DESIGNING THE VOTING SYSTEM FOR THE COUNCIL OF THE EUROPEAN UNION

Dennis Leech,

Department of Economics and CSGR, University of Warwick
and
Voting Power and Procedures programme, CPNSS, London School of Economics

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Dr D Leech
Department of Economics
University of Warwick
Coventry
CV4 7AL
UK
Phone: (+44)(0)(24)76523047 (direct)
(+44)(0)(24)76523027 (messages)
07712353201 (mobile)
Fax: (+44)(0)(24)76523032
Email: d.leech@warwick.ac.uk
http://www.warwick.ac.uk/fac/soc/Economics/leech/

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Title: Designing the Voting System for the Council of the European Union

Author: Dennis Leech

Affiliation: Department of Economics and CSGR University of Warwick and Voting Power and Procedures Programme, CPNSS, London School of Economics.

Abstract: This paper examines the system of Qualified Majority Voting, used by the Council of the European Union, from the perspective of enlargement of the Union. It uses an approach based on power indices due to Penrose, Banzhaf and Coleman to make two analyses: (1) the question of the voting power of member countries from the point of view of fairness, and (2) the question of how the threshold number of votes required for QMV should be determined. It studies two scenarios for change from 2005 onwards envisaged by the Nice Treaty: (1) no enlargement, the EU comprising 15 member countries, and (2) full enlargement to 27 members by the accession of all the present twelve candidates. The proposal is made that fair weights be determined algorithmically as a technical or routine matter as the membership changes. The analysis of how the threshold affects power shows the trade-offs that countries face between their blocking power and the power of the Council to act. The main findings are: (1) that the weights laid down in the Nice Treaty are close to being fair, the only significant discrepancies being the under-representation of Germany and Romania, and the over-representation of Spain and Poland; (2) the threshold required for a decision is set too high for the Council to be an effective decision making body.

Keywords: European Union; Nice Treaty; Qualified Majority Voting; Weighted Voting; Power Indices.

1. INTRODUCTION

The prospect of enlargement of the European Union by the accession of new member countries from Eastern Europe has posed fundamental questions about how its institutions of governance should change in response. The Intergovernmental Conference held in Nice in December 2000 was held to address these issues and produce an agreement on the basic structures of decision making as a framework for enlargement. However the Nice Treaty has been criticised and should be regarded as only a limited success.

The main focus of the conference was on the extension of the range of decisions taken by Qualified Majority Voting in the Council and on the technicalities of how this system would work from 2005 onwards. There was considerable discussion of how the weighted votes should be allocated to each member country and what the decision rule should be in terms of the threshold. The treaty made provision for new weights for the existing fifteen members and for twelve candidates. It also provided for changes to the decision rule in terms of the size of the majority required for a proposal to pass. These changes have been analysed rigorously in terms of *a priori* voting power by Felsenthal and Machover (2001b) who concluded that, while the allocation of voting weights is relatively fair in the sense that the system gives electors in different countries roughly similar voting power, the threshold agreed on is set too high for the Council to be an

effective democratic decision-making body. The present paper builds on that study, partly duplicating it, but also extending it.[1]

I investigate the properties of the voting systems laid down in the treaty to apply both before and after enlargement, duplicating the analysis of Felsenthal and Machover (2001b). I also consider the normative question of what the voting weights should be in order that the system is fair. I apply an algorithm for choosing the weights so as to achieve a given distribution of voting power among the members. This is proposed as a general procedure that could be applied in a more or less routine manner each time the membership changes: every time a new member country joins its voting weight can be calculated, and those of all existing members recalculated, by this algorithm in accordance with the agreed general criterion of fairness. The Nice Summit was held to determine the voting weights once and for all so that there would be no need to hold an Intergovernmental Conference every time new members joined. The general procedure proposed is an alternative that would have the advantage of giving fair weights in all cases. I also investigate how the choice of decision rule affects voting power given the Nice weights.

I address the following specific questions in terms of *a priori* voting power. Separate analyses are reported for the Union comprising the existing 15 and after enlargement to 27.

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[1] Other studies of voting power in the EU Council of Ministers include Hosli (1993, 1995, 1996, 1998), Widgren (1994), Felsenthal and Machover (1998, 2000), Laruelle and Widgren (1998), Nurmi and Meskanen (1999), Sutter (2000). A recent contribution on the Nice Treaty is Baldwin et al. (2001).

(1) What is the distribution of voting power among the member countries given by the voting system and weights in the Nice Treaty?

(2) How should the weights be chosen if the aim is to ensure that all citizens of the EU have equal voting power?

(3) What is the effect of the threshold required for a decision by weighted majority voting on the power of the Council to act and also the powers of the individual members?

The paper is organised as follows. Section 2 describes the system of Qualified Majority Voting, and the Nice Treaty is described in Section 3. Section 4 outlines the measurement of power under weighted voting, using the power indices due to Penrose, Banzhaf and Coleman. The idea of fair weighting and reweighting is defined and the algorithmic approach described in Section 5. Section 6 presents the analysis of the distribution of voting power and the fair weights under the Nice Treaty. Section 7 presents the analysis of the threshold for Qualified Majority Voting and Section 8 concludes.

2. QUALIFIED MAJORITY VOTING

The Council is the most senior decision-making body within the EU under the Treaty of Rome. It uses different decision rules for different matters, unanimity for certain matters affecting members' fundamental sovereignty, but qualified majority voting for others. Its key features are: (1) that all members have a seat but their respective

numbers of votes are different to reflect their different populations; and (2) decisions are taken by qualified majority voting with respect to a decision rule based on a supermajority requirement defined by a threshold. The threshold has always been set at about 71 percent of the total voting weight.

Table 1: Qualified Majority Voting: Weights and Populations

	1958-72			1973-80			1981-85			1986-94			1995-			
	Wt	%	% Pop	Wt	%	% Pop	Wt	%	% Pop	Wt	%	% Pop	Wt	%	% Pop	Pop (m)
Germany	4	**23.5**	**32.2**	10	**17.2**	**24.2**	10	**15.9**	**22.8**	10	**13.2**	**18.9**	10	**11.5**	**21.9**	82.0
UK		-	-	10	**17.2**	**21.8**	10	**15.9**	**20.5**	10	**13.2**	**17.6**	10	**11.5**	**15.8**	59.2
France	4	**23.5**	**26.6**	10	**17.2**	**20.3**	10	**15.9**	**20.0**	10	**13.2**	**17.2**	10	**11.5**	**15.7**	59.0
Italy	4	**23.5**	**29.1**	10	**17.2**	**21.4**	10	**15.9**	**20.9**	10	**13.2**	**17.6**	10	**11.5**	**15.3**	57.6
Spain		-	-		-	-		-	-	8	**10.5**	**12.0**	8	**9.2**	**10.5**	39.4
Netherlands	2	**11.8**	**6.6**	5	**8.6**	**5.2**	5	**7.9**	**5.3**	5	**6.6**	**4.5**	5	**5.7**	**4.2**	15.8
Greece		-	-		-	-	5	**7.9**	**3.6**	5	**6.6**	**3.1**	5	**5.7**	**2.8**	10.5
Belgium	2	**11.8**	**5.4**	5	**8.6**	**3.8**	5	**7.9**	**3.6**	5	**6.6**	**3.1**	5	**5.7**	**2.7**	10.2
Portugal		-	-		-	-		-	-	5	**6.6**	**3.1**	5	**5.7**	**2.7**	10.0
Sweden		-	-		-	-		-	-		-	-	4	**4.6**	**2.4**	8.9
Austria		-	-		-	-		-	-		-	-	4	**4.6**	**2.2**	8.1
Denmark				3	**5.2**	**2.0**	3	**4.8**	**1.9**	3	**3.9**	**1.6**	3	**3.4**	**1.4**	5.3
Finland		-	-		-	-		-	-		-	-	3	**3.4**	**1.4**	5.2
Ireland		-	-	3	**5.2**	**1.2**	3	**4.8**	**1.3**	3	**3.9**	**1.1**	3	**3.4**	**1.0**	3.7
Luxembourg	1	**5.9**	**0.2**	2	**3.4**	**0.1**	2	**3.2**	**0.1**	2	**2.6**	**0.1**	2	**2.3**	**0.1**	0.4
Total	17	100	100	58	100	100	63	100	100	76	100	100	87	100	100	375.3
Threshold	12	70.6		41	70.7		45	71.4		54	71.1		62	71.3		

Source: Felsenthal and Machover (2001b). The table shows, for each member, the number of its weighted votes, abbreviated to Wt, its percentage share and its percentage share of the total population. The final column, Pop(m), shows its current population in millions.

Table 1 provides an overview of the evolution of the system since its origin in 1958. It shows that larger countries have always received a smaller share of the voting weight than their share of the population, reflecting the need to ensure adequate representation of small countries as independent states. The inference has frequently been drawn from this that the larger countries are relatively under-represented. The response to this has been to keep the threshold very high so that the power of the large countries is

protected. This has meant that any decision has always required the support of at least two out of the big four countries (Germany, UK, France and Italy); therefore these members have had a considerable ability to prevent action.

Such arguments however are based on a simple comparison of population shares with vote shares and ignore relative voting power. The fact that decisions are taken by block-voting, each member country casting all its votes together – in contrast to the European Parliament where MEPs are not constrained to vote as national groups – means that the relationship between voting power and weight is complex. It is well known that, in general, in a legislature that uses weighted voting, relative voting powers of members are different from relative numbers of votes, sometimes substantially so.[2] In making this comparison it is necessary to examine the rules of the legislature in relation to all the possible voting outcomes that could conceivably occur.

While choosing a high level for the threshold has the advantage of protecting the large countries against being outvoted too easily, it has the disadvantage that it limits the effectiveness of the Council by making it difficult, *a priori*, to make a decision and therefore imparts a considerable bias in favour of the *status quo*[3]. It also restricts the ability of all members to get their own proposals accepted, including those of the large members themselves, whose sovereignty it is meant to protect. The choice of the threshold was on the agenda at the Nice Summit but it was not changed despite the

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[2] This is described in Felsenthal and Machover (1998).

[3] Different terms have been used in the literature for this characteristic of a decision rule which reflects the ease with which it responds to variations in the members' wishes. Felsenthal and Machover (1998) use the term sensitivity. I use the measure of it due to Coleman (1971) who called it the power of the collectivity to act.

substantial extension of the scope of qualified majority voting to cover a greater range of policy areas.

3. THE TREATY OF NICE

The Nice Treaty amended the system of Qualified Majority Voting to apply from 2005. It laid down the rules of decision making on different scenarios for enlargement. For each assumed scenario weighted voting is at the heart of the system but two additional conditions which must also be met have been added, in terms of the number of countries and population.[4] The system should therefore be thought of as one requiring a triple majority. For a proposal to pass three conditions must be satisfied: (1) the number of weighted votes equalling or exceeding the threshold; (2) a simple majority of the member countries; (3) a supermajority (62%) of the population must be represented. In fact, as Felsenthal and Machover (2001b) have shown, conditions (2) and (3) add little in that there are very few voting outcomes in which either is not met when (1) is. Nevertheless I allow for all three conditions in the following analysis.

Two scenarios are assumed for 2005:

(1) No enlargement. No new members have acceded and the EU comprises the same fifteen countries as at present. The triple-majority system in this case is

[4] See EU (2001). The provisions laying down the first scenario are in Article 3 of the *Protocol on the Enlargement of the European Union* (pp97-8). The second scenario is based on the *Declaration on the Qualified Majority Threshold and the Number of Votes for a Blocking Minority in an Enlarged Union* (p.167).

referred to as N15.[5] When I analyse the effect of the threshold I assume condition (1) only; this decision-making system based on weighted voting is referred to as W15.

(2) Maximum enlargement. All candidates have joined and the EU has expanded to 27 members. I refer to this case under the provisions of the treaty as N27, and the weighted majority system as W27, respectively.

Table 2 shows the member countries together with their voting weights, the threshold and the decision rule for N15. The big four countries all have 29 votes, 12.2% of the total, Spain slightly fewer with 27, the Netherlands,13, traditionally the same as, but now getting slightly more than, Belgium, 12, and so on, finally Luxembourg having 4. This allocation of weights represents a slight shift towards the larger countries compared with the present weights reported in Table 1. The threshold is set at 169 out of a total of 237 votes, representing 71.3 percent, the same as before. A decision under N15 therefore requires the support of: (1) at least 169 weighted votes, (2) eight member countries, and (3) member countries whose combined population is at least 62 percent of the total.

[5] This nomenclature was used by Felsenthal and Machover (2001b). They looked at two variants for the union of 27 members because of ambiguity in the text of the treaty, which they labelled N27 and N´27. The difference is that in the former the threshold is stated as 258 while in the latter the blocking minority is stated as 91, which means that the threshold is lowered to 255. I have ignored this distinction and analysed the second case on the assumption that this is the authoritative version; I have called this case N27.

Table 2:N15, No Enlargement

Member	Votes	Votes %	
Germany	29	12.2	
UK	29	12.2	Threshold =169 (71.3%)
France	29	12.2	
Italy	29	12.2	
Spain	27	11.4	
Netherlands	13	5.5	Decision Rule:
Greece	12	5.1	1. Combined weight = 169
Belgium	12	5.1	2. No. of members = 8
Portugal	12	5.1	3. Population = 62%
Sweden	10	4.2	
Austria	10	4.2	
Denmark	7	3.0	
Finland	7	3.0	
Ireland	7	3.0	
Luxembourg	4	1.7	
Total	237	100	

Table 3 shows the equivalent data after all the current candidates have been admitted. The total number of weighted votes is now 345 with the threshold set at 255. This represents a relative increase in the threshold to 73.9% of the total weighted votes. Despite this increase in the threshold, however, it is now no longer possible for three of the big four to block a decision. As I show below, this increase in the threshold cannot be said to benefit either the Council or the individual member countries concerned.

Table 3: N27, Enlargement by all 12 candidates

Member	Votes	Votes %	
Germany	29	8.4	
UK	29	8.4	
France	29	8.4	Threshold = 255 (73.9%)
Italy	29	8.4	
Spain	27	7.8	
Poland	27	7.8	Decision Rule:
Romania	14	4.1	1.Weight = 255
Netherlands	13	3.8	2. No. of members = 14
Greece	12	3.5	3. Population = 62%
Czech Rep	12	3.5	
Belgium	12	3.5	
Hungary	12	3.5	
Portugal	12	3.5	
Sweden	10	2.9	
Bulgaria	10	2.9	
Austria	10	2.9	
Slovakia	7	2.0	
Denmark	7	2.0	
Finland	7	2.0	
Ireland	7	2.0	
Lithuania	7	2.0	
Latvia	4	1.2	
Slovenia	4	1.2	
Estonia	4	1.2	
Cyprus	4	1.2	
Luxembourg	4	1.2	
Malta	3	0.9	
Total	345	100	

4. MEASUREMENT OF POWER IN WEIGHTED VOTING SYSTEMS

Now I describe the measures used to analyse power under weighted voting.[6] Two approaches will be used: first, analyses of relative voting power of members within a

[6] The use of power indices to study the EU has attracted a lot of criticism from Garrett and Tsebelis (1996, 1999). The reader who is interested in this debate is referred to the

given legislature using the Banzhaf power index, and second, analyses of absolute voting power using the Penrose index and three indices proposed by Coleman (the power to act, the power to prevent action and the power to initiate action).[7] The former, relative power analysis, is useful for making comparisons of *a priori* voting power between members *within a given voting body* defined by weights and decision rule, and also as the basis of a suitable choice of the weights using the algorithm I will describe below, but useless for making comparisons between *different* voting bodies with different weights and decision rules. On the other hand the absolute measures can be used for such comparisons and in particular to study the effect of the threshold. First it is necessary to give definitions.

A voting body has n members with voting weights, $w_1, w_2, \ldots, w_n$ and a decision rule in terms of a threshold, q.[8] The set of all members is N. All the indices are based on counting the number of swings, voting outcomes that can be changed from losing to winning by members changing how they cast their weighted vote. A particular voting outcome will be referred to as a division.

A swing for member i is a coalition (corresponding to a division) represented by a subset of members S_i, $N \supset S_i$, $i \notin S_i$, such that

$$\sum_{j \in S_i} w_j < q \quad \text{and} \quad \sum_{j \in S_i} w_j + w_i = q.$$

A swing is a coalition where the total votes cast in favour of a particular decision fall short of the threshold without those of member i, but equal or exceed it when member i

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symposium in the Journal of Theoretical Politics in 1999, especially Lane and Berg (1999) and to Felsenthal and Machover (2001a).

[7] Banzhaf (1965), Penrose (1946), Coleman (1971).

joins. Let the number of swings for i be η_i and the total number of swings be $\bar{h} = \sum h_i$. The total number of divisions, the number of subsets of N, is 2^n.

Five measures of power are used, defined as follows.[9].

(1) The Penrose Measure for i is the proportion of all possible divisions which are swings, denoted by π_i:

$$\pi_i = \eta_i / 2^{n-1} \qquad i=1,2,..,n$$

The denominator is the number of possible coalitions among n members which do not include i, and therefore the maximum number of swings. Finding this for all i provides an absolute measure of each member's voting power which is used directly in the analysis of the threshold and as the basis of the definition of the Banzhaf index below.[10]

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[8] It is usual in the theoretical literature to refer to q as the quota. However no confusion will result from retaining the official term, threshold.

[9] Other power indices than these have been used, in particular the well known index proposed in Shapley and Shubik (1954), which provides a measure of relative voting power often regarded as comparable with the Banzhaf index, but based on a completely different coalition model. The decision not to use the Shapley-Shubik index here is based on two considerations: first it was found not to perform well in a comparison of its empirical properties with those of the Banzhaf index, and second, criticism of its theoretical basis. See Leech (2000a), also Felsenthal and Machover (1998), Coleman (1971).

[10] This measure has a number of names. It is often called the Absolute (or Non-Normalised) Banzhaf index, or Banzhaf-Coleman index, most writers emphasising its relation with the Banzhaf index. However, since it was invented by Penrose (Penrose (1946)) and I am arguing that the distinction between a normalised and a non-normalised index is more than a technical detail, it seems sensible to attribute it historically correctly. Felsenthal and Machover (1998) restrict the use of the term "index" to one which is normalised, and refer to this measure as the Banzhaf Measure.

(2) The Banzhaf Index for member i is the member's relative number of swings, the normalised version of the Penrose measure, denoted by β_i:

$$\beta_i = \eta_i/\bar{h} = \pi_i/\Sigma\pi_j \qquad i=1,2,..,n$$

This has the property that the indices of all members sum to 1 and can be interpreted as giving the share of member i in the combined capacity of all members to influence decisions. This index is used to analyse relative powers of members under the Treaty of Nice and also as the basis of the approach to the fair choice of weights.

(3) The Power of the Body to Act measures the ease with which members' interests in a division can be translated into actual decisions. It is denoted by A. The measure is a property of the voting body itself, rather than any particular member. It is defined as the proportion of all the theoretically possible divisions that lead to a decision.

$$A = w/ 2^n,$$

where w is the number of winning divisions (i.e. divisions where the total number of votes cast for the decision at least equals the threshold). This measure is important when the decision rule requires a supermajority with a threshold in excess of 0.5.

(4) The Power of a Member to Prevent Action measures the ability of member i to prevent a decision being taken. It is denoted P_i. It is defined as the proportion of winning divisions that are swings for i:

$$P_i = \eta_i / w \qquad i=1,2,..,n.$$

(5) The Power of a Member to Initiate Action is complementary to this, measuring member i's power to get its proposals accepted and is denoted I_i. It is formally defined as the number of swings for i as a proportion of the total number of divisions that do not produce a decision without the support of i.

$$I_i = \eta_i / (2^n - w) \qquad i=1,2,..,n.$$

Both (4) and (5) can be regarded as rescalings of the Banzhaf index or Penrose measure. They are both identical to the latter when q=0.5[11], since then there is no difference between the power to prevent action and the power to initiate action. However there is a difference where there is a supermajority decision rule, and they are useful in enabling the analysis to focus on these two different aspects of members' voting power. The distinction is especially useful in the present context where discussions surrounding the choice of the threshold have centred on individual members' and groups of members' ability to block decisions.

The relationships among the indices are brought out by noting that we can write the Penrose measure as:

$$\pi_i = \eta_i / 2^{n-1} = 2(\eta_i / w).(w/2^n) = 2P_i.A,$$

and, $$\pi_i = \eta_i / 2^{n-1} = 2[\eta_i / (2^n - w)]. [(2^n - w). /2^n] = 2I_i.[1 - A].$$

[11] Strictly q must be slightly greater than 0.5.

Therefore the Penrose measure combines the individual member's power either to prevent action or to initiate action with the power of the voting body itself to act.[12]. These measures are used to compare the properties of different thresholds.

5. Fair Weighting and Reweighting

The first main question to be addressed using the measures of voting power defined in the last section is whether the weights agreed in the Nice Treaty are appropriate. I address this question using two different approaches. First I calculate the power indices and compare them in terms of a criterion of fairness, and second I use an iterative algorithm to determine what ideally they *should* be in order to be fair in this sense. I propose that the votes can be reweighed routinely in this way whenever a new member accedes.

Felsenthal and Machover (2000) propose basing the allocation of voting weight on the principle of equitability whereby citizens of all member countries should have equal voting power. Decision making is modelled as a two-stage voting system in which the first stage is the ordinary political process in each member country and the second stage is weighted voting in the Council.

[12] It is also of interest to note that the Penrose measure π_i is the harmonic mean of I_i and P_i: $1/\pi_? = (1/P_i + 1/I_i)/2$ (Dubey and Shapley (1979)).

Formally fair or equitable weights are defined by considering the following two stages of voting:

Stage 1. Citizen Voting in Member States. Each member country holds an election or plebiscite on the basis of One Person One Vote and a simple majority decision rule. Each citizen has formally the same voting power as any other within the same country but this is different in each country. Electors in a small country have a much greater chance of a swing than those in a large country; this was formally analysed by Penrose (1946) who showed the probability to be inversely proportional to the square root of the number of electors. I use population as a proxy for the number of electors.

Stage 2. Weighted Voting in the Council. Each member state casts all its weighted votes according to the result of Stage 1. Its Penrose measure is the probability of a swing within this weighted voting body.

The product of these two probabilities, at Stage 1 and Stage 2, for any member country, measures the power of one of its citizens, as the probability of his or her theoretically being able to determine the overall outcome. The principle of fairness suggests that this measure should be equal for all citizens in all member countries and therefore voting weights should be allocated such that the power indices of each member are proportional to the square roots of populations.

Let the population of member state i be denoted by m_i. Then the fair weights, $w_1^*, w_2^*, \ldots, w_n^*$, are determined by the property that the resulting Penrose measures satisfy

$$\pi_i = k\, m_i^{0.5} , \qquad \text{for some } k>0, \qquad i=1,2,\ldots,n.$$

Let the share of member i in the sum of the square roots of populations be t_i. That is, let $t_i = m_i^{0.5} / \sum m_j^{0.5}$, i =1,2,…, n. Then the t_i are the target values to which the Banzhaf indices should be set equal in the distribution of voting weights. This must be done by an iterative procedure, which entails successively computing the power indices and reweighting to bring them closer to the target values; iterations continue until convergence has occurred according to an appropriate stopping rule,[13]. The algorithm can be thought of as the determination of a fixed point of a mapping from the unit simplex to itself.

Let it be required that member i should possess a voting power of t_i, where $\sum t_i = 1$. The problem is to find weights $w_i^*, w_2^*, \ldots, w_n^*$ that have associated Banzhaf indices, β_i , such that $\beta_i = t_i$, for all i. For notational simplicity I denote the target, the weights and corresponding power indices, as functions of the weights, by the n-vectors t, w and $\beta(w)$.

Let the weights after d iterations be denoted by the vector $w^{(d)}$, and corresponding power indices by the vector of functions $\beta(w^{(d)})$. The iterative procedure consists of an initial guess $w^{(0)}$ and an updating rule:

$$w^{(d+1)} = w^{(d)} + \lambda(t - \beta(w^{(d)})) \qquad (1)$$

for some appropriate choice of scalar $\lambda>0$.

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[13] A similar approach has been proposed by Laruelle and Widgren (1998) and used by Sutter (2000). It has also been used in Leech (2000b). The question was discussed by Nurmi (1982).

If power indices are continuous functions of the weights, and (1) is a continuous point-to-point mapping of a compact convex set into itself; it therefore satisfies the conditions of the Brouwer fixed point theorem and has a unique fixed point.[14] If the procedure converges to a vector, w*, then that will be the desired weight vector, since then:

$$w^* = w^* + \lambda(t - \beta(w^*)) \text{ and so } t = \beta(w^*).$$

Convergence can be defined in terms of a measure of the distance between $\beta(w^{(d)})$ and t. The simple sum of squares $\sum (b_i^{(d)} - t_i)^2$ with a suitable stopping rule has been found to work well in practice.[15] The algorithm is set out graphically in Figure 1.

Figure 1 about here

[14] The continuity property does not strictly hold for small voting bodies. The Penrose measure is not continuous since it is a rational number and therefore the Banzhaf index is also a rational number. However for large n it seems reasonable to assume that the conditions hold approximately, and that the approximation improves as n increases.

It does not follow from this that a member's Banzhaf index necessarily increases when it is given more weight; in fact the opposite can occur. Felsenthal and Machover (1998, p 253) call this the "fattening paradox". It is a property of the normalised power index only and is not shared by the Penrose measure. How serious this is for the algorithm employed in this paper is unclear since the fixed-point theorem on which it is based requires the mapping to be point-to-point, that is to associate a unique vector of power indices with each vector of weights. Then, convergence guarantees finding a fixed point.

[15] For N15 the algorithm was found to converge to an accuracy, in terms of this criterion, of the order of 10^{-8}, but it was not possible to get full convergence with a smaller value. For N27 it easily converged with respect to a stopping rule of the order of10^{-10}. The power indices were computed exactly using the program ipnice (Leech (2001b)). In Leech (2000b) the same iterative algorithm was used to compute fair weights for the International Monetary Fund Board of Governors with n=178. In this case the power indices were calculated using a different program suitable for large n (described in Leech (2001c)); the accuracy achieved in terms of the sum of squares stopping rule was of the order of 10^{-17}.

6. VOTING POWER UNDER THE NICE TREATY

Tables 4 and 5 present the results of applying this approach to the Nice Treaty.[16] Table 4 shows the analysis for N15; the same information is displayed graphically in Figure 2. In the first column after the names of the countries are the weights expressed as percentages of the total, then the Banzhaf power indices (columns (3) and (4)). The effect of the 62% population condition is evident from the greater power of Germany than the other three of the big four despite its having the same weight. It appears from comparing these two columns that the allocation of weights is very close to being proportional: that is, that weight shares and power indices are almost the same. This can be seen from Figure 2(a), which shows these numbers for each country against a population scale, and Figure 2(b), which shows them for each country separately: the graphs for weight and power almost coincide for every member country.

That a member's power index is approximately proportional to its share of the weight does not mean that the weights are fair. Comparing the power indices with their target values (column (5)) shows that there are some discrepancies from fair weights, in particular Germans are under-represented and Spaniards over-represented: Germany's power index is 12.11% compared with a target of 13.97%, Spain's power index is 11.11% compared with its target of 9.68%. For all other countries the discrepancy is less than one percentage point.

[16] The computer program used was Leech (2001b).

Table 4: Voting Power in N15

N15	q_1=169	q_2=62%				
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Weight	Country	Weight%	Bz Index %	vPop%	Fair Weight %	Pop%
29	Germany	12.24	12.11	13.97	15.12	21.858
29	UK	12.24	11.99	11.87	12.06	15.786
29	France	12.24	11.99	11.84	12.05	15.711
29	Italy	12.24	11.99	11.70	11.99	15.350
27	Spain	11.39	11.11	9.68	9.34	10.496
13	Netherlands	5.49	5.50	6.12	5.98	4.199
12	Greece	5.06	5.16	5.00	4.64	2.806
12	Belgium	5.06	5.16	4.93	4.61	2.721
12	Portugal	5.06	5.16	4.87	4.58	2.659
10	Sweden	4.22	4.30	4.59	4.47	2.359
10	Austria	4.22	4.30	4.38	4.41	2.153
7	Denmark	2.95	3.09	3.55	3.22	1.416
7	Finland	2.95	3.09	3.50	3.20	1.375
7	Ireland	2.95	3.09	2.98	3.03	0.998
4	Luxembourg	1.69	1.96	1.01	1.29	0.114
237		100.00	100.00	100.00	100.00	100.00

Bz: Banzhaf; q_1= the threshold in terms of weighted votes, q_2 = the population condition.

Applying the iterative algorithm gives the fair weights, listed in column (6). The only member countries whose weights change substantially are Germany and Spain: Germany's weight has now increased to 15.12 and Spain's reduced to 9.34 percent of the votes. These results are shown in Figure 2(a) and 2(b) also.

Figure 2 about here

The equivalent analysis for N27 is presented in Table 5 and Figure 3. A broadly similar story emerges with power and weight being roughly proportional, although the discrepancy for the big four countries is now larger, more than half a percentage point.

The population condition no longer favours Germany, its power index being the same as that of the UK, France and Italy.

However these are not fair weights in that Germany is under-represented and both Spain and Poland are over-represented: Germany has a power index of 7.78% compared with a target of 9.54%, Spain and Poland have a power index of 7.42% compared with targets of 6.61% and 6.55%; also Romania is under-represented. Applying the algorithm to compute the fair weights adjusts these discrepancies (column (6)). The most substantial changes are that Germany should have 12.21 (instead of 8.41) percent of the weight, Spain's and Poland's weights should be reduced to 6.53 and 6.45 (instead of 7.83) percent and Romania's increased to 4.74 (instead of 4.06) percent. Some of the changes for other countries are large in relative terms, compared with their absolute weight, but they make little difference in absolute terms.

Table 5: Voting Power in N27

N27	q_1=255	q_2=62%				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Weight	Country	Weight %	Bz Index %	vPop.%	Fair Weight%	Pop.%	
29	Germany	8.41	7.78	9.54	12.21	17.049
29	UK	8.41	7.78	8.10	8.54	12.313
29	France	8.41	7.78	8.09	8.53	12.254
29	Italy	8.41	7.78	7.99	8.36	11.973
27	Spain	7.83	7.42	6.61	6.53	8.187
27	Poland	7.83	7.42	6.55	6.45	8.036
14	Romania	4.06	4.26	4.99	4.74	4.674
13	Netherlands	3.77	3.97	4.18	3.92	3.275
12	Greece	3.48	3.68	3.42	3.18	2.189
12	Czech Rep	3.48	3.68	3.38	3.14	2.138
12	Belgium	3.48	3.68	3.37	3.14	2.122
12	Hungary	3.48	3.68	3.35	3.12	2.097
12	Portugal	3.48	3.68	3.33	3.10	2.074
10	Sweden	2.90	3.09	3.13	2.91	1.840
10	Bulgaria	2.90	3.09	3.02	2.80	1.710
10	Austria	2.90	3.09	2.99	2.77	1.680
7	Slovakia	2.03	2.18	2.45	2.26	1.121
7	Denmark	2.03	2.18	2.43	2.24	1.104
7	Finland	2.03	2.18	2.39	2.21	1.072
7	Ireland	2.03	2.18	2.04	1.88	0.778
7	Lithuania	2.03	2.18	2.03	1.87	0.769
4	Latvia	1.16	1.25	1.64	1.51	0.507
4	Slovenia	1.16	1.25	1.48	1.36	0.411
4	Estonia	1.16	1.25	1.27	1.17	0.301
4	Cyprus	1.16	1.25	0.91	0.83	0.156
4	Luxembourg	1.16	1.25	0.69	0.63	0.089
3	Malta	0.87	0.94	0.65	0.60	0.079
345		100.00	100.00	100.00	100.00	100.000

Bz: Banzhaf; q_1= the threshold in terms of weighted votes, q_2 = the population condition.

Figure 3 about here

7. THE EFFECT OF THE CHOICE OF THRESHOLD

The analysis so far has been in terms of the relative voting power of each member country within a given decision-making system, defined by a particular threshold, and no consideration has been given to what that ought to be. For N15 q was equal to 71.3%, and for N27, 73.9%. Now in this section the decision rule becomes the main focus of the analysis and I invesitgate its effect using the power indices due to Coleman as well as the Penrose measure. I allow the decision rule as determined by the value of q to vary over its entire feasible range from a simple majority, q=50 %, to unanimity, q=100%. In order to define the problem to be analysed clearly, I assume qualified majority voting in terms of a single decision rule, for the two scenarios that have been previously defined as W15 and W27, with the weights fixed in the Nice Treaty.[17]

The analysis of this section uses member countries' powers to prevent action, P_i, and to initiate action,I_i, as properties of the voting system, reflecting countries' sovereignty, to interpret the effects of varying q. It also shows how the power to act of the Council itself, A, is affected. An important feature of this approach is that it allows us to study the tradeoff between members' powers to prevent action and the power of the Council to act, which is at the heart of the political development of the European Union.

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[17]. This means that for this purpose the other two conditions in N15 and N27 (a majority of countries and 62% of the population) are ignored. The reason for doing this is that, if the threshold for weighted voting is varied, there seems no particular reason not to vary the population threshold also and investigate whether its optimal value is 62%The analysis could just as easily be done for N15 and N27. In any case the power indices for W15 and W27 are almost identical to those for N15 and N27.

The results for W15 are presented first, in Figure 4.[18] Figure 4(a) shows the effect of the threshold on the power of the Council to act. Its maximum value is 0.5 when q=50% and its minimum value 2^{-15} = 3.05E-5 when q=100%. It is clear that the value of q set by the Nice Treaty makes it very difficult to make a decision in the Council, its power to act when q=71.3% being only 0.0826. This means that only 8.26% of divisions, *a priori*, would result in a decision. Therefore there is a very strong conservative bias.

Figure 4 about here

Figure 4(b) shows how the threshold affects the powers of members to prevent action, their capacities to block initiatives they do not like. Not surprisingly it increases monotonically for all countries as q increases until it reaches a maximum of 1 when q=100 and all members have a veto. For q=71.3% all members have a substantial power to prevent action: for each of Germany, the UK, France and Italy it is 0.735 (one of the big four countries can block 73.5% of divisions), for Netherlands 0.342, and even for the smallest member Luxembourg it is 0.125. Figure 4(c) shows the equivalent diagram for the power to initiate action. This measure of power falls very rapidly indeed for every member as q increases, showing that blocking power is bought at a high price in terms of loss of influence. Figure 4(d) shows the Penrose power measure, π_i, against q. It confirms that all members, most significantly the largest countries, suffer a loss of influence, in choosing too high a threshold.

Figure 5 shows the equivalent analysis for W27. The findings are substantially the same: Figure 5(a) shows the power to act falls very rapidly as q increases, reaching 10%

[18] The results of this section were obtained using the computer program Leech (2001a).

when q=65%. The effect of the Nice Treaty, which set q=73.9%, is even worse for this case, with the power to act falling to as low as 0.02: only 2% of divisions *a priori* lead to a decision. The diagrams showing the powers of member countries to prevent action, Figure 5(b), to initiate action, Figure 5(c), and Penrose voting power, Figure 5(d), give similar results to those for N15. Thus, again, the conclusion is that choosing too high a value of q is counterproductive to a member country's own sovereignty within the EU.

Figure 5 about here

Figures 6 and 7 show the same information for W15 and W27 as relationships between member countries' own absolute power measures and the power of the Council to act as q varies. These diagrams make explicit the tradeoffs involved in qualified majority voting. Figure 6(a) plots the power to prevent action for each member against the power of the Council to act. There would seem to be a fairly strong tradeoff for the largest five countries showing how much of their own blocking power they must give up in order to create an effective Council. Figure 6(b) shows a very direct relationship between the power to initiate action of each member and the power to act. Figure 6(c) shows the relationships between the Penrose powers of members and the Council's power to act. Again there is a direct relationship in every case suggesting that member countries have greater influence through higher power of the Council to act. Figure 7 shows the same analyses for W27, leading to the similar conclusions.

Figures 6 and 7 about here

8. CONCLUSIONS

This paper has reported on a study of the system of Qualified Majority Voting in the Council of the European Union using the methods of *a priori* voting power analysis. The perspective of the study has been that of an enlarging union to which new members accede from time to time, as envisaged in the Treaty of Nice to apply from 2005.

Two investigations have been carried out. First, a study of weighted voting using the Banzhaf power index aimed at discovering if the allocation of weights between member countries is fair in a relative sense. Fair weights are defined to be such as to equalise voting power of citizens in all member countries. Second, a study of the effects of varying the threshold both on the measures of absolute power of each member country and the power to act of the Council itself. For each investigation two extreme scenarios have been considered as envisaged by the Treaty: the union of 15, no new members having acceded, and the maximum expansion, with 12 candidates having joined.

From the perspective of enlargement considered in general terms, it is proposed that fair weights could be determined as a routine or simply technical matter, by means of an algorithm, for any changes to the membership that may occur. This is applied to the two extreme scenarios considered.

The findings of the analysis of fair weights are: first, that the weights laid down by the Nice Treaty are approximately proportional to the voting power they represent; second, that they are close to being fair for most members; third, that German and Romanian citizens will be under-represented, Spanish and Polish citizens over-represented.

The results of the analysis of the threshold are: first, that the power of the Council to act will be very small because of the high level at which the threshold has always been set (about 71%) and will continue to be set unless the treaty is amended; second, the Treaty's provision to raise it to almost 74% when many new members join makes this aspect considerably worse; third, there is a trade-off between individual member countries' blocking powers and the power of the Council to act, but a direct relation between a country's overall measure of power and the power of the Council to act. The main conclusion of this analysis is that decision making within the Council of the European Union is likely to remain rigid because of members' being overly concerned with their own blocking powers, and for this to get worse with every enlargement.

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Figure 1:Flowchart of an Iterative Algorithm to Determine Weights

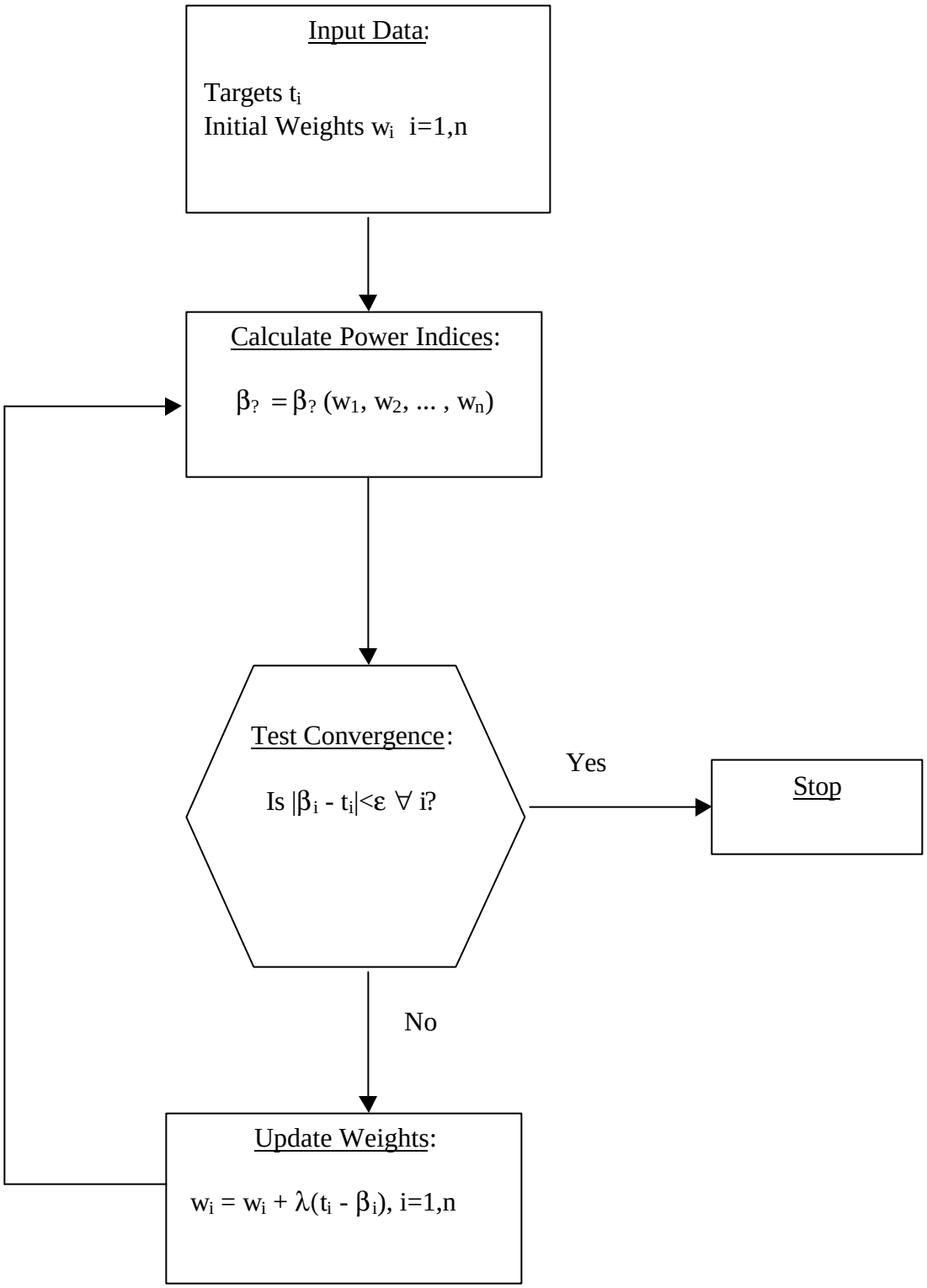

Figure 2(a): N15 Weight, Power and Fair Weight by Population

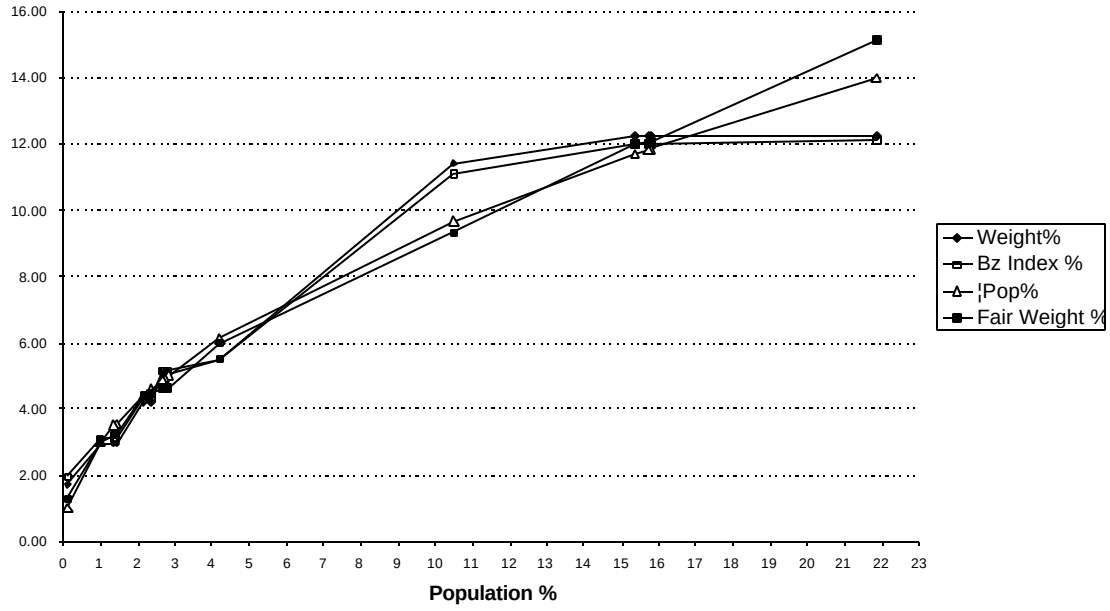

Figure 2(b): N15 Weight, Power and Fair Weight by Country

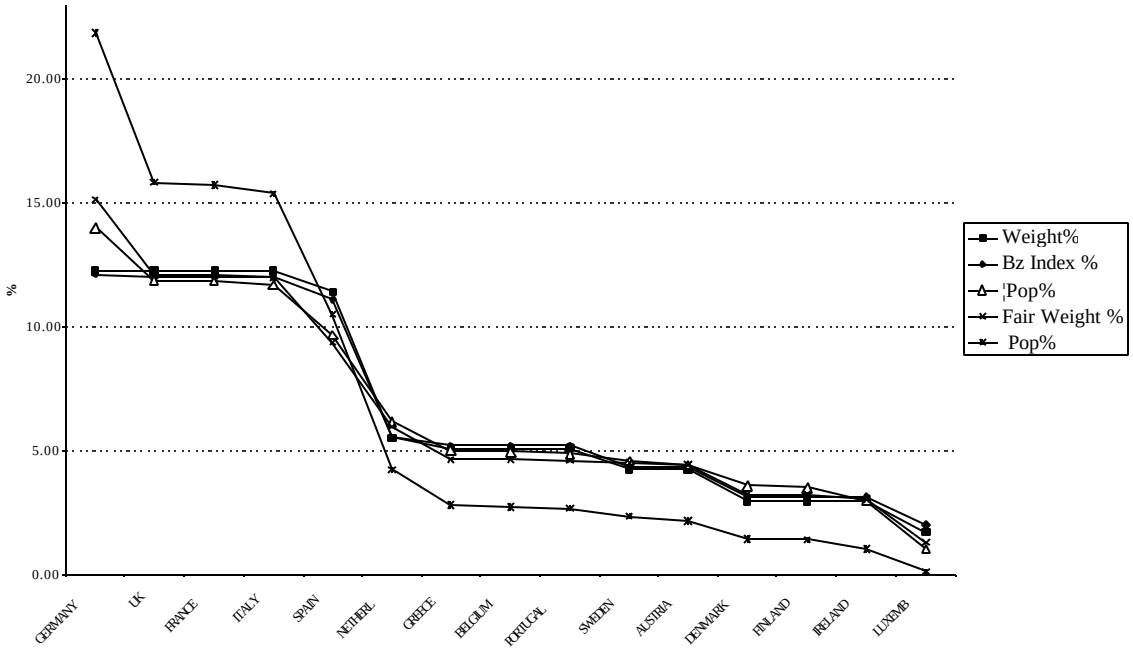

Figure 3(a): N27 Weight, Power and Fair Weight by Population

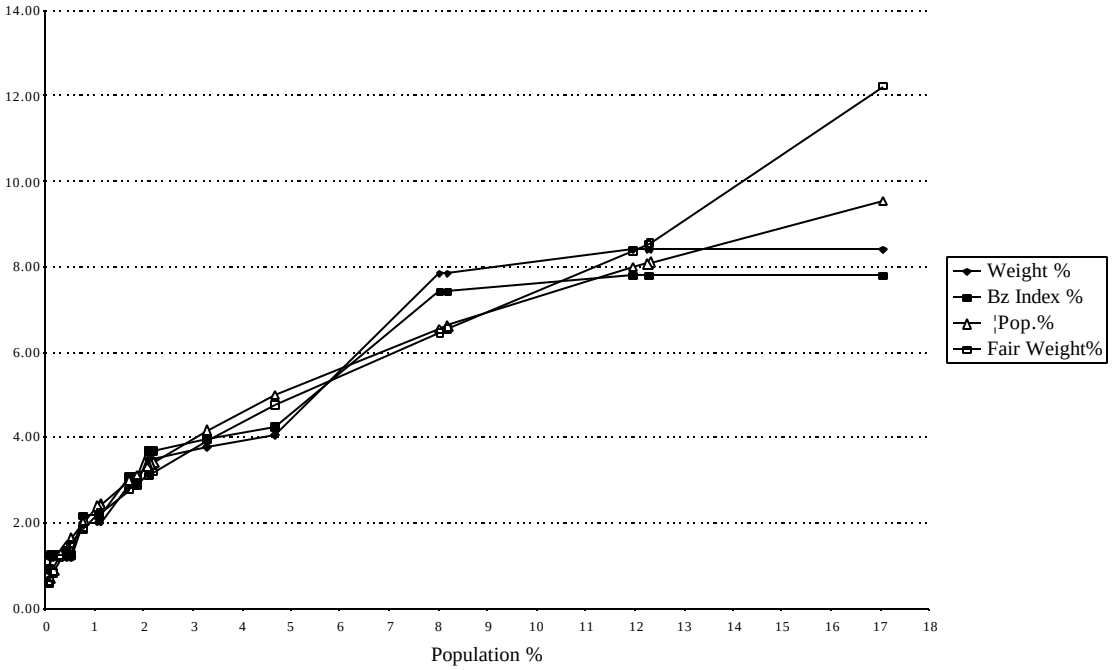

Figure 3(b): N27 Weight, Power and Fair Weight by Country

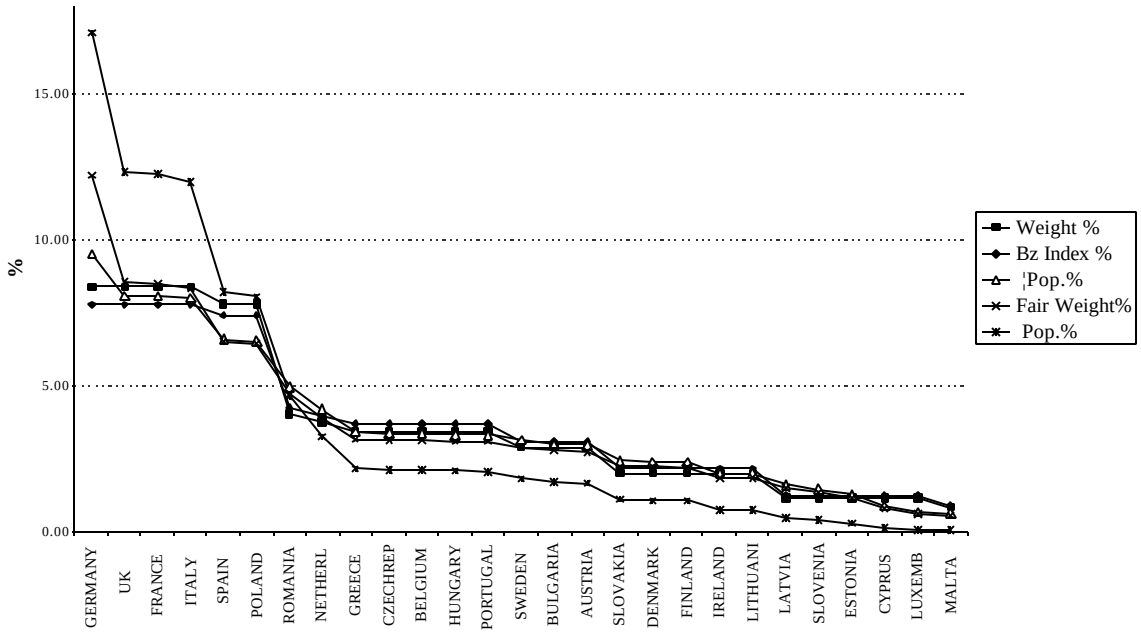

Figure 4(a): W15 Effect of the Threshold on the Council's Power to Act

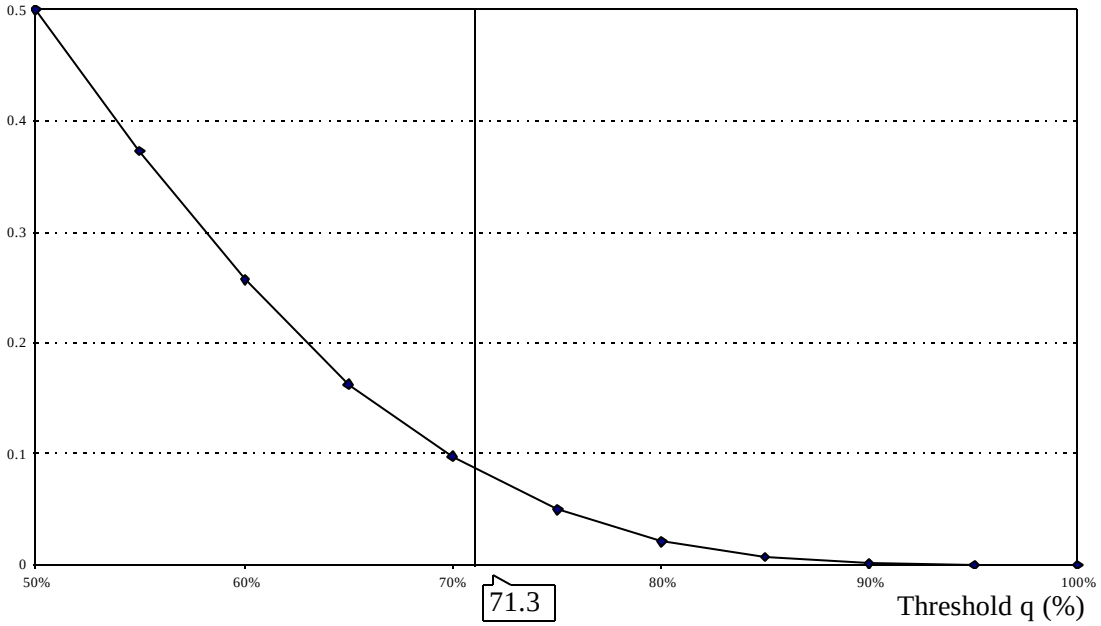

Figure 4(b): W15 Effect of the Threshold q on Power to Prevent Action

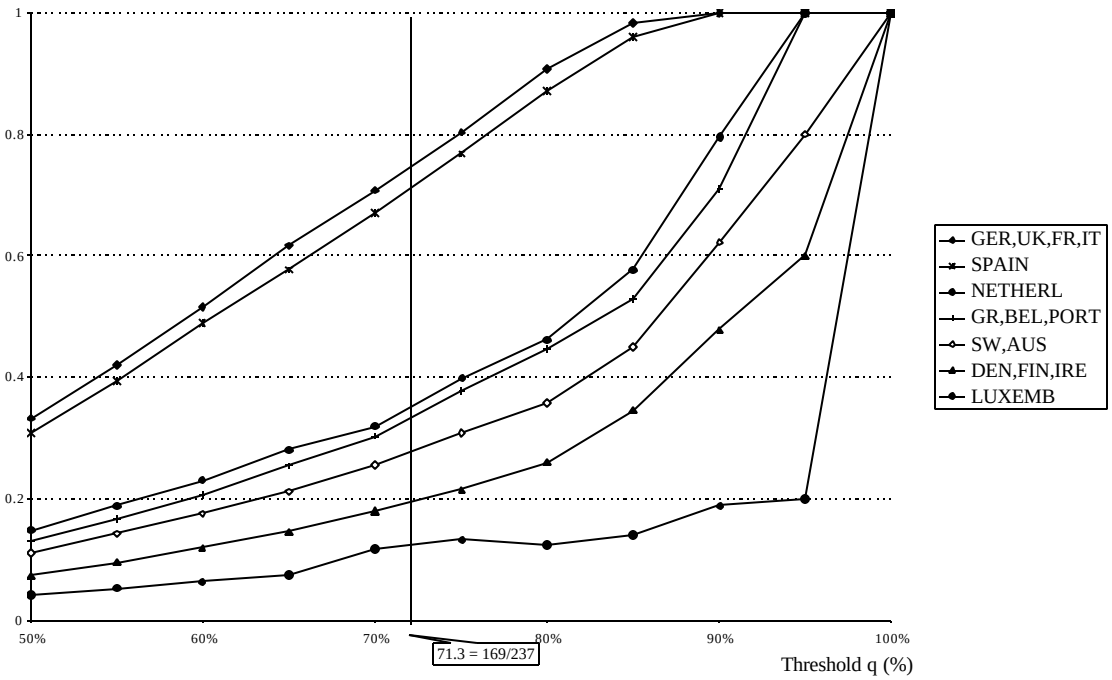

Figure 4(c): W15 Effect of the Threshold q on Power to Initiate Action

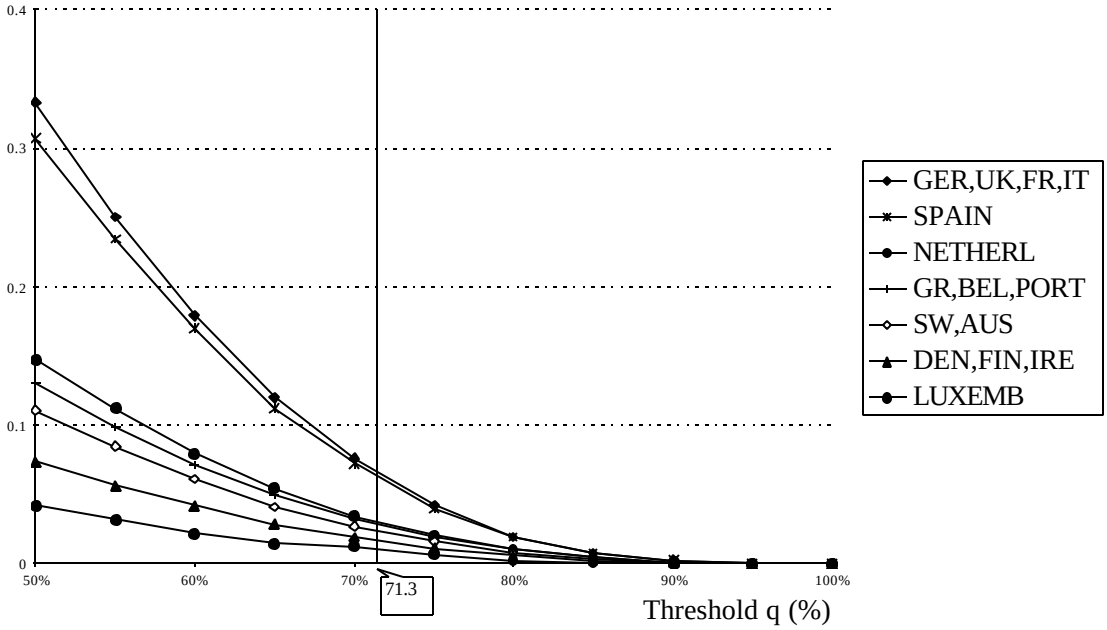

Figure 4(d): W15 Effect of the Threshold q on the Penrose Measure, π'

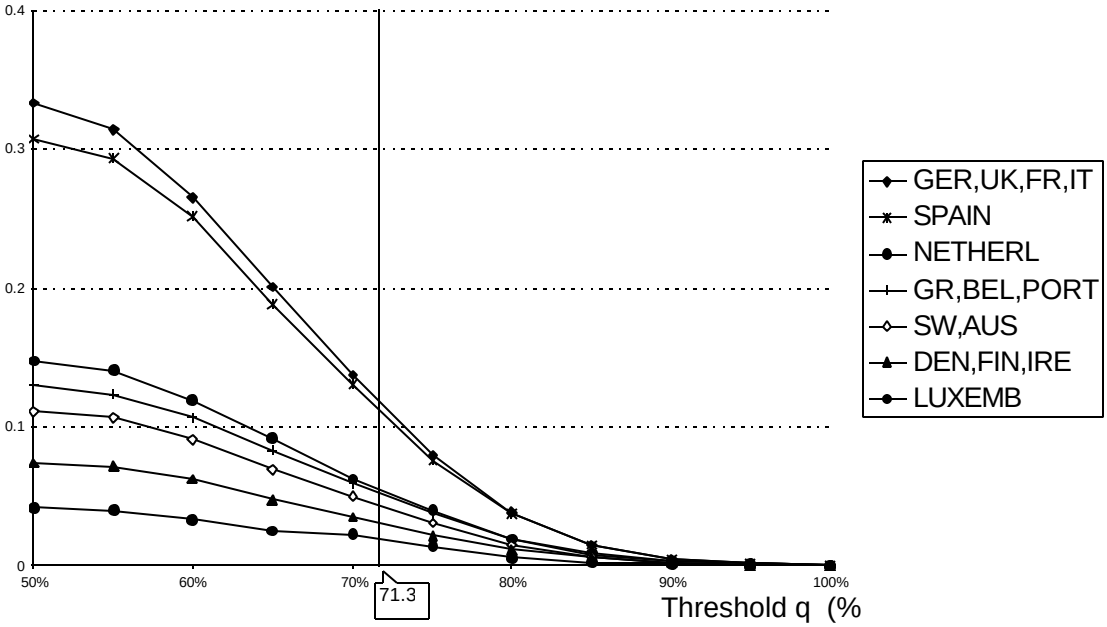

Figure 5(a): W27 Effect of the Threshold q on the Council's Power to Act

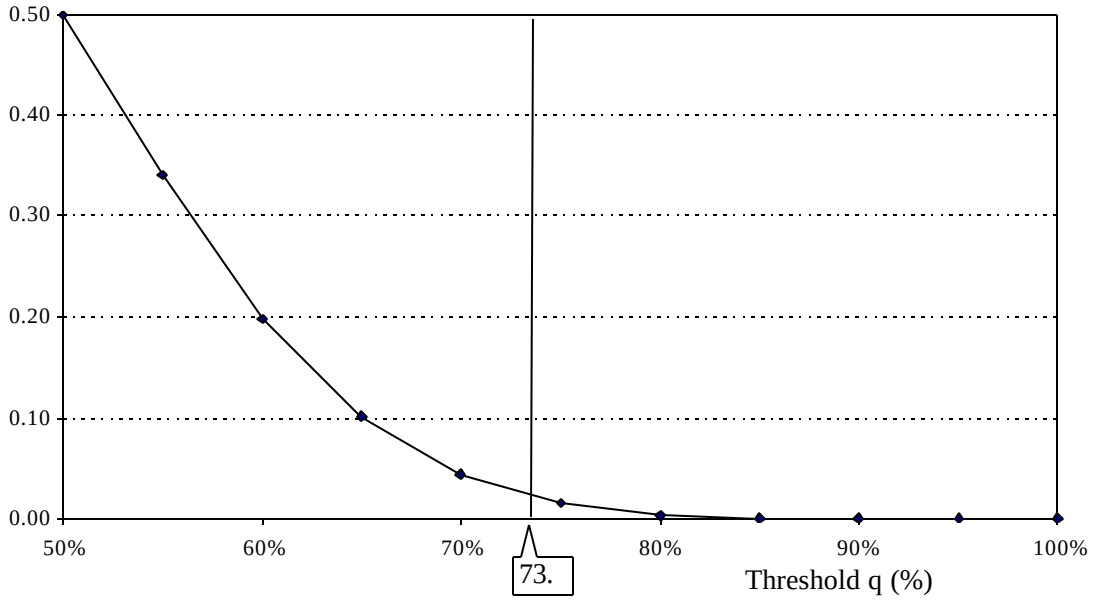

Figure 5(b): W27 Effect of the Threshold q on Power to Prevent Action

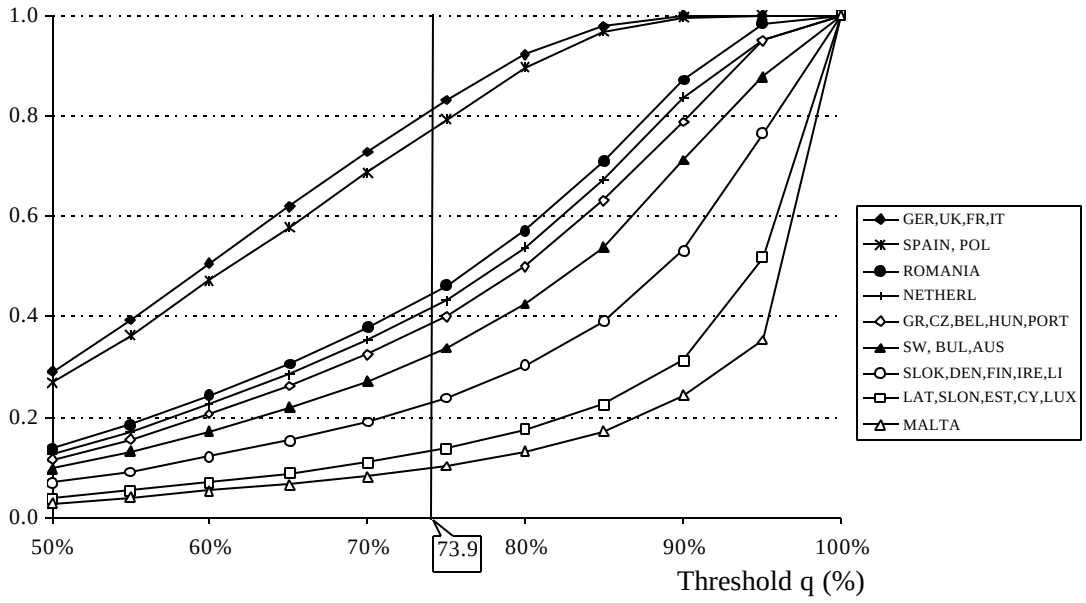

Figure 5(c): W27 Effect of the Threshold q on Power to Initiate Action

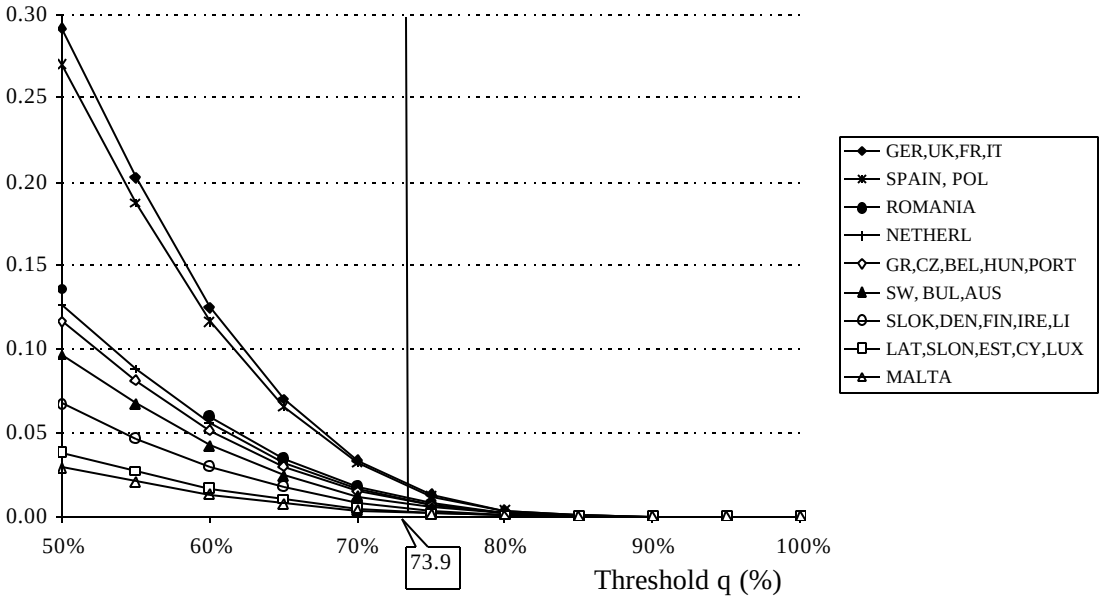

Figure 5(d): W27 Effect of the Threshold q on the Penrose Power Measure π_i

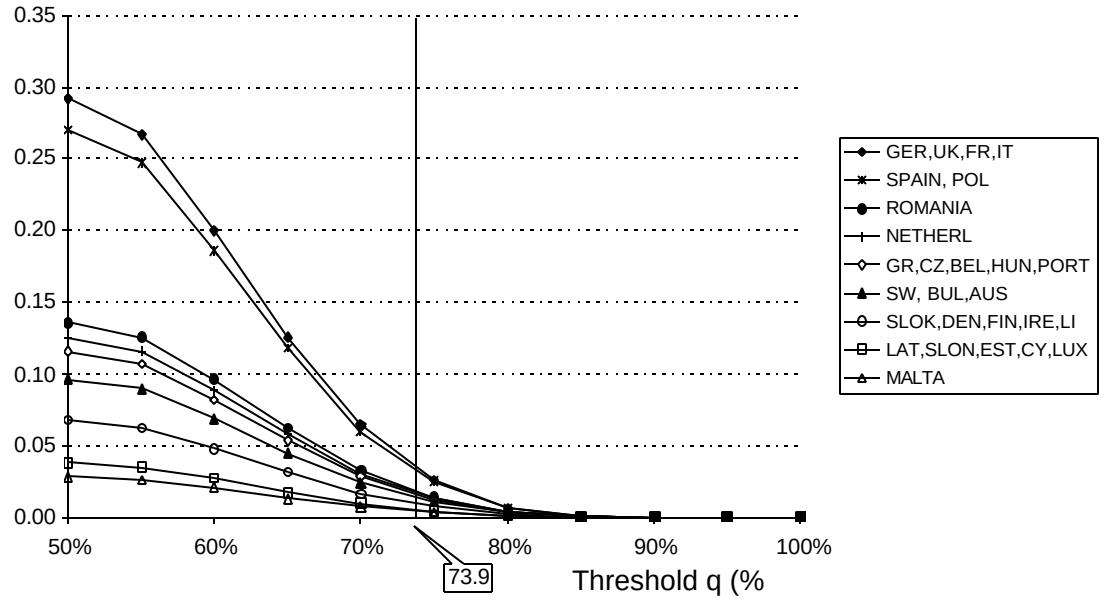

Figure 6(a): W15 Power to Prevent Action versus Power of the Council to A

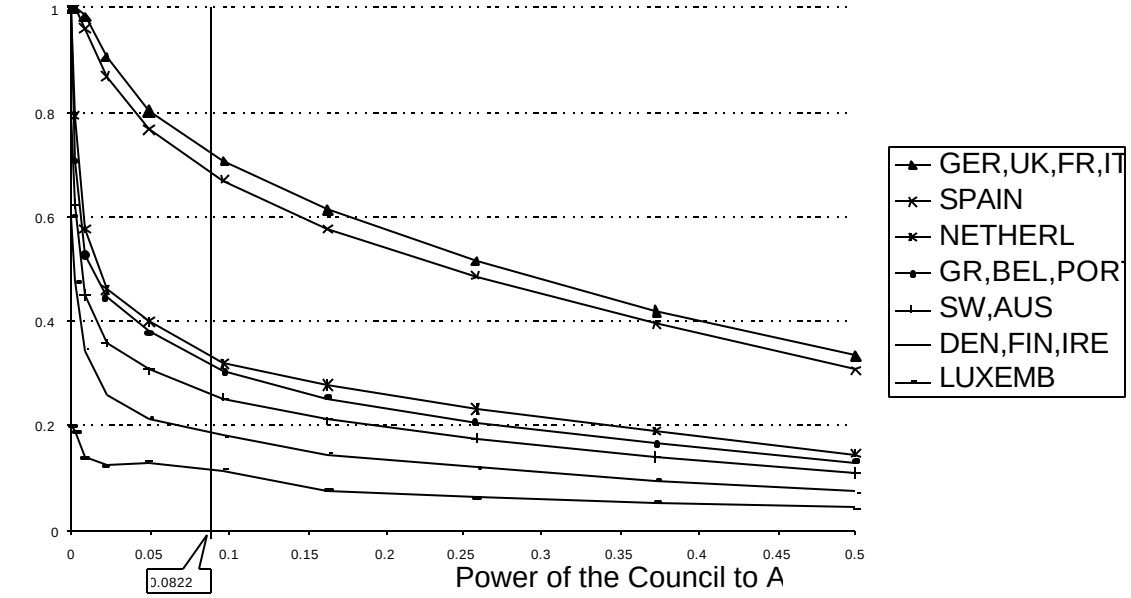

Figure 6(b): W15 Power to Initiate Action versus Power of the Council to Act

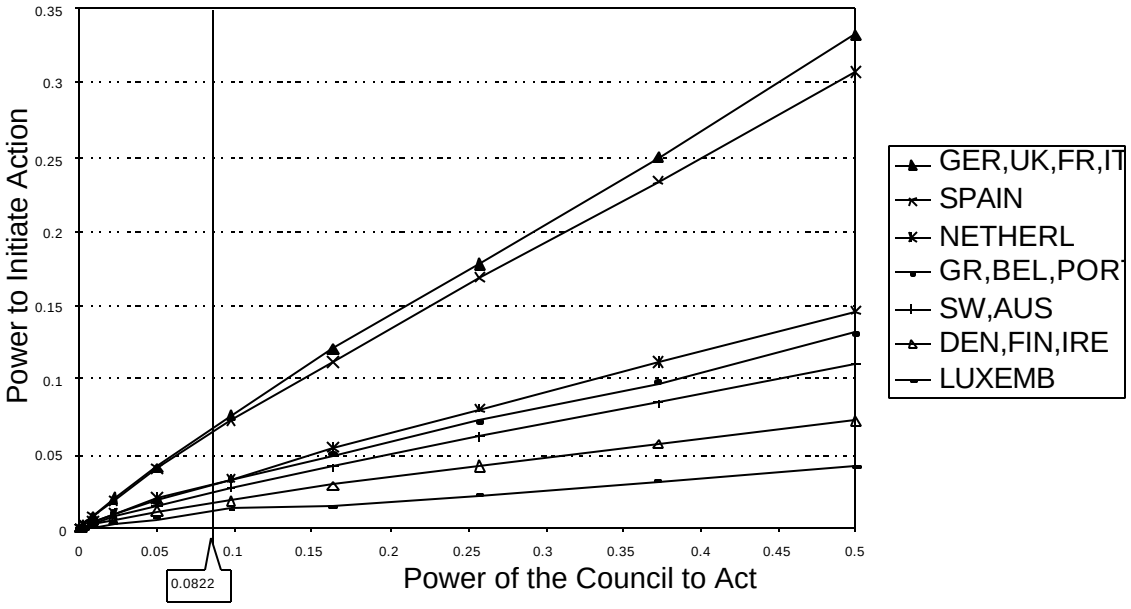

Figure 6(c): W15 Penrose Power Measure versus Power of the Council to Act

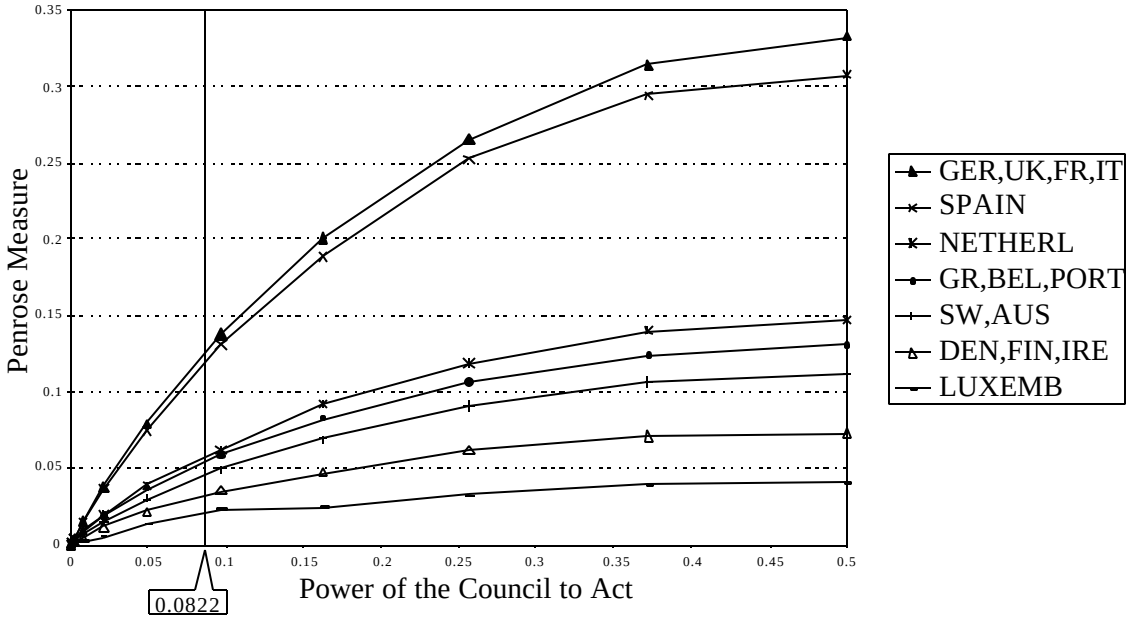

Figure 7(a):W27 Power to Prevent Action versus Power of the Council to Act

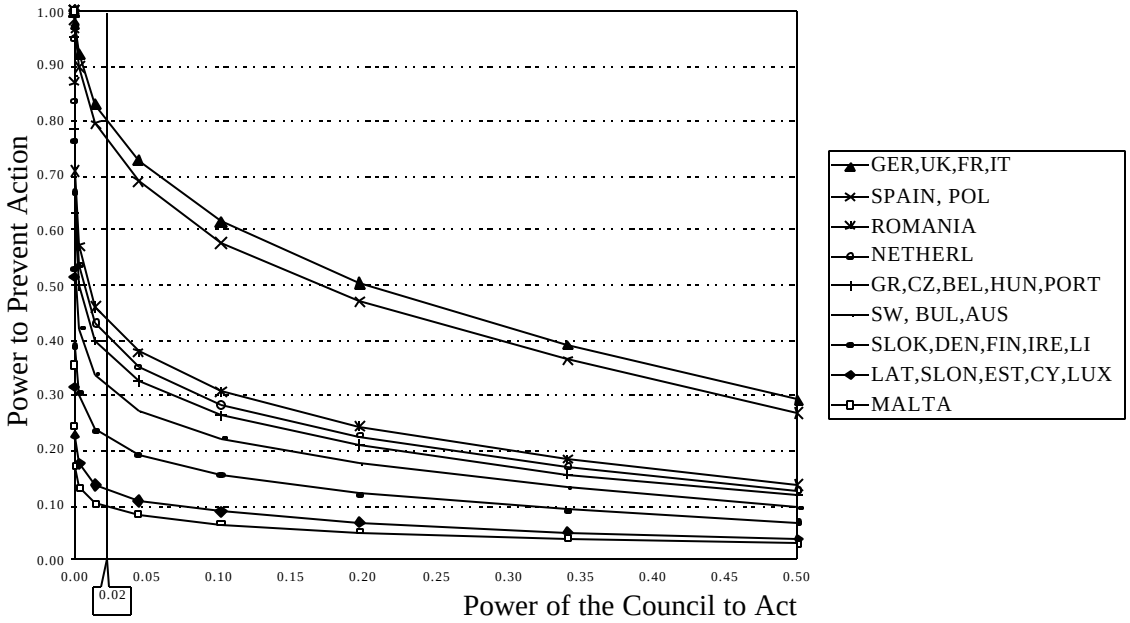

Figure 7(b): W27 Power to Initiate Action versus the Council's Power to Act

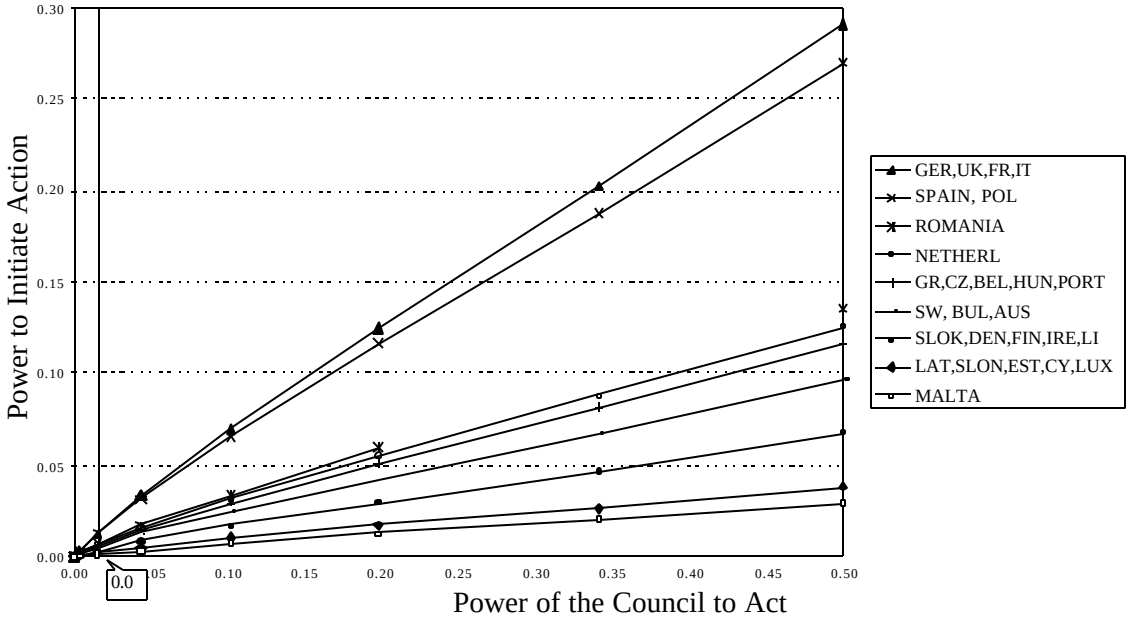
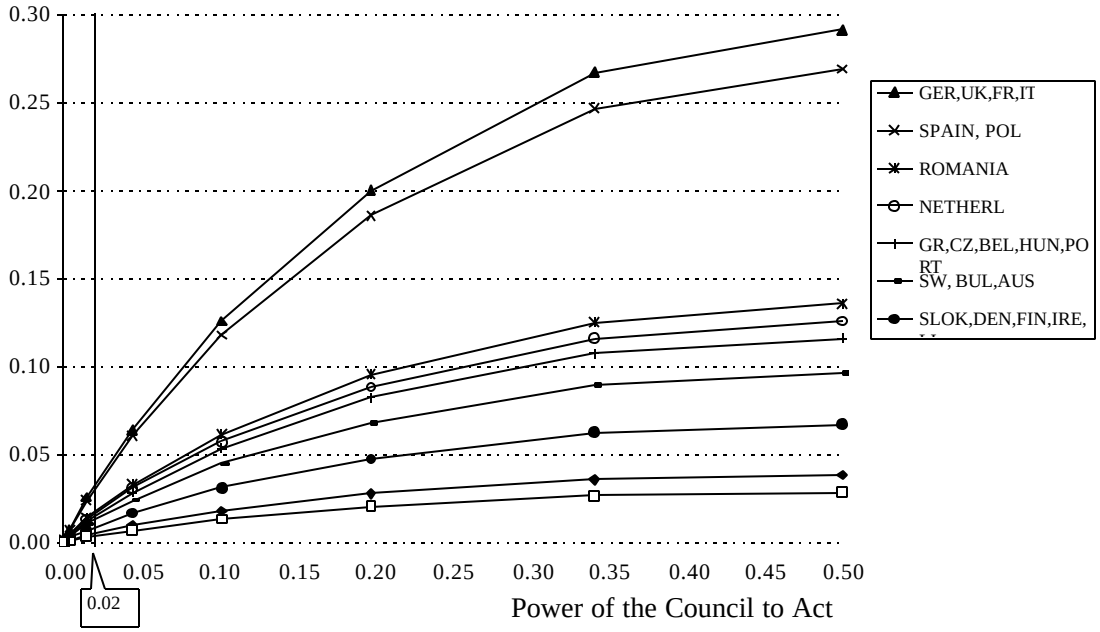
Figure 7(c): W27 Penrose Measure versus Power of the Council to Act