THE PERFORMANCE OF SETAR MODELS: A REGIME CONDITIONAL EVALUATION OF POINT, INTERVAL AND DENSITY FORECASTS

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The performance of SETAR models: a regime conditional evaluation of point, interval and density forecasts

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Abstract

The aim of this paper is to analyse the out-of-sample performance of SETAR models relative to a linear AR and a GARCH model using daily data for the Euro effective exchange rate. The evaluation is conducted on point, interval and density forecasts, unconditionally, over the whole forecast period, and conditional on specific regimes. The results show that overall the GARCH model is better able to capture the distributional features of the series and to predict higher-order moments than the SETAR models. However, from the results there is also a clear indication that the performance of the SETAR models improves significantly conditional on being on specific regimes.

Keywords: SETAR models, forecasting accuracy, point forecasts, MSFEs, interval forecasts, density forecasts, Euro effective exchange rate.

JEL: C22, C51, C53, E17

1. INTRODUCTION

In this study we focus on the dynamic representation of the euro effective exchange rate and on its short run predictability. The analysis is conducted in the context of univariate models, exploiting recent developments of nonlinear time series econometrics. The models that we adopt to describe the dynamic behaviour of the euro effective exchange rate series are the self-exciting threshold autoregressive (SETAR) models, which represent a stochastic process generated by the alternation of different regimes. Although there have been many applications of threshold models to describe the nonlinearities and asymmetries of exchange rate dynamics (Kräger and Kugler, 1993, Brooks, 1997, 2001), there are still few studies on the forecasting performance of the models, using historical time series data. Notoriously, the in-sample advantages of nonlinear models have only rarely provided better out-of-sample forecasts compared with a random walk or a simple AR model.

One reason for the poor forecast performance of nonlinear models lies in the different characteristics of the in-sample and out-of-sample periods. For example, nonlinearities may be highly significant in-sample but fail to carry over to the out-of-sample period (Diebold and Nason, 1990). In a recent application to the yen/US dollar exchange rate, Boero and Marrocu (2002b) show clear gains from the SETAR model over the linear competitor, on MSFEs evaluation of point forecasts, in sub-samples characterised by stronger non-linearities. On the other hand, the performance of the SETAR and AR models was indistinguishable over the sub-samples with weaker degrees of nonlinearity.

The oft-claimed superiority of the linear models has also been challenged by a number of recent studies suggesting that the alleged poor forecasting performance of nonlinear models can be due to the evaluation and measurement methods adopted. In a Monte Carlo study, Clements and Smith (2001) show that the evaluation of the whole forecast density may reveal gains to the nonlinear models which are systematically masked in MSFE comparisons. Boero and Marrocu (2002a, 2002b) confirm this result in various applications with actual data, and show that when the nonlinear models are evaluated on interval and density forecasts, they can exhibit accuracy gains which remain concealed if the evaluation is based only on MSFE metric. Some gains of the SETAR models have also been found, even in terms of MSFEs, when the forecast accuracy is evaluated conditional upon a specific regime (Tiao and Tsay, 1994, Clements and Smith, 2001, and Boero and Marrocu, 2002a). An interesting result, common to these studies, suggests that SETAR models can produce point forecasts that are superior to those obtained from a linear model, when the forecast observations belong to the regime with fewer observations.

In the present study we investigate further the possibility that the SETAR models are more valuable in terms of forecasting accuracy when the process is in a particular regime. We do this by extending the 'conditional' evaluation approach to interval and density forecasts, as well as point forecasts. By using daily data for the returns of the euro effective exchange rate (euro-EER), the performance of two and three-regime SETAR models is evaluated against that of a simple AR and a GARCH model. The evaluation of the models conditional on the regimes is possible because of the large number of data points available in our application. Point forecasts are evaluated by means of MSFEs and the Diebold and Mariano test. Interval forecasts are assessed by means of the likelihood ratio tests proposed by Christoffersen (1998), while the techniques used to evaluate density forecasts are those introduced by Diebold *et al*. (1998). For the evaluation of density forecasts we also use the modified version of the Pearson goodness-of-fit test and its components, as proposed by Anderson (1994) and recently discussed in Wallis (2002). These methods provide information on the nature of departures from the null hypothesis, with respect to specific characteristics of the distribution of interest - such as location, scale, skewness and kurtosis – and may offer valuable support in the evaluation of the models.

The rest of the paper is organised as follows. In section 2 we present the statistical properties of the data and the results of the linearity tests. In section 3 we report the results from the modelling and forecasting exercises. In section 4 we summarise the results and make some concluding remarks.

2. LINEARITY TESTS AND MODELS SPECIFICATION

In this study we analyse the dynamic behaviour of the returns of the daily euro nominal effective exchange rate over the period 30/1/1990-10/07/02 (3081 observations). The nominal effective exchange rate for the euro is calculated by the European Central Bank[1].

The log-levels and the returns of the series are depicted in figure 1. In table 1a we report the summary of the descriptive statistics of the returns series for three different periods: the entire sample period, the estimation period and the forecasting period. The estimation sample refers to the period 03/01/1990-30/12/1999 (2439 observations), while the forecasting sample extends to the period 03/01/2000-10/07/2002 (642 observations). The splitting of the entire sample between estimation and forecasting period allows us to withhold around 20% of the total number of observations in order to evaluate the forecasting performance of the nonlinear models, as suggested by Granger (1993)[2].

The data accord well with the stylised facts of exchange rate series which emerge from the empirical literature. The returns of the series are mean-stationary, periods of high volatility and tranquillity tend to cluster together, the sample moments suggest fat taildness of the return distribution. Kurtosis is particularly high in the estimation period. The forecasting period exhibits a larger variance and less kurtosis.

2.1 Linearity tests

In order to detect nonlinearities in the euro-EER returns we performed the RESET test and the S_2 test proposed by Luukkonen-Saikkonen-Teräsvirta (1988). Both tests are devised for the null hypothesis of linearity. While the RESET test is devised for a generic form of misspecification, the S_2 test is formulated for a specific alternative hypothesis, i.e. smooth transition autoregressive (STAR)-type nonlinearity. Luukkonen-Saikkonen-Teräsvirta, however, show that the S_2 test has reasonable power even when the true model is a SETAR one. The RESET test has been computed in the traditional version and in the modified version found to be superior by Thursby and Schmidt (1977)[3]. The S_2 test is performed assuming that the variable governing the transition from one regime to the other is y_{t-d} with the delay parameter d in the range [1,6][4].

Table 1b reports the results of the linearity tests computed for the whole sample period, the estimation period and the forecast period. The selected lag order p ranges from 3 to 5 in order to check for the effects of different dynamic structures. The tests applied to the entire sample period and to the estimation period lead to the rejection of the null in a large number of cases, indicating that there is strong evidence of nonlinear components for the data. However, when the tests are applied to the forecast period the evidence based on the RESET tests indicates that nonlinearities are present with less intensity. The S_2 test (for d=3), on the other hand, is highly significant at almost all lags.

2.2 MODELS SPECIFICATION

The forecasting models adopted in this study belong to the class of threshold autoregressive (TAR) models. These are compared with a simple AR model and with a GARCH model. The basic idea of the TAR models is that the behaviour of a process is described by a finite set of linear autoregressions[5]. The appropriate AR model that generates the value of the time series at each point in time is determined by the relation of a conditioning variable to the threshold values. If the conditioning variable is the dependent variable itself after some delay *d* (y_{t-d}), the model is known as *self-exciting threshold autoregressive* (SETAR) model.

The SETAR model is piecewise-linear in the space of the threshold variable, rather than in time. An interesting feature of SETAR models is that the stationarity of y_t does not require the model to be stationary in each regime, on the contrary, the limit cycle behaviour that this class of models is able to describe arises from the alternation of explosive and contractionary regimes[6].

In this study we choose a two-regime (SETAR-2) and a three-regime (SETAR-3) SETAR models, which can be represented as follows:

SETAR-2: $$y_t = \begin{cases} f_0^{(1)} + \sum_{i=1}^{p^{(1)}} f_i^{(1)} y_{t-i} + e_t^{(1)} & if\ y_{t-d} \le r \\ f_0^{(2)} + \sum_{i=1}^{p^{(2)}} f_i^{(2)} y_{t-i} + e_t^{(2)} & if\ y_{t-d} > r \end{cases}$$

SETAR-3: $$y_t = \begin{cases} f_0^{(1)} + \sum_{i=1}^{p^{(1)}} f_i^{(1)} y_{t-i} + e_t^{(1)} & if\ y_{t-d} \le r_1 \\ f_0^{(2)} + \sum_{i=1}^{p^{(2)}} f_i^{(2)} y_{t-i} + e_t^{(2)} & if\ r_1 < y_{t-d} \le r_2 \\ f_0^{(3)} + \sum_{i=1}^{p^{(3)}} f_i^{(3)} y_{t-i} + e_t^{(3)} & if\ y_{t-d} > r_2 \end{cases}$$

where $\varepsilon_t^{(j)}$ is assumed IID($0,\sigma^{2(j)}$) and r_j represent the threshold values.

The models are estimated, over the period 03/01/1990-30/12/1999, by following the three-stage procedure suggested by Tong (1983) for the case of a SETAR-2 (p_1, p_2; d) model. For given values of *d* and *r*, separate AR models are fitted to the appropriate subsets of data, the order of each model is chosen according to the usual AIC criteria. In the second stage *r* can vary over a set of possible values while d has to remain fixed, the re-estimation of the separate AR models allows the determination of the r parameter, as the one for which AIC(d) attains its minimum value. In stage three the search over d is carried out by repeating both stage 1 and stage 2 for $d=d_1, d_2, ..., d_p$. The selected value of d is, again, the value that minimises AIC(d).

The selected specifications are reported in table 2. The models show clear evidence that the euro-EER returns are strongly characterised by nonlinearities as the dynamic structure, the estimated coefficients and the error variance differ across regimes. In the forecasting exercise discussed in the next sections the performance of the estimated SETAR models is compared with that of a restricted AR(3) model and an AR(1)-GARCH(1,1). The latter turned out to be adequate in capturing the volatility displayed by the series and is expected to produce better calibrated density and interval forecasts than the simple AR model. It is of interest to see how the SETAR model compares with the GARCH model in predicting higher-order moments.

3. THE FORECASTING EXERCISE

In this section we conduct three different forecasting exercises intended to evaluate the models on their ability to produce point forecasts, density and interval forecasts. For each kind of forecasts the evaluation is conducted over the entire forecasting sample - *unconditional evaluation* - and over each regime of the SETAR models - *conditional on regime*. So far, regime-conditional evaluations of nonlinear models have focussed on point forecasts only (Clements and Smith, 1999, and Boero and Marrocu, 2002a). In the following analysis we explore whether a conditional evaluation extended to density and interval forecasts can add useful information on the relative quality of the forecasts of the models.

3.1. POINT FORECASTS EVALUATION

The forecasting sample covers the period 03/01/00-10/07/02; the models are specified and estimated over the first estimation period, 03/01/1990-30/12/1999, and the first set of 1 to 5 steps ahead forecast (h=1, 2,…5) computed. The models are then estimated recursively keeping the same specification but extending the sample with one observation each time. In this way 638 point forecasts are obtained for each forecast horizon. These forecasts can be considered *genuine forecasts* as in the specification stage we completely ignore the information embodied in the forecasting period. The computation of multi-step-ahead forecasts from nonlinear models involves the solution of complex analytical calculations and the use of numerical integration techniques, or alternatively, the use of simulation methods. In this study the forecasts are obtained by applying the Monte Carlo method with regime-specific error variances, so that each point forecast is obtained as the average over 500 replications (see Clements and Smith, 1997, 1999)[7].

In table 3 we report the MSFEs normalised with respect to the AR model (panel A) and the GARCH model (panel B). The values are calculated as the ratio $MSFE_{SETAR}/MSFE_{AR}$ and $MSFE_{SETAR}/MSFE_{GARCH}$, so that a value less than 1 denotes a better forecast performance of the SETAR model. We have also applied the Diebold and Mariano (DM) test for equality of forecasting accuracy, and indicated with stars the cases for which the MSFEs of the competing models are statistically significantly different[8]. From table 3 we can see that when the comparison is conducted with respect to the AR model (panel A), the assessment of the models by regime produces more cases in favour of the SETAR models than those obtained from the evaluation of the entire forecasting sample. This is particularly evident for the SETAR-2 model in regime 2. However, when the rival model is the AR(1)-GARCH(1,1) the differences between the MSFEs in terms of the Diebold and Mariano test are in most cases not significant (panel B).

3.2. DENSITY FORECASTS EVALUATION

Previous authors have found that an evaluation based on density forecasts may reveal greater discrimination over the linear models than evaluations based on the first moment (Clements and Smith, 2000, 2001, Boero and Marrocu, 2002a). In this section, we evaluate the one-step-ahead density forecasts of the models by applying the methods suggested by Diebold *et al*. (1998) and surveyed by Tay and Wallis (2000). We also apply the modified Pearson goodness-of-fit test and its components, proposed by Anderson (1994) and recently discussed in Wallis (2002) with applications to inflation forecasts.

Density forecasts

The evaluation of the density forecasts is based on the analysis of the probability integral transforms of the actual realisations of the variables with respect to the forecast densities of the models. These are defined as $z_t = F_t(y_t)$, where F(.) is the forecast cumulative distribution function and y_t is the observed outcome. Thus, z_t is the forecast probability of observing an outcome no greater than that actually realised. If the density forecasts correspond to the true density, then the sequence of probability integral transforms $\{z_t\}_{t=1}^{N}$ is i.i.d. uniform (0,1). To check whether the sequence of probability integral transforms departs from the i.i.d. uniform hypothesis, the distributional properties of the z_t series are examined by visual inspection of plots of the empirical distribution function of the z_t series, which are compared with those of a uniform (0,1). To supplement these graphical devices, the Kolmogorov-Smirnov test[9] can be used on the sample distribution function of the z_t series (see

Diebold *et al*., 1999, and Tay and Wallis, 2000). Alternatively, uniformity can be tested by applying the Pearson chi-squared goodness-of-fit test. These methods address the unconditional uniformity hypothesis. The independence part of the i.i.d. uniform (0,1) hypothesis can be assessed by studying the correlograms of the z_t series and of powers of this series (to establish the existence of dependence in higher moments) and applying formal tests of autocorrelation.

In our analysis below, we use both the Kolmogorov-Smirnov test and the Pearson X^2 test, in the modified version suggested by Anderson (1994), and the Ljung-Box test for autocorrelation on $(z_t - \overline{z})$, $(z_t - \overline{z})^2$, $(z_t - \overline{z})^3$, $(z_t - \overline{z})^4$. A well known limitation of this approach is that the effects of a failure of independence on the distribution of the tests for unconditional uniformity is unknown[10]. Moreover, failure of the uniformity assumption will affect the tests for autocorrelation. The use of alternative techniques is therefore recommended in practical applications as they can offer different insights into the relative quality of the forecasts and help discriminating between rival models.

The modified Pearson goodness-of-fit test and its components

The following description draws from Anderson (1994) and Wallis (2002). The standard expression for the chi-squared goodness-of-fit test is given by

$$X^2 = \sum_{i=1}^{k} \frac{(n_i - n/k)^2}{(n/k)}$$

where k is the number of equiprobable classes in which the range of the z_t series is divided, n_i are the observed frequencies, n the number of observations (in our case the number of forecasts). This test has a limiting χ^2 distribution with k-1 degrees of freedom under the null hypothesis.

Anderson (1994) proposed a rearrangement of the test, which can be decomposed in various components to test departures from specific aspects of the distribution of interest. For example, shifts in location, shifts in scale, changes in symmetry and in kurtosis can all be detected from these tests. The rearranged test, valid under equiprobable partitions (see Boero, Smith and Wallis, 2002) is written as:

$$X^2 = (\boldsymbol{x} - m)' \, [\boldsymbol{I} - \boldsymbol{ee}'/ k] \, (\boldsymbol{x} - m) / (n / k)$$

In this expression, $\boldsymbol{x}$ is a kx1 vector of observed frequencies (x_1, x_2, …, x_k), which, under the null hypothesis has mean vector m=(n / k, …, n / k)′ and covariance matrix $\mathbf{V} = (n / k) \, [\boldsymbol{I} - \boldsymbol{ee}'/ k]$, where $\boldsymbol{e}$ is a kx1 vector of ones. The asymptotic distribution of the test rests on the k-variate normality of the multinomial distribution of the observed frequencies. The test can also be written as

$$X^2 = \boldsymbol{y}'\boldsymbol{y} / (n / k)$$

where $\boldsymbol{y} = \boldsymbol{A}(\boldsymbol{x}-m)$ is a (k-1) column vector, and $\boldsymbol{A}$ is defined as a (k-1) x k transformation matrix such that

$$\boldsymbol{AA}' = \boldsymbol{I} \text{ and } \boldsymbol{A}'\boldsymbol{A} = [\boldsymbol{I} - \boldsymbol{ee}'/ k].$$

With k=4, one can test departures from three distributional aspects, namely shifts in location, shifts in scale and changes in skewness. The $\boldsymbol{A}$ matrix in this case is defined as

$$\mathbf{A} = \frac{1}{\sqrt{4}}\begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

Here, the first row relates to the location of the distribution, the second to the scale, and the third to skewness. The elements of the (3x1) vector $\boldsymbol{y}=\boldsymbol{A}(\boldsymbol{x}-m)$ are therefore given respectively by:

y_1 : ½[(x_1 + x_2) - (x_3 + x_4)]

y_2 : ½[(x_1 + x_4) - (x_2 + x_3)]

y_3 : $\frac{1}{2}[(x_1 + x_3) - (x_2 + x_4)]$

Thus, the total X^2 test $\boldsymbol{y}'\boldsymbol{y}/(n/4)$ is equal to the sum of the squared elements of y. The three components of the test, $y_i^2/(n/4)$, are independently distributed as χ^2 with one degree of freedom under the null hypothesis. The first component of this sum is given by:

$$(1/n)[(x_1 + x_2) - (x_3 + x_4)]^2$$

This component detects possible shifts in location, with reference to the median of the distribution (shifts from the first half of the distribution to the second half). The second component detects shifts from the tails to the centre (interquartile range). Finally, the third component detects possible asymmetries, that is shifts from the first and third quarters to the second and fourth.

With k=8, one can also focus on the fourth characteristic related to kurtosis. In this case the $\boldsymbol{A}$ matrix is defined as

$$\mathbf{A} = \frac{1}{\sqrt{8}}\begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \end{bmatrix}$$

Here, only the first four rows are related to features of the distribution that are familiar, therefore the last three rows are omitted. So, in this case, the total chi-squared goodness-of-fit test, computed with the standard formula, will not be obtained as the sum of seven individual components, but will be equal to the sum of the first four components plus a remaining aggregate component independently distributed as χ^2 with three degrees of freedom under the null hypothesis.

Model evaluation

The one-step-ahead density forecasts of the effective exchange rate returns are obtained under the assumption of Gaussian errors, with the appropriate regime-specific variances for the SETAR models. The evaluation of the forecasts is carried out unconditionally, over the forecast period as a whole, and separately for each regime. In figure 2 we report some selected plots of the empirical distribution function of the z_t series against the theoretical uniform distribution function. We omit the 45° line to avoid over-crowding the plots. The 95% confidence intervals along side the hypothetical 45° line are calculated using the critical values of the Kolmogorov Smirnov test, reported in Lilliefors (1967, Table 1, p. 400), in the presence of estimated parameters[11]. The results from the Pearson X^2 test and its components, computed with k=8 partitions, are presented in table 4. In table 5 we report the results of the Ljung-Box test for autocorrelation of the z_t series and its powers.

As we can see from table 4 and figure 2, the GARCH model seems to produce density forecasts which are unconditionally correct, as suggested by the overall goodness-of-fit test, by its individual components, and by the Kolmogorov Smirnov test. Moreover, the results in table 5 show that the GARCH forecasts also satisfy the independence part of the joint hypothesis, with the Ljung-Box test showing no significant dependencies in the first and higher moments of the z_t series. These results for the GARCH model are robust across the two types of evaluations conducted in this paper, that is for the entire forecast period and conditional on the regimes of the SETAR models. It is now interesting to see how the SETAR density forecasts compare with the GARCH forecasts.

We start by discussing the results for the SETAR model with 2 regimes. As shown by the results in table 4 and figure 2, the SETAR-2 model fails the unconditional uniformity test in the evaluation over the entire forecasting sample. However, when the forecast densities are evaluated separately for each regime, we find that the forecast performance of the SETAR model is clearly improved in regime 2, which is the regime with fewer observations (T=192). For this regime, in fact, we cannot reject the hypothesis that the forecasts are well calibrated (unconditional uniformity).

The plots of the cdf of the z_t series versus the uniform (0,1) distribution, in figure 2, confirm these results. The empirical cdf of the SETAR-2 model (figure 2) crosses the bounds in various regions of the distribution in the entire sample and for the observations in regime 1, while the cdf is inside the bounds for the observations in regime 2. Further information on the nature of departures from the null hypothesis can be obtained from the individual test components of the goodness-of-fit test. The results in table 4 show that the largest contribution for the failure of the SETAR forecasts over the entire forecast period and for the observations in regime 1 comes from the second (scale) and fourth (kurtosis) components. It is interesting to note that there is some weak evidence of departure from kurtosis also for the forecasts in regime 2, suggesting that the SETAR-2 density forecasts are not as well calibrated as the GARCH forecasts in the tails of the distribution.

In order to complete the evaluation of the density forecasts of the SETAR model, we now look at the results from the test for autocorrelation of the z_t series and their powers. It is in fact of interest to see to what extent the SETAR models are able to capture the dynamics in heteroschedasticity. Table 5 clearly shows that the density forecasts from the SETAR models violate the independence assumption, when they are evaluated over the entire forecast period and conditional on regime 1. Violations occur with respect to the second and fourth power of the z_t transforms. However, consistently with our findings so far, the quality of the density forecasts improves for the observations in regime 2, for which the independence part of the joint i.i.d. uniform hypothesis is also satisfied.

A similar pattern of results can be noticed for the SETAR model with 3 regimes, confirming that the ability to produce ‘good’ forecasts varies across regimes. The density forecasts of the SETAR-3 model are unconditionally incorrect, according to the chi-squared goodness-of-fit test (table 4) computed over the entire forecasting period, and violate the independence assumption (table 5). However, when the tests are computed conditionally on each regime, we find that the SETAR-3 model produces density forecasts which satisfy the joint i.i.d U(0,1) hypothesis for the observations in regime 1, and are unconditionally well calibrated (though not independent) in regime 3. The results from the chi-squared goodness-of-fit test are, in general, confirmed by the plots of the empirical distribution function of the z_t series, not reported here for space reasons.

By combining the information in table 4, table 5 and figure 2, overall the GARCH model has shown better able to capture the distributional aspects of the euro-EER returns. In particular we have found evidence that the SETAR models fail to capture the scale and leptokurtosis in the distribution of the series when the density forecasts are evaluated over the entire forecast period. However, a regime conditional evaluation of the models has consistently shown an improved performance of the SETAR forecasts when the forecast origin is conditioned on specific regimes. These regimes turned out to be those with fewer observations.

In the next section we will adopt methods that can be used to evaluate interval forecasts.

3.3. INTERVAL FORECASTS EVALUATION

In this section we extend the forecast comparison by evaluating the models on their ability to produce interval forecasts. An interval forecast, or prediction interval, for a variable specifies the probability that the future outcome will fall within a stated interval. The lower and upper limits of the interval forecast are given as the corresponding percentiles. We use central intervals, so that, for example, the 90 per cent prediction interval is formed by the 5^{th} and 95^{th} percentiles.

Although the evaluation of the entire forecast density is more general than one based on forecast intervals, the results may be affected by some regions of the density, which may be of less concern to the forecast user. For example, financial operators are mostly concerned with the ability to model and forecast the behaviour in the tails of the distribution. Evaluation of interval forecasts enables the forecast user to assess more directly the ability of the models to produce correct forecasts, focussing on levels of coverage of specific interest.

The evaluation of interval forecasts is conducted by means of the likelihood ratio test of correct conditional coverage as recently proposed by Christoffersen (1998). The forecasts are assessed, like in the previous evaluations, over the entire forecast period and by conditioning upon regimes.

Christoffersen (1998) shows that a correctly conditionally calibrated interval forecast will provide a hit sequence I_t (for t=1, 2, …, T), with value 1 if the realisation is contained in the forecast interval, and 0 otherwise, that is distributed i.i.d. Bernoulli, with the desired success probability *p*. However, as stressed by Christoffersen, a simple test for correct unconditional coverage (LR_{UC}) is insufficient in the presence of dynamics in higher-order moments (conditional heteroscedasticity, for example) because it does not have power against the alternative that the zeros and ones are clustered in time-dependent fashion. In order to overcome this limitation, Christoffersen proposes a test for independence (LR_{IND}) which assumes a binary first-order Markov chain for the indicator function I_t. Under the null, the test follows a χ^2 distribution with one degree of freedom. The joint test of correct conditional coverage, LR_{CC}, is obtained as the sum of LR_{UC} and LR_{IND}, and is asymptotically χ^2 distributed with two degrees of freedom. For a detailed description of the tests we refer the reader to Christoffersen (1998).

In this paper we have considered intervals with nominal coverage, p, in the range [0.95-0.20]. The results are presented in table 6, where, for each nominal coverage, we report the actual unconditional coverage (π) and the P-values of the three LR tests[12]. Table 6a reports the results for the entire forecast period, while tables 6b and 6c report the results for the individual regimes.

As expected from our previous findings, the interval forecasts obtained from the GARCH model are conditionally well calibrated, at every level of coverage, and in both unconditional and regime-conditional evaluations. The SETAR models fail the conditional coverage test, when they are evaluated over the entire forecast period, for all levels of coverage, mostly due to strong rejection of the unconditional coverage test. The empirical coverage (the sample frequency π) is in general less than the nominal coverage, p, that is a smaller number of outcomes are observed to fall within the stated intervals. This means that the models overestimate the probability that the variable will fall within the predicted interval. Thus, over the whole forecast period, the models produce interval forecasts that are too narrow, indicating that the variance of the predicted distribution is too small. These results find confirmation in those reported in table 4, suggesting a major departure with respect to the scale of the distribution.

With respect to the test for independence, an interesting result is that the SETAR-3 model seems more able to produce forecasts that are independent over the whole forecast period, while there is more evidence against the independence of the SETAR-2 forecasts.

Finally, from tables 6b and 6c we notice that the SETAR-2 model shows a substantial improvement in regime 2, delivering interval forecasts with correct conditional coverage for all intervals considered. Similarly the forecast performance of the SETAR-3 is improved in regime 1. The forecast intervals in this regime are all well calibrated, with the exception of the wider intervals in the range 0.95 - 0.85. This result may be interpreted as failure to correctly capture the behaviour in the tails of the distribution also for the observations in regime 1. For this range of intervals, in fact, p is significantly greater than π, that is fewer observations fall in the stated intervals, which also implies that more observations actually fall in the tails than those predicted.

4. CONCLUSIONS

In this paper we have studied the out-of-sample forecast performance of SETAR models in an application to daily returns from the euro effective exchange rate. The SETAR models have been specified with two and three regimes, and their performance has been assessed against that of a simple linear AR model and a GARCH model. The forecast exercise is genuine in the sense that for the specification and estimation of the models we have ignored any information contained in the forecasting period.

The models have been assessed, first of all, on their ability to produce point forecasts, measured by means of MSFEs accompanied by the Diebold-Mariano test. Then the evaluation of the models has been extended to interval and density forecasts, to see whether the SETAR models can accurately predict higher-order moments.

The evaluation of the models has been conducted not only on different measurement methods, but also at different levels. That is, we have looked at the relative performance of the models on average, over the forecast period as a whole, and also we have investigated whether the models are better at predicting future values when the process is in a particular regime. Evaluations of SETAR models conditional on regimes have been carried out in previous research, but on point forecasts only. In this paper we have moved a step forward by extending the conditional evaluation to density and interval forecasts.

By evaluating the SETAR models over the entire forecasting sample we have found that none of the models was able to produce ‘good’ density and interval forecasts in general, while the density and interval forecasts produced by the GARCH model were correctly conditionally calibrated at each level of the evaluation study. The correct calibration or not of the various regions of the density has been illustrated by cumulative probability plots of the probability integral transforms against the uniform (0,1), and also assessed by the X^2 goodness-of-fit test and its individual components. The decomposition of the goodness-of-fit test into individual components has enabled us to explore possible directions of departures more closely, indicating major departures for the SETAR models with respect to scale and kurtosis.

The assessment of the models conditional on regimes has indicated a significant improvement in the quality of the SETAR forecasts in correspondence of specific regimes. In particular, the SETAR specification with two regimes has shown a good performance in terms of point, intervals and density forecasts when the process was in regime 2. On the other hand, the three-regime SETAR has not shown any improvement in terms of point forecasts, while it has delivered better interval and density forecasts in regime 1. In all evaluations, the improved performance of the SETAR models has occurred conditional on the regimes with a relatively small number of observations. This is in line with suggestions from previous studies.

To conclude, the GARCH model has shown more able to capture the distributional features of the euro effective exchange rate returns and to predict higher-order moments than the SETAR models. However, both SETAR models have shown a substantially improved forecast performance when the forecast origin was conditioned on some specific regimes.

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TABLES AND FIGURES

TABLE 1A DESCRIPTIVE STATISTICS

	Entire sample *03/01/90-10/07/02* T=3081	Estimation sample *03/01/90-30/12/99* T=2439	Forecasting sample *03/01/00-10/07/02* T=642
Mean	-0.0001	-0.0001	0.0000
Median	-0.0001	-0.0001	0.0000
Maximum	0.0289	0.0214	0.0289
Minimum	-0.0382	-0.0382	-0.0179
Std. Dev.	0.0041	0.0037	0.0053
Skewness	-0.0703	-0.4387	0.3933
Kurtosis	7.6953	9.3357	4.5813
Jarque-Bera	2832.6670	4157.5370	83.4425
Probability	0.0000	0.0000	0.0000
Observations	3081	2439	642

TABLE 1B LINEARITY TESTS - P-VALUES

	Entire sample *03/01/90-10/07/02* n=3081			Estimation sample *03/01/90-30/12/99* n=2439			Forecasting sample *03/01/00-10/07/02* n=642		
p	*3*	*4*	*5*	*3*	*4*	*5*	*3*	*4*	*5*
RESET, *h*=2	**0.0024**	**0.0230**	**0.0401**	0.3952	0.4142	**0.0804**	0.2523	**0.0327**	0.1796
RESET, *h*=3	**0.0085**	**0.0528**	**0.0089**	**0.0002**	**0.0002**	**0.0006**	0.4965	0.1007	0.4062
RESET, *h*=4	**0.0227**	0.1174	**0.0229**	**0.0001**	**0.0001**	**0.0011**	0.6333	0.2043	0.2057
Mod. RESET, *h*=2	**0.0006**	**0.0016**	**0.0036**	**0.0250**	**0.0232**	**0.0306**	**0.0836**	0.1128	0.1209
Mod. RESET, *h*=3	**0.0003**	**0.0011**	**0.0007**	**0.0012**	**0.0016**	**0.0003**	**0.0933**	0.1467	0.1534
Mod. RESET, *h*=4	**0.0002**	**0.0011**	**0.0009**	**0.0001**	**0.0004**	**0.0001**	0.2521	0.3996	0.4067
S_2, *d*=1	0.1440	0.2586	0.2428	0.4585	0.4496	0.6018	0.4443	0.5831	0.6338
S_2, *d*=2	**0.0015**	**0.0000**	**0.0002**	**0.0004**	**0.0000**	**0.0000**	0.4949	0.1197	0.1944
S_2, *d*=3	**0.0001**	**0.0000**	**0.0002**	**0.0004**	**0.0013**	**0.0003**	**0.0123**	**0.0243**	**0.0223**
S_2, *d*=4	0.5433	0.6992	0.4608	**0.0134**	**0.0143**	**0.0247**	0.3077	0.2499	0.1145
S_2, *d*=5	**0.0454**	0.1218	**0.0883**	**0.0059**	**0.0014**	**0.0021**	**0.0872**	0.1268	0.1872
S_2, *d*=6	**0.0433**	0.1039	**0.0083**	**0.0601**	0.1136	**0.0402**	**0.0129**	**0.0485**	**0.0562**
p denotes the lag order under the null hypothesis of linearity									

TABLE 2 SETAR MODELS SPECIFICATION

		SETAR-2		SETAR-3	
		Coeff.	t-value	Coeff.	t-value
REGIME 1	$\phi_0^{(1)}$	-0.0001	-1.000	-0.0012	-3.0000
	$\phi_1^{(1)}$	0.0517	2.3716	-0.1446	-2.0569
	$\phi_2^{(1)}$	0.0402	1.8962		
	$\phi_3^{(1)}$	-0.0685	-3.1136		
	$\sigma^{(1)}$	0.0035		0.0044	
	$T^{(1)}$	1930		455	
REGIME 2	$\phi_0^{(2)}$	0.0000	0.0000	0.0000	0.0000
	$\phi_1^{(2)}$	-0.0869	-1.7345		
	$\sigma^{(2)}$	0.0045		0.0034	
	$T^{(2)}$	497		1539	
REGIME 3	$\phi_0^{(3)}$			-0.0001	-0.2000
	$\phi_1^{(3)}$			0.0134	0.1553
	$\phi_2^{(3)}$			0.1009	2.3037
	$\phi_3^{(3)}$			-0.1099	-2.1381
	$\sigma^{(3)}$			0.0042	
	$T^{(3)}$			440	
MODEL	$\sigma^{(model)}$	0.0037		0.0037	
	d	4		1	
	r_1	0.00248		-0.00279	
	r_2	--		0.00277	
	AIC	-11.206		-11.208	

For the *SETAR-2* model the transition variable is represented by y_{t-4} while the threshold is selected to be 0.00248; in regime 1 the series is described by an AR(3) process, while in regime 2 it follows an AR(1) process.

For the *SETAR-3* model the transition variable is represented by y_{t-1} while the thresholds values are approximately symmetric and equal to -0.00279 and 0.00277; in regime 1 the series is described by an AR(1) process, in regime 2 it is approximated just by a constant, while in regime 3 it follows an AR(3) process.

TABLE 3A FORECASTING PERFORMANCE - NORMALIZED MSFE

		Number of steps-ahead				
		1	2	3	4	5
($MSFE_{SETAR}/MSFE_{AR}$)		A				
SETAR-2	Entire sample, *T=638*	1.0025	1.0011	0.9948	0.9982	0.9991
	Regime 1	1.0097**	1.0065*	1.0015	1.0021	0.9991
	T_1	*446*	*446*	*446*	*446*	*638*
	Regime 2	0.9842	0.9875	0.9779*	0.9884**	na
	T_2	*192*	*192*	*192*	*192*	*0*
SETAR-3	Entire sample, *T=638*	1.0079	0.9984	0.9962	0.9989	0.9986
	Regime 1	1.0077	na	1.0021	0.9949	1.0022
	T_1	*186*	*0*	*128*	*165*	*158*
	Regime 2	0.9921	0.9984	0.9939	0.9987	0.9951
	T_2	*271*	*638*	*366*	*320*	*321*
	Regime 3	1.0244**	na	0.9985	1.0055	0.9995
	T_3	*181*	*0*	*144*	*153*	*159*
($MSFE_{SETAR}/MSFE_{GARCH}$)		B				
SETAR-2	Entire sample, *T=638*	1.0014	1.0059	0.9998	0.9984	0.9993
	Regime 1	1.0016	1.0049	1.0001	1.0016	0.9993
	T_1	*446*	*446*	*446*	*446*	*638*
	Regime 2	1.0008	1.0085	0.9990	0.9903	na
	T_2	*192*	*192*	*192*	*192*	*0*
SETAR-3	Entire sample, *T=638*	1.0068	1.0031	1.0012	0.9991	0.9987
	Regime 1	0.9966	na	1.0118	0.9960	1.0016
	T_1	*186*	*0*	*128*	*165*	*158*
	Regime 2	1.0020	1.0031	0.9980	0.9974	0.9952
	T_2	*271*	*638*	*366*	*320*	*321*
	Regime 3	1.0212	na	1.0020	1.0085	1.0009
	T_3	*181*	*0*	*144*	*153*	*159*

*, ** denotes significance of the Diebold-Mariano test at 10% and 5%

“na” refers to the cases for which the MSFE can not be computed as the relevant model does not produce any forecast for that particular regime/horizon.

TABLE 4 FORECASTING PERFORMANCE - χ^2 GOODNESS-OF-FIT TESTS - P-VALUES IN ITALICS (ANDERSON-WALLIS DECOMPOSITION, K=8)

		Models	location	scale	skewness	kurtosis	total
	Entire sample	GARCH	0.401	0.759	1.605	0.056	5.461
	(T=638)		*0.526*	*0.384*	*0.205*	*0.812*	*0.604*
		SETAR-2	0.100	14.445	0.157	6.828	26.301
			0.751	*0.000*	*0.692*	*0.009*	*0.000*
		SETAR-3	0.006	11.060	0.000	5.643	20.708
			0.937	*0.001*	*1.000*	*0.018*	*0.004*
	Regime1	GARCH	0.000	0.897	0.439	0.143	3.040
	(T_1=446)		*1.000*	*0.344*	*0.507*	*0.705*	*0.881*
		SETAR-2	0.036	19.812	0.000	3.955	32.601
SETAR-2			*0.850*	*0.000*	*1.000*	*0.047*	*0.000*
	Regime2	GARCH	1.333	0.021	1.688	0.021	10.417
	(T_2=192)		*0.248*	*0.885*	*0.194*	*0.885*	*0.166*
		SETAR-2	0.083	0.021	0.521	3.000	10.667
			0.773	*0.885*	*0.470*	*0.083*	*0.154*
	Regime1	GARCH	2.602	0.538	0.194	0.086	3.677
	(T_1=186)		*0.107*	*0.463*	*0.660*	*0.769*	*0.816*
		SETAR-3	0.052	0.052	0.468	1.671	5.081
			0.820	*0.820*	*0.494*	*0.196*	*0.650*
SETAR-3	Regime2	GARCH	0.624	0.446	0.446	0.033	5.044
	(T_2=271)		*0.430*	*0.504*	*0.504*	*0.855*	*0.655*
		SETAR-3	0.299	11.162	0.446	3.546	17.148
			0.585	*0.001*	*0.504*	*0.060*	*0.016*
	Regime3	GARCH	1.994	2.436	1.243	0.934	8.392
	(T_3=181)		*0.158*	*0.119*	*0.265*	*0.334*	*0.299*
		SETAR-3	1.243	2.923	0.138	0.934	9.807
			0.265	*0.087*	*0.710*	*0.334*	*0.200*

TABLE 5 P-VALUES OF THE LJUNG-BOX Q STATISTICS FOR SERIAL CORRELATION (FIRST SIX AUTOCORRELATIONS)

		Moments			
		$(z-\bar{z})$	$(z-\bar{z})^2$	$(z-\bar{z})^3$	$(z-\bar{z})^4$
Entire sample	GARCH	0.258	0.588	0.187	0.402
	SETAR-2	0.472	0.000	0.191	0.000
	SETAR-3	0.394	0.000	0.125	0.000
Regime 1	GARCH	0.424	0.998	0.411	0.989
	SETAR-2	0.382	0.000	0.177	0.000
Regime 2	GARCH	0.253	0.354	0.089	0.594
	SETAR-2	0.493	0.323	0.327	0.434
Regime 1	GARCH	0.438	0.325	0.707	0.391
	SETAR-3	0.337	0.276	0.342	0.690
Regime 2	GARCH	0.244	0.386	0.775	0.495
	SETAR-3	0.190	0.000	0.705	0.000
Regime 3	GARCH	0.387	0.772	0.496	0.425
	SETAR-3	0.290	0.002	0.429	0.003

TABLE 6A FORECAST INTERVAL EVALUATION FOR 1-STEP-AHEAD HORIZON – ENTIRE FORECAST PERIOD

	GARCH				SETAR-2				SETAR-3			
p	π	LR_{UC}	LR_{IND}	LR_{CC}	π	LR_{UC}	LR_{IND}	LR_{CC}	π	LR_{UC}	LR_{IND}	LR_{CC}
0.95	0.944	0.465	--	--	0.857	**0.000**	1.000	**0.000**	0.868	**0.000**	0.706	**0.000**
0.90	0.897	0.773	0.071	0.189	0.803	**0.000**	0.447	**0.000**	0.813	**0.000**	0.747	**0.000**
0.85	0.845	0.716	0.294	0.539	0.749	**0.000**	0.156	**0.000**	0.763	**0.000**	0.485	**0.000**
0.80	0.807	0.647	0.217	0.421	0.710	**0.000**	**0.007**	**0.000**	0.715	**0.000**	0.247	**0.000**
0.75	0.751	0.963	0.782	0.961	0.666	**0.000**	**0.003**	**0.000**	0.676	**0.000**	0.226	**0.000**
0.70	0.697	0.890	0.637	0.886	0.610	**0.000**	**0.023**	**0.000**	0.627	**0.000**	0.990	**0.000**
0.65	0.647	0.888	0.541	0.822	0.560	**0.000**	0.107	**0.000**	0.575	**0.000**	0.178	**0.000**
0.60	0.585	0.429	0.489	0.576	0.530	**0.000**	0.364	**0.001**	0.527	**0.000**	0.076	**0.000**
0.55	0.538	0.530	0.564	0.695	0.476	**0.000**	0.538	**0.001**	0.489	**0.002**	**0.012**	**0.000**
0.50	0.483	0.384	0.685	0.630	0.425	**0.000**	0.071	**0.000**	0.434	**0.001**	0.052	**0.001**
0.45	0.442	0.685	0.289	0.525	0.379	**0.000**	0.211	**0.001**	0.395	**0.005**	0.296	**0.011**
0.40	0.389	0.560	0.192	0.360	0.339	**0.001**	0.469	**0.005**	0.350	**0.009**	0.358	**0.021**
0.35	0.351	0.954	0.426	0.727	0.299	**0.007**	**0.024**	**0.002**	0.287	**0.001**	0.196	**0.001**
0.30	0.299	0.972	0.187	0.418	0.268	0.075	**0.004**	**0.003**	0.257	**0.016**	0.099	**0.014**
0.25	0.246	0.819	0.240	0.488	0.218	0.057	**0.025**	**0.013**	0.223	0.105	0.720	0.252
0.20	0.199	0.953	0.341	0.634	0.166	**0.029**	0.124	**0.028**	0.172	0.076	0.549	0.173

p indicates the nominal coverage, π indicates the actual unconditional coverage; numbers in bold represent rejections at 5% level of significance

TABLE 6B FORECAST INTERVAL EVALUATION FOR 1-STEP-AHEAD HORIZON – CONDITIONING ON REGIMES OF THE SETAR-2 MODEL

	REGIME 1 $T_1=446$								REGIME 2 $T_2=192$							
	GARCH				SETAR-2				GARCH				SETAR-2			
p	π	LR_{UC}	LR_{IND}	LR_{CC}	π	LR_{UC}	LR_{IND}	LR_{CC}	π	LR_{UC}	LR_{IND}	LR_{CC}	π	LR_{UC}	LR_{IND}	LR_{CC}
0.95	0.944	0.565	0.704	0.788	0.832	**0.000**	0.268	**0.000**	0.943	0.650	0.649	0.814	0.916	0.052	0.166	0.058
0.90	0.890	0.494	0.773	0.759	0.774	**0.000**	0.277	**0.000**	0.911	0.590	0.676	0.793	0.869	0.180	0.297	0.237
0.85	0.836	0.424	0.734	0.686	0.722	**0.000**	0.083	**0.000**	0.865	0.566	0.735	0.801	0.812	0.159	0.572	0.316
0.80	0.794	0.741	0.767	0.906	0.679	**0.000**	**0.001**	**0.000**	0.839	0.170	0.254	0.204	0.780	0.521	0.254	0.425
0.75	0.738	0.550	0.665	0.761	0.646	**0.000**	**0.002**	**0.000**	0.781	0.310	0.749	0.568	0.712	0.251	0.954	0.516
0.70	0.684	0.459	0.612	0.668	0.590	**0.000**	**0.001**	**0.000**	0.729	0.373	0.954	0.672	0.660	0.193	0.427	0.313
0.65	0.630	0.379	0.328	0.421	0.538	**0.000**	**0.016**	**0.000**	0.688	0.272	0.959	0.546	0.613	0.243	0.351	0.328
0.60	0.581	0.407	0.910	0.705	0.504	**0.000**	0.142	**0.000**	0.594	0.860	0.965	0.984	0.592	0.755	0.706	0.887
0.55	0.536	0.549	0.973	0.835	0.453	**0.000**	0.062	**0.000**	0.542	0.817	0.874	0.961	0.534	0.606	0.985	0.875
0.50	0.478	0.344	0.697	0.592	0.395	**0.000**	**0.008**	**0.000**	0.495	0.885	0.827	0.966	0.497	0.900	0.943	0.990
0.45	0.433	0.463	0.407	0.542	0.357	**0.000**	0.059	**0.000**	0.464	0.706	0.999	0.931	0.435	0.624	0.437	0.656
0.40	0.381	0.416	0.540	0.595	0.321	**0.001**	0.275	**0.001**	0.406	0.860	0.868	0.971	0.382	0.576	0.882	0.846
0.35	0.341	0.683	0.820	0.897	0.276	**0.001**	**0.020**	**0.000**	0.375	0.470	0.703	0.716	0.356	0.914	0.321	0.607
0.30	0.278	0.308	0.298	0.346	0.249	**0.016**	**0.011**	**0.002**	0.349	0.144	0.366	0.229	0.314	0.709	0.408	0.663
0.25	0.229	0.294	0.222	0.273	0.193	**0.004**	0.060	**0.003**	0.286	0.251	0.609	0.453	0.277	0.411	0.356	0.465
0.20	0.182	0.326	0.105	0.165	0.150	**0.007**	0.738	**0.023**	0.240	0.180	0.395	0.284	0.204	0.919	0.067	0.186

p indicates the nominal coverage, π indicates the actual unconditional coverage; numbers in bold represent rejections at 5% level of significance

TABLE 6C FORECAST INTERVAL EVALUATION FOR 1-STEP-AHEAD HORIZON – CONDITIONING ON REGIMES OF THE SETAR-3 MODEL

	REGIME 1 $T_1=186$								REGIME 2 $T_2=271$								REGIME 3 $T_3=181$							
	GARCH				SETAR-3				GARCH				SETAR-3				GARCH				SETAR-3			
p	π	LR_{UC}	LR_{IND}	LR_{CC}	π	LR_{UC}	LR_{IND}	LR_{CC}	π	LR_{UC}	LR_{IND}	LR_{CC}	π	LR_{UC}	LR_{IND}	LR_{CC}	π	LR_{UC}	LR_{IND}	LR_{CC}	π	LR_{UC}	LR_{IND}	LR_{CC}
0.95	0.925	0.140	--	--	0.887	**0.001**	0.662	**0.003**	0.948	0.901	--	--	0.838	**0.000**	0.426	**0.000**	0.956	0.715	--	--	0.895	**0.003**	0.996	**0.012**
0.90	0.876	0.298	0.160	0.217	0.823	**0.001**	0.955	**0.006**	0.889	0.563	0.835	0.828	0.790	**0.000**	0.478	**0.000**	0.928	0.186	--	--	0.840	**0.012**	0.220	**0.020**
0.85	0.833	0.530	0.518	0.666	0.774	**0.006**	0.822	**0.023**	0.834	0.466	0.825	0.748	0.738	**0.000**	0.301	**0.000**	0.873	0.377	0.511	0.546	0.790	**0.031**	0.086	**0.022**
0.80	0.796	0.884	0.930	0.986	0.747	0.081	0.683	0.201	0.812	0.624	0.509	0.713	0.686	**0.000**	0.317	**0.000**	0.812	0.680	0.478	0.715	0.724	**0.014**	0.211	**0.022**
0.75	0.763	0.670	0.850	0.897	0.704	0.158	0.355	0.240	0.745	0.861	0.599	0.857	0.661	**0.001**	0.280	**0.003**	0.746	0.898	0.766	0.949	0.669	**0.014**	0.434	**0.036**
0.70	0.731	0.348	0.582	0.554	0.667	0.326	0.465	0.473	0.686	0.625	0.434	0.653	0.601	**0.001**	0.417	**0.002**	0.680	0.551	0.414	0.600	0.624	**0.030**	0.071	**0.018**
0.65	0.688	0.271	0.537	0.451	0.634	0.657	0.751	0.861	0.624	0.365	0.062	0.116	0.531	**0.000**	0.161	**0.000**	0.641	0.797	0.492	0.764	0.580	0.052	0.185	**0.063**
0.60	0.608	0.834	0.952	0.976	0.597	0.928	0.760	0.951	0.572	0.348	0.065	0.117	0.483	**0.000**	0.230	**0.000**	0.580	0.586	0.919	0.858	0.519	**0.028**	0.054	**0.014**
0.55	0.570	0.585	0.859	0.848	0.538	0.735	0.660	0.857	0.535	0.621	0.289	0.504	0.443	**0.000**	0.603	**0.002**	0.508	0.260	0.884	0.525	0.508	0.260	0.101	0.139
0.50	0.527	0.463	0.856	0.752	0.484	0.660	0.930	0.904	0.480	0.504	0.885	0.792	0.399	**0.001**	0.478	**0.003**	0.442	0.118	0.788	0.284	0.436	0.087	**0.040**	**0.028**
0.45	0.484	0.354	0.836	0.637	0.446	0.918	0.814	0.968	0.443	0.812	0.511	0.783	0.369	**0.007**	0.197	**0.011**	0.398	0.156	0.263	0.196	0.381	0.061	0.215	0.080
0.40	0.430	0.404	0.802	0.684	0.382	0.610	0.724	0.825	0.395	0.862	0.243	0.498	0.339	**0.040**	0.128	**0.038**	0.337	0.081	0.440	0.161	0.331	0.057	0.654	0.148
0.35	0.382	0.368	0.506	0.534	0.306	0.208	0.796	0.438	0.347	0.914	0.733	0.938	0.280	**0.015**	0.050	**0.007**	0.326	0.495	0.823	0.773	0.276	**0.034**	0.819	0.103
0.30	0.333	0.326	0.883	0.611	0.280	0.540	0.808	0.805	0.295	0.863	0.505	0.789	0.251	0.073	**0.030**	**0.019**	0.271	0.385	0.612	0.603	0.243	0.088	0.835	0.229
0.25	0.274	0.451	0.771	0.721	0.253	0.933	0.611	0.875	0.251	0.972	0.548	0.834	0.214	0.164	0.367	0.253	0.210	0.205	0.353	0.291	0.204	0.148	0.858	0.345
0.20	0.220	0.491	0.708	0.736	0.215	0.611	0.130	0.279	0.207	0.785	0.385	0.661	0.159	0.080	0.170	0.084	0.166	0.238	0.583	0.429	0.149	0.076	0.276	0.115

p indicates the nominal coverage, π indicates the actual unconditional coverage; numbers in bold represent rejections at 5% level of significance

FIGURE 1

EURO EFFECTIVE EXCHANGE RATE

03/01/90-10/07/02

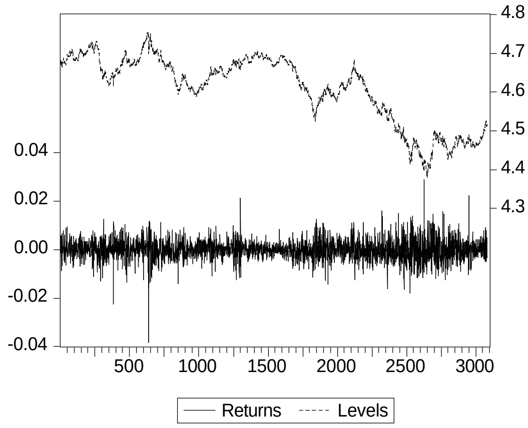
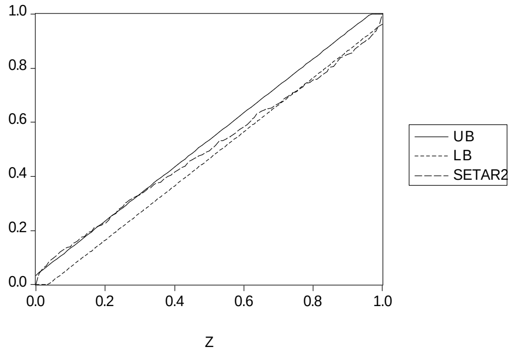


FIGURE2

DENSITY FORECASTS-SETAR-2 *VS* GARCH

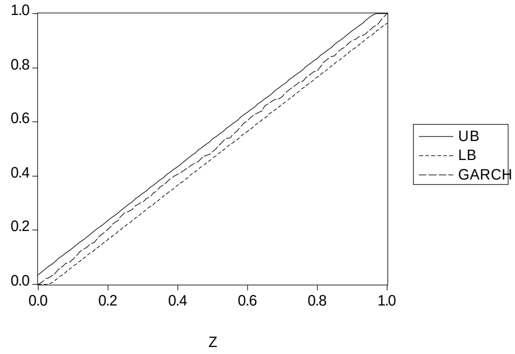
SETAR-2

Entire sample(T=638)

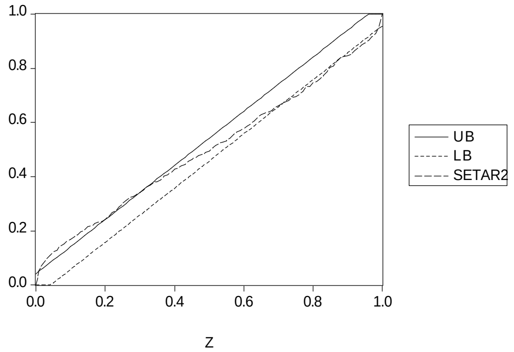


GARCH

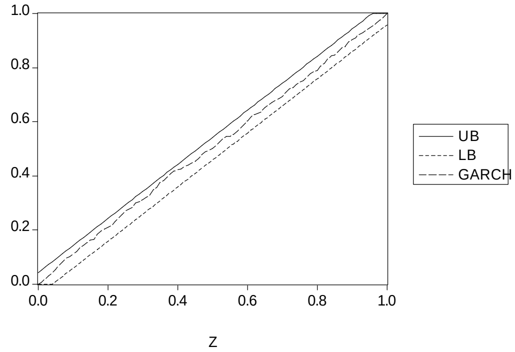
Entire sample



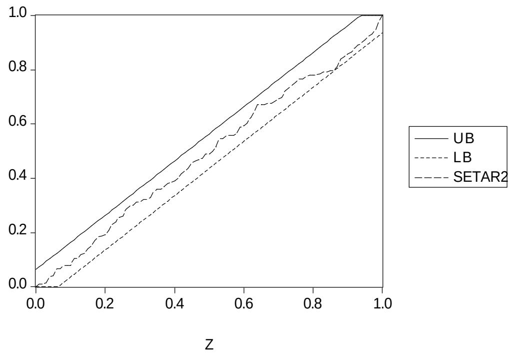
Regime1 (T_1=446)



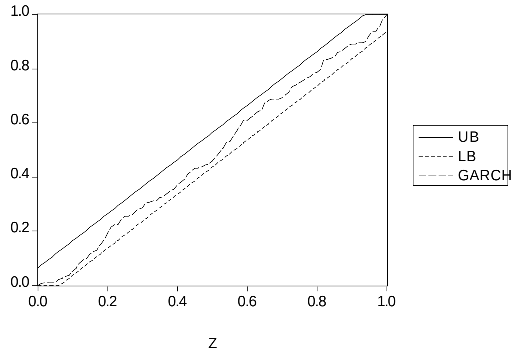
Regime1



Regime2 (T_2=192)



Regime2



NOTES

[1] See the European Central Bank website (http://www.ecb.int/stats/eer/eer.shtml) for a technical comment on the method adopted to construct the series of the Euro nominal effective exchange rate.
[2] We have carried out the forecasting evaluation exercise allowing for different divisions of the estimation and forecasting periods, and found qualitatively similar results in terms of the relative performance of the rival models (the results are available from the authors upon request).
[3] In the traditional form, the RESET test is computed by running a linear autoregression of order *p*, followed by an auxiliary regression in which powers of the fitted values obtained in the first stage are included along with the initial regressors. The modified RESET test requires that all the initial regressors enter linearly and up to a certain power *h* in the auxiliary regression; Thursby and Schimdt suggest using *h=4*. The Lagrange Multiplier form (Granger and Teräsvirta, 1993) of the test is adopted in this study, thus the test is distributed as a c^2 with up to 3*p* degrees of freedom for the modified version.
[4] The auxiliary regression for the LM S_2 test is computed as follows:
$\hat{e}_t = b_0 + \sum_{i=1}^{p} b_i y_{t-i} + \sum_{i=1}^{p} x_i y_{t-i} y_{t-d} + \sum_{i=1}^{p} y_i y_{t-i} y_{t-d}^2 + \sum_{i=1}^{p} k_i y_{t-i} y_{t-d}^3$ where ε_t are the estimated residuals from a linear regression of order *p*. Under the null hypothesis the test has a χ^2 distribution with 3*p* degrees of freedom.
[5] For a complete discussion of this class of models see Tong (1983).
[6] A variant of the TAR model can be obtained if the parameters are allowed to change smoothly over time, the resulting model is called a Smooth Transition Autoregressive (STAR) model (see Granger and Teräsvirta, 1993, and Teräsvirta, 1994).
[7] As suggested by one referee, we have also calculated the forecasts by bootstrapping the estimated regime-specific residuals. However, the multi-step-ahead forecasts did not show any significant difference across the two alternative methods.
[8] We also performed the modified version of the DM test proposed by Harvey et al. (1997), which corrects for the oversize shortcomings of the original DM tests in small samples and for h>1. The results, not reported here, do not differ appreciably from those presented in table 3.
[9] The maximum absolute difference between the empirical distribution function and the distribution function under the null hypothesis of uniformity.
[10] For a preliminary study of the size and power of alternative tests see Noceti, Smith and Hodges, "An evaluation of tests of distributional forecasts", Discussion paper FORC, University of Warwick, 2000, no. 102.
[11] The formula reported in Lilliefors (1967) for T>30, level of significance 0.05, is given by $0.886/\sqrt{T}$. The standard critical values of the Kolmogorov-Smirnov test are probably a conservative estimate of the 'correct' critical values when certain parameters of the distribution must be estimated from the sample.
[12] All the tests have been performed with Eviews codes, available from the authors upon request.