Relaxing Tax Competition through Public Good Differentation

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Relaxing Tax Competition through Public Good Differentiation[1]

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Abstract: This paper argues that, because governments are able to relax tax competition through public good differentiation, traditionally high-tax countries have continued to set taxes at a relatively high rate even as markets have become more integrated. The key assumption is that firms vary in the extent to which public good provision reduces costs. We show that Leviathan governments are able to use this fact to relax the forces of tax competition, reducing efficiency. When firms can 'vote with their feet' tax competition leads firms to locate in 'too many' jurisdictions. A 'minimum tax' further relaxes tax competition, further reducing efficiency.

Keywords: asymmetric equilibrium, core-periphery, non-renegotiable minimum tax, tax competition, tax harmonization.

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1 Introduction

A puzzle in public finance has come to light with the observation that even as markets for goods and mobile factors have become more integrated, some jurisdictions have continued to tax at a relatively high rate while others tax at low rates. Baldwin and Krugman (2004) were among the first to draw attention to this fact, commenting with reference to European nations that 'it has always been the case that tax rates have been higher in the core than the periphery.'[4] Baldwin and Krugman present data to show that corporate tax rates in the 'core' countries France, Germany, Italy and Benelux have always been higher than tax rates in the poorer periphery countries Ireland, Greece, Spain and Portugal. Data on the effective average tax rate (EATR) developed by Devereux and Griffith (2003) appears to confirm Baldwin and Krugman's assertions.[5] In 2003 the average tax rate in the periphery was significantly below that in the core, at 23 percent compared to 31 percent respectively.[6] Much of the previous literature has tended to focus on declining overall tax rates in both the core and periphery to motivate a 'race to the bottom'. In contrast, we focus on the fact that countries in the core have been able to maintain higher tax rates than have periphery countries.

The purpose of this paper is to present a novel view of tax competition in which governments are able to relax tax competition by providing public goods at different levels. Our view of tax competition helps to explain why core countries are able to maintain effective average tax rates on capital above those in the periphery. The idea that firms may be able to relax price competition by offering goods with different characteristics goes all the way back to Hotelling (1929), and has attracted attention more recently since the work of d'Aspremont, Gabszewicz, and Thisse (1979) and Shaked and Sutton (1982). But the idea that governments can relax tax competition by differential public good provision has been

[4]Baldwin and Krugman's (2004) paper originally appeared in 2000 as CEPR discussion paper number 2630.

[5]Devereux and Griffith (2003) collect data for all OECD countries, which are made available at http://www.ifs.org.uk/corptax/internationaltaxdata.zip. Devereux and Griffith work with these data to produce careful estimates of the effective average tax rates as well as effective marginal tax rates on capital across countries.

[6]In 1982 the average tax rate in the periphery was 31 percent compared to 42 percent in the core. While tax rates have fallen steadily from 1982 to 2003, convergence between the core and periphery rates has been limited.

overlooked.[7] Alongside differential public good provision, governments tax at different levels as well in equilibrium. Thus, we propose a simple explanation for why tax rates have not completely converged in Europe and elsewhere as markets have become more integrated. Our explanation is that the impact of public good provision on cost reduction varies across firms, and governments are able to use this fact to offset the forces of tax competition.

Our approach to examining tax competition is related to that of Kanbur and Keen (1993). Like Kanbur and Keen, our central focus is on the international policy-making environment, where the set of policy options is more limited than in a federal setting. Thus, while the issues that we investigate are similar to the problems of fiscal federalism investigated by Arnott and Grieson (1981), Gordon (1983) and Wilson (1986), the range of policy options that we consider are more limited than under federalism, mirroring more closely an international setting. We present a model that is simple enough to yield sharp insights into some key questions while being rich enough to capture some of the central features of the interaction between national tax systems in an integrated world.[8] However, unlike Kanbur and Keen, our purpose is not to analyze the effects of variation in country size on tax competition. While it is true that countries vary in size in our analysis, differing country size is a feature of equilibrium in our model, not an exogenous variable as in Kanbur and Keen's.

A central feature of our analysis that is not considered by Kanbur and Keen is the idea that firms vary in their requirements for public good provision and that this variation affects the equilibrium characteristics of jurisdictions. Building on Casella (2001), Casella and Feinstein (2002) describe the same variation in public good requirements that we have in mind: "[Public goods] can be given a physical interpretation - roads, airports, infrastructure - or ... they can be more abstract - laws and legal enforcement, rules and conventions, standards and regulations, currency and language. An important feature of the examples

[7]A related idea is explored by Hoyt and Jensen (2001). They too borrow the idea from the industrial organization literature that products can be differentiated and apply the analogy to the level of public good provision within the context of tax competition. However, their main focus is quite different from ours. They have a model of a metropolitan area in which the decision about where to reside is independent of the decision about where to work. House prices are then shown to depend on the level of public education provided. While tax competition is a feature of their model, they do not develop the idea of relaxed tax competition as we do here. Our focus is obviously at the country level, and firms cannot have a presence in more than one jurisdiction. (This aspect of our model set-up will be discussed at greater length below.)

[8]In contrast, Gordon (1983) and Mintz and Tulkens (1986) for example, obtain more general results on existence of equilibrium but the generality of their model precludes sharp characterization results. Moreover, any attempt to examine the impact on welfare of changes through comparative statics quickly becomes intractable.

we have in mind is that preferences over the specific realization of the public good are not homogeneous among all market participants, but depend on the individual's position within the market."

To model such variation in public good requirements, we have a continuum of firms uniformly distributed on a unit interval. The position of a firm on the interval reflects the extent to which public good provision reduces the firm's costs of production and delivery to market. For example, in the textiles and apparel industry, at one end of the interval there are the so called 'haute couture'; the leading designers in the creation of exclusive fashions. These firms make extensive use of international travel and communications networks; they employ highly educated and trained workers; they rely on intellectual property laws to safeguard returns on the designs that they create. At the other end of the interval there are sweat shops that produce copies of earlier designs, employ local and relatively low skilled workers, source inputs locally, and tend to copy rather than create the designs that they use, so do not rely on intellectual property protection. (More loosely, such firms may even be harmed by public good provision in the form of intellectual property protection if their intention is to produce copies of existing designs.) Casella (2001) and Casella and Feinstein (2002) analyze the effect of an expansion of the market on the incentive for firms or traders to form into jurisdictions (where the purpose of a jurisdiction is to provide local public goods that facilitate trade). Their concern, however, is not with how variation in firms' public good requirements affect tax competition.

We assume that governments are Leviathans, using the policy variables at their disposal to maximize the rents to office.[9] The governments play a two-stage game wherein each government aims to attract firms to its jurisdiction. In the first stage, governments simultaneously choose a level of public good provision. In the second stage, having observed the levels of public good provision chosen, governments simultaneously set taxes. This order of events is regarded to reflect the idea that taxes can be changed relatively easily once the level of public good provision has been chosen, while a change in the level of public good

[9]Although the assumption that governments are Leviathans is unsatisfactory, in that it leaves unmodelled the incentive structures that motivate politicians, it remains an influential approach in practical policy discussion. In addition to Kanbur and Keen (1993), see Hoyt (1995, 1999) and Keen and Katsogiannis (2003). The issue of how the objectives of policy makers should be modelled is reviewed comprehensively by Wildasin and Wilson (2001).

provision requires modification of the infrastructure through which it is provided.[10] Firms are not strategic. They simply take taxes and prices as given and locate in the jurisdiction where they make the highest profits.[11]. Our analysis is interesting from a theoretical standpoint because we find a unique asymmetric outcome even though jurisdictions are ex ante symmetric.[12] Asymmetric equilibria have been studied before in the tax competition literature but the emphasis of previous contributions differs from ours in two ways. First, in one branch of the literature the asymmetry of outcomes results from assumed asymmetry of jurisdictions while in our work, asymmetry of jurisdictions is a consequence of equilibrium.[13] Second, another branch of the literature obtains asymmetry of outcomes as a consequence of increasing returns to scale, where agglomeration brings about positive spillovers that can be captured through taxation.[14] Our model focuses purely on tax competition and we are able to draw clear-cut (analytically based) conclusions about the welfare effects of relaxing tax competition through public good differentiation.

In terms of welfare analysis, we are able to show that when tax competition is relaxed full efficiency is not achieved; there is under-provision of the public good. In the efficient solution of our model, all firms locate in the same jurisdiction, where the public good is provided at the efficient level.[15] Under relaxed tax competition, firms locate in one of two

[10] We are not the first to model interjurisdictional competition in tax and spending levels between Leviathan governments as a two stage game; this approach has been taken previously by Edwards and Keen (1996) among others. As Kreps and Scheinkman (1983) argue in their study of firm behavior, the appropriateness of the set-up, or the game context, is essentially an empirical matter. Certainly, it seems reasonable to argue that levels of public good provision are more difficult to change than taxes and so these are set in the first stage because governments can more easily commit to them. This parallels the familiar argument that firms can more easily commit to the capacity for production than prices. Then in the second stage governments announce taxes in the same way that firms announce prices.

[11] There are no multinational firms in our model. Each firm can locate in one and only one jurisdiction. We do not consider instances where a firm can avoid paying taxes by locating part of its production activity in a low tax jurisdiction. Below we suggest how our model could be extended in that direction.

[12] The equilibrium is unique in pure strategies up to a re-labelling of jurisdictions and their governments. There must also exist at least one mixed strategy Nash equilibrium. We do not consider mixed strategy Nash equilibria in our analysis for reasons discussed below.

[13] For example, Wilson (1987) studies a model of Heckscher-Ohlin trade and tax competition where one jurisdiction is endowed with more capital than the other. Consequently, public good provision is above the efficient level in one jurisdiction and inefficiently low in the other.

[14] See, for example, Kind et al (2000), Ludema and Wooton (2000) and Baldwin and Krugman (2004). These insightful models are complicated, however, and it is typically difficult to draw clear-cut conclusions about the welfare implications of tax competition.

[15] Our model could be extended by allowing for attachments to a jurisdiction. Conventional tax competition models do this by assuming that capital is mobile across jurisdictions while labor (or land) is attached to jurisdictions. It is obvious that extending the model in this way may cause some firms to locate in each jurisdiction under efficiency. However, this would obscure the welfare effects of relaxed tax competition.

jurisdictions, one of which provides no public good while the other provides the public good at a positive but still inefficiently low level. There is a marginal firm that is just indifferent between locating in either of the two jurisdictions. The jurisdiction that provides no public goods taxes at a relatively low level. The firms that locate there care more that taxes are low than that public good provision is high.

Moreover, we are able to show that the greater the cost-reducing impact of the public good on any given firm in the distribution, the more tax competition is relaxed. The more the public good reduces a firm's costs, the bigger the negative impact on the profit of any given firm of moving from the high-tax-high-public-good jurisdiction to the other one. Consequently, the more the public good reduces the cost of a given firm, the higher the high-tax government is able to set its tax without inducing that firm to switch to the other jurisdiction. But there is an additional effect. A higher tax in the high-tax jurisdiction makes it still less attractive to firms in the low-tax jurisdiction. So the low-tax government is able to set its tax at a higher level as well. This is the sense in which tax competition can be relaxed, and becomes increasingly relaxed the more the public good reduces the costs of firms across the distribution. Moving in the other direction, as the impact of the public good on firms' profits is reduced tax competition becomes more intense. As the cost-reducing impact of the public good converges to zero the outcome converges towards standard Leviathan tax competition (Brennan and Buchanan 1980), where the public good is not provided, but nor is it valued by firms, and there is efficiency in equilibrium.[16]

[16] While the impact on cost of the public good may be smaller on one firm than another, we have a parameter in the model that varies the impact of the public good on costs across all firms in the same proportion.

Our model follows the branch of the tax competition literature where competition promotes efficiency. Tiebout (1956) was the first to discuss the idea that competition between jurisdictions may promote efficiency. As mentioned above, Brennan and Buchanan (1980) discuss how competition ties the hands of Leviathans. Oates and Schwab (1988) show that majority rule can select the efficient outcome when there is interjurisdictional competition for mobile resources. Black and Hoyt (1989) show how the process by which jurisdictions bid for firms may promote efficiency. The efficiency promotion of tax competition has also been discussed by Boadway, Cuff and Marceau (2002), Boadway, Pestieau and Wildasin (1989), Wildasin (1989) and Wooders (1985). Another branch of the literature focuses on situations where policy makers tend to be benevolent and competition brings about inefficiency; see Gordon and Wilson (1986), Wilson 1986), Zodrow and Miezkowski (1986), Wildasin (1988), Wooders, Zissimos and Dhillon (2002). Rothstein (2004) examines the existence of equilibrium in such tax competition models. In a broader context, Gordon and Wilson (1999) examine how the benefits derived by government officials from the size of the tax base can affect the design of the tax system itself. Besley and Smart (2001) argue that the issue of whether tax competition raises or lowers efficiency depends on whether politicians are more likely to be benevolent or rent-seeking. Gordon and Wilson (2002) show that efficiency is promoted by competition when 'officials benefit by taking a smaller piece from a larger pie'. See Wilson (1999) for a comprehensive review of the earlier literature.

As far as we know, our paper is the first to establish links between the literature on tax competition and the literature on the number and size of countries. The fact that all firms locate in one jurisdiction under the efficient outcome arises because the good provided by governments is a nonrival public good and because there are no congestion externalities in our model. These features of our model are common to the literature on 'the number and size of countries' initiated by Alesina and Spolaore (1997).[17] Alesina and Spolaore show that the democratic process leads to an inefficiently large number of countries. We show that when firms are able to 'vote with their feet' then again the number of jurisdictions in which they choose to locate is inefficiently large.

Our paper is also the first of which we are aware to show that models of vertical product differentiation can be adapted to yield useful insights about tax competition. Our approach to the modelling of tax competition between governments is similar in some respects to that of Shaked and Sutton's (1982) to the modelling of price competition between firms. Shaked and Sutton have a model of vertical product differentiation in which firms first decide whether to enter the market, then decide on a quality level for the good that they produce, and then finally set prices. As consumers vary by willingness to pay for quality, which depends in turn on income, it is possible to determine the income level of a consumer who is just indifferent between purchasing two goods at differing quality levels in the same way that our model yields a firm that is just indifferent between location in the two jurisdictions. Shaked and Sutton have an extra stage in their game where entry is modelled, which seems natural when modelling firm behavior. In the present setting of competition between governments it seems more natural to hold the number of jurisdictions fixed. The implications of the fact that our tax competition model is based on the analytical structure of a vertical product differentiation model are discussed further in the conclusions.[18]

[17] See also Goyal and Staal (2004), who look beyond country formation to examine regionalism.

[18] A framework of horizontal (as opposed to vertical) product differentiation has also been adapted in previous work to the context of tax competition. Justman, Thisse and van Ypersele (2001) treat a local public good and contrast efficiency under complete information with inefficiency under incomplete information. Hohaus, Konrad and Thum (1994) and Wooders and Zissimos (2003) have models that are public good versions of Hotelling's original (1929) model. They borrow the idea from the 'horizontal product differentiation' literature that preferences or profit functions are single-peaked in the public good. While these earlier studies provide interesting insights into the nature of such tax competition, they are not well suited to address the question that concerns us here; that of why one jurisdiction would have more (or 'better') public goods and higher taxation than another in equilibrium. Moreover, the assumption embodied in the present paper, that profit functions are everywhere increasing and concave in the public good, is arguably valid for a wider class of public good than those captured by the assumption that preferences are single-peaked in the public good.

The paper proceeds as follows. In Section 2 we present the basic model. In Section 3 we solve for the efficient solution under the assumption that taxes and levels of public good provision are set by a planner. This may also be thought of as a 'federal' solution in which a federal government sets taxes and levels of public good provision across states. In Section 4 we model a game of tax competition between jurisdictions. In Section 4.1 we solve the second stage of the game, finding a unique subgame perfect equilibrium under tax competition, taking levels of the public good across jurisdictions as given. In Section 4.2, we solve the first stage and show (using backwards induction) that in equilibrium public good provision is below the efficient level. Section 5 then considers policies of tax coordination, focusing on tax harmonization and the imposition of a minimum tax.[19] When looking at a minimum tax, we are able to identify a 'non-renegotiable minimum tax frontier' for the minimum tax both when the minimum tax is anticipated before levels of public good provision are set and when the minimum tax is not anticipated. The *non-renegotiable minimum tax frontier* is the set of minimum taxes for which neither government can obtain higher rent by a change in the minimum tax without the other government having to accept lower rent. This is the only research that we are aware of to study the effects of a minimum tax in a setting where taxes and the level of public good provision are determined.[20] Section 6 provides a summary and conclusions.

2 The Model

There are two jurisdictions, A and B, each of which has a government that sets the level of public good provision, x_A and x_B respectively, and the tax level , τ_A and τ_B respectively for its jurisdiction. There is a set of firms each of which is able to sell a single unit of a good in the market.[21] We think of the jurisdictions as being countries and the market as being the world market. We now specify the behavior of firms, after which we will turn to governments. Finally, we set out the sequence of events in the policy setting game.

In the absence of the public good each firm incurs a private cost c to produce a unit of the

[19] A minimum tax is a floor below which neither government is allowed to set its tax.

[20] Kanbur and Keen (1993) study the effects of a minimum tax but not in a setting where the level of public good provision is determined.

[21] This could be generalized so that there are many firms at each point on the interval, but this would not add insight. Note that the location on $[0, 1]$ reflects technological requirement for the public good and not geographical location.

good that it sells and deliver it to market. But the public good provides a technology which reduces a firm's cost of production (or delivery to market).[22] The size of x_i captures the extent of public good provision in jurisdiction i; $i \in \{A, B\}$. The expression $kx_i{}^{\theta}$ captures the overall cost reducing impact across all firms in jurisdiction i, where $k > 0$ and $0 < \theta < 1$ are parameters. The parameter θ ensures that the impact of the public good is declining at the margin as we should generally expect.[23] The parameter k determines the overall impact of public good provision on profitability. Note that use of the public good generates no congestion externalities within the jurisdiction and no spillovers to other jurisdictions.

Firms take public good provision and tax levels as given and choose to locate in the jurisdiction where they make the highest profits.[24] Each firm is able to sell its single unit for price p.[25] The profit function for the firm at $s \in [0, 1]$ is given by

$$\begin{aligned} \pi_s &= p - c - \tau_i + skx_i{}^{\theta},\ i \in \{A, B\}, \\ 1 &> \theta > 0,\, k > 0. \end{aligned}$$

To focus the analysis on location decisions we shall assume that $p - c$ is fixed at a sufficiently high level so that in the analysis to follow there is an interior solution in which all firms make non-negative profits.[26]

The technological positions of firms are distributed uniformly on the interval $[0, 1]$. The (technological) position of a firm $s \in [0, 1]$ reflects the extent to which public good provision reduces the firm's costs. Thus the cost-reducing impact of the public good on an individual firm is given by $skx_i{}^{\theta}$. For a given increase in public good provision, the further a firm is to the right of the interval the greater is the cost-reducing impact of the public good on the

[22]For some types of public good such as intellectual property protection it is more appropriate to think of the public good reducing the ex ante expected cost of production. This is consistent with our analytical framework although our model is deterministic.

[23]The parameter θ determines the elasticity of profit with respect to public good provision. But in the present model (unlike in 'standard' tax competition models) it is the net contribution of the *level* of taxation and public good provision to profits (or returns on capital) that is compared across jurisdictions when the location decision is made. This is because the entire firm is mobile in the present set-up, whereas in standard tax competition models only one factor is mobile while the other is fixed; see Devereux, and Griffith (2003) for further discussion of this point.

[24]Each firm must choose between one jurisdiction or the other. Our model could be extended to allow one firm to purchase the output of another and use that output as an intermediate input in its own production. In this way our model could be extended to consider certain types of multinational enterprise. However, such an extension would not change the basic insights of our model.

[25]The price that each firm receives for the good that it sells could be made to vary across firms without affecting the results.

[26]Because $p - c$ is assumed to be the same across jurisdictions, it does not affect a firm's location decision.

firm's production. If the firm at s locates in jurisdiction i it must pay a tax τ_i. The tax can be thought of as a lump sum tax or a sales tax (as each firm produces and sells only a single unit of the good).

We could think of all firms being in the same industry and s could reflect variation in adoption efficiency of the public good across firms. But our preferred interpretation is to think of the interval $[0,1]$ spanning firms in different industries, with the public good having a greater impact on costs in some industries than others. For example, at the left hand end of the interval we might have textiles manufacturers who require only fairly rudimentary levels of public good provision in the form of basic roads and unsophisticated communications networks to produce their products and bring them to market. At the right hand end of the interval we might have firms in the information and technology industries, which benefit more from the availability of good communications networks and roads as well as a more educated work-force.

Each firm takes τ_A, τ_B, x_A and x_B as given, choosing between A and B on the basis of where it makes the highest profits. If $x_A \neq x_B$ then without loss of generality we assume that $x_A < x_B$.[27] In that case a firm may find it profitable to locate in the jurisdiction with higher taxes if the cost reducing effect of the public good dominates.

For given τ_A, τ_B, x_A and x_B (taking $x_A < x_B$) we can calculate the position in $[0,1]$ of the marginal firm $\hat{s}$ that is just indifferent between locating in A and B. That is, the firm $\hat{s}(\tau_A, \tau_B)$ makes the same profits in either jurisdiction;

$$\tau_A - \hat{s}k{x_A}^\theta = \tau_B - \hat{s}k{x_B}^\theta.$$

Then $\hat{s}$ gives the share of firms in A and $1-\hat{s}$ gives the share of firms in B. We impose the necessary restrictions to ensure that the marginal firm must belong to the $[0,1]$ interval. First, solve the above expression for $\hat{s}$ and hence define the function[28]

$$\hat{s}(\tau_A, \tau_B, x_A, x_B) = \frac{\tau_B - \tau_A}{k\left({x_B}^\theta - {x_A}^\theta\right)} \tag{1}$$

Then $\hat{s}$, the share of firms in Jurisdiction A, is defined as follows:

$$\hat{s} = \begin{cases} \hat{s}(\tau_A, \tau_B, x_A, x_B) \text{ if } \hat{s}(\tau_A, \tau_B, x_A, x_B) \in [0,1]; \\ 1 \text{ if } \hat{s}(\tau_A, \tau_B, x_A, x_B) > 1; \\ 0 \text{ if } \hat{s}(\tau_A, \tau_B, x_A, x_B) < 0. \end{cases}$$

[27] In Section 4.2 we show that there exists a unique subgame perfect equilibrium in which one government must set a higher tax than the other. Then $x_A < x_B$ is just a choice of labelling.

[28] Parameter values k and θ will be suppressed throughout from general functional notation.

If $(\tau_B - \tau_A)/k\left(x_B{}^\theta - x_A{}^\theta\right) \in [0,1]$ it is easy to check that all firms $s \in [0,\hat{s})$ make higher profits in A than in B and all firms $s \in (\hat{s},1]$ make higher profits in B than in A. For the firms $s \in (\hat{s},1]$, the difference in the tax $\tau_B - \tau_A$ is dominated by the lower costs brought about by higher public good provision. Clearly, the higher is τ_B the smaller is the share of firms that finds it profitable to locate in jurisdiction B.

If $x_A = x_B$ then $\hat{s}$ as given by (1) is undefined. However, $x_A = x_B$ implies that the public good offered by the governments is homogeneous, and so firms can be thought of as responding in the manner of consumers in a Bertrand price setting game. So we borrow the usual Bertrand assumptions to define the distribution of firms between jurisdictions.[29] If $x_A = x_B$ then all firms locate in the jurisdiction with the lowest taxes:

$$\hat{s} = \begin{cases} 0 \text{ if } \tau_A < \tau_B; \\ 1 \text{ if } \tau_A > \tau_B; \\ \frac{1}{2} \text{ if } \tau_A = \tau_B. \end{cases}$$

We turn now to the governments. The *rents to office*, r_A, of Government A are given by the function $r_A = \tau_A \hat{s} - x_A$. The rents to office, r_B, of Government B are given by $r_B = \tau_B(1-\hat{s}) - x_B$. From the rent functions it is evident that the level of public good provision by a government also determines its cost; a level of public good provision x_i costs x_i to provide, $i \in \{A, B\}$. In cases where $\hat{s}$ is defined by (1), $r_A(\tau_A, \tau_B)$ and $r_B(\tau_A, \tau_B)$ are given as follows:

$$\begin{aligned} r_A(\tau_A, \tau_B, x_A, x_B) &= \frac{\tau_A(\tau_B - \tau_A)}{k\left(x_B{}^\theta - x_A{}^\theta\right)} - x_A; \\ r_B(\tau_A, \tau_B, x_A, x_B) &= \tau_B\left(1 - \frac{(\tau_B - \tau_A)}{k\left(x_B{}^\theta - x_A{}^\theta\right)}\right) - x_B. \end{aligned}$$

Otherwise, in situations were $\hat{s} = 0$, $r_A = -x_A$ and $r_B = \tau_B - x_B$ and where $\hat{s} = 1$, $r_A = \tau_A - x_A$ and $r_B = -x_B$. Of course, for the overall game we require that the governments be individually rational; each must make non-negative rents. The appropriate feasibility conditions will be imposed in due course.

To summarize, in terms of their technological requirements for public good provision, firms' positions are fixed in the interval $s \in [0,1]$, but each firm is able to choose its preferred

[29] Even when the levels of public good provision are not identical, the form of competition between governments conforms more generally to Bertrand competition. Sutton (1991) points out that the characterization of equilibrium holds for Bertrand or Cournot competition if the fixed cost (here the level of public good provision) is set in the first stage of the game and the price (here the tax) is set in the second stage.

jurisdiction to maximize profits.[30] Each government, on the other hand, is able to choose its level of taxation and public good provision but obviously its jurisdiction (A or B) is fixed.

The specific sequence of events is as follows. The public good is chosen in the first stage and taxation is chosen in the second stage; this holds whether taxes and public good levels are chosen by a planner or by competing governments. Effectively, we assume that the value of s for each firm cannot be observed by policy makers but that policy makers do know the distribution of firms, and can use this knowledge in setting taxes. In Section 3 we examine efficiency under the usual assumption that a planner chooses public goods and taxation. This establishes a benchmark with which the output for competition between governments can be compared. In Section 4 we consider a two-stage game in which policy decisions are taken by competing governments. In this case, at each stage the policy decisions are taken simultaneously and noncooperatively. Under both regimes, firms take public goods and taxes as given and choose location to maximize profits.

3 Efficiency

In this section we adapt a standard definition of efficiency to the context of the present model. We make the standard assumption that a planner chooses taxes τ_A and τ_B and public good levels x_A and x_B on behalf of the governments to maximize the combined government rents and firm profits across jurisdictions. Firms take τ_A, τ_B, x_A and x_B as given and locate in the jurisdiction where they make the highest profits as specified above. We maintain the assumption, without loss of generality, that when $x_A \neq x_B$ it is the case that $x_A < x_B$. Consequently it is possible to use (1) to solve for the marginal firm $\hat{s}$, and $\hat{s}$ can be used in the definition of efficiency.

[30] In principle a firm at $s \in [0, 1]$ could change its position in the interval. Perhaps it could make an investment that enabled it to make better use of the public good. However, this possibility is beyond the scope of the present paper, though we intend to take it up in future research.

Definition 1. A *plan*, consisting of a pair of taxes $\boldsymbol{\tau}^E = \left(\tau_A^E, \tau_B^E\right) \in \mathbb{R}_+^2$ and a public good allocation $\mathbf{x}^E = \left(x_A^E, x_B^E\right) \in \mathbb{R}_+^2$, is *efficient* if for all other pairs of taxes $\boldsymbol{\tau} = (\tau_A, \tau_B) \in \mathbb{R}_+^2$ and public good allocations $\mathbf{x} = (x_A, x_B) \in \mathbb{R}_+^2$, it holds that

$$\begin{aligned} & r_A\left(\tau_A^E, \tau_B^E, x_A^E\right) + r_B\left(\tau_A^E, \tau_B^E, x_B^E\right) + \int_0^{\hat{s}} \pi_s\left(\tau_A^E, x_A^E\right) ds + \int_{\hat{s}}^1 \pi_s\left(\tau_B^E, x_B^E\right) ds \\ \geq \;& r_A\left(\tau_A, \tau_B, x_A\right) + r_B\left(\tau_A, \tau_B, x_B\right) + \int_0^{\hat{s}} \pi_s\left(\tau_A, x_A\right) ds + \int_{\hat{s}}^1 \pi_s\left(\tau_B, x_B\right) ds. \end{aligned}$$

Under Definition 1, a pair of taxes and a public good allocation is efficient if it entails the largest possible surplus for division between the two governments and the firms. The planner's problem can be represented in the form

$$\begin{aligned} \max_{\tau_A, \tau_B, x_A, x_B} \Omega\left(\tau_A, \tau_B, x_A, x_B\right) &= r_A\left(\tau_A, \tau_B, x_A\right) + r_B\left(\tau_A, \tau_B, x_B\right) \\ &\quad + \int_0^{\hat{s}} \pi_s\left(\tau_A, x_A\right) ds + \int_{\hat{s}}^1 \pi_s\left(\tau_B, x_B\right) ds \\ &= (p - c) - x_A - x_B + \frac{1}{2}\left(k x_B^{\theta} - \hat{s}^2 k\left(x_B^{\theta} - x_A^{\theta}\right)\right) \quad (2) \end{aligned}$$

The first term, $(p - c)$, measures the net private revenues across all firms that are independent of public good provision under the planner. The terms $-x_A$ and $-x_B$ reflect the costs (to society) of providing the public good in each of the jurisdictions. The first term in the parentheses, $k x_B^{\theta}/2$, reflects the impact on total output across all firms if all firms locate in B. The second term in the parentheses reflects the loss of total output that results if a proportion $\hat{s}$ of firms locates in A. This loss comes about because, for all firms, output is increasing in public good provision and public good provision is lower in A than in B.

The following result provides all the efficient solutions.

Proposition 1. *There exists an efficient plan* $\boldsymbol{\tau}^E = \left(\tau_A^E, \tau_B^E\right)$, $\mathbf{x}^E = \left(x_A^E, x_B^E\right)$ *where* $\tau_A^E \geq \tau_B^E$, $x_A^E = 0$, $x_B^E = \left(\frac{1}{2}\theta k\right)^{\frac{1}{1-\theta}}$ *and* $\hat{s} = 0$.

Proposition 1 confirms the intuition that output is efficient when all firms locate in the same jurisdiction. From the proof we learn that efficiency is achieved when $\tau_A^E \geq \tau_B^E$. The fact that $\tau_A^E \geq \tau_B^E$ ensures, by (1), that $\hat{s} = 0$ for $x_B > x_A$. So all firms choose to locate in B, the jurisdiction that provides the high level of public goods. Moreover, the tax has no distortionary effect within the jurisdiction, as it is effectively a transfer from the firms to the government.[31]

The choice of $x_B^E > x_A^E = 0$ is then efficient for two reasons. First, as pointed out above, total output is higher if all firms locate in the jurisdiction where public good provision is highest. Second, efficiency is maximized when the public good is provided in only one jurisdiction. This follows from the fact that the public good is nonrival in our model and the fact that there are no congestion externalities.

The result also provides a unique solution for the level of x_B^E. The solution is unique because the direct impact of the public good on costs skx_B^θ is declining at the margin while the cost of the public good to society x_B is linear. This efficient solution will be useful as a benchmark for comparison against levels of provision under competition between jurisdictions. We can see from the solution for the efficient level of provision $x_B^E = \left(\frac{1}{2}\theta k\right)^{\frac{1}{1-\theta}}$ that x_B^E is increasing in k. The bigger the cost-reducing impact of the public good, the greater the marginal value to society of having more of the public good and so the greater the efficient level of provision. The impact of θ on x_B^E is similar but it is complicated by the fact that a change in θ may affect the marginal and average returns to x_B in opposite directions. This issue will be taken up in more detail below.

4 Competition in Taxes and Public Good Provision

In this section we examine the outcome of competition for firms between governments using public good provision and taxation. We will see that, by providing different levels of public good provision, governments can relax the forces of tax competition, taxing at different levels in equilibrium. In attempting to induce firms to locate in its jurisdiction, the government in each of the two jurisdictions, A and B, competes over taxes and the level of public good provision. These governments are assumed to be Leviathans, maximizing the rents to office through taxation and public good provision. We solve for an equilibrium in taxes and public good provision using backwards induction. In Stage 1 of the game, the two governments, A and B, noncooperatively and simultaneously choose (as pure strategies) levels of public good provision $x_A \in \mathbb{R}_+$ and $x_B \in \mathbb{R}_+$ respectively. Then in Stage 2 the two governments

[31] If we had assumed ad valorem or specific taxation then the planner's solution would have to take account of the marginal effect of the tax on production within each jurisdiction as well.

choose (as pure strategies) levels of taxation $\tau_A \in \mathbb{R}_+$ and $\tau_B \in \mathbb{R}_+$ respectively.[32] Once the governments' decisions have been taken, firms take taxes and levels of public good provision as given and choose their geographical locations (i.e. A or B) to maximize profits. We refer to this whole process, including both stages, as a *tax competition game.*

4.1 Stage 2: The Tax Subgame

The purpose of this subsection is to solve for Stage 2, where the levels of public good provision by the two governments are taken as fixed at (non-negative) levels x_A and x_B.

For given levels of public good provision x_A and x_B, a strategy τ_A^* of Government A is a *best response tax* against a strategy τ_B when it maximizes $r_A(\tau_A, \tau_B)$. A *Nash equilibrium in taxes* is a pair (τ_A^*, τ_B^*) for which (i) τ_A^* is a best response to τ_B^* and vice-versa (ii) $\tau_A^* \hat{s} \geq x_A$ and $\tau_B^*(1-\hat{s}) \geq x_B$.

The case where $x_A = x_B$ is analyzed as a straight-forward application of Bertrand equilibrium in homogeneous goods. The case where $x_A < x_B$ is less straight-forward and we need the following lemma to establish best response taxes in this situation.

Lemma 1. *Assume that x_A and x_B are fixed, with $0 \leq x_A < x_B$. For given τ_B, the unique tax that maximizes $r_A(\tau_A, \tau_B)$ is*

$$T_A(\tau_B) = \frac{\tau_B}{2}.$$

For given τ_A, the unique tax τ_B that maximizes $r_B(\tau_A, \tau_B)$ is

$$T_B(\tau_A) = \frac{\tau_A}{2} + \frac{k\left({x_B}^\theta - {x_A}^\theta\right)}{2}.$$

Lemma 1 determines tax reaction functions, which are illustrated in Figure 1. We see that for fixed levels of public goods optimal tax rates are strategic complements. Government A's reaction function is derived in a very straight forward manner by rearranging the first order condition for the maximization of r_A. The reaction function shows that Government A's best response depends only on the level of τ_B.

[32]It will be assumed throughout that mixed strategies in tax rates are not available to governments. This is generally deemed to be an acceptable assumption in the applied literature on policy setting in a perfect-information environment.

Government B's reaction function is more interesting. For any τ_A, the level of τ_B that maximizes r_B is increasing in k. To see why, look at the first order condition for maximization of r_B;

$$\begin{aligned}\frac{dr_B}{d\tau_B} &= 1-\hat{s}-\tau_B\frac{\partial\hat{s}}{\partial\tau_B} \\ &= 1-\frac{\tau_B-\tau_A}{k\left(x_B^{\theta}-x_A^{\theta}\right)}-\frac{\tau_B}{k\left(x_B^{\theta}-x_A^{\theta}\right)}=0.\end{aligned}$$

From the first order condition it is easy to see that r_B is strictly concave. It also becomes clear that $dr_B/d\tau_B$ is increasing in k. Look first at $\hat{s}$. Assuming values of τ_A, τ_B, and $x_A < x_B$ that imply $\hat{s} \in (0,1)$,

$$\frac{\partial\hat{s}}{\partial k}=-\frac{\tau_B-\tau_A}{k^2\left(x_B^{\theta}-x_A^{\theta}\right)}=-\frac{\hat{s}}{k}<0.$$

An increase in k results in a decrease in $\hat{s}$. Intuitively, the greater the positive impact of the public good on profits, the higher Government B can set its tax τ_B above τ_A and still attract a given share of firms $1-\hat{s}$ to its jurisdiction.[33]

Looking now at the third term of the first order condition and differentiating with respect to k we see that

$$\frac{\partial^2\hat{s}}{\partial\tau_B\partial k}=-\frac{1}{k^2\left(x_B^{\theta}-x_A^{\theta}\right)}<0. \tag{3}$$

So if Government B increases its tax this induces firms to move to A, i.e. $\partial\hat{s}/\partial\tau_B = \left(k^2\left(x_B^{\theta}-x_A^{\theta}\right)\right)^{-1}$, but this effect is dampened by an increase in k. For higher k, Government B's loss in share of firms due to an increase in τ_B is more limited. It is due to these two combined effects that an increase in k increases Government B's best response tariff for any given τ_A. As we shall see, it is through these two effects that governments are able to relax tax competition, and tax competition is increasingly relaxed as a result of an increase in k.

We now characterize equilibrium taxes and the equilibrium share $\hat{s}$ of firms between jurisdictions.

[33] The parameter θ affects the impact of the public good on profits in a similar but more complex way. This will be discussed further below.

Proposition 2. *(Relaxed Tax competition). Assume that x_A and x_B are fixed.*

For $x_A = x_B$, both governments provide the same level of public good and there exists a unique equilibrium in which $\tau_A^* = \tau_B^* = 0$.

For $x_A \neq x_B$ assume that $x_A < x_B$. Then there exists a unique subgame equilibrium point determined by the taxes

$$\begin{aligned}
\tau_A^*(x_A, x_B) &= \frac{1}{3}k\left(x_B^{\theta} - x_A^{\theta}\right); \\
\tau_B^*(x_A, x_B) &= \frac{2}{3}k\left(x_B^{\theta} - x_A^{\theta}\right).
\end{aligned}$$

At $\tau_A^*(x_A, x_B; k)$ and $\tau_B^*(x_A, x_B; k)$, the share of firms locating in Jurisdiction A is given by $\hat{s} = 1/3$.

We will say that *tax competition is relaxed* when $\tau_B^* > \tau_A^*$. We see from Proposition 2 that tax competition is more relaxed (that is, the larger the gap between τ_B^* and τ_A^*) the larger is x_B relative to x_A, and the higher is k; $\tau_B^* - \tau_A^* = k\left(x_B^{\theta} - x_A^{\theta}\right)/3$. These features of the equilibrium can be seen quite clearly from Figure 1, which shows that the intercept of Government B's reaction function $T_B(\tau_A)$ is increasing in $x_B^{\theta} - x_A^{\theta}$ and k. Consequently, the point where both reaction functions $T_A(\tau_B)$ and $T_B(\tau_A)$ cross, which determines the equilibrium tax levels τ_A^* and τ_B^* moves away from the origin as either $x_B^{\theta} - x_A^{\theta}$ or k are increased.

As x_A is reduced relative to x_B, Jurisdiction A becomes less attractive to firms that locate in B. So Government B is able to raise its tax, making higher rents from each firm while holding its share of firms constant. At the same time, this makes Jurisdiction B less attractive to firms in A, so Government A is able to raise its tax and make higher rents from each firm while holding its share of firms constant.

The fact that tax competition becomes more relaxed the greater the difference between x_B and x_A suggests that Government A has an incentive to reduce x_A relative to x_B in Stage 1 so that it can raise taxes in Stage 2. When we look at Stage 1 in the next subsection we will see that this incentive is further reinforced by the fact that reducing x_A reduces the cost for Government A of public good provision. For Government B these forces pull in opposite directions. Tax competition is more relaxed when x_B is increased, enabling Government B to raise τ_B while holding its share of firms constant, potentially increasing rents. But of course this increases the cost of provision, which works on rents in the opposite direction. The balance of these effects will be analyzed in Section 4.2.

The fact that $\tau_B^* > \tau_A^*$ does not depend on budget balance requirements. Indeed, note that no balanced budget constraints are imposed on best response outcomes in Stage 2. We will see in the next section that when governments choose levels of provision in Stage 1 using these best response tax functions, rents in equilibrium are always positive. How can the government budget constraint be satisfied as equilibrium taxes tend to zero? We shall see in the next section that equilibrium levels of public good provision tend towards zero at a faster rate.

If $x_A = x_B$ then public good provision is the same across jurisdictions and we effectively have Bertrand tax competition which leads to an outcome in which $\tau_A^* = \tau_B^* = 0$. Because x_A is sunk, for any positive tax level it is a dominant strategy for each government to undercut the other in setting taxes and in doing so attract all firms to its jurisdiction. Recall that the share of firms that locates in each jurisdiction is indeterminate in such an equilibrium, but because taxes are zero the share of firms that locates in each jurisdiction makes no difference to rents; thus $r_A = r_B = -x_A$.

One point worth clarifying is that in our framework it is not possible to conclude that tax competition necessarily leads taxes to be set 'too high' or 'too low.' This is because, while under tax competition the level of taxation is determined by the level of public good provision, under the efficient solution it is indeterminate. Recall that efficiency does not stipulate a level for taxation, only that the tax in Jurisdiction A is at least as high as the tax in Jurisdiction B. Moreover, Government B may make positive rents under the efficient solution and this is perfectly consistent with efficiency. The presence of government rents is sometimes associated with inefficiency but, as we shall see, it is the fact that governments compete for rents that brings about an inefficient outcome here, not the fact that they make rents per se.

Proposition 2 shows that $\hat{s} \neq 0$ in equilibrium and from this we can conclude immediately that relaxed tax competition is inefficient. We know from Proposition 1 that the unique efficient outcome has all firms in a single jurisdiction with the public good provided at the efficient level.

Corollary to Proposition 2. *Under relaxed tax competition $\hat{s} > 0$; thus relaxed tax competition is inefficient.*

This inefficiency is created by the relaxation of tax competition because a positive share of firms ($\hat{s} = 1/3$) is lured to Jurisdiction A by low taxation, despite the fact that in equilibrium no public goods are provided in A. This result is reminiscent of Alesina and Spolaore's (1997) finding that in a democracy there is an inefficiently large number of countries. In their model, it is the citizens furthest away from the government under the efficient solution who find the formation of a new nation most appealing. Similarly, in our model it is obviously the firms at the bottom of the interval who are attracted to the low-tax jurisdiction under tax competition.[34]

The result that location over more than one jurisdiction is inefficient is somewhat stark. It contrasts with the standard tax competition models of Wilson (1986) and Zodrow and Mieskowski (1986). In those models, one factor is immobile while the other is not, and increasing opportunity cost of factor substitution in the production function means that the marginal product of the mobile factor becomes high when it is scarce. If the assumption that one factor is immobile were dropped then a result like this corollary could be obtained under standard tax competition as well. Other factors could be brought into the model such as congestion effects and attachments to location, and if significant enough these would cause firms to locate across more than one jurisdiction in the efficient outcome.

It is interesting to note from Proposition 2 that for our example the share of firms locating in Jurisdiction B is relatively large, at $1 - \hat{s} = 2/3$, even though B sets a higher tax in equilibrium. We might have expected to see the high-tax jurisdiction attracting a relatively small share of firms but this is not the case. Our example shows that a higher level of public good provision can have a cost-reducing impact sufficiently large as to make location in Jurisdiction B more profitable for a majority of firms, despite higher taxation there.[35]

Here in this section we have seen that inefficiency is created by the relaxation of tax

[34] In some situations like this the inefficiency disappears as the number of jurisdictions becomes large. But Shaked and Sutton (1987) show that when a fixed cost (here public good provision) is sunk in the first stage, the fixed cost is escalated to prevent the number of jurisdictions becoming large.

[35] In a more general specification we would expect $\hat{s}$ to be a function of x_A and x_B in the equilibrium of Stage 2.

competition because a positive share of firms locates in A where no public good is provided. In the next section we will see that further inefficiency arises under relaxed tax competition because the public good is under-provided in Jurisdiction B.

4.2 Stage 1: Level of public good provision

We now solve Stage 1, which determines the level of public good provision by the respective governments. To do this, we must drop the assumption that $x_A \leq x_B$. In looking for Government A's best response to x_B, we must evaluate $r_A(x_A, x_B)$ for $x_A < x_B$, $x_A = x_B$ and $x_A > x_B$.

Recall that, by Proposition 2, if $x_A < x_B$ then $\tau_A^*(x_A, x_B) = \frac{1}{3}k\left({x_B}^\theta - {x_A}^\theta\right)$, $\tau_B^*(x_A, x_B) = \frac{2}{3}k\left({x_B}^\theta - {x_A}^\theta\right)$ and $\hat{s} = \frac{1}{3}$, and if $x_A = x_B$, $\tau_A^* = \tau_B^* = 0$ and $\hat{s}$ is indeterminate.[36] It also follows from Proposition 2 that if $x_A > x_B$ then $\tau_A^*(x_A, x_B) = \frac{2}{3}k\left({x_A}^\theta - {x_B}^\theta\right)$ and $\tau_B^*(x_A, x_B) = \frac{1}{3}k\left({x_A}^\theta - {x_B}^\theta\right)$. Using these equilibrium values in $r_A = \tau_A\hat{s} - x_A$, Government A's rent function is defined as follows:

$$r_A(x_A, x_B) = \begin{cases} k\left({x_B}^\theta - {x_A}^\theta\right)/9 - x_A \text{ if } 0 \leq x_A < x_B; \\ -x_A \text{ if } 0 \leq x_A = x_B; \\ 4k\left({x_A}^\theta - {x_B}^\theta\right)/9 - x_A \text{ if } 0 \leq x_B < x_A. \end{cases} \tag{4}$$

For Government B,

$$r_B(x_A, x_B) = \begin{cases} 4k\left({x_B}^\theta - {x_A}^\theta\right)/9 - x_A \text{ if } 0 \leq x_A < x_B; \\ -x_B \text{ if } 0 \leq x_A = x_B; \\ k\left({x_A}^\theta - {x_B}^\theta\right)/9 - x_B \text{ if } 0 \leq x_B < x_A. \end{cases} \tag{5}$$

A level of public good provision x_A^* of Government A is a *best response against a level of public good provision* x_B, denoted $BR_A(x_B)$, when it maximizes $r_A(x_A, x_B)$. A *Nash equilibrium in levels of public good provision* is a pair (x_A^*, x_B^*) where (i) x_A^* is a best response against x_B^* and vice-versa; (ii) $r_A(x_A^*, x_B^*) \geq 0$, $r_B(x_A^*, x_B^*) \geq 0$.

We will now state our existence and characterization of equilibrium result, Proposition 3, followed immediately by the statement of Proposition 4 which compares the equilibrium level of public good provision to the efficient level. A discussion of both propositions then follows. Proposition 3 shows that while Jurisdiction B provides the public good at a positive level, A provides none at all. Also note that, although taxation is higher in B than in A, taxation in A is nevertheless positive. Thus Jurisdiction A has a degree of monopoly power

[36]Recall that in this case $r_A = r_B = -x_A = -x_B$; rents are well defined even though $\hat{s}$ is indeterminate.

and is able to collect rents due to the fact that firms must locate in one jurisdiction or the other in order to produce. Proposition 4 then shows that the equilibrium level of public good provision is inefficient.

Proposition 3. *Assume that governments play a tax competition game.*

1. *There exists a unique subgame perfect Nash equilibrium in pure strategies.*

2. *The equilibrium has the property that one jurisdiction, say A, provides a smaller amount of the public good than the other, B.*

3. *The subgame perfect equilibrium is determined by the levels of public good provision*

$$\begin{aligned} x_A^* &= 0, \\ x_B^* &= \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}, \end{aligned}$$

   and taxes are (uniquely)

$$\begin{aligned} \tau_A^* &= \frac{1}{3}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}, \\ \tau_B^* &= \frac{2}{3}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}. \end{aligned}$$

Proposition 4. *In the (pure strategies) subgame perfect Nash equilibrium, public good provision in Jurisdiction B is inefficiently low:* $x_B^E = \left(\frac{1}{2}\theta k\right)^{\frac{1}{1-\theta}} > x_B^* = \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}$.

Discussion of Proposition 3. In the proof we show that the equilibrium in pure strategies must be asymmetric in that one government sets public good provision above the level of the other. We prove that this equilibrium exists and is unique subject to a re-labelling of jurisdictions. We then choose to label Jurisdictions A and B as before, as the jurisdictions of low and high level public good provision respectively.

In Section 4.1 we argued that from any positive level of public good provision Government A always has an incentive to reduce its provision. This is so both because reducing public good provision relaxes tax competition, enabling taxes to be raised (by both jurisdictions) and because it saves costs. Both effects work in the same direction to increase rents. Proposition 3 shows formally that this effect does indeed operate to the point where Government A provides no public goods at all. It seems reasonable to argue that such an effect would operate under more general specifications than ours, although in more complex models public good provision may not be driven all the way to zero.

For Government B, on the other hand, it was observed in Section 4.1 that the incentive to raise public good provision in order to relax tax competition and the incentive to lower provision in order to save costs operate in opposite directions. Proposition 3 shows that these effects are balanced at a positive level $x_B^* = \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}$ in equilibrium. The effect of a change in k is clear. As k is increased this increases x_B^* because public good provision has a bigger impact on firms' profits and therefore on government rents through taxation.

The effect of θ on x_B^* is less obvious. While for k relatively large, x_B^* is monotonically increasing in θ, for k relatively small the effect on x_B^* of an increase in θ is ambiguous. To show the ambiguity, in Figure 2 we illustrate r_B under the assumption that $k = 1$ (i.e. relatively small) and that all equilibrium values other than x_B^* hold; $\tau_A = \tau_A^*$, $\tau_B = \tau_B^*$, $x_A^* = 0$ and consequently $\hat{s} = \frac{1}{3}$. Using these values, it is easy to work out that $r_B = \frac{4}{9}kx_B^{\theta} - x_B$. Figure 2 illustrates how r_B varies with x_B for $\theta = \frac{1}{10}$, $\theta = \frac{1}{4}$ and $\theta = \frac{2}{3}$. We see that for each value of θ there is a unique value x_B^* that maximizes r_B. Moreover, x_B^* *increases* as θ is increased from $\theta = \frac{1}{10}$ to $\theta = \frac{1}{4}$ but x_B^* *decreases* as θ is increased form $\theta = \frac{1}{4}$ to $\theta = \frac{2}{3}$. The reason can be seen most clearly by inspection of the first derivative of the rent function, $dr_B/dx_B = \frac{4}{9}\theta k x_B^{\theta-1} - 1$. An increase in θ has two conflicting effects on the first term. While an increase in θ tends to increase $\frac{4}{9}\theta k x_B$, an increase in θ tends to decrease $x_B^{\theta-1}$ (for fixed k and x_B). Moreover, the negative second effect increases non-linearly with θ. To put this another way, an increase in θ reduces the curvature of r_B everywhere but also reduces the initial gradient of r_B in the neighborhood of $x_B = 0$. Thus x_B^* may be first increasing then decreasing in θ. However, it is also easy to see that k may be set large enough so that the first term is monotonically increasing in k for $\theta \in (0, 1)$. In that case x_B^* is monotonically increasing in θ just as it is monotonically increasing in k.

The effect of θ on x_B^E is very similar, for reasons that are closely related. Observe, by differentiating the planner's problem (2), that $d\Omega/dx_B = \frac{1}{2}\theta k x_B^{\theta-1} - 1$. We can see by analogy that, for relatively low k, x_B^E is first increasing then decreasing in θ. As for x_B^*, it is possible to set k sufficiently large so that x_B^E is monotonically increasing in θ.

It is easy to check that both governments make positive rents in equilibrium. For Jurisdiction A this is immediately obvious because it collects taxes from a positive share of firms but has no costs of public good provision. For Jurisdiction B we use the equilibrium values for τ_B^* and x_B^* in the expression for Government B's rents to obtain, in reduced form, $r_B = \left(\frac{4k}{9}\right)^{\frac{1}{1-\theta}} \left[\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right]$. To see that $r_B > 0$ for all $\theta \in (0,1)$ note that $\lim_{\theta\to 0} \theta^{\frac{\theta}{1-\theta}} = 1$ while $\lim_{\theta\to 0} \theta^{\frac{1}{1-\theta}} = 0$ and $\lim_{\theta\to 1} \theta^{\frac{\theta}{1-\theta}} = \lim_{\theta\to 1} \theta^{\frac{1}{1-\theta}} = 1/e$, with $\theta^{\frac{\theta}{1-\theta}}$ decreasing monotonically from 1 to $1/e$ as θ is varied from 0 to 1, and $\theta^{\frac{1}{1-\theta}}$ increasing monotonically from 0 to $1/e$ as θ is varied from 0 to 1. This makes intuitive sense if we think of the outcome as oligopolistic, where both governments are able to choose quantities and prices (here taxes) at which they make non-negative rents.[37]

We can now determine which government makes higher rents. Using equilibrium values from Proposition 3, we know that $r_A = \frac{k}{9}\left(\frac{4\theta k}{9}\right)^{\frac{\theta}{1-\theta}}$ and $r_B = \left(\frac{4k}{9}\right)^{\frac{1}{1-\theta}} \left[\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right]$. From this we have that $r_A \gtreqless r_B$ if and only if $\frac{1}{4\theta}\left(\frac{4\theta k}{9}\right)^{\frac{1}{1-\theta}} \gtreqless \left(\frac{1}{\theta} - 1\right)\left(\frac{4\theta k}{9}\right)^{\frac{1}{1-\theta}}$ or, equivalently, if and only if $\theta \gtreqless \frac{3}{4}$.

Discussion of Proposition 4. Proposition 4 shows that the level of public good provision is suboptimal under relaxed tax competition. In the Corollary to Proposition 2 we showed that inefficiency arises under relaxed tax competition because firms locate in more than one jurisdiction. Here we have a second component to the inefficiency that arises under relaxed tax competition in that the public good is under-provided in the high-tax jurisdiction. This suboptimality arises because some firms locate in Jurisdiction A and so the marginal benefit to a policy maker is lower, whether this policy market is the planner or the Leviathan government. To see this, first recall from the efficient solution that if $\tau_A = \tau_B$ and $x_A = x_A^E = 0$ and $x_B = x_B^E$ then all firms locate in Jurisdiction B. It is easy to show that if $\tau_A = \tau_B$ were fixed (or 'harmonized') at Stage 2, then Government B's incentive to set x_B is identical to that of the planner, and it would set $x_B = x_B^E$. Consequently, all firms would be attracted to B. Conversely, if the planner were somehow constrained to set taxes $\tau_A^* = \frac{1}{3}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$ and $\tau_B^* = \frac{2}{3}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$, the outcome of relaxed tax competition, then the planner's solution to the level of public good provision would be $x_B = x_B^*$.

[37]We conjecture that this property, governments making positive rents in equilibrium, would hold for a more general specification for the profit function in that the term $k{x_i}^\theta$ could be replaced by a general function $b(x_i; \theta, k)$, with $b(\cdot)$ concave in x_i and $\partial b/\partial k > 0$.

We are also able to see quite clearly the effect of θ on the suboptimality of public good provision. We do this by calculating the ratio of the level of public good provision at equilibrium and efficient levels in Jurisdiction B; $x_B^*/x_B^E = \frac{8}{9}^{\frac{1}{1-\theta}}$. Observe that $x_B^*/x_B^E \to \frac{8}{9}$ as $\theta \to 0$ and $x_B^*/x_B^E \to 0$ as $\theta \to 1$. We noted above that the effect of an increase in θ on x_B^* and x_B^E may be ambiguous. Recall from Figure 2, for example, that an increase in θ could bring about an increase in x_B^* and x_B^E at θ relatively close to 0 but a decrease in x_B^* and x_B^E at θ relatively close to 1. From Proposition 4 it becomes evident that there is a systematic effect of θ on x_B^* relative to x_B^E in spite of the ambiguous effect of θ on the levels of x_B^* and x_B^E.

As a final point note that both x_B^E and x_B^* go to 0 as k goes to zero. This is plausible since the public good does not save costs as k tends to zero so no firm will pay for it, and so no jurisdiction will (or should) provide it.

5 Policies of Tax Coordination

The two most commonly advanced proposals for tax policy coordination are tax harmonization and the setting of a minimum tax. Most of this section will be concerned with analysis of a minimum tax, as tax harmonization in the context of our model is very straight forward. We consider tax harmonization first.

5.1 Tax Harmonization

Tax harmonization at its simplest imposes the requirement that both jurisdictions set the same tax rate. Within the present model, the outcome of tax harmonization is obvious. Recall from the discussion following Proposition 4 that if taxes are harmonized then Government B's incentive to set x_B is identical to that of the planner, and it sets $x_B = x_B^E$. The outcome from tax harmonization is that all firms locate in Jurisdiction B and that Government B sets public good provision at the efficient level. Without the imposition of a revenue-sharing requirement, Government B would collect all rents under tax harmonization. Thus, Government B's rents would certainly rise under harmonization relative to relaxed tax competition and Government A's rents would certainly fall.

5.2 A Minimum Tax

If governments agree to set a minimum tax, denoted by μ, then they agree to a common lower bound for taxes. Within the context of our model there is no unique minimum tax on which governments will agree. Therefore we characterize the *non-renegotiable minimum tax frontier* as the set of minimum taxes for which, given a minimum tax on the frontier: (i) neither government can obtain higher rent by a change in the minimum tax without the other government having to accept lower rent; (ii) both governments obtain higher rents than with no minimum tax.[38] Given any minimum tax on the frontier, the two governments would not jointly agree to renegotiate to any other minimum tax or to abolish the minimum tax. The determination of the specific minimum tax that is implemented on the frontier would depend on factors beyond the scope of our model.

A minimum tax only imposes a binding constraint if $\mu \geq \tau_A^*$. On the other hand, μ can be set sufficiently high to ensure that tax rates are harmonized. By inspection of (1), it is clear that if the constraint sets a minimum such that $\tau_A = \tau_B$ then all firms locate in Jurisdiction B. Since rents for A are zero if the share of firms that locates in A is zero, a value of μ higher than the value required to ensure $\tau_A = \tau_B$ cannot yield higher rents for A than with no minimum tax. Therefore, we may restrict attention to μ that lies between τ_A^* and a value that ensures $\tau_A = \tau_B$.[39]

An issue that arises is whether a minimum tax should be applied when jurisdictions are ex-ante symmetric; that is, when $x_A = x_B$. Here we take the view that the primary motivation for a minimum tax is to reduce the difference between tax levels only when jurisdictions would otherwise set different taxes in equilibrium, motivated by the fact that they provide public goods at different levels. When jurisdictions provide public goods at the same level, arguably this motivation for a minimum tax does not apply.[40] Thus, we maintain

[38]The notion of the non-negotiable minimum tax frontier is related to the Pareto efficient frontier. The key difference is that the non-renegotiable minimum tax frontier is defined by the outcome of strategic interactions between the two governments and, as we shall see, is not Pareto efficient.

[39]For reasons that will become clear, τ_B^* does not impose the upper bound on μ, unlike in Kanbur and Keen (1993).

[40]It could also be argued that the primary purpose of a minimum tax is to limit competition between the governments, and that this applies when jurisdictions are ex ante symmetrical as well. A competition limiting effect will be a feature of the non-renegotiable minimum tax frontier. But it will arise when the minimum tax is designed to limit the extent to which a low-public-good jurisdiction can undercut a high-public-good jurisdiction.

the approach taken throughout the paper that if $x_A = x_B$ in Stage 1 then tax competition between governments in Stage 2 is characterized by standard Bertrand competition, and taxes are competed to zero.

We now formalize a minimum tax under the assumption that $x_A < x_B$.[41] Tax setting under the minimum tax is unaffected by whether or not the constraint is anticipated. Let μ be set at a level ε above A's equilibrium tax under relaxed tax competition;[42]

$$\mu = \tau_A^* + \varepsilon = \frac{1}{3}k\left(x_B{}^\theta - x_A{}^\theta\right) + \varepsilon.$$

Let τ_A^μ be the tax that Government A sets in the presence of the minimum tax. By the concavity of r_A in τ_A, the best Government A can do in the presence of the minimum tax is to set $\tau_A^\mu = \mu$. The tax set by Government B is determined by the reaction function $T_B(\tau_A) = \left(\tau_A + k\left(x_B{}^\theta - x_A{}^\theta\right)\right)/2$ as $\tau_B^\mu = \frac{2}{3}k\left(x_B{}^\theta - x_A{}^\theta\right) + \frac{1}{2}\varepsilon$. We can now see that if $\varepsilon = \frac{2}{3}k\left(x_B{}^\theta - x_A{}^\theta\right)$, then $\tau_A^\mu = \tau_B^\mu$. Therefore, we restrict attention to $\varepsilon \in \left[0, \frac{2}{3}k\left(x_B{}^\theta - x_A{}^\theta\right)\right]$. To agree upon a minimum tax, the governments must effectively agree upon a value for ε.

There are similarities here to Kanbur and Keen's (1993) approach to the analysis of a minimum tax. However, an issue that Kanbur and Keen do not need to address is how the introduction of the minimum tax affects the sequence of events. Their game only has a single period. The minimum tax is imposed before tax setting takes place within that period, bringing about a constrained equilibrium. In the model of this present paper, the imposition of a minimum tax constraint raises the extra issue of whether the constraint is anticipated before the level of public good provision is fixed. From a purely theoretical standpoint, it seems natural to argue that the imposition of the constraint is fully anticipated when levels of public good provision are determined. In an applied context, on the other hand, it might be argued that proposals for a minimum tax could take place after public good provision has been fixed. The context we have in mind here is the current call for a minimum tax in the newly expanded EU. In the following we will examine both assumptions in two separate subsections. We will examine the assumption that the minimum tax is not anticipated first, in Section 5.2.1, because it is analytically easier to deal with. In Section 5.2.2 we

[41] The case where $x_B < x_A$ is analogous. In demonstrating equilibrium we take the same approach as in Section 4.2, initially dropping the assumption that $x_A < x_B$. After it is established that in equilibrium one government must set public good provision at a higher level than the other then the assumption that $x_A < x_B$ may be adopted without loss of generality.

[42] See Proposition 2, Section 4.1, for the determination of τ_A^*.

assume instead that the minimum tax is anticipated. In fact, our findings are qualitatively independent of whether the constraint is anticipated or not.

5.2.1 Minimum Tax Unanticipated

In this subsection we assume that the governments set the levels of public good provision simultaneously and noncooperatively at Stage 1 as if no minimum tax were to be imposed, anticipating instead that the game would proceed straight to Stage 2 in which tax setting would take place. After levels of public good provision are fixed in Stage 1, the governments are then unexpectedly granted the opportunity to agree upon a minimum tax. After the minimum tax is agreed upon, the game then proceeds to Stage 2, at which point governments set taxes simultaneously and noncooperatively (but now subject to the minimum tax).

Writing the respective levels of public good provision under the unanticipated minimum tax constraint as x_A^μ and x_B^μ we therefore have $x_A^\mu = x_A^* = 0$ and $x_B^\mu = x_B^* = \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}$. Using $x_A^\mu = x_A^* = 0$, $x_B^\mu = x_B^* = \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}$, $\tau_A^\mu = \mu = \tau_A^* + \varepsilon = \frac{1}{3}k\left(x_B^\mu\right)^\theta + \varepsilon$ and $\tau_B^\mu = \frac{2}{3}k\left(x_B^\mu\right)^\theta + \frac{1}{2}\varepsilon$ in the expressions for $\hat{s}$, r_A, and r_B, (that is 1, 4 and 5), we obtain the following reduced form expressions for government rents. To emphasize that rents are being derived under the minimum tax, we shall write these as $r_A^\mu(\varepsilon)$ and $r_B^\mu(\varepsilon)$ respectively:

$$\begin{aligned} r_A^\mu(\varepsilon) &= \frac{1}{9}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}} + \frac{1}{6}\varepsilon - \frac{\varepsilon}{2k\left(\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}\right)}; \\ r_B^\mu(\varepsilon) &= \frac{4}{9}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}} - \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}} + \frac{2}{3}\varepsilon + \frac{\varepsilon^2}{4k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}}. \end{aligned}$$

We now characterize the non-renegotiable minimum tax frontier.

Proposition 5. *Fix $x_A^\mu = 0$ and $x_B^\mu = \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}$ and fix a minimum tax $\mu = \frac{1}{3}k\left(x_B^{\theta} - x_A^{\theta}\right) +$ ε. Then Government A maximizes $r_A(\varepsilon)$ by setting $\tau_A^\mu = \frac{1}{3}k\left(\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}\right) + \varepsilon$ and Government B maximizes $r_B(\varepsilon)$ by setting $\tau_B^\mu = \frac{2}{3}k\left(\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}\right) + \frac{1}{2}\varepsilon$. A minimum tax is on the non-renegotiation minimum tax frontier if $\varepsilon \in \left(\frac{1}{6}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}, \frac{1}{3}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}\right)$.*

If the minimum tax is set such that $\varepsilon \in \left[\frac{1}{6}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}, \frac{1}{3}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}\right)$ then both governments make higher rents than with no minimum tax, and any change in the minimum tax will yield strictly higher rents for one government but strictly lower rents for the other government. To see this, first note by inspection that $r_B^\mu(\varepsilon)$ is monotonically increasing in ε. A little more work tells us that $r_A^\mu(\varepsilon)$ is strictly concave in ε, with maximum at $\varepsilon = \frac{1}{6}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}$. Thus $r_A^\mu(\varepsilon)$ is strictly decreasing, and $r_B^\mu(\varepsilon)$ is strictly increasing, for all $\varepsilon > \frac{1}{6}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}$. Given that $\varepsilon = 0$ corresponds to a situation where there is no minimum tax, it is easy to see that both A and B must make higher rents at $\varepsilon = \frac{1}{6}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}$ than with no minimum tax. Also, since $r_A^\mu(\varepsilon)$ is strictly decreasing in ε for $\varepsilon > \frac{1}{6}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}$, it must be possible to find a value for ε at which Government A makes the same rent as with no minimum tax; $r_A^\mu(\varepsilon) = r_A^\mu(0)$. This value is $\varepsilon = \frac{1}{3}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}$, which defines the upper bound to the frontier.

The reason both governments are able to make higher rents is because the minimum tax further relaxes tax competition. While obviously the assumption that the minimum tax is unanticipated is restrictive, we see now why it is useful. By holding the level of public good provision constant, we are able to see the direct effect on taxes and hence rents of introducing the minimum tax. Using (1) it is possible to check that while A benefits from being able to set higher taxes, it loses firms as ε is increased. As ε is increased above $\varepsilon = \frac{1}{6}k\left(\frac{4}{9}k\theta\right)^{\frac{\theta}{1-\theta}}$, the loss to A from the migration of firms to B is greater than the gain from being able to tax each firm at a higher level. We shall see in Section 5.2.2 that this effect carries over to the situation where governments anticipate the introduction of the minimum tax.

5.2.2 Minimum Tax Anticipated

In the following, we show that even when the minimum tax is anticipated, rents for the respective governments have the same qualitative characterization as in Section 5.2.1 where public good provision was fixed. Specifically, $r_B(0, x_B^\mu(\varepsilon))$ is monotonically increasing in ε while $r_A(0, x_B^\mu(\varepsilon))$ is concave in ε with a unique optimum that defines the lower bound of the non-renegotiable minimum tax frontier.

We assume that the imposition of the minimum tax is anticipated before the start of Stage 1, so each government takes the minimum tax into account when determining the level of public good provision. Thus, the minimum tax is agreed upon after which the sequence of events is exactly as in Section 4. Best response taxes with the minimum tax are as follows: if $x_B > x_A$ then $\tau_A^\mu = \frac{1}{3}k\left(x_B^\theta - x_A^\theta\right) + \varepsilon$ and $\tau_B^\mu = \frac{2}{3}k\left(x_B^\theta - x_A^\theta\right) + \frac{1}{2}\varepsilon$; on the other hand if $x_A > x_B$ then $\tau_A^\mu = \frac{2}{3}k\left(x_A^\theta - x_B^\theta\right) + \frac{1}{2}\varepsilon$ and $\tau_B^\mu = \frac{1}{3}k\left(x_A^\theta - x_B^\theta\right) + \varepsilon$. If $x_A = x_B$ then $\tau_A^\mu = \tau_B^\mu = 0$. But now the levels of public good provision x_A and x_B are determined optimally in Stage 1. Using these expressions for τ_A^μ and τ_B^μ, Government A's rent function is defined as follows for $\varepsilon \in \left[0, \frac{2}{3}k\left|x_B{}^\theta - x_A{}^\theta\right|\right]$:

$$r_A^\mu(x_A, x_B; \varepsilon) = \begin{cases} \frac{1}{9}k\left(x_B{}^\theta - x_A{}^\theta\right) - x_A + \frac{1}{6}\varepsilon - \frac{1}{2k\left(x_B{}^\theta - x_A{}^\theta\right)}\varepsilon^2 \text{ if } 0 \le x_A < x_B; \\ -x_A \text{ if } 0 \le x_A = x_B; \\ \frac{4}{9}k\left(x_A{}^\theta - x_B{}^\theta\right) - x_A + \frac{2}{3}\varepsilon + \frac{1}{4k\left(x_A{}^\theta - x_B{}^\theta\right)}\varepsilon^2 \text{ if } 0 \le x_B < x_A. \end{cases} \quad (6)$$

For Government B,

$$r_B^\mu(x_A, x_B; \varepsilon) = \begin{cases} \frac{4}{9}k\left(x_B{}^\theta - x_A{}^\theta\right) - x_B + \frac{2}{3}\varepsilon + \frac{1}{4k\left(x_B{}^\theta - x_A{}^\theta\right)}\varepsilon^2 \text{ if } 0 \le x_A < x_B; \\ -x_B \text{ if } 0 \le x_A = x_B; \\ \frac{1}{9}k\left(x_A{}^\theta - x_B{}^\theta\right) - x_B + \frac{1}{6}\varepsilon - \frac{1}{2k\left(x_A{}^\theta - x_B{}^\theta\right)}\varepsilon^2 \text{ if } 0 \le x_B < x_A. \end{cases} \quad (7)$$

As was the case for when there was no minimum tax, when it maximizes $r_A^\mu(x_A, x_B; \varepsilon)$ a level of public good provision $x_A^\mu(\varepsilon)$ of Government A is a *best response against a level of public good provision* $BR_A(x_B; \varepsilon)$.[43] A *Nash equilibrium in levels of public good provision* is a pair $(x_A^\mu(\varepsilon), x_B^\mu(\varepsilon))$ where (i) $x_A^\mu(\varepsilon)$ is a best response against $x_B^\mu(\varepsilon)$ and vice-versa; (ii) $r_A^\mu(x_A^\mu(\varepsilon), x_B^\mu(\varepsilon); \varepsilon) \ge 0$, $r_B^\mu(x_A^\mu(\varepsilon), x_B^\mu(\varepsilon); \varepsilon) \ge 0$.

The characterization of equilibrium is technically the same as discussed in Section 4.2 for the case with no minimum tax; see the appendix for details. The equilibrium is asymmetric, with one government providing no public good and the other providing the public good at a positive level. As before, w.o.l.o.g. we let A be the jurisdiction with no public good provision in equilibrium; $x_A^\mu(\varepsilon) = 0$. The top panel of Figure 3 shows a plot of $x_B^\mu(\varepsilon)$ while the bottom panel shows $\hat{s}(0, x_B^\mu(\varepsilon))$ for $k = 1$ (and $\theta = \frac{1}{2}$) as ε is varied.[44] Note from the bottom panel that $\hat{s}(0, x_B^\mu(\varepsilon))$ is increasing in ε and $\hat{s}(0, x_B^\mu(\varepsilon)) = 1$ for $\varepsilon = \frac{2}{3}k\left(x_B{}^\theta - x_A{}^\theta\right) = \frac{5}{36}$. Also note that all values for $\varepsilon = 0$ correspond to equilibrium values given in Proposition 3. Thus, at $\varepsilon = 0$, $x_B^\mu(0) = x_B^\mu = \left(\frac{2}{9}\right)^2$. The top panel shows that $x_B^\mu(\varepsilon)$ decreases monotonically with ε until the point where $\hat{s}(0, x_B^\mu(\varepsilon)) = 1$. Government B's incentive to compete in public good provision (by offering the public good at a higher level than Government A) is reduced by the fact that Government A is limited in the extent to which it is allowed to set its tax lower than B's.

Turning to Figure 4, we see that for $k = 1$, $\theta = \frac{1}{2}$, rents for the respective governments have the same qualitative characterization as in Section 5.2.1 where public good provision was

[43] Note the distinction we make between the best response function and rent function with and without the minimum tax; the functions are shown to be dependent on the parameter ε in the former case.

[44] We have written $\hat{s}(\tau_A, \tau_B, x_A, x_B)$ in the form $\hat{s}(0, x_B^\mu(\varepsilon))$ to represent the fact that taxes $\tau_A = \tau_A^\mu$ and $\tau_B = \tau_B^\mu$ have been determined as functions of $x_A^\mu(\varepsilon) = 0$ and $x_B^\mu(\varepsilon)$.

fixed. Thus, as claimed, $r_B(0, x_B^\mu(\varepsilon))$ is monotonically increasing in ε while $r_A(0, x_B^\mu(\varepsilon))$ is concave in ε. The non-renegotiable minimum tax frontier is shown in Figure 4 as the interval $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon})$. The upper and lower bounds, $\underline{\varepsilon}$ and $\overline{\varepsilon}$, are defined in the same way as in Proposition 5. For $\varepsilon < \underline{\varepsilon}$ both governments would agree to implement a higher minimum tax. But for $\varepsilon > \overline{\varepsilon}$, Government A makes higher rent with no minimum tax.

6 Conclusions

The main point of this paper has been to argue that when the value placed by firms on public good provision varies, governments are able to use this fact to relax tax competition. The fact that governments are able to relax tax competition may explain why governments in the core of Europe, that have historically provided public goods at a relatively high level, have been able to continue to tax at a higher level than those in the periphery even as markets for goods and capital have become more integrated.

We show that public good provision in the high-tax jurisdictions is sub-optimally low under tax competition, which accords with the conventional model of the standard tax competition/'race to the bottom' literature. But, unlike with standard tax competition, our model can also explain the often heard complaint that taxes are set too high, in the sense that governments expropriate rents from firms in equilibrium. This arises out of the monopolistic power that governments have because firms must locate in one jurisdiction or another in order to produce.

The imposition of a minimum tax further relaxes the forces of tax competition. Both governments make higher rents on the non-renegotiable minimum tax frontier than without a minimum tax. When the minimum tax is unanticipated, taxation is unambiguously higher in both jurisdictions. When the minimum tax is anticipated our example shows that, where provided, public good provision is even further below the efficient level than without the minimum tax. Our results on the minimum tax contrast with those of Kanbur and Keen (1993) which suggest that countries are likely to gain from the imposition of a minimum tax.

Sutton (1991 Chapter 3) discusses the way that technology affects equilibrium market structure in models of vertical product differentiation. We can use Sutton's discussion to put our findings in context because the structure of our tax competition model conforms to the general structure of a model of vertical product differentiation. In this sense one might say that we have a model of vertical public good differentiation, in which expenditure on public goods may be thought of as a sunk cost.[45]

Sutton observes that if an increase in (perceived) product quality is achieved mainly by an increase in a sunk cost then market concentration may increase with the size of the market. As a stylized characterization of this, Shaked and Sutton (1982) show conditions under which equilibrium will support no more than two firms because sunk costs are escalated with the size of the market. The correspondence of product quality to the size of a sunk cost in a model of vertical production differentiation is exactly analogous to the correspondence of the level of public good provision to the size of the cost of provision in our tax competition model. (In particular, note our assumption that the level of public good provision, and therefore its cost, is determined in the first stage of the game.) Therefore, extending Sutton's conclusions to the present context, even if we allowed jurisdictions to form endogenously as elsewhere in the literature the basic characterization of the equilibrium that we demonstrate, being based on just two jurisdictions, would not change.[46]

While we relate our model to recent European experience for which we present data constructed by Devereux and Griffith (2003), our model may help to understand patterns of taxation elsewhere as well. Mintz and Smart (2001) present and examine evidence that corporate income tax rates have remained the same (or even increased slightly since 1986) across provinces in Canada. More loosely, the variation in tax rates across states in the US has attracted significant media attention, with the spotlight focused on discrepancies between jurisdictions where taxes and public good provision are relatively high, like Massachusetts, and those where taxes and public good provision are at low levels, such as Alabama. Our model, while focused on international taxation, puts forward a way of understanding these patterns of variation in taxation across states as well, where federal transfers between states

[45] This is not to be confused with vertical tax competition discussed by Keen and Kotsogiannis (2003), for example, which relates to competition between governments at the 'federal' and 'state' levels.

[46] Sutton (1991) observes that if product quality is determined by a variable (and not a fixed) cost, then the number of firms becomes large with the size of the market, with the outcome converging to efficiency. Analogously, the literature on Tiebout tax competition allows the number of jurisdictions to be endogenously determined, with the outcome tending towards efficiency as the economy becomes large. See Wilson (1999) and Wooders (1999) for comprehensive reviews. The link is made between the two literatures by noting that in the Tiebout literature the level of public good provision is characterized by a variable cost, and not a fixed (sunk) cost as in the present model.

are imperfect.

Although our model can explain in static terms why taxes and public good provision may be higher in one jurisdiction than another, it is silent on the dynamics of how taxes have evolved over time. While some commentators have taken evidence of falling taxes across all countries to suggest that tax rates will eventually converge, our model suggests that the long run equilibrium will exhibit differentiation in tax levels across countries. An agenda for future research is to explain how average tax rates fall over time as markets become more integrated while still maintaining a stable differential between the core and the periphery.

Our analysis may yield insights concerning the number and size of countries as well. A feature of Alesina and Spolaore's model is that the level of public good provision (or 'the government' in their terminology) is the same across all jurisdictions, leading all countries to be the same size in equilibrium.[47] In the model of this present paper, by contrast, the level of public good provision is determined endogenously and varies across the two jurisdictions in the equilibrium under tax competition. Consequently, the sizes of the jurisdictions are different in equilibrium as well. The jurisdiction that provides the public good at the higher level attracts a larger share of firms in equilibrium. This result is interesting because it might have been expected that the low-tax jurisdiction would have attracted most of the firms. It would be interesting to take this analysis further, and investigate how the relative size of jurisdictions changes under alternative model specifications to get a better sense of what determines the relative size of countries.

A Appendix

Proof of Proposition 1. We first derive the efficient solution under the assumption that $x_A < x_B$. We will then show that the efficient solution cannot arise when $x_A = x_B > 0$.

Differentiate the planner's problem (2) to obtain the first and second order conditions for an interior efficient solution; that is, a solution in which $x_A < x_B$ and $\hat{s} \in (0, 1)$ by (1). We shall see from these first and second order conditions that the efficient solution is in fact obtained at $\hat{s} = 0$, and it will be obvious that the efficient solution cannot occur at $\hat{s} = 1$.

[47] The result of Alesina and Spolaore that all countries are the same size in equilibrium rests partly on the fact that individuals are uniformly distributed. They discuss informally the way in which their result would change if individuals were not uniformly distributed.

First, substitute the right hand side of (1) into (2) to obtain

$$\max_{\tau_A,\tau_B,x_A,x_B} \Omega\left(\tau_A,\tau_B,x_A,x_B\right) = (p-c) - x_A - x_B + \frac{1}{2}\left(k{x_B}^{\theta} - \frac{\left(\tau_B-\tau_A\right)^2}{k\left({x_B}^{\theta}-{x_A}^{\theta}\right)}\right)$$

Then, under the assumption that $x_A < x_B$, it is easy to see that the first and second order conditions for τ_A are as follows:

$$\begin{aligned} \frac{\partial\Omega\left(\tau_A,\tau_B,x_A,x_B\right)}{\partial\tau_A} &= \frac{\tau_B-\tau_A}{k\left({x_B}^{\theta}-{x_A}^{\theta}\right)} = 0; \\ \text{and } \frac{\partial^2\Omega\left(\tau_A,\tau_B,x_A,x_B\right)}{\partial\tau_A^2} &= -\frac{1}{k\left({x_B}^{\theta}-{x_A}^{\theta}\right)} < 0. \end{aligned} \tag{8}$$

Admitting corner solutions in taxes also requires that $\tau_B < \tau_A$. But in that case the outcome is the same as for $\tau_B = \tau_A$ because, by definition, $\hat{s} = 0$.

Next we have the same thing for τ_B:

$$\begin{aligned} \frac{\partial\Omega\left(\tau_A,\tau_B,x_A,x_B\right)}{\partial\tau_B} &= -\frac{\tau_B-\tau_A}{k\left({x_B}^{\theta}-{x_A}^{\theta}\right)} = 0; \\ \text{and } \frac{\partial^2\Omega\left(\tau_A,\tau_B,x_A,x_B\right)}{\partial\tau_A^2} &= -\frac{1}{k\left({x_B}^{\theta}-{x_A}^{\theta}\right)} < 0. \end{aligned} \tag{9}$$

Again, admitting corner solutions in taxes also requires that $\tau_B < \tau_A$. The second order conditions in (8) and (9) show that $\Omega\left(\tau_A,\tau_B,x_A,x_B\right)$ is concave in τ_A (holding τ_B constant) and τ_B (holding τ_A constant). From the first order condition, the efficient solutions for taxes is $\tau_A^E = \tau_B^E$.

Now we introduce the efficient condition for x_A and x_B. Take x_A first and so fix $x_B > x_A$:

$$\begin{aligned} \frac{\partial\Omega\left(\tau_A,\tau_B,x_A,x_B\right)}{\partial x_A} &= -1 - \frac{\theta x_A^{\theta-1}\left(\tau_B-\tau_A\right)^2}{2k\left({x_B}^{\theta}-{x_A}^{\theta}\right)^2} < 0 \\ \text{and } \frac{\partial^2\Omega\left(\tau_A,\tau_B,x_A,x_B\right)}{\partial x_A^2} &= -\frac{x_B^{\theta-2}\theta((\theta-1)\,{x_B}^{\theta}+(\theta+1)\,{x_A}^{\theta})(\tau_B-\tau_A)^2}{2k\left({x_B}^{\theta}-{x_A}^{\theta}\right)^3}. \end{aligned} \tag{10}$$

Next take x_B and so fix x_A. Then for any $x_B > x_A$:

$$\frac{\partial\Omega\left(\tau_A,\tau_B,x_A,x_B\right)}{\partial x_B} = -1 + \frac{\theta x_B^{\theta-1}}{2k}\left(k^2 + \frac{\left(\tau_B-\tau_A\right)^2}{\left({x_B}^{\theta}-{x_A}^{\theta}\right)^2}\right) = 0 \text{ and}$$

$$\frac{\partial^2\Omega\left(\tau_A,\tau_B,x_A,x_B\right)}{\partial x_B^2} =$$

$$-\frac{\theta\left(x_B\right)^{\theta-2}\left(\left(1-\theta\right)k^2\left({x_B}^{\theta}-{x_A}^{\theta}\right)^3 + \left(\left(1+\theta\right){x_B}^{\theta} - \left(1-\theta\right){x_A}^{\theta}\right)\left(\tau_B-\tau_A\right)^2\right)}{2k\left({x_B}^{\theta}-{x_A}^{\theta}\right)^3} < 0. \tag{11}$$

Condition (10) shows that $\Omega(\tau_A, \tau_B, x_A, x_B)$ is everywhere declining in x_A and therefore achieves a maximum when $x_A = 0$ given $x_B > 0$. The second order condition cannot be signed unambiguously but this does not matter given that the first order condition is unambiguously negative.

Condition (11) shows that $\Omega(\tau_A, \tau_B, x_A, x_B)$ is concave in x_B and ensures a unique efficient level. It is immediate from (8) and (9) that the efficient level of taxation is obtained when $\tau_A^E = \tau_B^E$. Using $\tau_A^E = \tau_B^E$ in (10), $\partial\Omega(\tau_A, \tau_B, x_A, x_B)/\partial x_A = -1$ and $\partial^2\Omega(\tau_A, \tau_B, x_A, x_B)/\partial x_A^2 = 0$ so $\Omega(\tau_A, \tau_B, x_A, x_B)$ is maximized with respect to x_A at $x_A = 0$. Using $\tau_A^E = \tau_B^E$ in (11), setting $\partial\Omega(\tau_A, \tau_B, x_A, x_B)/\partial x_A = 0$ and solving in terms of x_B^E we have that $x_B^E = \left(\frac{1}{2}\right)^{\frac{1}{1-\theta}}(\theta k)^{\frac{1}{1-\theta}}$. In addition, it is clear by inspection that $\partial^2\Omega(\tau_A, \tau_B, x_A, x_B)/\partial x_B^2 < 0$ for any $x_A < x_B$. Thus we have characterized the efficient solution as $\tau_A^E = \tau_B^E$, $x_A = 0$ and $x_B^E = \left(\frac{1}{2}\right)^{\frac{1}{1-\theta}}(\theta k)^{\frac{1}{1-\theta}}$ under the assumption that $x_A < x_B$.

It remains to show that efficiency cannot be increased by setting $x_A = x_B > 0$. In that case, the value of $\hat{s}$ depends on the value of τ_A relative to τ_B: If $\tau_A > \tau_B$ then, by (1), $\hat{s} = 0$; if $\tau_A < \tau_B$ then $\hat{s} = 1$; if $\tau_A = \tau_B$ then by assumption $\hat{s} = \frac{1}{2}$. Take each case in turn.

Suppose first that efficiency is achieved for $x_A = x_B$ and $\tau_A > \tau_B$. By (1), $\hat{s} = 0$ and so by (2),

$$\Omega(\tau_A, \tau_B, x_A, x_B) = p - c - x_A - x_B + \frac{1}{2}\left(k{x_B}^{\theta}\right).$$

But efficiency could be increased by reducing x_A; contradiction.

Next suppose that efficiency is achieved for $x_A = x_B$ and $\tau_A < \tau_B$. By (1), $\hat{s} = 1$ and so by (2),

$$\Omega(\tau_A, \tau_B, x_A, x_B) = p - c - x_A - x_B + \frac{1}{2}\left(k{x_A}^{\theta}\right).$$

But now efficiency could be increased by reducing x_B; contradiction.

Finally, suppose that efficiency is achieved for $x_A = x_B$ and $\tau_A = \tau_B$. By (1), $\hat{s} = \frac{1}{2}$ and so by (2),

$$\begin{aligned}\Omega(\tau_A, \tau_B, x_A, x_B) &= p - c - x_A - x_B + \frac{1}{2}\left(\frac{1}{2}k{x_A}^{\theta} + \frac{1}{2}k{x_B}^{\theta}\right)\\ &= p - c - x_A - x_B + \frac{1}{2}\left(k{x_B}^{\theta}\right)\end{aligned}$$

where the second equality follows because $x_A = x_B$. But this is the same outcome as for $x_A = x_B$ and $\tau_A > \tau_B$, and for that case we saw that it was possible to increase efficiency by reducing x_A; contradiction. $\square$

Proof of Lemma 1. Fix $0 \leq x_A < x_B$. To solve for τ_A^*, fix $\tau_B \geq 0$. We want to solve

$$\max_{\tau_A} r_A(\tau_A, \tau_B) = \frac{\tau_A(\tau_B - \tau_A)}{k\left(x_B^\theta - x_A^\theta\right)} - x_A.$$

First, looking at the second order condition, we see that

$$\partial^2 r_A / \partial \tau_A^2 = -2/\left(k\left(x_B^\theta - x_A^\theta\right)\right) < 0,$$

so $r_A(\tau_A, \tau_B)$ is everywhere concave with respect to τ_A. Setting the first order condition $\partial r_A / \partial \tau_A = (-2\tau_A^* + \tau_B)/\left(k\left(x_B^\theta - x_A^\theta\right)\right)$ equal to zero and rearranging in terms of τ_A^* obtains $\tau_A(\tau_B; x_A, x_B, k) = \tau_B/2$.

To solve for τ_B^*, fix $\tau_A \geq 0$. Now we want to solve

$$\max_{\tau_B} r_B(\tau_A, \tau_B) = \tau_B\left(1 - \frac{(\tau_B - \tau_A)}{k\left(x_B^\theta - x_A^\theta\right)}\right) - x_B.$$

Again, looking at the second order condition first, we see that

$$\partial^2 r_B / \partial \tau_B^2 = -2/\left(k\left(x_B^\theta - x_A^\theta\right)\right) < 0,$$

so $r_B(\tau_A, \tau_B)$ is concave with respect to τ_B. Setting the first order condition $\partial r_B / \partial \tau_B = 1 + (\tau_A - 2\tau_B^*)/\left(k\left(x_B^\theta - x_A^\theta\right)\right)$ equal to zero and rearranging in terms of τ_B obtains the result. □

Proof of Proposition 2. For $x_A = x_B$ both governments provide the same level of public goods and we effectively have a standard Bertrand equilibrium in homogeneous products. Then $\hat{s} = 1/2$.

For $x_A < x_B$, by Lemma 1 for given τ_B, $r_A(\tau_A, \tau_B)$ is maximized by $\tau_A^* = \tau_B/2$. For given τ_A, $r_B(\tau_A, \tau_B)$ is maximized by $\tau_B^* = \tau_A/2 + k\left(x_B^\theta - x_A^\theta\right)/2$. Solving simultaneously obtains the reduced form expressions for $\tau_A^*(x_A, x_B; k)$ and $\tau_B^*(x_A, x_B; k)$.

Using $\tau_A^*(x_A, x_B; k) = k\left(x_B^\theta - x_A^\theta\right)/3$ and $\tau_B^*(x_A, x_B; k) = 2k\left(x_B^\theta - x_A^\theta\right)/3$ in $\hat{s} = (\tau_B - \tau_A)/k\left(x_B^\theta - x_A^\theta\right)$ obtains $\hat{s} = 1/3$. □

Proof of Proposition 3: To determine Government A's set of best responses, we investigate the properties of $r_A(x_A, x_B)$. It is clear by inspection of (4) that $r_A(x_A, x_B)$ achieves a minimum at $x_A = x_B$. So we can rule out $x_A = x_B$ from $BR_A(x_B)$. Now observe that if $0 \leq x_A < x_B$ then $r_A = k\left(x_B^\theta - x_A^\theta\right)/9 - x_A$ so $r_A(x_A, x_B)$ is everywhere downward sloping and convex over this range. Consequently, $x_A = 0$ maximizes $r_A(x_A, x_B)$ for $0 \leq x_A < x_B$. If on the other hand $0 \leq x_B < x_A$, then $r_A = 4k\left(x_A^\theta - x_B^\theta\right)/9 - x_A$, and $r_A(x_A, x_B)$

is everywhere strictly concave. Differentiating once, setting the result equal to zero and rearranging, we find that $r_A(x_A, x_B)$ has a unique maximum at $x_A = \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}$. Thus $BR_A(x_B) \in \left\{0, \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}\right\}$. Because $r_B(x_A, x_B)$ has the same functional form as $r_A(x_A, x_B)$, it follows that $BR_B(x_A) \in \left\{0, \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}\right\}$; see (5). Recall that $r_A(x_A, x_B)$ and $r_B(x_A, x_B)$ achieve a minimum at $x_A = x_B$. So $\left(0, \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}\right)$ is the only set of mutual best responses and must therefore be a Nash equilibrium. Clearly, there are two Nash equilibria; $\left(0, \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}\right)$ and $\left(\left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}, 0\right)$. But we may now assume, without loss of generality, that $x_A < x_B$. Then $(x_A^*, x_B^*) = \left(0, \left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}\right)$ is the unique Nash equilibrium. Using these values to solve for equilibrium taxes from Proposition 2, we have that $\tau_A^* = \frac{1}{3}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$ and $\tau_B^* = \frac{2}{3}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$. Thus we have proved the proposition. □

Proof of Proposition 4. Differentiate the planner's problem (2), to obtain the first order condition; $d\Omega/dx_B = \frac{1}{2}\theta k x_B^{\theta-1} - 1$. Setting this equal to 0 and solving for x_B obtains x_B^E. □

Proof of Proposition 5. To see that $r_A^\mu(\varepsilon)$ is concave in ε, differentiate $r_A^\mu(\varepsilon)$ once with respect to ε to obtain

$$\frac{dr_A^\mu(\varepsilon)}{d\varepsilon} = \frac{1}{6} - \frac{\varepsilon}{k\left(\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}\right)}.$$

Clearly, $dr_A^\mu(\varepsilon)/d\varepsilon > 0$ as $\varepsilon \to 0$ and $dr_A^\mu(\varepsilon)/d\varepsilon < 0$ as ε becomes large. Also, $dr_A^\mu(\varepsilon)/d\varepsilon$ declines monotonically with ε. The unique value of ε that maximizes r_A^μ is $\varepsilon = \frac{1}{6}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$. By definition, a minimum tax for which $\varepsilon < \frac{1}{6}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$ cannot be on the frontier because both governments make higher rents by increasing ε to $\varepsilon = \frac{1}{6}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$; thus we have defined the lower bound of the non-renegotiable minimum tax frontier.

By definition, the minimum tax on the frontier must yield higher rents for both governments than no minimum tax. Because $r_B^\mu(\varepsilon)$ increases monotonically with ε, B makes higher rent with any minimum tax than with no minimum tax. However, $r_A^\mu(\varepsilon)$ declines monotonically with ε for $\varepsilon > \frac{1}{6}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$. Therefore, a level of $\varepsilon > \frac{1}{6}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$ must exist at which $r_A^\mu(\varepsilon) = r_A^\mu(0)$. It is easy to establish that $r_A^\mu(0) = \frac{2}{3}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$. Then $\varepsilon = \frac{1}{3}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$ is the unique level of $\varepsilon > \frac{1}{6}k\left(\frac{4}{9}\theta k\right)^{\frac{\theta}{1-\theta}}$ at which $r_A^\mu(\varepsilon) = r_A^\mu(0)$; thus we have defined the upper bound of the non-renegotiable minimum tax frontier. □

Minimum Tax Anticipated: Characterization of Equilibrium. To determine Government A's set of best responses with the minimum tax, we investigate the properties of $r_A^\mu(x_A, x_B; \varepsilon)$. First we check the range $0 \leq x_A < x_B$, over which $r_A^\mu(x_A, x_B; \varepsilon) = \frac{1}{9}k\left(x_B^\theta - x_A^\theta\right) - x_A + \frac{1}{6}\varepsilon - \varepsilon^2/2k\left(x_B^\theta - x_A^\theta\right)$. Taking the first derivative, we have $dr_A/dx_A =$

$-1-\theta x_A^{\theta-1}\left(2k^2+9\varepsilon^2/\left(x_B^{\theta}-x_A^{\theta}\right)^2\right)/18k<0$; $r_A^{\mu}\left(x_A,x_B;\varepsilon\right)$ is everywhere downward sloping over the range $0\le x_A<x_B$. Now note that $r_A^{\mu}\left(x_A,x_B;\varepsilon\right)>-x_A$ at $x_A=x_B>0$ for $\varepsilon\in\left(0,\frac{2}{3}k\left(x_B^{\theta}-x_A^{\theta}\right)\right)$, and $r_A^{\mu}\left(x_A,x_B;\varepsilon\right)=-x_A$ at $x_A=x_B>0$ for $\varepsilon=\frac{2}{3}k\left(x_B^{\theta}-x_A^{\theta}\right)$. We can conclude that $r_A^{\mu}\left(x_A,x_B;\varepsilon\right)\ge -x_A$ for all ε as $x_A\to x_B$ from below. Consequently, $x_A=0$ maximizes $r_A^{\mu}\left(x_A,x_B;\varepsilon\right)$ for the range $0\le x_A<x_B$ and $x_A=0$ dominates $x_A=x_B$. Thus $x_A^{\mu}\left(\varepsilon\right)=0$ is the best response over the range $0\le x_A<x_B$.

If on the other hand $0\le x_B<x_A$, then $r_A^{\mu}\left(x_A,x_B;\varepsilon\right)=\frac{4}{9}k\left(x_A^{\theta}-x_B^{\theta}\right)-x_A+\frac{2}{3}\varepsilon+\varepsilon^2/4k\left(x_A^{\theta}-x_B^{\theta}\right)$. Taking the first derivative, we have

$$dr_A/dx_A=-1+\frac{4}{9}k\theta x_A^{\theta-1}\left(1-9\varepsilon^2/16\left(x_A^{\theta}-x_B^{\theta}\right)^2\right).$$

We cannot solve explicitly for $x_A^{\mu}\left(\varepsilon\right)$ over the range $0\le x_B<x_A$ without specifying θ. However, by specifying values of ε we can obtain a characterization of $x_A^{\mu}\left(\varepsilon\right)$. To illustrate, fix ε at its maximum admissible value $\varepsilon=\overline{\varepsilon}=\frac{2}{3}k\left(x_B^{\theta}-x_A^{\theta}\right)$, and substitute this into the first derivative. We have $dr_A/dx_A=-1+\frac{5}{12}k\theta x_A^{\theta-1}$. Setting the result equal to zero and solving, we have $x_A^{\mu}\left(\overline{\varepsilon}\right)=\left(\frac{5}{12}\theta k\right)^{\frac{1}{1-\theta}}$. Then, following the same logic as in Section 4.2 preceding Proposition 3, and using the assumption that $x_A<x_B$, we have that $\left(x_A^{\mu}\left(\overline{\varepsilon}\right),x_B^{\mu}\left(\overline{\varepsilon}\right)\right)=\left(0,\left(\frac{5}{12}\theta k\right)^{\frac{1}{1-\theta}}\right)$ is the unique Nash equilibrium. Taxes are obviously the same across jurisdictions for $\varepsilon=\overline{\varepsilon}$, at $\tau_A^{\mu}=\tau_B^{\mu}=k\left(\frac{5}{12}\theta k\right)^{\frac{\theta}{1-\theta}}$ and $\hat{s}=1$.

Notice that $x_A^{\mu}\left(\overline{\varepsilon}\right)=\left(\frac{5}{12}\theta k\right)^{\frac{1}{1-\theta}}<x_A^{\mu}\left(0\right)=\left(\frac{4}{9}\theta k\right)^{\frac{1}{1-\theta}}$. More generally, by the implicit function theorem we know that $x_A^{\mu}\left(\varepsilon\right)$ may be treated as a continuous function of ε. It can be established that $x_A^{\mu}\left(\varepsilon\right)$ is inversely related to ε as ε is varied over the interval $\varepsilon\in[0,\overline{\varepsilon}]$ for $0\le x_B<x_A$. Following, once again. the same logic as in Section 4.2 preceding Proposition 3, we have that $\left(x_A^{\mu}\left(\varepsilon\right),x_B^{\mu}\left(\varepsilon\right)\right)=\left(0,BR_B\left(0;\varepsilon\right)\right)$ is the unique Nash equilibrium (given that $x_A<x_B$).[48]

We want to go one step further, and investigate the behavior of $r_A\left(0,BR_B\left(0;\varepsilon\right);\varepsilon\right)$ and $r_B\left(0,BR_B\left(0;\varepsilon\right);\varepsilon\right)$ as ε is varied in order to determine the non-renegotiable minimum tax frontier. While this cannot be done at a general level, it can be done for the specific value $\theta=\frac{1}{2}$, which we believe to be generally illustrative. We maintain the assumption that $x_A<x_B$ and solve for $x_B^{\mu}\left(\varepsilon\right)$. This root is very cumbersome to write down, and since $x_A^{\mu}\left(\varepsilon\right)=0$ is a dominant strategy for Government A we jump straight to the equilibrium

[48]By the same arguments as in Section 4.2, $BR_B\left(0;\varepsilon\right)\neq 0$.

value:

$$x_B^*(\varepsilon) = 512(-2)^{1/3} k^8 - (-2)^{2/3} \phi^{2/3} + 16k^4 \left(-2187(-2)^{1/3} \varepsilon^2 + 2\phi^{1/3}\right) / \left(1944\phi^{1/3}\right)$$

where

$$\phi = -8192k^{12} + 839808k^8\varepsilon^2 - 14348907k^4\varepsilon^4 + 59049\sqrt{-768k^{12}\varepsilon^6 + 59049k^8\varepsilon^8}.$$

This solution for $x_B^\mu(\varepsilon)$ is illustrated for $k = 1$ in Figure 3 and used to define the non-renegotiable minimum tax frontier illustrated in Figure 4.

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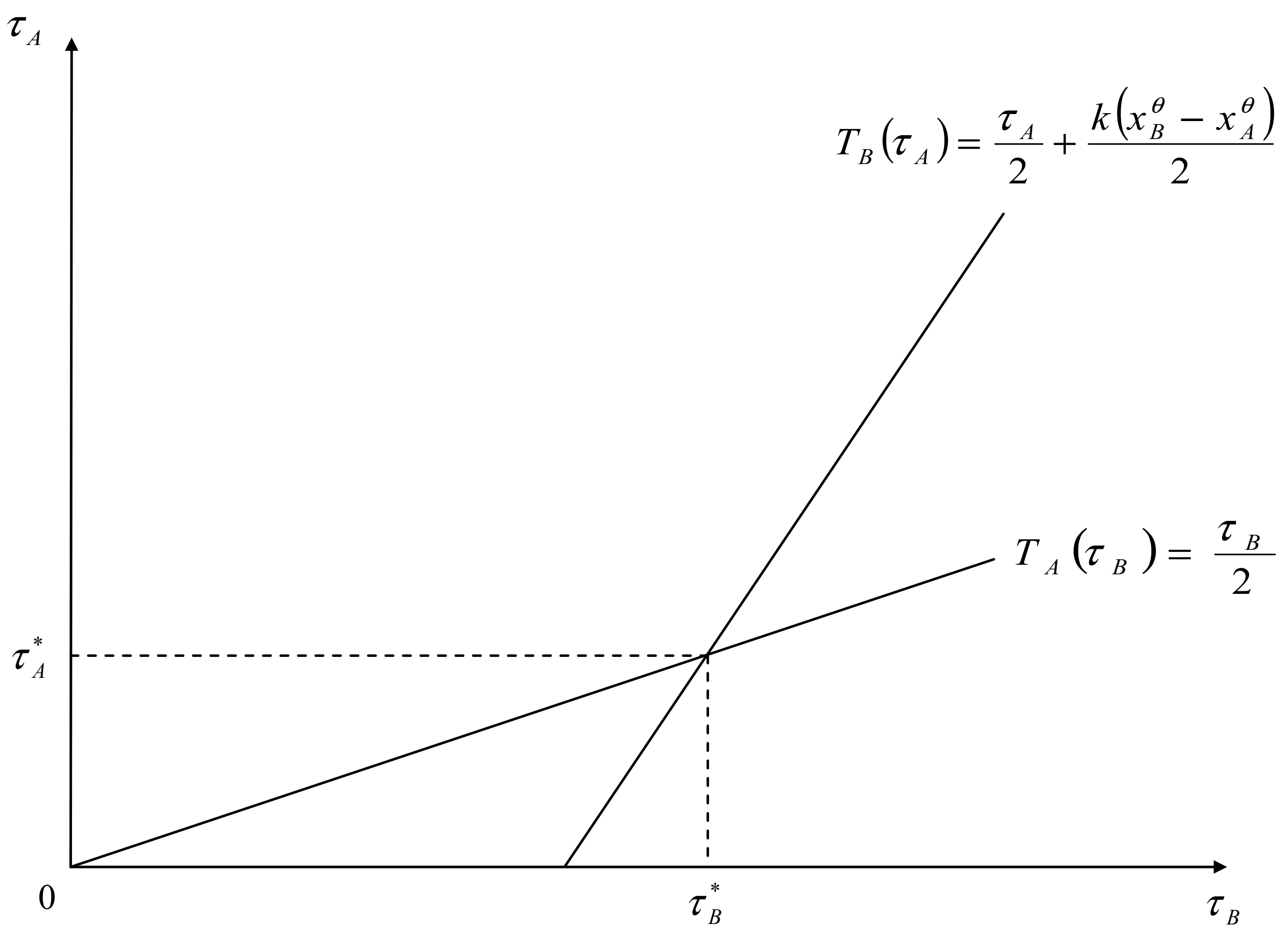
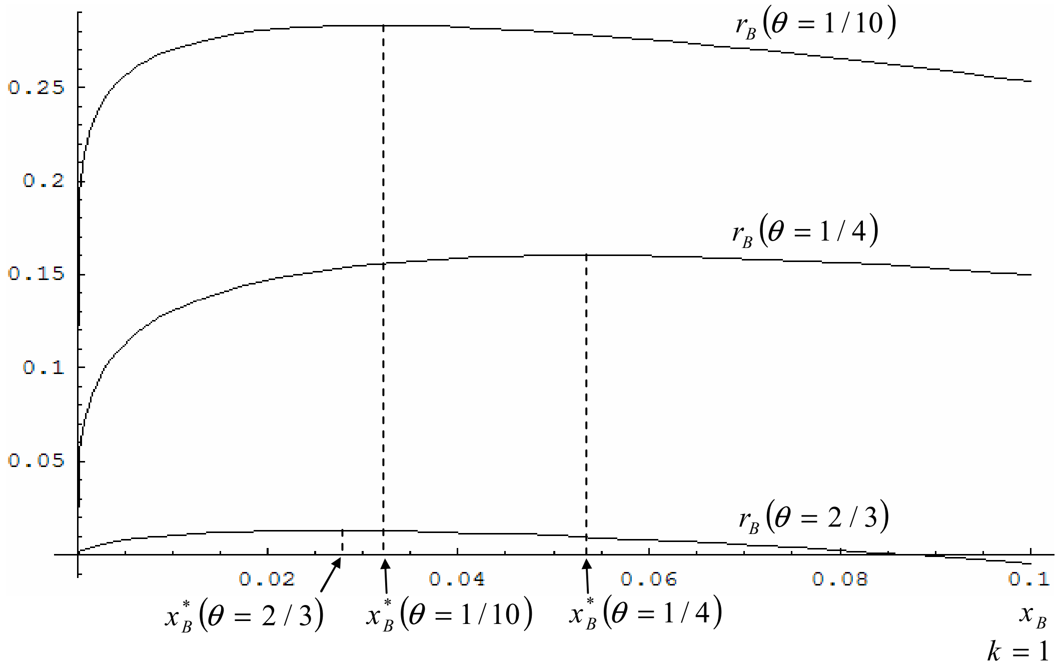
Figure 1



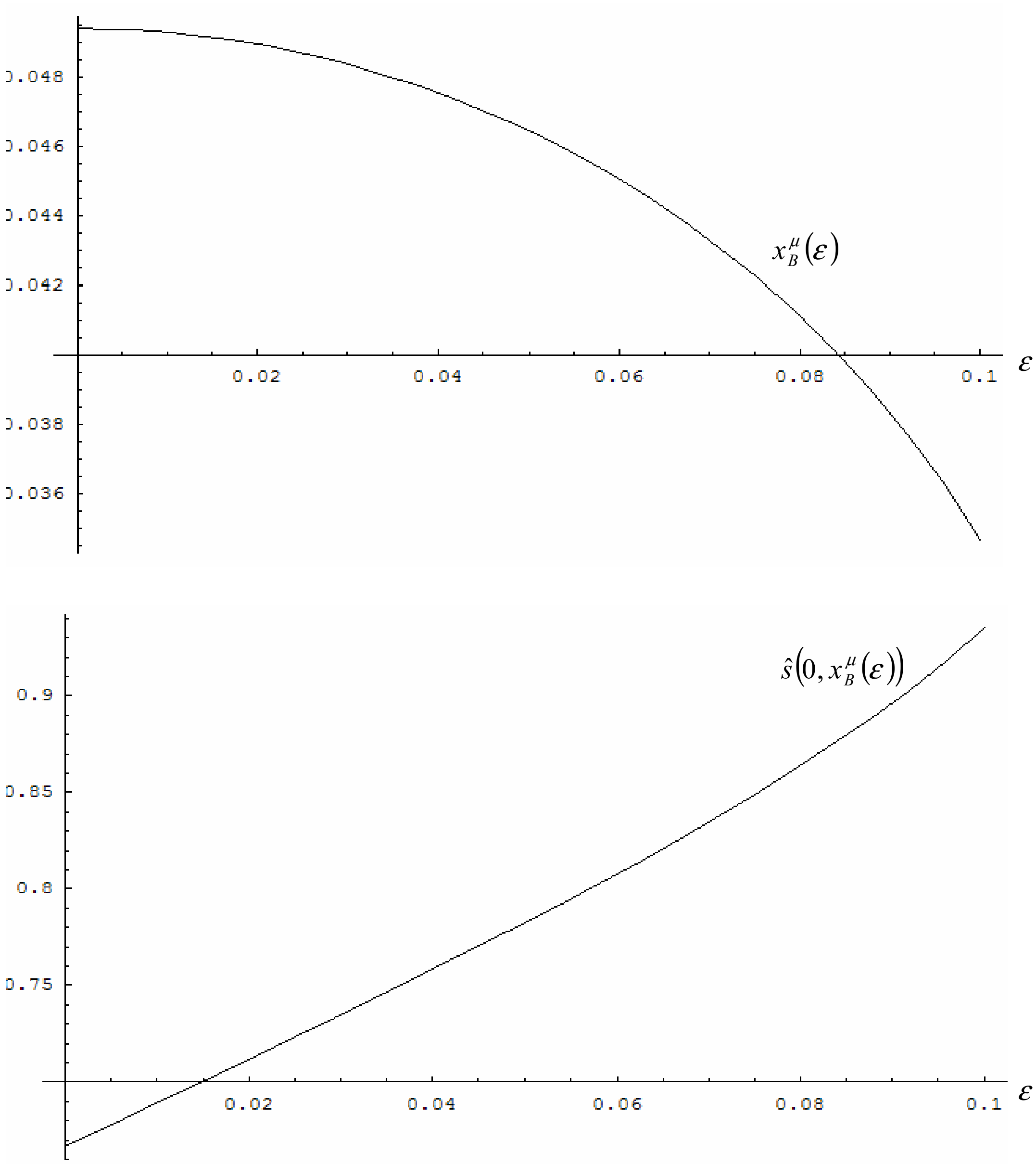
τ_A
$T_B(\tau_A) = \frac{\tau_A}{2} + \frac{k\left(x_B^\theta - x_A^\theta\right)}{2}$
$T_A(\tau_B) = \frac{\tau_B}{2}$
τ_A^*
0
τ_B^*
τ_B

Figure 2



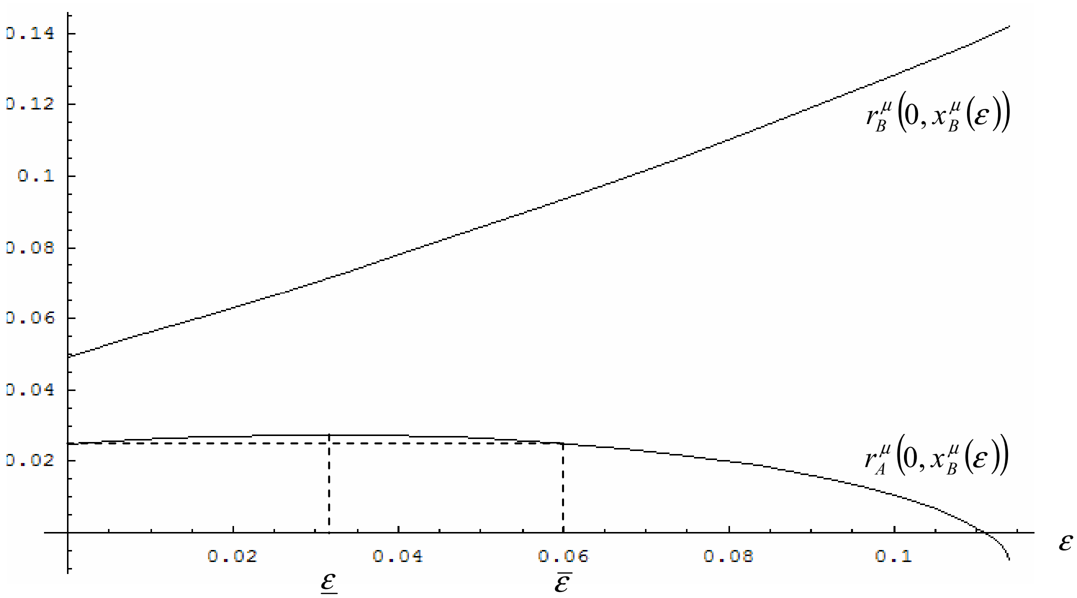
$r_B(\theta = 1/10)$
$r_B(\theta = 1/4)$
$r_B(\theta = 2/3)$
0.25
0.2
0.15
0.1
0.05
0.02
0.04
0.06
0.08
0.1
$x_B^*(\theta = 2/3)$
$x_B^*(\theta = 1/10)$
$x_B^*(\theta = 1/4)$
x_B
$k = 1$

Figure 3



0.048
0.046
0.044
0.042
0.038
0.036
0.02
0.04
0.06
0.08
0.1
ε
$x_B^{\mu}(\varepsilon)$
0.9
0.85
0.8
0.75
0.02
0.04
0.06
0.08
0.1
ε
$\hat{s}(0, x_B^{\mu}(\varepsilon))$

Figure 4


0.14
0.12
0.1
0.08
0.06
0.04
0.02
0.02
0.04
0.06
0.08
0.1
$r_B^{\mu}\left(0, x_B^{\mu}(\varepsilon)\right)$
$r_A^{\mu}\left(0, x_B^{\mu}(\varepsilon)\right)$
ε
$\underline{\varepsilon}$
$\bar{\varepsilon}$