination currency denominated share $\alpha_{l,m}$. If both $\alpha_{l,l}$ and $\alpha_{l,m}$ are sufficiently small and thus $\alpha_{l,k}$ is sufficiently big, then country-l firms tend to choose VCP.

Moreover, if $\sigma_{l,m} < 0$ as in Figure 7, then VCP is the optimal choice for a wider set of parameters because a negative covariance $\sigma_{l,m}$ implies that $e_{l,k}$ and $e_{l,m}$ are positively correlated and that from the perspective of country-l firms, the currencies of countries k and m tend to move in the same direction. Similarly, Figure 8 illustrates for the numerical

[9] See footnote 6.

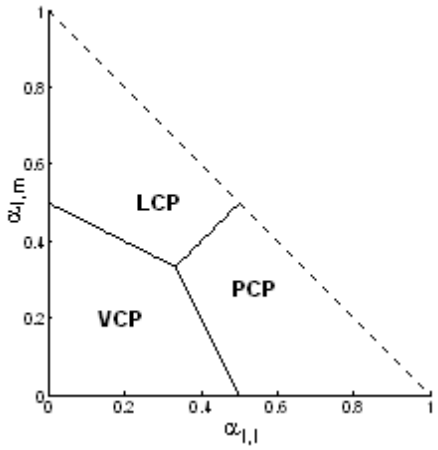

Figure 6: Invoicing from l to m for the numerical example of $\sigma_{l,m} = 1/2$ and $\sigma^2_{k,l} = \sigma^2_{k,m} = 1$.

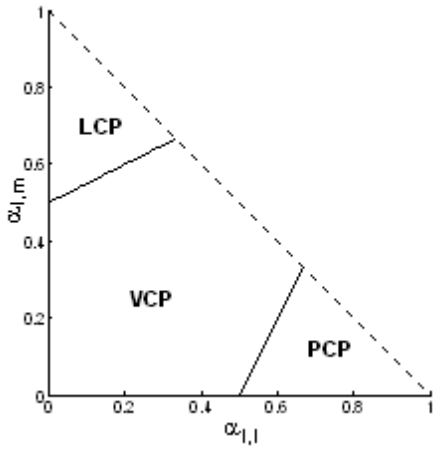

Figure 7: Invoicing from l to m for the numerical example of $\sigma_{l,m} = -1/2$ and $\sigma^2_{k,l} = \sigma^2_{k,m} = 1$.

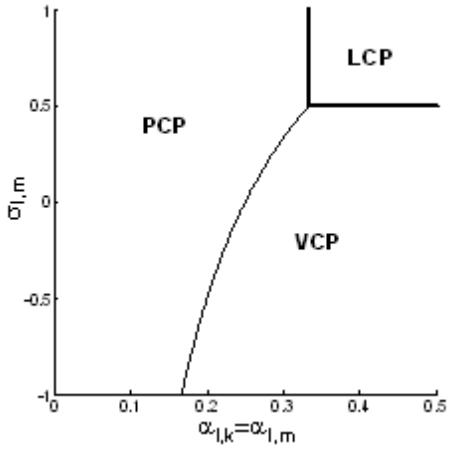

Figure 8: Invoicing from l to m for the numerical example of $\sigma^2_{k,l} = \sigma^2_{k,m} = 1$ and $\alpha_{l,k} = \alpha_{l,m}$.

example of $\alpha_{l,k} = \alpha_{l,m}$ that for sufficiently high $\alpha_{l,k}$ VCP is more prevalent when $e_{l,k}$ and $e_{l,m}$ are positively correlated (i.e. $\sigma_{l,m} < 0$).

2.3.4 Summary of the Invoicing Conditions

The invoicing decisions encapsulated in conditions (8)-(12) are all driven by the desire of firms to hedge their costs but also by their desire to avoid exchange rate volatility. If firms face a big fraction of their costs in a particular currency, they can hedge their costs by invoicing in that currency. If that currency is highly correlated with another currency, those two currencies tend to be substitutes. But firms also try to avoid exchange rate volatility. A conflict arises if firms face a large fraction of their costs in a certain currency and would therefore like to invoice in that currency, but that currency happens to be especially volatile. It then depends on parameter values which motive prevails.

2.4 Aggregation of Invoicing Decisions

Empirical invoicing data are typically available as invoicing currency shares of total exports for a particular country and year.[10] For example, in 2001 the UK invoiced 29 percent of its total exports in U.S. dollars. In the same year 16 percent of total UK exports were sent to the U.S. such that the ratio of invoicing currency share and export share is 29/16 in this particular case. Given this format of available data, in order to empirically test the model of Sections 2.1-2.3 it becomes necessary to establish the theoretical invoicing currency shares arising under the distinct options of PCP, LCP and VCP.

[10] Invoicing data relating to imports are less frequent and will therefore not be considered.

For any h let inv_j^h denote the invoicing currency share of currency h as a fraction of all exports from country j (29 percent in the example). Moreover, let $exp_{j,i}$ denote exports from j to i and let exp_j be total exports from country j. The country-i export share as a fraction of total country-j exports is thus given by $exp_{j,i}/exp_j$. In the above example when j is the UK and i is the U.S., this share is 16 percent. Now the ratio of invoicing currency share over export share with respect to h-currency can be defined as

$$invexp_{j,i}^h \equiv \frac{inv_j^h}{exp_{j,i}/exp_j} \quad \text{for } j \neq i$$

(29/16 in the example). Note that given the available data, $invexp_{j,i}^h$ can be computed for $h = i$ (as in the example) and usually also for $h = j$. But when the invoicing currency is neither the exporter's nor the importer's currency (i.e. for $h \neq j, i$), the ratio $invexp_{j,i}^h$ is typically unknown.

2.4.1 Invoicing from the Vehicle Country

As all firms within one country are symmetric, the aggregate invoicing share can be obtained without difficulty. When selling to foreign consumers, vehicle country-k firms can invoice in either domestic (i.e. vehicle) or foreign currency. Under VCP=PCP to all foreign customers the invoicing share of the vehicle currency is $inv_k^k = 1$ and the invoicing share of the foreign currency is $inv_k^i = 0$. When country-k firms invoice in foreign local currency under LCP, the inv_k^i share corresponds to the export share $inv_k^i = exp_{k,i}/exp_k$ and the invoicing share of the vehicle currency is $inv_k^k = 0$. The invoicing/export ratios therefore follow as

$$\begin{array}{lll}
invexp_{k,i}^k = exp_k/exp_{k,i} > 1 & \text{for } i = l, m & \text{under VCP=PCP} \\
invexp_{k,i}^i = 0 & \text{for } i = l, m & \text{under VCP=PCP} \\
invexp_{k,i}^k = 0 & \text{for } i = l, m & \text{under LCP} \\
invexp_{k,i}^i = 1 & \text{for } i = l, m & \text{under LCP}
\end{array}$$

The ratio $invexp_{k,i}^i$ for invoicing in non-vehicle currency i is therefore bounded by 0 under VCP=PCP and 1 under LCP.

2.4.2 Invoicing from a Non-Vehicle Country

For exports from a non-vehicle country $j = l, m$, the invoicing currency shares of the vehicle currency are $inv_j^k = 0$ under PCP, $inv_j^k = exp_{j,k}/exp_j$ under LCP and $inv_j^k = 1$ under VCP such that the corresponding invoicing/export ratios $invexp_{j,k}^k$ for the vehicle

currency are given by

$$\begin{aligned} invexp^k_{j,k} &= 0 && \text{for } j = l, m \quad \text{under PCP} \\ invexp^k_{j,k} &= 1 && \text{for } j = l, m \quad \text{under LCP} \\ invexp^k_{j,k} &= exp_j/exp_{j,k} > 1 && \text{for } j = l, m \quad \text{under VCP} \end{aligned}$$

$invexp^k_{j,k} > 1$ is therefore a necessary and sufficient condition for VCP by a non-vehicle country firm. The invoicing behavior of UK exporters to the U.S. falls into this category. As $invexp^k_{j,k}$ under VCP is bounded by 1 at the lower end but unbounded from above, it is referred to in Section 3 as the *extent of VCP*.[11]

Finally, if country-l firms export to the other non-vehicle country m, then the invoicing/export ratio $invexp^m_{l,m}$ for the non-vehicle currency m is bounded by 0 under PCP and VCP, and $invexp^m_{l,m}$ is bounded by 1 under LCP such that it can be referred to as the *fraction of LCP*

$$\begin{aligned} invexp^m_{l,m} &= 0 \quad \text{under PCP and VCP} \\ invexp^m_{l,m} &= 1 \quad \text{under LCP} \end{aligned}$$

3 Empirical Evidence

3.1 The Invoicing Data

Data on currency invoicing are scarce. Only recently have some government agencies and central banks started to collect them systematically. For example, the European Central Bank (2005) has compiled data on the use of the euro as an international invoicing currency. Goldberg and Tille (2005) give an excellent overview of the data currently available.

Making use of as big a cross section of data as possible, I consider altogether 56 observations of invoicing relationships for exports. To avoid double counting each invoicing relationship is included for the most recent observation year only. The 56 observations are reported by altogether 24 countries. They are the UK, seven eurozone countries (Belgium, France, Germany, Greece, Luxembourg, Portugal and Spain), the ten new EU members (Cyprus, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Slovakia and Slovenia) plus Bulgaria as well as Australia, Japan, Korea, Malaysia and Thailand. Invoicing data gathered by U.S. authorities are not publicly available (cf. Goldberg and Tille 2005, Appendix Table 1).

In total, the observations involve ten currencies. Apart from the U.S. dollar and the euro the data report invoicing in the Canadian dollar, pound sterling, the Deutschmark,

[11] As a rather contrived scenario, VCP could also occur with $invexp^k_{j,k} < 1$ if firms from a non-vehicle country did VCP with respect to other non-vehicle countries but PCP with respect to the vehicle country. In the theory developed in Sections 2.1-2.3, however, this scenario cannot arise since conditions (9) and (10) are the same. It will therefore be ignored since it would reflect non-optimal pricing.

the Swiss franc, the Swedish krona, Japanese yen, the New Zealand dollar and the Singapore dollar. The years of observation vary between 1996 and 2004. The data appendix gives the precise data sources.

3.2 Output Shares and Export Shares

The weights α in the production function (3) represent the shares of the currency denomination of firms' costs. In order to test the model, a breakdown of firms' costs into currencies would be ideal as α-regressors but such data are not available, certainly not as macroeconomic data. Instead, I will suggest two alternative measures that are consistent with the model presented in Section 2.

Assume that the continuum $[0,1]$ encompasses all input factors in the world. Similar to the continuum of final goods, the range $[0,n_k]$ of inputs is associated with country k, the range $[n_k,n_l]$ with country l and the range $[n_l,n_m]$ with country m. To allow for the possibility of nontradable inputs such as internationally immobile labor, assume that for each country $h=k,l,m$ the share s_h of inputs is tradable such that $[n_{h-1},n_{h-1}+s_h(n_h-n_{h-1})]$ represents the range of all tradable inputs from country h with $n_{k-1}=0$, $n_{l-1}=n_k$ and $n_{m-1}=n_l$.

The first measure of the α's can be motivated by assuming a perfect world without trade frictions in which all inputs are tradable such that $s_h=1$ for $h=k,l,m$. In this case the α's simply follow as relative country sizes and for the empirical analysis, $\alpha_{j,h}$ will be taken as the country-h share of world output for $j,h=k,l,m$.

In contrast, the second measure of the α's arises in a world with trade frictions where some inputs are nontradable and thus only available to domestic firms such that from the perspective of country j, $s_j=1$ but $0\leq s_h\leq 1$ for $h\neq j$. The input range $[n_{h-1},n_{h-1}+s_h(n_h-n_{h-1})]$ will now be proxied by total exports of country $h\neq j$ and the range $[n_{j-1},n_j]$ will be interpreted as output of country j. The whole range of inputs available to country-j firms is therefore given by total exports in the world plus country-j output. For the empirical analysis $\alpha_{j,h}$ then follows as the ratio of total country-h exports over total exports in the world plus country-j output and $\alpha_{j,j}$ follows as the ratio of country-j output over total exports in the world plus country-j output.

To summarize, if there are no trade frictions, the $\alpha_{j,h}$'s are determined by output shares and are the same for all $j=k,l,m$. In the presence of nontradable inputs the $\alpha_{j,h}$'s are represented by export shares and generally differ across $j=k,l,m$. The empirical $\alpha_{j,h}$'s are computed using data from the IMF International Financial Statistics (IFS) as well as data from the IMF Direction of Trade Statistics (DOTS). Details can be found in the data appendix.

3.3 Explaining the Extent of Vehicle Currency Pricing

From Section 2.4.2 it follows that an invoicing/export ratio bigger than one ($invexp^k_{j,k} > 1$) is a necessary and sufficient condition for VCP by firms from non-vehicle currency country $j \neq k$. 36 out of the 56 observations fulfill this condition. Since $invexp^k_{j,k}$ under VCP has a lower bound of 1 but is unbounded from above, it is referred to as the *extent of VCP*.

As might be expected, virtually all of the 36 VCP observations have the U.S. dollar or the euro as invoicing currencies, meaning that vehicle currency use can be associated almost exclusively with these two currencies. The extent of VCP is considerably higher for the U.S. dollar, the average invoicing/export ratio being 6.5. The biggest value is in fact 19.9 for Cyprus. In contrast, the average invoicing/export ratio for pricing in euros is only 1.4 with no single value exceeding 2. Two observations associate VCP with the Deutschmark for the year 1996, i.e. before the introduction of the euro. The only surprise is one observation that associates VCP with the Swedish krona for exports from Bulgaria.

Firms from non-vehicle country l can use the vehicle currency as invoicing currency both when selling to customers from vehicle country k and when selling to customers from the other non-vehicle country m. The model's corresponding theoretical predictions stem from condition (9) for selling to vehicle country k and from conditions (10) and (11) for selling to non-vehicle country m. But as explained in Section 2.4, $invexp^k_{l,m}$ is typically unknown so that the distinction between selling to the vehicle country as opposed to selling to a non-vehicle country cannot be made. For the empirical analysis I will therefore focus on conditions (9) and (10), which are the same, because this VCP condition applies to both selling to vehicle country customers (as a necessary and sufficient condition) and selling to non-vehicle country customers (as a necessary condition).

Proposition 1 summarizes the model's predictions about the extent of VCP ($invexp^k_{l,k}$). As the model assumes symmetry amongst all firms within one country, for given values of relevant regressors it yields the extreme prediction of either no or total VCP in the aggregate. In practice, of course, firms are heterogeneous and for given regressor values, one would expect a more diverse aggregate outcome. The share $\alpha_{l,l}$ in condition (9) is below 1/2 for all output share and export share observations in the sample. As implied by Proposition 1, one would expect a positive coefficient for $\alpha_{l,k}$ and a negative coefficient for $\sigma_{l,m}/\sigma^2_{k,l}$ in a regression of the invoicing/export ratio $invexp^k_{l,k}$.[12] For the share $\alpha_{l,l}$ one would expect a negative coefficient because the requirement $\sigma_{l,m}/\sigma^2_{k,l} < 1$ is met for the mean of $\sigma_{l,m}/\sigma^2_{k,l}$ ($= 0.68$) and for 31 out of the 36 single observations.

Note that while $\sigma_{l,m}$ is a clear-cut variable in the three-country model, its interpretation is more difficult for the empirical analysis. In a multi-country world, m represents the rest of the world with a range of currencies. In order to reflect the use of various cur-

[12] As explained in the data appendix, the σ's are computed on the basis of demeaned logarithmic exchange rate series, which is consistent with the lognormal distribution.

Table 1: Invoicing in Vehicle Currency

	Output shares		*Export shares*		*Export shares*	
Regressors	OLS	Heckman	OLS	Heckman	OLS	Heckman
Vehicle country share ($\alpha_{l,k}$)	27.32^{***} (3.82)	34.84^{***} (5.08)	-45.24 (-1.28)	-51.57 (-1.52)	52.26^{***} (3.10)	15.30 (0.60)
Exporter's share ($\alpha_{l,l}$)	-45.93^{***} (-3.03)	-46.88^{***} (-3.02)	-13.93^{*} (-1.98)	-14.96^{**} (-2.28)	-12.26^{**} (-2.55)	-14.58^{***} (-3.23)
Curr. comovement ($\sigma_{l,m}/\sigma^2_{k,l}$)	-0.66^{**} (-2.43)	-0.61^{***} (-2.69)	-0.62 (-1.66)	-0.36 (-0.82)	-0.42^{*} (-1.78)	-0.35^{*} (-1.65)
Constant	-1.99 (-1.10)	-4.68^{***} (-2.65)	10.96^{**} (2.35)	11.88^{***} (2.77)	1.71 (1.26)	5.68^{**} (2.37)
Euro as vehicle currency					-8.05^{***} (-4.68)	-5.51^{**} (-2.48)
R^2	0.347		0.161		0.363	

Dependent variable: the extent of VCP ($invexp^k_{l,k}$) with $l \neq k$
Sample size: 36 (all observations for which $invexp^k_{l,k} > 1$)
The Heckman procedure uses FIML.
t-statistics given in parentheses, based on robust standard errors.
***, ** and * indicate significance at the 1, 5 and 10 percent levels.

rencies, m is interpreted as an entity that uses the IMF's Special Drawing Right (SDR) as a currency. The SDR is a basket of the world's major currencies currently containing the U.S. dollar (45 percent), the euro (29 percent), Japanese yen (15 percent) and pound sterling (11 percent). Although not perfect, the SDR still serves as a suitable benchmark for an assessment of the exchange rate properties of the currencies of countries k and l. More generally, if the model is extended to multiple countries, the qualitative effects of the σ's do not change. Intuitively, an additional currency will merely reinforce the importance of exchange rates whose properties with respect to the vehicle currency are similar to its own, but it will not create qualitatively new insights.

Table 1 reports regression results for the extent of VCP. $\alpha_{l,k}$ is referred to as the vehicle country share and $\alpha_{l,l}$ is referred to as the exporter's share.[13] The first pair of columns uses α-regressors based on output shares and the remaining columns use α-regressors based on export shares, as explained in Section 3.2.

When output shares are used, all regressors have the expected signs and are significant. In addition to an OLS regression, a Heckman sample selection procedure is estimated as a robustness check controlling for the fact that only observations are considered for which the invoicing/export ratio $invexp^k_{l,k}$ is greater than 1. Apart from the regressors of the regression equation, the selection equation also includes a dummy variable that indicates whether the U.S. is a destination country since the U.S. dollar is a likely vehicle currency

[13] $\alpha_{l,m}$ as the share representing the rest of the world cannot be included because it would be collinear with $\alpha_{l,k}$ and $\alpha_{l,l}$ due to the assumption of constant returns to scale.

candidate. The Heckman estimation procedure yields similar results.

When export shares are used in otherwise the same regressions, the currency comovement coefficient has the correct sign but is no longer significant. Moreover, the coefficient of the vehicle country share $\alpha_{l,k}$ has the wrong sign but is insignificant. The reason for the wrong sign appears to be the fact that given its size, the eurozone is a relatively open economic entity that exports disproportionately many goods.

Indeed, if a dummy variable indicating whether the eurozone is a destination country is included (see the last pair of columns), the coefficient of the vehicle country share has the correct sign and is significant in the OLS regression.[14] The currency comovement variable is significant, too, and the R^2 of the OLS regression is raised to roughly the same level as in the output share regression, confirming the importance of the underlying heterogeneity. The finding that the dummy itself has a negative and significant coefficient might be related to the fact that the euro as a young currency is not entirely established yet and that invoicing in euros is expected to rise. Time-series evidence reported by the European Central Bank (2005) in fact shows that the use of the euro as an invoicing currency has continually risen since 2000.

Further robustness checks, albeit unreported here, corroborate the results of Table 1. Almost half of the 36 observations involve the ten new Eastern EU member states as exporters and thus the sample might not be representative. But including a dummy variable as a fixed effect for those countries hardly alters the results. Furthermore, the sample includes Estonia and Bulgaria which peg their currencies against the euro. Adding a suitable dummy or removing those observations from the sample does not have any substantial effect on the results.

In conclusion, Table 1 confirms the predictions for the extent of VCP that emanate from the model developed in Section 2. In particular, currency comovements appear to be an important determinant for the decision to invoice in vehicle currency. Note that in the output share regressions, the absolute magnitude of coefficients is higher for the exporter's share $\alpha_{l,l}$ than for the vehicle country share $\alpha_{l,k}$ (i.e. the importer's share). This result indicates that the economic strength of the exporting country has a stronger impact on the extent of VCP than the economic strength of the destination country. It is consistent with Grassman's (1973) well-known finding that among developed countries exports of manufactured goods are more often invoiced in domestic currency than imports.

[14] The $\alpha_{l,k}$ coefficient is not significant in the Heckman regression reported in the last column because the selection equation includes two dummy variables that indicate whether the U.S. or the eurozone are destination countries, respectively. These dummy variables effectively pick up the vehicle country share $\alpha_{l,k}$. Indeed, if the two dummy variables are dropped from the selection equation, the $\alpha_{l,k}$ coefficient is estimated at 49.82 with a t-statistic of 3.05 (significant at the 1 percent level).

3.4 Explaining the Fraction of Local Currency Pricing

The aggregation in Section 2.4.2 shows that the invoicing/export ratio $invexp^m_{l,m}$ for invoicing from non-vehicle country l to non-vehicle country m is bounded by 0 in the case of PCP and 1 in the case of LCP. The ratio $invexp^m_{l,m}$ is therefore referred to as the *fraction of LCP* and Proposition 2 provides the relevant theoretical predictions. 20 out of the 56 observed invoicing/export ratios lie in between 0 and 1.[15]

No single $\alpha_{l,m}$ or $\alpha_{l,l}$ observation in the sample is larger than 1/2. According to Proposition 2 one would therefore expect a positive coefficient for $\sigma^2_{k,l}/\sigma^2_{k,m}$ in a regression of the invoicing/export ratio $invexp^m_{l,m}$. As the requirement $\sigma_{l,m}/\sigma^2_{k,m} < 1$ is met for the mean of $\sigma_{l,m}/\sigma^2_{k,m}$ ($= 0.61$) and for 13 out of the 20 single observations, one would expect a positive coefficient for the importer's share $\alpha_{l,m}$. For the exporter's share $\alpha_{l,l}$ one would expect a negative coefficient because the requirement $\sigma_{l,m}/\sigma^2_{k,m} < \sigma^2_{k,l}/\sigma^2_{k,m}$ is met for the mean and for 14 out of the 20 individual observations.[16] For the coefficient of $\sigma_{l,m}/\sigma^2_{k,m}$ the expected sign depends on the relative sizes of $\alpha_{l,l}$ and $\alpha_{l,m}$. As for the majority of observations $\alpha_{l,l}$ is smaller than $\alpha_{l,m}$, one might expect a negative coefficient. The U.S. dollar is now regarded as the vehicle currency k because it is used considerably more than any other vehicle currency identified in Section 3.3.

Table 2 reports regression results for the fraction of LCP. The first pair of columns uses α-regressors based on output shares and the second pair of columns uses α-regressors based on export shares. The Heckman sample selection procedure controls for the fact that observations are only included in the regressions if the invoicing/export ratio $invexp^m_{l,m}$ is smaller than 1. Apart from the regressors of the regression equation, the selection equation includes a dummy indicating whether the U.S. is a destination country. This dummy takes into account that exporting to the U.S. typically results in the use of the U.S. dollar as invoicing currency and thus in an invoicing/export ratio that is greater than 1.

All α-coefficients have the expected signs and are significant. Again note that in the output share regressions, the coefficient of the exporter's share $\alpha_{l,l}$ is bigger in absolute magnitude than the coefficient of the importer's share $\alpha_{l,m}$, consistent with Grassman's (1973) finding.

In contrast, the σ-regressors are not significant. The relative variance $\sigma^2_{k,l}/\sigma^2_{k,m}$ has the expected sign but the currency comovement $\sigma_{l,m}/\sigma^2_{k,m}$ does not. The latter finding might arise because the requirement $\alpha_{l,l} < \alpha_{l,m}$ for $\sigma_{l,m}/\sigma^2_{k,m}$ to have a negative coefficient is not very clearly met. In addition, as pointed out by the European Central Bank (2005),

[15] None of those 20 observations are associated with exporters that are vehicle countries as identified in Section 3.3 such that the invoicing/export ratio $invexp^i_{k,i}$ for $i = l, m$ from Section 2.4.1 does not apply. Invoicing data for exports from eurozone countries are not available for non-vehicle currencies. Invoicing data for exports from the U.S. are not available at all.

[16] $\alpha_{l,k}$ cannot be included as a regressor because it would be collinear with $\alpha_{l,m}$ and $\alpha_{l,l}$ due to the assumption of constant returns to scale.

Table 2: Invoicing in Local Currency

	Output shares		*Export shares*	
Regressors	OLS	Heckman	OLS	Heckman
Importer's share ($\alpha_{l,m}$)	1.33^{**}	1.22^{***}	3.30^{***}	2.53^{***}
	(2.14)	(2.64)	(3.00)	(3.39)
Exporter's share ($\alpha_{l,l}$)	-6.22^{**}	-4.48^{**}	-1.35^{**}	-0.97^{**}
	(-2.43)	(-2.21)	(-2.59)	(-2.21)
Rel. variance ($\sigma^2_{k,l}/\sigma^2_{k,m}$)	0.01	0.01	0.01	0.01
	(1.11)	(1.33)	(1.33)	(1.49)
Curr. comovement ($\sigma_{l,m}/\sigma^2_{k,m}$)	0.11	0.08	0.08	0.06
	(1.35)	(1.11)	(1.13)	(0.90)
Constant	0.23^{***}	0.19^{***}	0.16^{**}	0.16^{**}
	(3.47)	(3.49)	(2.29)	(2.49)
R^2	0.453		0.572	

Dependent variable: the fraction of LCP ($invexp^m_{l,m}$) with $l \neq m$
Sample size: 20 (all observations for which $0 < invexp^m_{l,m} < 1$)
The Heckman procedure uses FIML.
t-statistics given in parentheses, based on robust standard errors.
***, ** and * indicate significance at the 1, 5 and 10 percent levels.

the invoicing data might be noisy inasmuch as some observations refer to the currency of settlement rather than the currency of invoicing. These results hold up if the regressions are based on two alternative sets of σ-regressors. The first set is computed on the basis of detrended exchange rate series in order to filter out secular exchange rate trends. The second set is computed on the basis of exchange rates series that cover a longer time window.[17] Furthermore, the findings of Table 2 do not change qualitatively if dummy variables are added for Eastern EU members as well as for Estonia and Bulgaria, which peg their currencies against the euro. The findings do not change either if these observations are dropped.

In summary, the results of Table 2 do not point to a prominent role of exchange rate variances and comovements in determining the choice of invoicing currency in the case of local currency pricing. But given the low number of observations and given that the estimated σ-coefficients are small, a definite conclusion can hardly be drawn.

4 Discussion

4.1 The Special Role of the U.S. Dollar

The trade flows of many countries are heavily invoiced in U.S. dollars although some of them do not trade much with the U.S. at all. Poland is a typical case in point. In 2002

[17] See the data appendix for an exact description.

roughly 30 percent of total exports from Poland and an equal percentage of total imports to Poland were invoiced in U.S. dollars but only about 3 percent of Polish trade during that year was conducted with the U.S.[18] Except for the U.S. one cannot find such an asymmetry between invoicing and trade flows for any other country including the eurozone. What can explain this asymmetry?

One explanation for this phenomenon is that invoicing in a vehicle currency can be efficient by minimizing transaction costs. The three-country model by Rey (2001) is a recent contribution to this literature.

An alternative explanation is that a large proportion of primary and intermediate goods are traditionally priced in U.S. dollars, in particular raw materials such as oil products, metals and other fairly homogeneous commodities. The model developed in Section 2 emphasizes the currency denomination of inputs and the hedging of exchange rate risk as major driving forces behind invoicing decisions. If firms are dependent on inputs that are denominated in U.S. dollars, they have an incentive to price in U.S. dollars even if their products are no raw materials and no homogeneous goods.

In fact, the emphasis on the currency denomination of inputs can potentially also explain why invoicing involving the U.S. is qualitatively different from invoicing that does not involve the U.S. As documented, for instance, by Mann (1986) and Knetter (1989 and 1993), U.S. exporters follow PCP significantly more than non-U.S. exporters, and importers to the U.S. follow LCP significantly more than importers to other countries. This asymmetry implies a disproportionately heavy use of the U.S. dollar as an invoicing currency for trade that involves the U.S. American firms tend to price in U.S. dollars because their costs are mainly denominated in U.S. dollars. Non-U.S. firms typically face a smaller share of their costs in U.S. dollars but when trading with the U.S., they are nevertheless inclined to price in U.S. dollars because it provides them with an automatic hedge. If they priced in their domestic currencies, they would not have this automatic hedge.

4.2 Questions for Future Research

Data on invoicing are still rare to find. The empirical literature so far has naturally focused on invoicing in vehicle currency because of better data availability. But we still hardly know for which type of trading partner countries invoice in vehicle currency. Japan is currently one of the very few countries to provide at least a rough breakdown of vehicle currency use into destination countries and regions. Apart from invoicing in vehicle currency, it is also important to collect more observations on invoicing in non-vehicle currencies.

[18] Similarly, Friberg (1998) points out that 50 percent of world trade is invoiced in U.S. dollars while the U.S. share of world trade in manufactured goods is only 14 percent.

In addition, there is a need for industry-specific data on invoicing behavior. Goldberg and Tille (2005) demonstrate the theoretical role of different demand elasticities across industries for herding in invoicing decisions. In the context of the currency denomination of inputs it would matter whether an industry is labor-intensive and therefore faces a bigger share of costs in domestic currency. The European Central Bank (2005) provides a first breakdown of invoicing currency use into goods and services. Presuming that services are more intensively produced with domestic inputs such as labor, one would expect that service industries are more prone to price in producer currency.

In order to predict invoicing patterns, it would be instructive to examine time series data. According to the model of Section 2, if the U.S. share of world output keeps on falling, then the use of the U.S. dollar as an invoicing currency will diminish.[19] But time series data on invoicing are scarce. Korea is exceptional in that it reports invoicing data for the U.S. dollar, the yen, Deutschmark and pound sterling from 1976 until 2001. Some few time series data have also been collected by the European Central Bank.

Finally, it has not been studied empirically what the effects of financial hedging products are on invoicing decisions. Friberg (1998) develops a model in which exporters have access to a forward currency market and predicts that the expansion of forward markets should lead to more invoicing in the importer's currency.

5 Conclusion

The choice of invoicing currency is fundamental for the international transmission of macroeconomic policy and it is therefore essential to understand the factors that drive the choice of invoicing currency. This paper develops a three-country model of monopolistic competition in which firms preset prices under exchange rate risk. They can invoice either in producer currency, in local currency or in a third vehicle currency and endogenously choose the invoicing currency that maximizes their expected profits. The model is partial equilibrium but the same invoicing decisions would arise in a general equilibrium framework since the monopolistic firms take aggregate variables as given.

The key feature of the model is that firms face some of their production costs in foreign currency, an assumption which is consistent with the empirical evidence of the high pass-through of nominal exchange rates to import and wholesale prices. A forthright hedging intuition arises in that whenever a firm faces a high proportion of its costs denominated in a particular currency, it has an incentive to invoice in that currency.

In addition to the hedging intuition, invoicing decisions are also driven by exchange rate characteristics. If a certain currency is volatile relative to others, it is less suitable as an invoicing currency. Exchange rate correlations also play an eminent role. If a third

[19] The fact that the U.S. has continuously become a more open economy since the end of World War II does not alter this prediction because the rest of the world has expanded trade even more quickly.

currency moves in lockstep with the vehicle currency, firms are less inclined to invoice in their domestic currency because the high correlation accentuates the importance of the vehicle currency. The model can account for the disproportionate empirical prevalence of U.S. dollar invoicing for all trade involving the U.S. The disproportionate use can be attributed to the fact that the dollar is heavily used for pricing intermediate goods like oil so that it becomes optimal for firms to hedge their costs by invoicing in U.S. dollars.

Furthermore, the model is tested empirically with a comprehensive data set that encompasses 24 reporting countries and ten invoicing currencies. Vehicle currency pricing is distinguished from local currency pricing using a criterion based on the model. The results confirm the importance of currency comovements for the decision to invoice in vehicle currency. The findings also imply that if the U.S. share of world output continues to fall, other currencies will increasingly replace the U.S. dollar as an international vehicle currency.

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Technical Appendix

In order to derive cost function (5), solve the production function (3) for the individual input factor $N_{j,m}$ and plug the solution into cost function (4). Then take the derivative with respect to $N_{j,k}$, set the derivative to zero and solve for $N_{j,k}$ to obtain a first-order condition for $N_{j,k}$. Repeat this last procedure for $N_{j,l}$ to obtain a first-order condition for $N_{j,l}$. Combine the first-order conditions to obtain the optimal input demand functions

$$N_{j,k} = \left(\frac{e_{j,k}R_k}{\alpha_{j,k}}\right)^{\alpha_{j,k}-1}\left(\frac{e_{j,l}R_l}{\alpha_{j,l}}\right)^{\alpha_{j,l}}\left(\frac{e_{j,m}R_m}{\alpha_{j,m}}\right)^{\alpha_{j,m}} Y_j^T \tag{13}$$

$$N_{j,l} = \left(\frac{e_{j,k}R_k}{\alpha_{j,k}}\right)^{\alpha_{j,k}}\left(\frac{e_{j,l}R_l}{\alpha_{j,l}}\right)^{\alpha_{j,l}-1}\left(\frac{e_{j,m}R_m}{\alpha_{j,m}}\right)^{\alpha_{j,m}} Y_j^T \tag{14}$$

A first-order condition for $N_{j,m}$ can be derived by first solving the production function (3) for the individual input factor $N_{j,k}$ and then following the above steps analogously. The resulting optimal input demand function is

$$N_{j,m} = \left(\frac{e_{j,k}R_k}{\alpha_{j,k}}\right)^{\alpha_{j,k}}\left(\frac{e_{j,l}R_l}{\alpha_{j,l}}\right)^{\alpha_{j,l}}\left(\frac{e_{j,m}R_m}{\alpha_{j,m}}\right)^{\alpha_{j,m}-1} Y_j^T \tag{15}$$

Finally, plug (13)-(15) into (4) and make use of the assumption of constant returns to scale ($\alpha_{j,k}+\alpha_{j,l}+\alpha_{j,m}=1$) to yield cost function (5).

In order to derive invoicing condition (8), set $h=j=k$ and $i=l$ in the generic expected profits (6) and in the generic optimal price (7) to obtain $\mathrm{E}\left[\pi_{k,l}^k\right]$ and the PCP and VCP price $p_{k,l}^k$ (PCP=VCP), respectively. Set $j=k$ and $h=i=l$ to obtain profits $\mathrm{E}\left[\pi_{k,l}^l\right]$ and the LCP price $p_{k,l}^l$. Then set up the necessary and sufficient condition that for PCP=VCP to be chosen over LCP it must be

$$\mathrm{E}\left[\pi_{k,l}^k\right] > \mathrm{E}\left[\pi_{k,l}^l\right] \tag{16}$$

Based on (6) inequality (16) is given by

$$\begin{aligned}&\mathrm{E}\left[\left(p_{k,l}^k - B_k R_k^{\alpha_{k,k}} R_l^{\alpha_{k,l}} R_m^{\alpha_{k,m}} e_{k,l}^{\alpha_{k,l}} e_{k,m}^{\alpha_{k,m}}\right)\left(\frac{e_{l,k}p_{k,l}^k}{P_l^T}\right)^{-\rho} C_l^T\right]\\ &> \mathrm{E}\left[\left(e_{k,l}p_{k,l}^l - B_k R_k^{\alpha_{k,k}} R_l^{\alpha_{k,l}} R_m^{\alpha_{k,m}} e_{k,l}^{\alpha_{k,l}} e_{k,m}^{\alpha_{k,m}}\right)\left(\frac{p_{k,l}^l}{P_l^T}\right)^{-\rho} C_l^T\right]\end{aligned} \tag{17}$$

Since a monopolistic firm takes the price level P_l^T and consumption C_l^T as given, they can be dropped on both sides of inequality (17). The factor prices R are also taken as given. Plugging the prices $p_{k,l}^k$ and $p_{k,l}^l$ into (17), noting that $e_{l,k}=e_{k,l}^{-1}$ by definition and

rearranging yields

$$\begin{aligned}
&\tfrac{1}{\rho-1}B_k R_k^{\alpha_{k,k}} R_l^{\alpha_{k,l}} R_m^{\alpha_{k,m}} \,\mathrm{E}\left[e_{k,l}^{\alpha_{k,l}+\rho} e_{k,m}^{\alpha_{k,m}}\right] \left(\tfrac{\rho}{\rho-1}B_k R_k^{\alpha_{k,k}} R_l^{\alpha_{k,l}} R_m^{\alpha_{k,m}} \frac{\mathrm{E}\left[e_{k,l}^{\alpha_{k,l}+\rho} e_{k,m}^{\alpha_{k,m}}\right]}{\mathrm{E}\left[e_{k,l}^{\rho}\right]}\right)^{-\rho} \\
&> \tfrac{1}{\rho-1}B_k R_k^{\alpha_{k,k}} R_l^{\alpha_{k,l}} R_m^{\alpha_{k,m}} \,\mathrm{E}\left[e_{k,l}^{\alpha_{k,l}} e_{k,m}^{\alpha_{k,m}}\right] \left(\tfrac{\rho}{\rho-1}B_k R_k^{\alpha_{k,k}} R_l^{\alpha_{k,l}} R_m^{\alpha_{k,m}} \frac{\mathrm{E}\left[e_{k,l}^{\alpha_{k,l}} e_{k,m}^{\alpha_{k,m}}\right]}{\mathrm{E}\left[e_{k,l}\right]}\right)^{-\rho}
\end{aligned}$$

which simplifies to

$$\left(\mathrm{E}\left[e_{k,l}^{\alpha_{k,l}+\rho} e_{k,m}^{\alpha_{k,m}}\right]\right)^{1-\rho} \left(\mathrm{E}\left[e_{k,l}^{\rho}\right]\right)^{\rho} > \left(\mathrm{E}\left[e_{k,l}^{\alpha_{k,l}} e_{k,m}^{\alpha_{k,m}}\right]\right)^{1-\rho} \left(\mathrm{E}\left[e_{k,l}\right]\right)^{\rho} \tag{18}$$

To solve inequality (18) the moment-generating function of the joint lognormal distribution for $e_{k,l}$ and $e_{k,m}$ is required. Under the assumption of $\mu_{k,l} = \mu_{k,m} = 0$ as in Section 2.2.3, it is given by

$$\mathrm{E}\left[e_{k,l}^{r} e_{k,m}^{s}\right] = \exp\left(\frac{1}{2}[r^2\sigma_{k,l}^2 + 2rs\sigma_{l,m} + s^2\sigma_{k,m}^2]\right) \quad \text{for } r, s \in \mathbb{R} \tag{19}$$

A good introduction to lognormal distributions is provided by Kleiber and Kotz (2003, Chapter 4). Apply the moment-generating function (19) to inequality (18) and take natural logarithms to obtain

$$\begin{aligned}
&(1-\rho)\,\tfrac{1}{2}\left[\left(\alpha_{k,l}+\rho\right)^2\sigma_{k,l}^2 + 2\left(\alpha_{k,l}+\rho\right)\alpha_{k,m}\sigma_{l,m} + \alpha_{k,m}^2\sigma_{k,m}^2\right] + \rho^3\tfrac{1}{2}\sigma_{k,l}^2 \\
&> (1-\rho)\,\tfrac{1}{2}\left[\alpha_{k,l}^2\sigma_{k,l}^2 + 2\alpha_{k,l}\alpha_{k,m}\sigma_{l,m} + \alpha_{k,m}^2\sigma_{k,m}^2\right] + \rho\tfrac{1}{2}\sigma_{k,l}^2
\end{aligned}$$

Now rearrange and also use $\alpha_{k,m} = 1 - \alpha_{k,k} - \alpha_{k,l}$ to yield invoicing condition (8). Note that the parameter ρ drops out of the inequality.

In order to derive invoicing condition (9), generate the LCP and VCP price $p_{l,k}^k$ (LCP=VCP) and the PCP price $p_{l,k}^l$ from the generic optimal price (7) and plug them into the necessary and sufficient condition for LCP=VCP to be chosen over PCP that

$$\mathrm{E}\left[\pi_{l,k}^k\right] > \mathrm{E}\left[\pi_{l,k}^l\right]$$

Follow the steps of the previous paragraph analogously, noting that $e_{l,k} = e_{k,l}^{-1}$ by definition and $e_{l,m} = e_{k,m}/e_{k,l}$ due to triangular arbitrage, to arrive at the inequality

$$\left(\mathrm{E}\left[e_{k,l}^{-\alpha_{l,k}-\alpha_{l,m}} e_{k,m}^{\alpha_{l,m}}\right]\right)^{1-\rho} \left(\mathrm{E}\left[e_{k,l}^{-1}\right]\right)^{\rho} > \left(\mathrm{E}\left[e_{k,l}^{-\alpha_{l,k}-\alpha_{l,m}-\rho} e_{k,m}^{\alpha_{l,m}}\right]\right)^{1-\rho} \left(\mathrm{E}\left[e_{k,l}^{-\rho}\right]\right)^{\rho}$$

Use the moment-generating function (19) and $\alpha_{l,k} + \alpha_{l,l} + \alpha_{l,m} = 1$ to yield invoicing condition (9).

For the derivation of invoicing condition (10) set up the condition that for VCP to be

chosen over PCP it must be

$$\mathrm{E}\left[\pi_{l,m}^{k}\right] > \mathrm{E}\left[\pi_{l,m}^{l}\right] \tag{20}$$

Generate the VCP price $p_{l,m}^{k}$ and the PCP price $p_{l,m}^{l}$ from (7) as well as $\mathrm{E}\left[\pi_{l,m}^{k}\right]$ and $\mathrm{E}\left[\pi_{l,m}^{l}\right]$ from (6). Plug the prices and expected profits into inequality (20) and use $e_{l,k} = e_{k,l}^{-1}$, $e_{m,k} = e_{k,m}^{-1}$, $e_{l,m} = e_{k,m}/e_{k,l}$ and $e_{m,l} = e_{k,l}/e_{k,m}$ to solve for

$$\left(\mathrm{E}\left[e_{k,l}^{-\alpha_{l,k}-\alpha_{l,m}} e_{k,m}^{\alpha_{l,m}+\rho}\right]\right)^{1-\rho}\left(\mathrm{E}\left[e_{k,l}^{-1} e_{k,m}^{\rho}\right]\right)^{\rho} > \left(\mathrm{E}\left[e_{k,l}^{-\alpha_{l,k}-\alpha_{l,m}-\rho} e_{k,m}^{\alpha_{l,m}+\rho}\right]\right)^{1-\rho}\left(\mathrm{E}\left[e_{k,l}^{-\rho} e_{k,m}^{\rho}\right]\right)^{\rho}$$

Use the moment-generating function (19) and $\alpha_{l,k} + \alpha_{l,l} + \alpha_{l,m} = 1$ to yield invoicing condition (10).

For the derivation of invoicing condition (11) set up the condition that for VCP to be chosen over LCP it must be

$$\mathrm{E}\left[\pi_{l,m}^{k}\right] > \mathrm{E}\left[\pi_{l,m}^{m}\right]$$

and generate the LCP price $p_{l,m}^{m}$ from (7). Follow the procedure outlined in the preceding paragraphs to arrive at the inequality

$$\left(\mathrm{E}\left[e_{k,l}^{-\alpha_{l,k}-\alpha_{l,m}} e_{k,m}^{\alpha_{l,m}+\rho}\right]\right)^{1-\rho}\left(\mathrm{E}\left[e_{k,l}^{-1} e_{k,m}^{\rho}\right]\right)^{\rho} > \left(\mathrm{E}\left[e_{k,l}^{-\alpha_{l,k}-\alpha_{l,m}} e_{k,m}^{\alpha_{l,m}}\right]\right)^{1-\rho}\left(\mathrm{E}\left[e_{k,l}^{-1} e_{k,m}\right]\right)^{\rho}$$

which can be solved to obtain invoicing condition (11).

Invoicing condition (12) follows from the initial inequality for LCP to be chosen over PCP

$$\mathrm{E}\left[\pi_{l,m}^{m}\right] > \mathrm{E}\left[\pi_{l,m}^{l}\right]$$

which can be equivalently expressed as

$$\left(\mathrm{E}\left[e_{k,l}^{-\alpha_{l,k}-\alpha_{l,m}} e_{k,m}^{\alpha_{l,m}}\right]\right)^{1-\rho}\left(\mathrm{E}\left[e_{k,l}^{-1} e_{k,m}\right]\right)^{\rho} > \left(\mathrm{E}\left[e_{k,l}^{-\alpha_{l,k}-\alpha_{l,m}-\rho} e_{k,m}^{\alpha_{l,m}+\rho}\right]\right)^{1-\rho}\left(\mathrm{E}\left[e_{k,l}^{-\rho} e_{k,m}^{\rho}\right]\right)^{\rho}$$

As stated in Section 2.3.3, for the case of $\sigma_{k,l}^2 > \sigma_{l,m} > 0$ and $\sigma_{k,m}^2 > \sigma_{l,m} > 0$ particularly simple sufficient invoicing conditions can be derived for pricing between non-vehicle countries. In this case $\alpha_{l,k} > 1/2$ is a sufficient condition for VCP. In order to derive this result, use $\alpha_{l,k} + \alpha_{l,l} + \alpha_{l,m} = 1$ to rewrite invoicing conditions (10) and (11) as

$$\begin{aligned}\left(\alpha_{l,k} - \frac{1}{2}\right) &> \alpha_{l,m}\frac{\sigma_{l,m} - \sigma_{k,l}^2}{\sigma_{k,l}^2} \\ \left(\alpha_{l,k} - \frac{1}{2}\right) &> \alpha_{l,l}\frac{\sigma_{l,m} - \sigma_{k,m}^2}{\sigma_{k,m}^2}\end{aligned}$$

Similarly, $\alpha_{l,m} > 1/2$ is a sufficient condition for LCP. In order to derive this result, switch

the inequality sign of (11) to obtain

$$\left(\alpha_{l,m} - \frac{1}{2}\right) > -\alpha_{l,l}\frac{\sigma_{l,m}}{\sigma^2_{k,m}}$$

and rewrite (12) as

$$\left(-\frac{1}{2} + \alpha_{l,m} + \alpha_{l,k}\right)\left(\sigma^2_{k,l} - \sigma_{l,m}\right) > \left(\frac{1}{2} - \alpha_{l,m}\right)\left(\sigma^2_{k,m} - \sigma_{l,m}\right)$$

Finally, $\alpha_{l,l} > 1/2$ is a sufficient condition for PCP, which can be seen by switching the inequality sign of (10)

$$\alpha_{l,m}\frac{\sigma_{l,m}}{\sigma^2_{k,l}} > \left(\frac{1}{2} - \alpha_{l,l}\right)$$

and switching inequality (12) and rewriting it as

$$\left(\frac{1}{2} - \alpha_{l,m}\right)\left(\sigma^2_{k,m} - \sigma_{l,m}\right) > \left(\frac{1}{2} - \alpha_{l,l}\right)\left(\sigma^2_{k,l} - \sigma_{l,m}\right)$$

Data Appendix

All invoicing data refer to the invoicing of exports unless indicated otherwise. The years of observation vary between 1996 and 2004. Whenever data are available for multiple years, the most recent observations are chosen. Goldberg and Tille (2005, Appendix Table 1) give an overview of data availability.

The UK invoicing data are taken from the currency of invoicing press release by HM Revenue & Customs that can be downloaded from `http://www.hmrc.gov.uk/`. The data are for the year 2001, released in July 2002. The UK-Japan and UK-Canada observations have been computed with the data given in Table 4c. The UK-U.S. and the UK-eurozone data are taken from Table 4a. The invoicing data for the seven eurozone countries for exports in U.S. dollars are for the year 2002 and taken from Goldberg and Tille (2005, Appendix Table 2). The data for the ten new EU members for invoicing in U.S. dollars and euros are taken from Goldberg (2005, Table 1). Most of them are reported for the year 2002. For Latvia the data are a combination of invoicing of exports and imports. For Malta only the invoicing share of imports is available. The Bulgarian invoicing data are downloaded from the Bulgarian National Bank website at `http://www.bnb.bg/`, using the annual export invoicing data. The Australian invoicing data are downloaded from the Australian Bureau of Statistics website at `http://www.abs.gov.au/` and are reported in the feature article "Export and Import Invoice Currencies." The data are taken from Table 1 for the March quarter of 2004. The Japanese invoicing data are downloaded from the Japanese Ministry of Finance website at `http://www.mof.go.jp/english/`. They can be found in the report by the "Study Group for the Promotion of the Internationalization of the Yen," released in June 2001. The Korean invoicing data are taken from Table 2 (1) in Fukuda and Ono (2004). Their paper can be downloaded at `http://www.e.u-tokyo.ac.jp/cirje/`. The Korea-Germany observation is for 1998, the other observations are for 2001. The Malaysia and Thailand data are taken from an unpublished monograph by Chirathep Senivongs (1997), "Currency Internationalization in Selected ASEAN Countries," International Monetary Fund. The data are reproduced in Ngiam Kee Jin (2002, Table 1), "Financial and Monetary Cooperation in East Asia: The Singapore Perspective," Institute of Southeast Asian Studies. The data are for 1996.

The export data are taken from the IMF Direction of Trade Statistics (DOTS) through `http://www.esds.ac.uk/`. All export data are reported in U.S. dollars. The eurozone is treated as one country such that total exports from the eurozone include exports to non-eurozone destinations only and no intra-eurozone exports.

The GDP data for individual countries except for Bulgaria are taken from the IMF International Financial Statistics (IFS) through `http://www.esds.ac.uk/` for the same years as the corresponding invoicing observations. Lines 99B.CZF and 99B..ZF are used

for nominal GDP, lines 99BIRZF and 99BIPZF are used for the GDP deflator with the base year 2000. The RF.ZF period average exchange rate is used to convert real GDP into U.S. dollars. The Bulgarian and world real GDP data are taken from the United Nations Statistics Division available at `http://unstats.un.org/`.

The raw exchange rate data are taken from the IMF IFS through `http://www.esds.ac.uk/`, using the monthly end of period market exchange rate series (line ..AE.ZF). For each invoicing observation the exchange rate variances and covariances are computed by considering exchange rate data for the five years prior to the observation year and for the observation year itself, i.e. for six years in total. For a number of variances and covariances involving the euro the calculations have to be based on time series of less than six years because the euro was only launched in 1999. The exchange rate variances and covariances are computed in line with the assumption of the joint lognormal distribution in Section 2.2.3 in that the natural logarithm of the exchange rate series is taken and their means are subtracted, consistent with the assumption $\mu_{k,l} = \mu_{k,m} = 0$. The σ-variances and covariances are then computed with the demeaned logarithmic series.

As a robustness check of the results reported in Table 2, two alternative sets of σ-regressors are used. The first set is based on variances and covariances that are computed with the demeaned logarithmic values of detrended exchange rate series. As a simple linear detrending method, the linear trend between the first and the last observations is deducted from the individual observations of each exchange rate series so that the first and last values of the resulting series are equal. The second set is based on variances and covariances that are computed by considering exchange rate data for the ten years prior to the observation year and for the observation year itself, i.e. for eleven years in total. Exchange rate data for the European Currency Unit (ECU) are used as euro observations prior to 1999, provided by the Federal Reserve Economic Data database (FRED) at `http://research.stlouisfed.org/fred/`. These data report monthly averages of daily figures as opposed to end of period observations.