Vol XXXVII
No. 1

ISSN 0019-5014

JANUARY-
MARCH
1982

INDIAN JOURNAL OF AGRICULTURAL ECONOMICS

INDIAN SOCIETY OF
AGRICULTURAL ECONOMICS,
BOMBAY

ARTICLES

CHANGES IN RELATIVE FACTOR SHARES IN UNITED STATES AGRICULTURAL AND MANUFACTURING SECTORS

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The purpose of this paper is to examine how relative factor shares change when different factor proportions are varied in the United States agricultural and manufacturing sectors. The relative factor share is defined as the share of return to a particular factor of production in the value of output produced. The factor proportion measures the ratio of quantities of two inputs used in production.

The paper focuses upon the relative factor shares and factor proportions for certain key inputs such as land, fertilizer, labour and machinery in agriculture, and capital, energy, raw materials and labour in the manufacturing sector. By examining the changes in the factor shares one can get some idea about changes in income distribution among key classes such as labour and owners of capital. In view of the growing concern about energy scarcity around the world one can draw useful lessons from the United States experience about the impact of changes in energy intensity in production upon income distribution. These lessons are particularly important to developing countries due to their interest in reducing income disparity, on the one hand, and, on the other hand, due to their current policies for reducing the use of modern energy inputs in production in various sectors.

The paper uses the theoretical framework used by Hicks for two-input production technology and later generalised by Ranade for multiple input production technology.[1] In 1932, Hicks explored a way to relate the changes in the relative factor shares with factor proportions by using the elasticity of substitution when production involves only two inputs. In a recent paper, Ranade has shown that Hicks coefficient can be used to examine the changes in the relative factor shares for many input production technology. He showed that only for two input production functions, the Hicks coefficient is identical with the elasticity of substitution.

Researchers have widely used the elasticity of substitution for examining factor shares for two input production functions. In their well-known paper on inter-country CES production functions, Arrow, Chenery, Minhas and Solow showed that the relative factor share of labour increases when capital-

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I am grateful to Timothy D. Mount and the anonymous referee of this *Journal* for their critical comments on the previous draft. I am also grateful to John Mellor for his encouragement and for providing financial support for writing this paper.

1. See John R. Hicks: The Theory of Wages, Macmillan and Co. Ltd., London, 1932, and C. G. Ranade, "Hicks Coefficient to Depict Direction of Movements in Relative Factor Shares in Agricultural Production", *The Southern Economic Journal*, Vol. XLIII, No. 3, 1977, pp. 1588-1593.

labour ratio increases.[2] Their conclusion was based upon their estimate of elasticity of substitution between capital and labour which was less than unity. Except for some recent empirical work done by Ranade on rice production in the Philippines, the criterion of Hicks coefficient has remained theoretical.[3] It is expected that this paper will contribute to the empirical application of Hicks coefficient.

The paper does not estimate any fresh cost or production functions for this purpose. Instead, it uses the results in two recent articles of Binswanger, and Berndt and Wood which estimate factor demand functions by using the cost function approach.[4] The former article examines derived demand for five inputs, namely, land, labour, fertilizer, machinery and other inputs in the agricultural sector for the period 1949 and 1964. The latter article examines derived demand for four inputs, namely, capital, labour, energy and intermediate inputs called materials in the manufacturing sector for the period 1947 to 1971. These two papers provide results and data useful for accomplishing the purpose of this paper.

Indeed, the purpose of this paper is not merely to estimate Hicks coefficient for the above two sectors. Instead, it is intended that the implications of this paper would add to our existing knowledge on changes in distribution of output among some key participants in U.S. production such as labourers and owners of land and capital as the quantities of two key inputs, energy and fertilizer, are varied with respect to the quantities of other inputs. Very little research is done in this area.

METHODOLOGY

First a brief sketch is given of the methodology for using Hicks coefficient to examine the changes in the relative factor shares. Then Hicks coefficients are derived from the translog cost function used by Binswanger, and Berndt and Wood.

Assume that both agricultural and manufacturing sectors operate under conditions of perfect competition in both product and factor markets. Let the production function for a sector be

$$Q = F(X_1, \ldots, X_i, \ldots, X_n) \qquad \ldots. \ (1)$$

$$\text{with } F_i = \frac{\partial Q}{\partial X_i} > 0 > \frac{\partial^2 Q}{\partial X_i^2} = F_{ii};\ i = 1, \ldots, n,$$

2. Kenneth J. Arrow, Hollis B. Chenery, Begicha S. Minhas and R. M. Solow, "Capital-Labor Substitution and Economic Efficiency", *The Review of Economics and Statistics*, Vol. XLIII, No. 3, August 1961, pp. 225-250.

3. For the survey of literature on this subject, see C. G. Ranade: "Distribution of Benefits from New Agricultural Technologies: A Study at Farm Level", Unpublished Ph.D. Thesis, Cornell University, Ithaca, New York, U.S.A., 1977. See also C. G. Ranade and Robert Herdt, "Shares of Farm Earnings from Rice Production" in Randolph Barker and Yujiro Hayami (Eds.): Economic Consequences of the New Rice Technology, International Rice Research Institute, Los Banos, Philippines, 1978.

4. Hans P. Binswanger, "A Cost Function Approach to the Factor Demand and Elasticity of Substitution", *American Journal of Agricultural Economics*, Vol. 56, No. 2, May 1974, pp. 377-386; and Ernst D. Berndt and David D. Wood, "Technology, Prices and Derived Demand for Energy", *The Review of Economics and Statistics*, Vol. LVII, No. 3, August 1975, pp. 259-268.

where Q is output and X_i is the ith input. The production function is assumed to be quasi-concave, that is, the contour set $C(Q) = \{X_i, \ldots, X_n | F \geqslant Q\}$ is convex for every Q.

Hicks coefficient, σ_{ij}, between ith and jth input is defined as follows:

$$\sigma_{ij} = \frac{F_i F_j}{F F_{ij}} \qquad \ldots. \quad (2)$$

and due to perfect competition the relative share of ith input S_i, is given as,

$$S_i = \frac{X_i F_i}{Q} \qquad \ldots. \quad (3)$$

Earlier Ranade had shown that the relationship between the rate of change in the relative share of ith factor and changes in the quantitites of all inputs is as follows:[5]

$$\frac{\triangle S_i}{S_i} = \sum_j S_j \left(\frac{1}{\sigma_{ij}} - 1\right) \left(\frac{\triangle X_j}{X_j} - \frac{\triangle X_i}{X_i}\right), \ i, j = 1, \ldots, n. \qquad \ldots. \quad (4)$$

Thus, if the pairwise Hicks coefficients between ith and other inputs are all less than unity and if the quantity of ith input grows slower than the quantity of other inputs, its relative share in output would increase over time. Thus, given the magnitudes of Hicks coefficients and changes in input quantities, the changes in the relative factor shares can be inferred.

Now let us see how to derive Hicks coefficient from a cost function. The cost function is derived from the first order conditions of the minimization of the cost of production, $C = \Sigma P_i X_i$, subject to a given output given by (1); where P_i = price of the ith input. Let such cost function be written as follows:

$$C = g\,(P_i, \ldots, P_n, Q). \qquad \ldots. \quad (5)$$

Using the Shephard's lemma it can be shown that

$$S_i = \frac{g_i P_i}{C}, \qquad \ldots. \quad (6)$$

where $g_i = \dfrac{\partial C}{\partial P_i} = X_i$. Furthermore, Sato and Koizumi[6] have shown that the partial elasticity of complementarity between ith and jth inputs c_{ij}, is

$$c_{ij} = \frac{F F_{ij}}{F_i F_j} = \frac{C}{P_i P_j} \frac{G_{ij}}{G} = \frac{1}{\sigma_{ij}} \qquad \ldots. \quad (7)$$

where $g_{ij} = \dfrac{\partial^2 C}{\partial P_i \, \partial P_j}$, and G and G_{ij} are the determinant and (i, j)th cofactor of the following bordered Hessian matrix:

5. See Ranade, "Hicks Coefficient to Depict Direction of Movements in Relative Factor Shares in Agricultural Production", *op. cit.*

6. R. Sato and T. Koizumi, "Production Function and Theory of Distributive Shares", *The American Economic Review*, Vol. LXIII, No. 3, June 1973, pp. 484-489.

$$\begin{vmatrix} 0 & g_1 & \cdots & g_n \\ g_1 & g_{11} & \cdots & g_{1n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ g_n & g_{n1} & \cdots & g_{nn} \end{vmatrix}$$

Note that Hicks coefficient σ_{ij} is simply the reciprocal of c_{ij}.

In particular, Binswanger, and Berndt and Wood considered the following linear homogeneous translog cost function:[7]

$$\ln C = \alpha_o + \Sigma\alpha_j \ln P_j + 1/2\ \Sigma\Sigma\gamma_{ij} \ln P_i \ln P_j \quad (i,j=1,\ldots,n) \qquad \ldots.(8)$$

subject to

$$\sum_j \alpha_j = 1,\ \sum_j \gamma_{ij} = 0 \text{ for all i, and } \gamma_{ij} = \gamma_{ji} \text{ for } i \neq j \qquad \ldots.(9)$$

where α's and γ's are the parameters to be estimated. It can be shown that for the translog cost function (8) subject to conditions (9) the elements of the bordered Hessian are the following:[8]

$$g_i = \frac{C}{P_i} S_i \qquad \ldots.(10)$$

$$g_{ii} = \frac{C}{P_i^2} (S_i^2 - S_i + \gamma_{ii}) \qquad \ldots.(11)$$

$$g_{ij} = \frac{C}{P_i P_j} (S_i S_j + \gamma_{ij}) \text{ for } i \neq j, \qquad \ldots.(12)$$

Thus once the values of C, P_i, S_i, α's and γ's are known, by substituting g_i and g_{ij} in the bordered Hessian, Hicks coefficient σ_{ij} can be estimated from equation (7). However, Binswanger's paper does not provide the estimates of P_i's and therefore the formula (7) is not directly applicable. This problem can be overcome as follows:

Let $y_i = \frac{P_i}{C} g_i$ and $y_{ij} = \frac{P_i P_j}{C} g_{ij}$. Substituting these values in the bordered Hessian it can be shown that

$$\sigma_{ij} = \frac{Y}{Y_{ij}} \qquad \ldots.(13)$$

where Y and Y_{ij} are the determinant and cofactor of the following matrix:

$$\begin{vmatrix} 0 & y_1 & \cdots & y_n \\ y_1 & y_{11} & \cdots & y_{1n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ y_n & y_{n1} & \cdots & y_{nn} \end{vmatrix}$$

7. In their paper the logarithm of output also appeared on the right hand side of (8). However, in their actual estimation output was held constant and therefore in deriving Hicks coefficient for the translog cost function output is not explicitly mentioned.

8. See the Appendix for these derivations.

Thus by substituting the estimates of S_i, a_i and γ_{ij}, given in the two papers, into equation (13), the changes in the relative factor shares are examined by making some alternative assumptions about the changes in factor proportions.

Note that for the translog cost function (8) the relative share of ith input can be computed from equation (6) as follows:

$$S_i = a_i + \sum_j \gamma_{ij} \ln P_j \quad (i, j, = 1, \ldots, n) \qquad \ldots (14)$$

From the above equation the changes in the relative factor shares due to changes in factor prices can be deduced from the estimates of parameters a_i and γ_{ij} presented in the papers of Binswanger, and Berndt and Wood. However, the same is not true about the changes in the relative factor shares according to changes in factor proportions. Hicks coefficient is useful for this purpose.

RESULTS AND IMPLICATIONS

For the agricultural production there are five factor inputs, namely, land, machinery, fertilizer, labour, and other inputs such as livestock, poultry, feeds, seeds and plants.[9] For the manufacturing sector there are four factor inputs, namely, capital, labour, energy, and materials such as agricultural goods, non-fuel mining, and construction; manufacturing excluding petroleum products; and transportation.[10]

Hicks coefficients between different pairs of the above inputs in each sector are presented in Tables I and II. These tables also present the direction of

TABLE I—HICKS COEFFICIENT AND DIRECTION OF CHANGES IN RELATIVE FACTOR SHARES IN THE UNITED STATES AGRICULTURAL SECTOR*

Pair of inputs		Hicks coefficient	Change in relative share of				
ith	jth	σ_{ij}	Land	Fertilizer	Machinery	Labour	Other inputs
Land	Fertilizer	—0·1879	+	—	0	0	0
Land	Machinery	0·7403	—	0	+	0	0
Land	Labour	0·2373	—	0	0	+	0
Land	Other	0·2812	—	0	0	0	+
Fertilizer	Machinery	0·4301	0	—	+	0	0
Fertilizer	Labour	0·2396	0	—	0	+	0
Fertilizer	Other	1·0131	0	+	0	0	—
Machinery	Labour	1·3009	0	0	+	—	0
Machinery	Other	2·1482	0	0	+	0	—
Labour	Other	—5·0454	0	0	0	+	—

* Hicks coefficient is computed from the average relative factor shares for the period 1947-64 (Binswanger, *op. cit.*, p. 383).

9. Binswanger, *op. cit.*, p. 385.
10. Berndt and Wood, *op. cit.*, p. 262.

TABLE II—HICKS COEFFICIENT AND DIRECTION OF CHANGES IN RELATIVE FACTOR SHARES IN THE UNITED STATES MANUFACTURING SECTOR*

Pair of inputs		Hicks coefficient σ_{ij}	Change in relative share of			
ith	jth		Capital	Labour	Energy	Intermediate inputs
Capital	Labour	3·8052	+	—	0	0
Capital	Energy	0·0544	—	0	+	0
Capital	Materials	0·5018	—	0	0	+
Labour	Energy	0·4718	0	—	+	0
Labour	Materials	0·5909	0	—	0	+
Energy	Materials	0·9808	0	0	—	+

* In computing σ_{ij} the estimates of the relative share for 1959 are used. The directions of changes in relative factor shares, however, do not change if the estimates of relative factor shares for other years during 1949 to 1971 are used.

changes in the relative factor shares as a particular ratio of input quantities increases, holding all other ratios of quantities of inputs constant. Such directions are derived by using the estimated Hicks coefficient in equation (4). These tables can be read as follows: Consider the first pair of inputs land (ith) and fertilizer (jth) in Table I. Then the signs in the first row of this table mean that as the quantity of land per unit of the quantity of fertilizer (land-fertilizer ratio) is decreased and if all other factor proportions are held constant, the relative share of land would decrease, that of fertilizer would increase while the relative shares of labour, machinery and other inputs would remain unchanged. In the following discussion, unless specified, a particular ratio of the input quantities is varied holding all other ratios of input quantities constant.

Consider first the relative share of labour. In both the sectors this share decreases as capital (machinery) per unit of labour increases. In contrast, this share increases when fertilizer and energy grow faster than labour. Thus increase in capital intensity, on the one hand, distributes less output to labour while increase in fertilizer and energy intensity is beneficial to labourers in terms of their relative share in output in each sector.

Increase in land per unit of machinery, labour or other inputs results in a decline in the relative share of land in the agricultural output. Increase in fertilizer per unit of land also results in a decline in the relative share of land. Increase in materials in terms of all other inputs in the manufacturing sector results in their decline in the share in output. Note that the bulk of these materials is composed of agricultural goods. Thus it appears that a relative increase in the use of agricultural goods in manufacturing production results in a decline in their relative benefits from the manufacturing sector production.

Increase in energy per unit of capital increases the relative share of capital in manufacturing while increase in fertilizer per unit of machinery increases the relative share of machinery in agriculture.

Thus in the agricultural sector, if fertilizer and land grow faster than labour the relative share of labour increases, while in the manufacturing sector, if energy and materials grow faster than labour the relative share of labour increases.

From equation (4) it can be verified that if the magnitudes of Hicks coefficients are closer to unity, the relative factor shares are less sensitive to changes in factor proportions. Since the estimates of the majority of pairwise Hicks coefficients are much different from unity (Tables I and II), technology in both the sector seems to have highly flexible relative factor shares with respect to changes in factor proportions.

In their well-known paper on inter-country CES production functions, Arrow, Chenery, Minhas and Solow concluded that the relative share of labour would increase over time if capital-labour ratio increases over time.[11] Their conclusion was based upon their estimate of elasticity of substitution between capital and labour which was less than unity. In contrast, this paper finds that as capital-labour ratio increases, holding other factor proportions constant, the relative share of labour would decrease over time.

CONCLUSIONS

This paper examines the changes in the relative factor shares by using Hicks coefficient for the United States agricultural and manufacturing sectors with many input production. It shows that the distribution of output would be skewed against labourers and capital owners if the use of energy input grows slower than the growth in employment and capital respectively. Similar is the effect of fertilizer on the relative share of labour and machinery in agriculture. Thus in the U.S.A. as energy intensity increased income distribution became favourable to labourers. In view of the recent energy crisis around the world this finding is important in setting policies for controlling energy use in production and for increasing employment. The finding would be operationally meaningful especially when the relationship between changes in the relative factor shares and changes in personal income distribution is known.

The findings show that the relative factor shares are highly flexible with respect to factor proportions in modern technology such as in the United States and there is substantial scope for altering the distribution of output favourable to labourers by altering factor proportions.

In contrast to this, in traditional agricultures of less developed countries (LDCs) where there are usually two relatively less substitutable inputs such as land and labour and where labour input grows faster than land, the relative share of labour invariably declines over time. However, in view of the current technological change in LDCs which has brought fertilizer and new cultural

11. Arrow *et al.*, *op. cit.*

practices in agriculture, it would be useful to study how the relative share of labour varies as the amount of fertilizer per unit of land is changed.

In this respect, contrary to the changes in the relative share of labour in United States agriculture, Ranade has found that in the Philippine rice production the relative share of labour declines as the quantity of fertilizer per unit of land is increased using modern technology.[12] Is this because farmers apply less than optimum amounts of fertilizer in Philippine rice production ? How do the relative factor shares vary as the amount of factor proportions are varied in production in other LDCs ?

APPENDIX

(A) To Derive the Elements of Bordered Hessian

Differentiating the translog cost function partially with respect to P_i we get

$$\frac{\partial C}{\partial P_i} = \frac{C}{P_i}\left(\alpha_i + \Sigma\gamma_{ij}\ \ln P_j\right) i, j = 1, \ldots, n \qquad \ldots.(1)$$

Thus

$$S_i = \frac{P_i}{C}\ g_i = \left(\alpha_i + \Sigma\gamma_{ij}\ \ln P_j\right),$$

and therefore

$$g_i = \frac{C}{P_i}\ S_i \qquad \ldots.(2)$$

Differentiating (1) partial with respect to P_i,

$$\frac{\partial^2 C}{\partial P_i^2} = \left(\frac{\partial C}{\partial P_i}\ \frac{1}{P_i} - \frac{C}{P_i^2}\right)\left(\alpha_i + \Sigma\gamma_{ij}\ \ln P_j\right) + \frac{C}{P_i^2}\ \gamma_{ii}$$

Substituting (2) and S_i in the above equation and rearranging the terms on the right hand side,

$$\frac{\partial^2 C}{\partial P_i^2} = \frac{C}{P_i^2}\left(S_i - S_{ii}^2 + \gamma_{ii}\right) = g_{ii} \qquad \ldots.(3)$$

Differentiating (1) partially with respect to P_j,

$$\frac{\partial^2\ C}{\partial P_i\ \partial P_j} = \frac{\partial C}{\partial P_j}\ \frac{1}{P_i}\left(\alpha_i + \Sigma\gamma_{ij}\ \ln P_j\right) + \frac{C}{P_i P_j}\ \gamma_{ij}$$

$$\frac{\partial^2\ C}{\partial P_i\ \partial P_j} = \frac{C}{P_i P_j}\left(S_i S_j + \gamma_{ij}\right) = g_{ij} \qquad \ldots.(4)$$

12. Ranade, "Distribution of Benefits from New Agricultural Technologies: A Study at Farm Level", *op. cit.*

(B) To Show $\sigma_{ij} = \frac{Y}{Y_{ij}}$

Substitute $g_i = \frac{C}{P_i} \gamma_i$ and

$g_{ij} = \frac{C}{P_i P_j} \gamma_{ij}$ in the bordered Hessian matrix.

Then the determinant of that matrix (G) and (i, j)th cofactor (G_{ij}) can be written as follows:

$$G = Y \; C^{n+1} / \prod_{i=1}^{n} P_i^2 \qquad \ldots.(5)$$

$$G_{ij} = Y_{ij} \; C^n / \left(P_i P_j \prod_{k=1}^{n} P_k^2 \right) \text{ where } k \neq 1 \neq j \qquad \ldots.(6)$$

It was shown that the (i, j)th Hicks coefficient is

$$\sigma_{ij} = \frac{G}{G_{ij}} \; \frac{P_i P_j}{C} \qquad \ldots.(7)$$

Substituting (5) and (6) in (7),

$$\sigma_{ij} = \frac{Y}{Y_{ij}} \; \frac{C^{n+1}}{\prod_{i=1}^{n} P_i^2} \; \frac{P_i P_j \prod_{k=1}^{n} P_k^2 \; P_i P_j}{C^n \qquad C} = \frac{Y}{Y_{ij}}$$