STRATEGIC DELAY IN A REAL OPTIONS MODEL OF R&D COMPETITION

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Abstract

This paper considers irreversible investment in competing research projects with uncertain returns under a winner-takes-all patent system. Uncertainty takes two distinct forms: the *technological* success of the project is probabilistic, while the *economic* value of the patent to be won evolves stochastically over time. According to the theory of real options uncertainty generates an option value of delay, but with two competing firms the fear of preemption would appear to undermine this approach. In non-cooperative equilibrium two patterns of investment emerge depending on parameter values. In a preemptive leader-follower equilibrium firms invest sequentially and option values are reduced by competition. A symmetric outcome may also occur, however, in which investment is more delayed than the single-firm counterpart. Comparing this with the optimal cooperative investment pattern, investment is found to be more delayed when firms act non-cooperatively, as each holds back from investing in the fear of starting a patent race. Implications of the analysis for empirical and policy issues in R&D are considered.

JEL classification numbers: C61, D81, L13, O32.

Keywords: real options, preemption, R&D, investment.

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Strategic Delay in a Real Options Model of R&D Competition

1 Introduction

When a firm has the opportunity to make an irreversible investment facing future uncertainty there is an option value of delay. By analogy with a financial call option it is optimal to delay exercising the option to invest, even when it would be profitable to do so at once, in the hope of gaining a higher payoff in the future. Using this insight the real options approach improves upon traditional NPV-based investment appraisal methods by allowing the value of delay and the importance of flexibility to be quantified and incorporated explicitly into the analysis.

Real world investment opportunities, unlike financial options, are rarely backed by legal contracts which guarantee the holder's rights in precise terms. Most real options are non-proprietary investment opportunities whose terms are somewhat vague or subjective, and far from guaranteed. In particular, a firm's ability to hold the option is frequently influenced by the possibility that another firm may exercise a related option, which affects the value of the first firm's investment. In a few instances a legal right such as an oil lease or a patent gives a firm a proprietary right similar to that granted by a financial option. Or occasionally a firm has such a strong market position, as in a natural monopoly or network industry, that its investment opportunities are *de facto* proprietary. However, in most industries some degree of competition exists, either actual or potential, and the option to invest cannot be held independently of strategic considerations.

When a small number of firms are in competition with an advantage to the first mover, each one's ability to delay is undermined by the fear of preemption. Consider a situation in which two firms have the ability to exercise an option and the first to do so obtains the underlying asset in its entirety, leaving the second mover empty-handed. Each firm would like to exercise the option just before its rival does so, giving rise to discontinuous Bertrand-style reaction functions. With symmetric firms the value of delay is eliminated and the option will be exercised as soon at the payoff from doing so becomes marginally positive. Under such circumstances the real options approach becomes irrelevant and the traditional NPV rule resurfaces as the appropriate method of investment appraisal.

In order to study in detail the tension between real options and strategic competition, the continuous time framework of Fudenberg and Tirole (1985) is adapted in two important respects to apply to the specific context of rival investment in R&D. The firms' profit functions are specified so as to include two distinct forms of uncertainty: *economic* uncertainty over the future profitability of the project, and *technological* uncertainty over the success of R&D investment itself. Economic uncertainty gives rise to option values and a tendency for delay, which would not arise in a deterministic framework. Technological uncertainty, combined with a winner-takes-all patent system, generates a preemption effect that counteracts the incentive to delay. The instantaneous probability of success, or hazard rate, of rival firms captures in a simple form the strength of the first-mover advantage, allowing outcomes for varying degrees of preemption to be readily compared. In effect, technological uncertainty drives a wedge between a firm's decision to invest and the out-turn of that investment, giving some scope for the follower to leapfrog the leader and preserving its option value to some extent. It should be noted that the advantage gained by the first mover is not necessarily a persistent one: if the breakthrough is not achieved before the follower invests, the two firms are equally likely to succeed from then on.

In fact, the hazard rate has two distinct effects in this model. The direct effect of the rival's hazard rate is to reduce the expected value of investment to the second mover, since there is some probability that the leader will make the discovery first. This effect is analogous to the impact of rival investment in product market duopoly models such as Smets (1991): with the option value of delay unchanged, the reduction in the value of investment causes the follower to act later. In this paper, however, there is also a second effect: the hazard rate of rival innovation reduces the option value itself, tending to hasten investment. Thus option values and preemption interact in this model. This contrasts with existing contributions in the area, where the roles of option values and competition are additive: in these models the only effect of rivalry is to reduce the value of investment, while the option value of delay remains unchanged.

Focusing on Markov perfect equilibria, the outcome of the non-cooperative two-player game takes one of two forms depending upon parameter values. The first is a preemptive leader-follower outcome in which one firm invests strictly earlier than the other and option values are undermined by competition. The second has a multiplicity of equilibria, including a continuum of symmetric equilibria in which both firms invest at the same trigger point. The Pareto-dominant equilibrium coincides with the optimal joint-investment rule which would be chosen by firms that agree to adopt a common trigger point. This outcome entails greater delay than the single-firm counterpart.

The role of the hazard rate in non-cooperative equilibrium can be understood as follows. Its impact in lowering the expected value of investment to the second-mover, relative to the firm that invests first, creates a first-mover advantage that will tend to induce preemptive action. However, when the first firm invests the value of its rival's option to delay is also reduced, speeding up the competitive reaction to its investment. Thus, preemption is double-edged: the leader gains a privileged position for a time, but the option value effect tends to speed up the reaction of its competitor. Anticipating this reaction, a firm may instead choose to delay its own investment. In effect, an investing firm chooses the time at which the patent race will begin and it is better for each firm if this is delayed until the optimal joint-investment point is reached. A good analogy is the behaviour of contestants in a long-distance race, who typically remain in a pack proceeding at a moderate pace for most of the distance, until near the end when someone attempts to break away and the sprint for the finish begins. Compared with existing duopoly models of real options the cooperative joint-investment outcome is achievable as a non-cooperative equilibrium.

The fully optimising cooperative investment rule is derived as a benchmark for comparison. This is shown to involve sequential investment of the two units, so that research efforts are phased in over time. Compared with the non-cooperative leader-follower equilibrium, the cooperative trigger points are higher than their non-cooperative counterparts since option values are no longer undermined by preemption. The non-cooperative joint-investment equilibrium, although preferable to the preemptive leader-follower outcome, is seen not to be the fully-optimising choice of cooperating firms. It may, however, be interpreted as the second-best optimum of firms that are constrained to choose a symmetric investment rule, given the difficulty of agreeing an asymmetric investment pattern or making side-payments to support the fully-optimising solution. It is interesting to note that when simultaneous investment is the equilibrium outcome, the time to first investment is increased by strategic interactions between non-cooperative firms, compared with the cooperative solution.

By combining irreversible investment under uncertainty with strategic interactions in the presence of technological uncertainty, the paper brings together three strands of economics literature. Real options models have been used to explain delay and hysteresis arising in a number of contexts, but these are mostly set in a monopolistic or perfectly competitive framework. McDonald and Siegel (1986) and Pindyck (1988) consider irreversible investment opportunities available to a single firm. Dixit (1989, 1991) considers product market entry and exit in, respectively, monopolistic and perfectly competitive settings. The second branch of related literature analyses timing games of entry and exit in a deterministic framework. Timing games are straightforward examples of stopping time games where the underlying process is simply time itself. Papers analysing preemption games include Fudenberg et al. (1983) and Fudenberg and Tirole (1985), while wars of attrition have been modelled by Ghemawat and Nalebuff (1985) and Fudenberg and Tirole (1986). Finally, technological uncertainty in R&D, with discovery modelled as a Poisson arrival, is considered in papers by, *inter alia*, Loury (1979), Dasgupta and Stiglitz (1980), Lee and Wilde (1980), Reinganum (1983) and Dixit (1988). These papers, however, assume the return to successful R&D (or demand in the product market from which it is derived) to be deterministic, thus ruling out any option value of delay and related timing issues.

Existing literature combining real options with strategic interactions is as yet relatively limited. Smets (1991; summarised in Dixit and Pindyck 1994, pp. 309-314), examines irreversible market entry for a duopoly facing stochastic demand. Non-cooperative behaviour results in an asymmetric leader-follower equilibrium. When the leadership role is exogenously pre-assigned so that the follower is unable to invest until after the designated leader has done so, the cooperative symmetric outcome may then be attained. Grenadier (1996) considers the strategic exercise of options applied to real estate markets. Joint investment arises only when the underlying stochastic process starts at a sufficiently high initial value and, even then, is not necessarily undertaken at the optimal point. In a two-player game where each player's exercise cost is private information, Lambrecht and Perraudin (1997) find trigger points located somewhere between the monopoly and simple NPV outcomes. In a two-period model, Kulatilaka and Perotti (1998) consider the value of strategic investment as the degree of uncertainty increases.

The paper is structured as follows. The model is described in section 2. The optimisation problem of a single firm facing no actual or potential competition is solved in section 3. Section 4 derives the optimal cooperative investment plan for two firms. Non-cooperative equilibrium in the two-player game is found in section 5. The findings are discussed in section 6; section 7 then concludes.

2 The model

Two risk-neutral firms, i = 1, 2, have the opportunity to invest in competing research projects. Research is directly competitive: the firms strive for the same patent and successful innovation by one eliminates all possible profit for the other. The firms face both technological and economic uncertainty. Discovery by an active firm is a Poisson arrival, while the value of the patent received by the successful inventor evolves stochastically over time.[1] The decision to invest in a research project is assumed to be irreversible. The possible states of firm i are denoted $q_i \in \{0,1\}$ for the idle and active states respectively.

The value of the patent, p, evolves exogenously and stochastically according to a geometric Brownian motion (GBM) with drift given by the following expression

$$dp_t = mp_t dt + s p_t dW \quad (1)$$

where $m \in [0, r)$ is the drift parameter measuring the expected growth rate of p,[2] r is the risk-free interest rate, assumed to be constant over time, $s > 0$ is the instantaneous standard deviation or volatility parameter, and dW is the increment of a standard Wiener process where $dW \sim N(0, dt)$.

Each firm has the opportunity to invest in a research project. Following Loury (1979), firm i sets up a research project by investing an amount $K_i > 0$.[3] From the time of this investment, discovery takes place randomly according to a Poisson distribution with constant hazard rate $h_i > 0$. Thus the hazard rate is independent of the duration of research and the number of firms investing; possible variations on this assumption are discussed in section 7. The probabilities of discovery by each firm are independent. We focus on the symmetric case where $h_i = h$ and $K_i = K$ for i = 1, 2. All parameter values and actions are common knowledge, thus the game is one of complete information.

The following assumptions are made

Assumption 1. $E_0\left(\int_0^\infty e^{-(r+h)t} hp_t \, dt\right) - K < 0$.

Assumption 2. $If\ q_i(t) = 1\ then\ q_i(t) = 1\ \forall t \geq t$.

Assumption 1 states that the initial value of the patent, p_0, is sufficiently low that the expected return from immediate investment is negative, ensuring that neither firm will invest at once. Assumption 2 formalises the irreversibility of investment and constrains the strategy of the firm accordingly: if firm i has already invested by date t then it remains active at all dates subsequent to t until the game ends with a discovery.

In a multi-agent setting the firm's investment problem can no longer be solved using the optimisation techniques typically employed in real options analysis. Instead, the optimal control problem becomes a stopping time game (for a detailed analysis see Dutta and Rustichini (1991)). In a stopping time game each player has an irreversible action such that, following this action by one or more players, expected payoffs in the subsequent subgame are fixed. Dutta and Rustichini allow for the possibility that the stochastic process continues to evolve after the leader's action and that the follower still has a move to make, as is the case in this paper. The stopping time game is described by the stochastic process p along with the payoff functions for the leader and follower; these are derived in section 5 below.

The game proceeds as follows. In the absence of action taken by either firm, the stochastic process evolves according to (1). If firm i has not commenced research at any time $t < t$ its action set is $A_t^i = \{\text{invest, don't invest}\}$. If, on the other hand, i has invested at some $t < t$, then A_t^i is the null action set $\{\text{don't move}\}$. Thus each firm faces a control problem in which its only choice is when to choose the action 'stop' – or rather, in this case, to start research. After taking this action the firm can make no further moves to influence the outcome of the game. The game ends when a discovery is made by either firm.

A strategy for firm i is a mapping from the history of the game H_t to the action set A_t^i as follows: $s_t^i : H_t \to A_t^i$. At time $t \geq 0$, the history of the game has two components, the sample path of the stochastic state variable p and the actions of the two firms up to date t. With irreversible investment the history of actions in the game at t is summarised by the fact that the game is still continuing at t (i.e. $q_i = 0 \quad \forall i$). However, the history of the state variable is more complex since its current value could have been reached by any one of a huge number of possible paths.

Firms are assumed to employ stationary Markovian strategies: actions are functions of the current state alone and the strategy formulation itself does not vary with time. Since the state variable p follows a Markov process, Markovian strategies incorporate all payoff-relevant factors in the game. Furthermore, if one player uses a Markovian strategy then its rival has a best response that is Markovian as well. Hence a Markovian equilibrium remains an equilibrium when history-dependent strategies are also permitted, although other non-Markovian equilibria may then also exist. For further explanation see Maskin and Tirole (1988) and Fudenberg and Tirole (1991, chapter 13). With the Markovian restriction a player's strategy is a stopping rule specifying a critical value or "trigger point" for the stochastic variable p at which the firm invests.[4]

As Fudenberg and Tirole (1985) point out, the use of continuous time complicates the formulation of strategies as there is a loss of information inherent in taking the limit of a discrete time mixed strategy equilibrium. To deal with this problem they extend the strategy space to include not only the cumulative probability that a player has adopted, but also the "intensity" with which a player adopts "just after" the cumulative probability has jumped to one. Although this formulation uses symmetric mixed strategies, equilibrium outcomes are equivalent to those in which firms employ pure strategies and may adopt asymmetric roles.[5] Thus, although the underlying framework is an extended space with symmetric mixed strategies, the analysis will proceed as if each firm uses a (possibly asymmetric) pure Markov strategy.

3 Optimal investment timing for a single firm

We start by deriving the optimal stopping time for a single firm investing in the absence of competition. The firm's investment rule is found by solving the stochastic optimal stopping problem

$$V(p_t) = \max_T E_t \left\{ e^{-rT} \left(\int_T^{\infty} e^{-(r+h)t} hp_t \, dt - K \right) \right\} \tag{2}$$

where E_t denotes expectations conditional on information available at time t and T is the unknown future stopping time at which the investment is made. The value function $V(p)$ has two distinct components which hold over different intervals of p. Let $V_0(p)$ denote the value function before the firm invests, and $V_1(p)$ denote the value function after investment has taken place.

Prior to investment the firm holds the option to invest. It has no cashflows but may experience a capital gain or loss on the value of this option. Hence, in the continuation region (values of p for which it is not yet optimal to invest) the Bellman equation for the value of the investment opportunity is given by

$$rV_0 dt = E(dV_0). \tag{3}$$

Using Itô's lemma and the GBM equation (1) yields the ordinary differential equation

$$\frac{1}{2} s^2 p^2 V_0''(p) + mpV_0'(p) - rV_0 = 0. \tag{4}$$

From (1) it can be seen that if p ever goes to zero it stays there forever. Therefore the option to invest has no value when $p = 0$ and $V_0(p)$ must satisfy the boundary condition

$V_0(0) = 0$. Solving the differential equation (4) subject this boundary condition gives the value of the option to invest in research

$$V_0(p) = B_0 p^{b_0} \tag{5}$$

where $B_0 \geq 0$ is an unknown constant and b_0 is the positive root of the characteristic equation $e^2 - \left(1 - \frac{2m}{s^2}\right)e - \frac{2r}{s^2} = 0$, i.e. $b_0 = \frac{1}{2}\left\{1 - \frac{2m}{s^2} + \sqrt{\left(1 - \frac{2m}{s^2}\right)^2 + \frac{8r}{s^2}}\right\} > 1$.

We next consider the value of the firm in the stopping region (values of p for which is it optimal to invest at once). Since investment is irreversible the value of the firm in the stopping region, $V_1(p)$, is given by the project expected value alone with no option value terms. Recalling that discovery is a Poisson arrival, the expected value of the active project when the current value of the stochastic process is p_t is given by

$$V_1(p_t) = E_t\left(\int_t^\infty e^{-(r+h)t} hp_t \; dt\right). \tag{6}$$

Recalling that p is expected to grow at rate m and suppressing time subscripts we can write

$$V_1(p) = \frac{hp}{r + h - m}. \tag{7}$$

Note that the hazard rate h enters the denominator in this expression in the form of an 'augmented discount rate' $r + h$. This result is typical of models involving a Poisson arrival function: for other examples of this characteristic in the context of R&D see, *inter alia*, Loury (1979), Dasgupta and Stiglitz (1980), Lee and Wilde (1980) and Dixit (1988).

The optimal investment rule is found by solving for the boundary between the continuation and stopping regions. This boundary is given by a critical value of the stochastic process, or trigger point, p_U such that continued delay is optimal for $p < p_U$ and immediate investment is optimal for $p \geq p_U$. The optimal stopping time T_U is then defined as being the first time that the stochastic process p hits the interval $[p_U, \infty)$. By arbitrage, the critical value must satisfy the value-matching condition

$$V_0(p_U) = V_1(p_U) - K. \tag{8}$$

Optimality requires a second condition known as smooth-pasting to be satisfied. This condition requires the value functions $V_0(p)$ and $V_1(p)$ to meet smoothly at p_U with equal first derivatives[6]

$$V_0'(p_U) = V_1'(p_U). \tag{9}$$

Conditions (8) and (9) together imply that

$$p_U = \frac{b_0}{(b_0 - 1)} \frac{(r + h - m)}{h} K; \tag{10}$$

and

$$B_0 = \frac{h p_U^{1-b_0}}{(r + h - m) b_0}. \tag{11}$$

The optimal investment time at which the single firm invests is thus defined as

$$T_U = \inf \{t \geq 0 : p \geq p_U\}. \tag{12}$$

Briefly considering the properties of the trigger point p_U, as economic uncertainty is eliminated (i.e. as $s \rightarrow 0$), $b_0 \rightarrow r/m$ and the optimal stopping point approaches the breakeven value of the patent calculated on a simple NPV basis. As uncertainty rises b_0 falls towards unity, raising p_U and increasing the expected stopping time. Thus greater uncertainty over patent value delays investment, as expected from the papers by McDonald and Siegel (1986), Pindyck (1988) and Dixit (1989).

4 The cooperative benchmark

We next consider the benchmark case in which the two firms (or research units) plan their investments cooperatively.[7] The cooperative investment pattern may (in theory at least) take one of two possible forms: either both units invest at a single trigger point, or they invest sequentially at distinct trigger points. We start by deriving the optimal joint-investment rule when firms invest at the same trigger point, which follows straightforwardly from the analysis of section 3. The optimal sequential investment plan is then derived and compared with the optimal joint-investment strategy in order to determine which investment pattern forms the cooperative optimum.

The analysis of the preceding section can be readily extended to the case of two cooperating firms (or research units under common ownership) which agree to adopt a common investment rule. The decision is equivalent to a single firm optimisation problem with an investment cost of $2K$ and arrival rate $2h$. Denoting the optimal joint-investment trigger point by p_C, value-matching and smooth-pasting conditions are used as before to yield

$$p_C = \frac{b_0}{(b_0 - 1)} \frac{(r + 2h - m)}{h} K . \tag{13}$$

The optimal joint investment time T_C is analogous to expression (12). As before, the value of a (single) firm under this scenario has two parts. Prior to investment the firm holds the option to invest; after (joint) investment the value of the active project is given by its expected NPV to the firm, taking account of the fact that the other firm is also active, which is

$$NPV(p) = \frac{hp}{r + 2h - m} . \tag{14}$$

Thus, the value of an individual firm under this scenario is described by the following value function (i.e. the combined firm consisting of two research units has twice this value)

$$V_C(p) = \begin{cases} B_C p^{b_0} & \text{for } p < p_C \\ \\ NPV(p) - K & \text{for } p \geq p_C \end{cases} \tag{15}$$

where $B_C = \dfrac{hp_C^{1-b_0}}{(r+2h-m)b_0}$.

Comparing the cooperative trigger point (13) with (10) for the single firm, it can readily be seen that $p_C > p_U$. Given that the initial value p_0 is sufficiently low that immediate investment is unprofitable, the ranking of trigger points entails that investment takes place strictly later when two firms agree a common investment rule than when a single firm acts alone. Note that this result is due to the indirect effect of the hazard rate on the implicit discount rate faced by the firm after it invests, which is now $r+2h$ rather than $r+h$. Since both the cost and hazard rate of research are doubled, there is no direct effect on the efficiency of R&D.

We now characterise the optimal sequential investment plan, on the assumption (for now) that investment takes this form. Suppose that one unit invests at a trigger point p_1 and the other when a second trigger $p_2 > p_1$ is reached. The value of the combined entity under this investment plan is described by

$$V_{L+F}(p) = \begin{cases} A_0 p^{b_0} & \text{for } p < p_1 \\ \\ \dfrac{hp}{r+h-m} + A_1 p^{b_1} - K & \text{for } p \in [p_1, p_2) \\ \\ 2NPV(p) - 2K & \text{for } p \geq p_2 \end{cases} \tag{16}$$

where $b_1 = \dfrac{1}{2}\left\{1 - \dfrac{2m}{s^2} + \sqrt{\left(1 - \dfrac{2m}{s^2}\right)^2 + \dfrac{8(r+h)}{s^2}}\right\} > b_0$ for $h > 0$.

The optimal choice of p_1 and p_2, along with the option value terms A_0 and A_1, is determined by imposing value-matching and smooth-pasting conditions between the relevant components of the value function at each point. (By contrast, as will be seen in the next section, no smooth-pasting obtains at the leader's investment trigger in the non-cooperative case.) Solving value-matching and smooth-pasting conditions at p_2 yields

$$p_2 = \frac{b_1}{(b_1 - 1)} \frac{(r + 2h - m)}{h} \frac{(r + h - m)}{(r - m)} K \tag{17}$$

and

$$A_1 = p_2^{-b_1} \frac{K}{(b_1 - 1)} > 0. \tag{18}$$

Imposing value-matching and smooth-pasting conditions at p_1 and substituting the above expression for A_1 yields the following implicit expression for p_1

$$(b_0 - 1) \frac{h p_1}{(r + h - m)} - \frac{(b_1 - b_0)}{(b_1 - 1)} K \left(\frac{p_1}{p_2} \right)^{b_1} - b_0 K = 0. \tag{19}$$

Lemma 1 completes the proof that the optimal sequential investment plan (p_1, p_2) is uniquely defined.

Lemma 1. Equation (19) has a unique root p_1 in the interval $(0, p_2)$.

Proof. See appendix.

Which of the two investment patterns, sequential or simultaneous, is optimal is determined by comparing V_{L+F} with $2V_C$ (the combined value of the two firms when both invest at the optimal joint investment point). Note that, in each case, prior to the point at which both firms have invested the value functions are strictly convex with continuous first derivatives,[8] converging to zero as $p \to 0$ and smooth-pasting to the linear function $2NPV(p)$ at some point as p becomes large. Thus, the value functions cannot cross in the relevant range and the ranking of the value functions is given by the relative magnitudes of p_C and p_2. Lemma 2 proves that $p_2 > p_C$, thus demonstrating that staggered investment at (p_1, p_2) dominates joint investment.

Lemma 2. $p_2 > p_C$ for $h > 0$.

Proof. See appendix.

Proposition 1 follows directly from the preceding analysis.

Proposition 1. *The cooperative optimum is uniquely defined as a sequential investment pattern in which one research unit invests at* p_1 *and the other at* p_2*, where these trigger points satisfy (19) and (17) respectively.*

Next we compare the trigger points in the optimal cooperative investment pattern with the optimal joint-investment trigger p_C.

Lemma 3. $p_C > p_1$ for $h > 0$.

Proof. See appendix.

Proposition 2. *The ranking of trigger points in the optimal cooperative investment plan relative to the optimal joint-investment trigger point is given by* $p_1 < p_C < p_2$.

Proof. Follows directly from lemmas 2 and 3.

Thus, we have demonstrated that two cooperating firms which jointly optimise their investments would choose to phase their R&D investments progressively over time, rather than invest both research units at once. This is despite the fact that the cost function for research displays constant returns to scale, albeit with a given minimum size of a research unit. The sequential investment pattern gives the possibility of some return even when the value of innovation is fairly low (though NPV-positive), reducing the opportunity cost of delay while holding back from committing all R&D costs at once and retaining an option to increase the scale of investment in the future. The phasing of investment gives the cooperating firms a higher probability of gaining a high-valued patent, and the overall value of the (combined) investment opportunity is thereby maximised.

5 Non-cooperative equilibrium

We turn now to the non-cooperative two-player game. We start by assuming, without loss of generality, that one firm (the leader) invests strictly before its rival (the follower). As usual in dynamic contexts the stopping time game is solved backwards; thus we start by considering the optimisation problem of the follower.

5.1 The follower's investment problem

Given that the leader has already invested and this investment is irreversible, the follower faces a conditional probability hdt that its rival will make the breakthrough in a (short) time interval dt. Moreover, this probability is independent of whether the follower itself has or has not invested. Thus the follower's investment problem is equivalent to that of a single firm with the augmented discount rate $r+h$. This decision problem can be solved using the method described in section 3, simply replacing r by $r+h$ throughout, to yield the follower's trigger point

$$p_F = \frac{b_1}{(b_1 - 1)} \frac{(r + 2h - m)}{h} K \tag{20}$$

where b_1 is as defined following expression (16). The follower's value function is described by

$$V_F(p) = \begin{cases} B_F p^{b_1} & \text{for } p < p_F \\ NPV(p) - K & \text{for } p \geq p_F \end{cases} \tag{21}$$

where $B_F = \dfrac{h p_F^{1-b_1}}{(r + 2h - m) b_1} > 0.$

Denoting the leader's investment time by T_L (this being the first time that the leader's trigger point p_L is reached, to be defined in section 5.3 below), the follower's optimal investment time can be written as

$$T_F = \inf\left\{t \geq T_L : p \geq p_F\right\}. \tag{22}$$

Note that p_F is independent of the point at which the leader invests: given that the firm invests second, the precise location of the leader's trigger point is irrelevant. Comparing p_F with the trigger points derived in sections 3 and 4, it can readily be seen that $p_F < p_C$. However p_F and p_U cannot be ranked in general since, as discussed in the introduction, the leader's hazard rate has two conflicting effects on the follower. The direct effect of the leader's research activity is to reduce the expected value of investment to the follower, which is now given by $hp/(r+2h-m)$ as opposed to $hp/(r+h-m)$ in the single-firm case.[9] However, there is also a second effect via the option value mark-up factor, which is now given by $b_1/(b_1-1)$ rather than $b_0/(b_0-1)$ for the single firm. As explained in the introduction, the hazard rate of rival innovation reduces the follower's option value of delay. This can be seen clearly from the impact of h on the mark-up factor, which is reduced by its presence. This indirect effect tends to speed up the competitive reaction to the leader's investment, mitigating its preemptive advantage.

5.2 The leader's payoff

We now derive the payoff to a firm that invests as the leader, given that the follower acts optimally in the future in accordance with the stopping rule derived above. After the leader has sunk the investment cost K it has no further decision to make and its payoff is given by the expected value of its research project. However, this payoff is affected by the subsequent action of the rival firm investing at p_F. Taking account of investment by the follower, the leader's post-investment payoff is given by

$$V_L(p_t) = E_t\left(\int_t^{T_F} e^{-(r+h)t} hp_t\, dt + \int_{T_F}^{\infty} e^{-(r+2h)t} hp_t\, dt\right). \tag{23}$$

Two separate value functions must be considered for the leader: its value before the follower invests, denoted $V_{L(1)}(p)$, and its value after this investment takes place, $V_{L(2)}(p)$. Subsequent to investment by the follower the leader's (as well as the follower's) value is given by the expected value of the active research project taking account of the probability of rival discovery, which is simply $NPV(p)$ given by (14) above. Prior to investment by the follower the leader's value function consists of two components: the expected flow payoff from research and an option-like term that anticipates subsequent investment by the follower. Solving the Bellman equation for the leader's value over this interval, noting that as the value of the patent approaches zero the follower's option to invest becomes worthless and the follower will never enter the race, the following function is derived

$$V_{L(1)}(p) = \frac{hp}{r+h-m} - B_L p^{b_1} \tag{24}$$

where $B_L > 0$ is an unknown constant and $b_1 > 1$ is as previously defined.

The value of the unknown constant B_L is found by considering the impact of the follower's investment on the payoff to the leader. When p_F is first reached the follower invests and the leader's expected flow payoff is reduced, since there is now a positive probability that its rival will make the discovery instead. The first section of the leader's value function anticipates the effect of the follower's action with a value-matching condition holding at p_F (for further explanation see Harrison (1985)). However, since there is no optimality on the part of the leader there is no corresponding smooth-pasting condition in this case. This yields the following value function for a firm investing as the leader (which also takes account of the sunk cost *K* incurred when the firm invests)

$$V_L(p) = \begin{cases} \dfrac{hp}{r+h-m} - B_L p^{b_1} - K & \text{for } p < p_F \\ NPV(p) - K & \text{for } p \geq p_F \end{cases} \tag{25}$$

where $B_L = \dfrac{h^2 p_F^{1-b_1}}{(r+h-m)(r+2h-m)} > 0.$

5.3 Solving the game

Without the ability to precommit to trigger points at the start of the game (in contrast with the precommitment strategies used by Reinganum (1981), for example) the leader's stopping point p_L cannot be derived as the solution to a single-agent optimisation problem. Whether a firm becomes a leader, and the trigger point at which it invests if it does so, is determined by the firm's incentive to preempt its rival and the point at which it is necessary to do so to prevent itself from being preempted.

As in Fudenberg and Tirole (1985), the form of the non-cooperative equilibrium depends on the relative magnitudes of the leader's value, V_L, and the value when both delay until the optimal joint-investment point, V_C. Depending upon whether or not the functions intersect somewhere in the interval $(0, p_F)$, two investment patterns arise. [Since $V_L(p) = NPV(p) - K$ for $p \geq p_F$ while $V_C(p) > NPV(p) - K$ for $p < p_C$, the functions cannot intersect anywhere in the interval $[p_F, p_C)$.] If V_L *ever* exceeds V_C preemption incentives are too strong for a joint-investment equilibrium to be sustained and the only possible outcome is a leader-follower equilibrium in which one firm invests strictly earlier than its rival and both invest strictly prior to the optimal joint-investment time. If, on the other hand, V_L *never* exceeds V_C a joint-investment outcome may be sustained, although the leader-follower outcome is also an equilibrium in this case.

At the leader's investment point, p_L, the expected payoffs of the two firms must be equal. The reason for this follows Fudenberg and Tirole's rent equalisation principle: if this were not the case, one firm would have an incentive to deviate and the proposed outcome could not be an equilibrium. By investing earlier than its rival the leader gains the advantage of a temporary monopoly in research and has a greater likelihood of making the discovery. However the value of the prize it stands to win is likely to be lower than for the follower. Hence, when viewed from the start of the game, there is a trade-off between the probability of being first to make the discovery and the likely value of the prize that is gained. At p_L

the two effects are in balance and the firms' expected payoffs are equal. Thus, in contrast with many other games where asymmetric equilibria arise (such as Reinganum (1981)), the agents in this model are indifferent between the two roles.

Before formally describing the equilibria we must first define, and demonstrate the existence of, the leader's trigger point, p_L. From the rent equalisation principle described above, it follows directly that $V_L(p_L) = V_F(p_L)$. Using this equality, an implicit expression for p_L can be derived; this is given by expression (A4.1) in the appendix evaluated at zero. Thus it is necessary to prove the existence of a root of this expression other than, and strictly below, p_F.

Lemma 4. There exists a unique point $p_L \in (0, p_F)$ such that

$$V_L(p) < V_F(p) \text{ for } p < p_L$$
$$V_L(p) = V_F(p) \text{ for } p = p_L$$
$$V_L(p) > V_F(p) \text{ for } p \in (p_L, p_F)$$
$$V_L(p) = V_F(p) \text{ for } p \geq p_F.$$

Proof. See appendix.

The stopping time of the leader can thus be written as

$$T_L = \inf\{t \geq 0 : p \in [p_L, p_F]\}. \tag{26}$$

Proposition 3. (Case 1.) *If* $\exists p \in (0, p_F)$ *such that* $V_L(p) > V_C(p)$, *then there exist two asymmetric leader-follower equilibria differing only in the identities of the two firms. In one equilibrium firm 1 (the leader) invests when* p_L *is first reached with firm 2 (the follower) investing strictly later at* $p_F > p_L$*; in the other equilibrium the firms' identities are reversed.*

Proof. The proof is illustrated with reference to figure 1. As p rises from its low initial value, we know from the premise that a point (labelled *A*) will eventually be reached where V_L first exceeds V_C. At this point each firm has a unilateral incentive to deviate from the continuation strategy to become the leader. However, if one firm were to succeed in preempting its rival at A the payoff to the leader would be strictly greater than that of the follower, since $V_L > V_F$ at this point. From Lemma 4 we know that the leader's payoff is strictly greater than that of the follower everywhere in the interval (p_L, p_F). Thus preemption incentives rule out any putative trigger point in this range. We know also that $V_L < V_F$ for all $p < p_L$; thus before p_L is reached each firm prefers to let its rival take the lead. We know from Lemma 4 that p_L is unique. Once the leader has invested the follower faces a single-agent optimisation problem, the solution to which was derived in section 5.1. Thus, there exists a unique equilibrium configuration in which one firm (the leader) invests when p_L is first reached and the other (the follower) invests strictly later at p_F. Since the firms' identities are interchangeable there are two equilibria of this type. Q.E.D.

We next consider the alternative case where V_C always exceeds V_L and a joint investment equilibrium is sustainable. At p_C it is a dominant strategy to invest even though the rival will follow at once, thus there can be no equilibrium trigger point above p_C. Before describing the set of joint-investment equilibria we must first define p_S, the lowest joint-investment point such that there is no unilateral incentive to deviate. Note that the critical value p_S does not necessarily exist; this depends upon the relative positions of V_L and V_C.

$$p_S = \inf \left\{ p_J \in (0, p_C] : V_J(p; p_J) \geq V_L(p) \ \forall p \in (0, p_J] \right\} \tag{27}$$

where $V_J(p; p_J)$ is the firm's pre-investment value function when both invest jointly (but not necessarily optimally) at an arbitrary point p_J. This function, derived from the value-matching condition at p_J, is given by

$$V_J(p; p_J) = B_J p^{b_0} \tag{28}$$

where $B_J = p_J^{-b_0}\left(\frac{hp_J}{r+2h-m} - K\right)$; this function is defined over the range $(0, p_J]$.

Lemma 5.

(a) p_S exists and is unique whenever $V_C(p) \geq V_L(p) \;\; \forall p \in (0, p_C)$.

(b) $[p_S, p_C]$ forms a connected set such that $V_J(p; p_J) \geq V_L(p) \;\; \forall p \in (0, p_J)$, $p_J \in [p_S, p_C]$.

Proof. See appendix.

Proposition 4. (Case 2). *If* $V_C(p) \geq V_L(p) \;\; \forall p \in (0, p_C)$, *two types of equilibria exist. The first is the leader-follower equilibrium described in Proposition 3; two equilibria of this type exist as before. The second is a joint-investment equilibrium in which both firms invest at the same trigger point* $p_J \in [p_S, p_C]$; *there is a continuum of equilibrium trigger points over this interval.*

Proof. The proof is illustrated with reference to figure 2. As before, fear of preemption by one's rival in the interval (p_L, p_F) over which $V_L > V_F$ entails that the asymmetric leader-follower outcome is also an equilibrium configuration in this case. From the premise, however, there is no *unilateral* incentive to deviate from the continuation strategy anywhere in the interval $(0, p_C)$. For $p \geq p_C$ it is a dominant strategy to invest, despite the knowledge that the rival will follow at once. Thus the joint-investment outcome in which both firms invest at p_C is also an equilibrium. From lemma 5 any joint-investment point $p_J \in [p_S, p_C]$ has the property that no unilateral deviation is profitable and is therefore an equilibrium. Q.E.D.

Fudenberg and Tirole (1985) argue that if one equilibrium Pareto-dominates all others it is the most reasonable outcome to expect. Using the Pareto criterion the multiplicity of equilibria described in Proposition 4 can be reduced to a unique outcome.

Proposition 5. *Using the Pareto criterion, the multiplicity of equilibria arising in case 2 can be reduced to a unique outcome. This is the Pareto-optimal joint-investment equilibrium in which both firms invest when p_C is first reached.*

Proof. The proof consists of two parts.

(i) All joint-investment equilibria, if these exist, Pareto-dominate the asymmetric leader-follower equilibrium. From the definition of p_S any joint-investment trigger point $p_J \in [p_S, p_C]$ has the property that no unilateral deviation is profitable; thus $V_J(p) \geq V_L(p)\ \forall p \in (0, p_J]$. Thus, the value of continuation is at least as great as the amount that a firm would gain from preemption at any point. Furthermore, in the leader-follower equilibrium the payoffs of both firms are strictly lower than the maximum amount obtainable, since the optimal preemption strategy is not an equilibrium of the non-cooperative game.

(ii) The joint-investment equilibria are Pareto-ranked by their respective trigger points, with trigger points closer to p_C Pareto-dominating all lower ones. This follows directly from the derivation of p_C. Q.E.D.

The asymmetric equilibria arising in case 2 are situations where the leader preempts purely because of the fear that its rival will do so first. Such instances of 'attack as a means of defence' are somewhat irrational, as both firms achieve higher payoffs by coordinating on any one of the symmetric equilibria. The Pareto-dominant equilibrium, by contrast, preserves option values and entails that investment is more delayed than in the single-firm counterpart.

Comparing non-cooperative trigger points with those comprising the cooperative solution, derived in the previous section, the following comparisons can be drawn. It is already known from Proposition 2 that $p_2 > p_C > p_1$ for $h > 0$. A comparison of (13) and (20) shows that $p_C > p_F$, while lemma 4 yields $p_F > p_L$. Lemma 6 compares first investment points in the non-cooperative and cooperative solutions, p_L and p_1.

Lemma 6. $p_L < p_1$ for $h > 0$.

Proof. See appendix.

These comparisons are summarised in Proposition 6.

Proposition 6. Trigger points in the various cases are ranked as follows

(a) $p_L < p_1 < p_C < p_2$;

(b) $p_L < p_F < p_C < p_2$;

(c) the ranking of p_1 and p_F is ambiguous.

Whether equilibrium follows case 1, resulting in a preemptive leader-follower outcome with investment at p_L and p_F respectively, or case 2, with simultaneous investment at p_C, depends on parameter values. This can be determined numerically as follows. The question of whether a firm has an incentive to deviate from joint investment is identical to that of whether a designated leader (which, unlike the firms in this model, can choose its investment point optimally in the knowledge that its rival cannot invest until after it has done so) would *choose* to adopt the leadership role. The investment point of the designated leader, and the option value of its investment, is defined by value-matching and smooth-pasting conditions between the first component of the leader's value function $V_{L(1)}$ and the pre-investment option value $B_D p^{b_0}$. This problem has no closed-form solution; implicit expressions are presented in the appendix.

Once a value for B_D has been obtained, the equilibrium investment pattern can be determined by comparing this with B_C (defined following expression (15) above). If $B_D > B_C$ the value of becoming the leader at some point exceeds that of optimal joint investment and the only possible outcome is a preemptive leader-follower equilibrium. On the other hand, if $B_C \geq B_D$ the leader's value V_L does not exceed V_C at any point in the relevant interval and joint investment at p_C is the Pareto dominant equilibrium. Although this condition cannot be written down in an explicit form, numerical solutions can be found for any set of parameter values; some results are discussed in section 6.

6 Discussion

Comparing cooperative and non-cooperative outcomes, the inefficiency of non-cooperative behaviour can readily be seen. When non-cooperative behaviour results in a leader-follower equilibrium, preemption and business-stealing incentives prevent the option to invest from being held for long and both firms invest too soon. Although the leader gains the first-mover advantage of a temporary monopoly in research, this is subsequently undermined by the follower's investment. The firms' payoffs are equal, and low compared with the other outcomes.

The alternative joint-investment equilibrium, if achievable, is more favourable for both firms. It is identical to the outcome that would be seen if the firms agreed to adopt a common investment rule and chose this optimally. Although it is not the cooperative optimum – as section 4 has shown, simultaneous investment is dominated by the optimal sequential investment pattern – it could be seen as the best achievable cartel given the difficulty in agreeing asymmetric investment rules and the need for side-payments implicit in the cooperative outcome.

Interestingly, when equilibrium involves simultaneous investment the effect of competition is to *increase* the time to first investment: the non-cooperative trigger p_C exceeds the lower trigger in the cooperative plan, p_1. Investment occurs too late in this case due to the strategic behaviour of the firms who delay their investment in the fear of setting off a patent race. Hence, in this case, delay is due to strategic interactions between firms, not just the usual option effect of uncertainty. Investment is also more delayed than in the single firm counterpart. When investment does occur, however, a burst of research activity is seen which is then excessive – under the cooperative plan the second investment would be delayed until a later date.

The type of equilibrium that emerges in any particular case depends on the balance between two opposing forces, the option value of delay and the expected benefit of preemption. The simultaneous investment equilibrium becomes more prevalent as the option value of delay is increased or the preemptive effect of earlier investment is reduced. Numerical analysis indicates that simultaneous investment becomes more likely as, *ceteris paribus*, volatility s rises, the hazard rate h falls,[10] or the pure discount rate r increases.[11] (As with financial options, an increase in pure discounting reduces the current value of the investment cost, or strike price, paid at some date in the future, raising option values.)

Limiting results as h becomes insignificant or very large are informative. As h tends to zero all trigger points (expressed as an expected flow return, hp) converge to the same value. This is intuitively obvious: as the business-stealing effect of h becomes negligible, the investment opportunities available to the firms approximate stand-alone options unaffected by competition. As h becomes large on the other hand, the following results are found: p_U, $p_1 \to b_0/(b_0-1)K$; $p_C \to b_0/(b_0-1)2K$; $p_L \to K$; $p_F \to 2K$ and $p_2 \to \infty$. Again, the results are fairly intuitive: p_U and p_C are the standard trigger points when the return to investment is gained immediately, for investments of scale K and $2K$ respectively. In the non-cooperative leader-follower equilibrium, extreme preemption entirely removes the option to delay and firms invest at the simple NPV breakeven points taking account of their respective roles. In the cooperative solution, the first unit invests at the optimal stand-alone trigger point and the second unit is redundant and never invests.

These findings have a number of implications for the understanding and assessment of empirical investment behaviour. Since strategic interactions, in addition to uncertainty, have significant effects on the timing and pattern of investment, empirical studies of investment may be improved by including measures of industry concentration and strategic advantages as explanatory variables. If preemption effects are strong competition tends to speed up investment, which then takes place sequentially as firms avoid competing head-to-head. Greater volatility, on the other hand, increases the likelihood that a patent race will occur, with a sudden burst of competitive activity ending a prolonged period of stagnation – a phenomenon similar to that described by Choi (1991) but arising for different reasons.

Some welfare implications can also be drawn. Although a full welfare assessment requires a value function for consumers to be specified so that the social optimum can be determined, implications can be drawn straightforwardly from the existing analysis for one simple case. If the consumer surplus arising from the innovation remains in fixed proportion to p as this varies over time (i.e. the patent-holder extracts the same proportion of the social surplus of the innovation at all times),[12] the social optimum coincides with the cooperative solution.[13] In this case the social planner would phase investment progressively over time, choosing the same trigger points as the cooperating firms.

Although patent races are not socially optimal, they may nonetheless be preferable to the alternative non-cooperative equilibrium. Assuming that the welfare optimum is aligned with the cooperative solution as described above, a patent race commencing at the optimal joint-investment time is preferable to the preemptive leader-follower outcome in which both firms invest too soon and valuable options for the future are destroyed. Only if for some reason early investment has significant external benefits for consumers – and the mere existence of consumer surplus is not sufficient for this – would the social planner prefer the preemptive equilibrium.

Turning next to policy issues, the analysis has implications for the assessment of R&D joint ventures. It provides a possible further justification for adopting a liberal approach to cooperative R&D, in addition to the existing arguments concerning the use of complementary skills, spillover effects, the scale and riskiness of R&D investments. Again, on the broad assumption that the option to delay is socially as well as privately beneficial, the creation of an R&D joint venture with the freedom to choose the timing and scale of R&D investment cooperatively is strongly supported by this analysis. Of course, this and other benefits of cooperation must be balanced against its possible detriments, especially the weakening of efficiency incentives and the extension of cooperation to product market collusion.

It is interesting to note that in the case where a joint venture would be the most desirable, namely that in which a preemptive leader-follower equilibrium would otherwise occur, the joint venture would choose to *delay* R&D investment. This is in stark contrast with the usual policy approach whereby firms are required to demonstrate that the joint venture will invest in projects that would *not* otherwise be undertaken (at the present time). A significant change in approach on the part of competition authorities might be required to take account of this point! When non-cooperative equilibrium takes the simultaneous investment form, however, no such conflict arises: the joint venture will undertake the first investment earlier than would otherwise be the case, and further investment will be phased in at a later date as and when this becomes optimal.

7 Concluding remarks

This paper has shown that, in contrast to initial expectations, competition between firms does not necessarily undermine the option to delay. Instead, the fear of sparking a patent race may internalise the effect of competition, further raising the value of delay. When firms invest simultaneously in equilibrium, investment occurs later than when the firms plan their investments cooperatively. When this point is reached, however, a patent race ensues as the firms compete to achieve the breakthrough.

The paper has implications for empirical and policy issues. In situations where both option values and strategic interactions are important it is necessary to give careful consideration to precise industry conditions, particularly the degree of uncertainty and strength of preemption, in order to predict and assess the pattern of investment. The analysis suggests that empirical studies of the impact of uncertainty on investment should also include industry concentration and first-mover advantages as explanatory variables in their models. On the policy side, the paper provides a possible additional justification for adopting a permissive view of cooperative R&D joint ventures.

The results are robust to changes in the precise structure of the model. Although geometric Brownian motion is a convenient and tractable form, alternative stochastic processes, such as ones exhibiting mean-reversion or intermittent jumps, would generate similar qualitative results. More sophisticated research technologies could also be considered. For example, the hazard rate may increase with cumulative R&D spending as a result of learning-by-doing. Note that in this case the leader has a permanent rather than a temporary advantage, strengthening preemption incentives. Alternatively, if the probability of discovery is not known *a priori* and the hazard rate is thus an expectation, updating from fruitless research experience will cause this to fall over time.

The model could be extended in a number of ways. This paper has focused on the symmetric two-firm case. If the firms' research technologies are instead allowed to differ such that one is more efficient, the identities of the leader and follower will be uniquely defined and the more efficient firm will receive a strictly greater expected payoff. An increase in the number of firms, however, is more problematic. As explained by Fudenberg & Tirole (1985, section 5), rent equalisation holds only in the two-firm case; with three or more symmetric firms equilibrium behaviour is more complicated and asymmetric payoffs are possible.

The impact of rival investment in research may also have more complicated effects than those considered in this model. Congestion effects, such as a shortage of skilled workers, may reduce the efficiency of research as more firms invest, raising the advantage of earlier investment. Informational spillovers between firms, on the other hand, would cause a firm's hazard rate to rise when its rival invests. This generates an additional motive for delay, as a firm gains by free-riding on the research efforts of its rival.

Appendix

Lemma 1. Equation (19) has a unique root p_1 in the interval $(0, p_2)$.

Proof. We write

$$Y(p) = (b_0 - 1)\frac{hp}{(r+h-m)} - \frac{(b_1 - b_0)}{(b_1 - 1)} K \left(\frac{p}{p_2}\right)^{b_1} - b_0 K \,. \qquad \text{(A1.1)}$$

p_1 is the root of this function which lies in the interval $(0, p_2)$. For existence and uniqueness of such a root, it is sufficient to show that the continuous function $Y(p)$ has the following properties

(i) $Y''(p) = -b_1(b_1 - b_0)K p_2^{-b_1} p^{b_1 - 2} < 0$ for $p > 0$, thus $Y(p)$ is strictly concave over $(0, \infty)$;

(ii) $Y(0) = -\beta_0 K < 0$;

(iii) $Y(p_2) > 0$. This is demonstrated by writing the function evaluated at this point in the form

$$Y(p_2) = \frac{K}{(b_1 - 1)}\{b_0 b_1 l + 2b_0 - b_1(2 + l)\} \qquad \text{(A1.2)}$$

where $l = \dfrac{2h}{r-m} > 0$. When $h = 0$, $b_1 = b_0$, $l = 0$ and $Y(p_2) = 0$. Evaluating the first derivative

$$\frac{\partial Y(p_2)}{\partial l} = \frac{b_1}{(b_1 - 1)}(b_0 - 1)K > 0 \qquad \text{(A1.3)}$$

it is clear that $Y(p_2) > 0$ for $\forall h > 0$. Q.E.D.

Lemma 2. $p_2 > p_C$ for $h > 0$.

Proof. The objective is to compare expressions (13) and (17). This reduces to a comparison between

$$\frac{b_0}{(b_0 - 1)} \qquad \text{(A2.1)}$$

and

$$\frac{b_1}{(b_1-1)}\frac{(r+h-m)}{(r-m)}. \tag{A2.2}$$

Recall that b_0 is independent of h and b_1 increasing in h. When $h = 0$, $b_1 = b_0$ and the two expressions are identical. Expressing (A2.2) in the form $M(h)/N(h)$, where $M(h) = b_1(r+h-m)$ and $N(h) = (b_1-1)(r-m)$, differentiation with respect to h yields

$$\frac{\partial M}{\partial h} = \frac{\partial N}{\partial h} + h\frac{\partial b_1}{\partial h} + b_1 > \frac{\partial N}{\partial h}. \tag{A2.3}$$

Thus, (A2.2) is strictly increasing in h and therefore $p_2 > p_C$ for $h > 0$. Q.E.D.

Lemma 3. $p_C > p_1$ for $h > 0$.

Proof. From lemma 1, to show that $p_C > p_1$ it is necessary and sufficient to demonstrate that $Y(p_C) > 0$ (given that it is already known from lemma 2 that $p_C < p_2$). Substituting for p_C we can write

$$Y(p_C) = \frac{Kb_0h}{(r+h-m)} - \frac{K(b_1-b_0)}{(b_1-1)}\left[\frac{b_0}{(b_0-1)}\frac{(b_1-1)}{b_1}\frac{(r-m)}{(r+h-m)}\right]^{b_1}. \tag{A3.1}$$

As a corollary of lemma 2 we know that the term in square brackets is less than unity (as this is p_C/p_2). Since $b_1 > 1$, we know that

$$Y(p_C) > \frac{Kb_0h}{(r+h-m)} - \frac{K(b_1-b_0)}{(b_1-1)}\left[\frac{b_0}{(b_0-1)}\frac{(b_1-1)}{b_1}\frac{(r-m)}{(r+h-m)}\right] = \frac{Kb_0}{(r+h-m)}Z(h)$$

where $Z(h) = h - \frac{(r-m)}{(b_0-1)}\left[1 - \frac{b_0}{b_1}\right]$. (A3.2)

Thus, to prove lemma 3 it is sufficient to show that $Z(h) > 0 \ \forall h > 0$. This follows from the following facts:

(i) $Z(0) = 0$;

(ii) $Z(h)$ is strictly convex;

(iii) $Z'(h)$ evaluated at $h = 0$ is strictly positive.

(i) is straightforward. To demonstrate (ii) and (iii) we start by taking partial derivatives with respect to h

$$\frac{\partial Z}{\partial h} = 1 - \frac{b_0}{(b_0 - 1)} \frac{(r-m)}{b_1^2} \frac{2}{\{(2b_1 - 1)s^2 + 2m\}} \tag{A3.3}$$

and

$$\frac{\partial^2 Z}{\partial h^2} = \frac{4(r-m)}{b_1^3} \frac{b_0}{(b_0 - 1)} \frac{\{(3b_1 - 1)s^2 + 2m\}}{\{(2b_1 - 1)s^2 + 2m\}^2} > 0 . \tag{A3.4}$$

Recalling that $b_1 = b_0$ when $h = 0$, after some manipulation we can write

$$\left.\frac{\partial Z}{\partial h}\right|_{h=0} = 1 - \frac{(r-m)}{(r-m) + (b_0 - 1)G(r)} \tag{A3.5}$$

where $G(r) = 2r - (b_0 + 1)m$. As $r \to m$, $b_0 \to 1$ and so $G(r) \to 0$. Taking partial derivatives we can write

$$\frac{\partial G}{\partial r} = 2 - \frac{2m}{2m + (2b_0 - 1)s^2} > 0 . \tag{A3.6}$$

Thus $G(r) > 0 \ \forall r > m$ and therefore $Z'(h)$ evaluated at $h = 0$ is strictly positive. Hence $Z(h) > 0 \ \forall h > 0$, which is sufficient to demonstrate that $Y(p_C) > 0$. Thus, $p_C > p_1$ for $h > 0$. Q.E.D.

Lemma 4. There exists a unique point $p_L \in (0, p_F)$ such that

$$V_L(p) < V_F(p) \text{ for } p < p_L$$
$$V_L(p) = V_F(p) \text{ for } p = p_L$$
$$V_L(p) > V_F(p) \text{ for } p \in (p_L, p_F)$$
$$V_L(p) = V_F(p) \text{ for } p \geq p_F .$$

Proof. We start by defining the function $P(p) = V_L(p) - V_F(p)$ describing the gain to preempting one's opponent as opposed to being preempted. Expanding using equations (25) and (21) we can write

$$P(p) = \frac{hp}{r+h-m} - K - \left(\frac{p}{p_F}\right)^{b_1} \left(\frac{hp_F}{r+h-m} - K\right) \text{ for } p \in (0, p_F) . \tag{A4.1}$$

The following steps are sufficient to demonstrate the existence of a root somewhere in the interval $(0, p_F)$.

(i) Evaluating $P(p)$ at zero yields $P(0) = -K < 0$.

(ii) Evaluating $P(p)$ at p_F yields $P(p_F) = 0$.

(iii) Evaluating the derivative $P'(p)$ at p_F it can be shown that

$$\operatorname{sgn}\left\{\frac{dP}{dp}\bigg|_{p_F}\right\} = \operatorname{sgn}\left\{-\frac{h}{(r+h-m)}(r+h)(r+2h)K\right\} < 0\,. \tag{A4.2}$$

Thus, $P(p)$ must have at least one root in the interval (0, p_F).

Uniqueness of the root p_L and the validity of the two inequalities can be proven by demonstrating strict concavity of $P(p)$ over $(0, p_F)$. By differentiation we can derive

$$P''(p) = -b_1(b_1-1)p_F^{-b_1}\left(\frac{hp_F}{r+h-m} - K\right)p^{b_1-2} < 0 \text{ for } p > 0. \tag{A4.3}$$

Thus the root is unique, with $P(p) < 0$ for $p \in (0, p_L)$ and $P(p) > 0$ for $p \in (p_L, p_F)$.

The final equality is demonstrated by considering the follower's optimal behaviour over the range $[p_F, \infty)$. This interval is the follower's stopping region over which its best response to investment by the leader is to invest at once. Thus, the values of the leader and follower are equal over this range. Q.E.D.

Lemma 5.

(a) p_S exists and is unique whenever $V_C(p) \ge V_L(p)\ \ \forall p \in (0, p_C)$.

(b) $[p_S, p_C]$ forms a connected set such that $V_J(p; p_J) \ge V_L(p)\ \ \forall p \in (0, p_J)$, $p_J \in [p_S, p_C]$.

Proof.

(a) To demonstrate existence we start by showing that $V_J(p; p_C) = V_C(p)$. With some simplification, the expressions for V_J and p_C yield

$$V_J(p; p_C) = \frac{hp_C^{1-b_0}}{(r+2h-m)b_0}p^{b_0} = B_C p^{b_0} = V_C(p). \tag{A5.1}$$

It then follows from the premise that there exists at least one $p_J \in (0, p_C]$ such that $V_J(p; p_J) \ge V_L(p)\ \forall p \in (0, p_J]$: at the very least p_C itself satisfies this condition. p_S is then defined to be the smallest element of the set of joint investment points satisfying the condition.

With p_S defined as the lowest joint investment point such that the two functions $V_L(p)$ and $V_J(p; p_S)$ just touch one another, a sufficient condition for uniqueness of p_S is that $V_J(p; p_J)$ is strictly increasing in p_J for $p_J \in (0, p_C)$. We derive

$$\frac{\partial V_J}{\partial p_J} = \left[b_0 K - (b_0 - 1)\frac{hp_J}{(h + 2r - m)} \right] p_J^{-(b_0+1)} p^{b_0} > 0 \text{ for } p_J \in (0, p_C).$$

(A5.2)

Thus, for any $p_J < p_C$ a higher value of p_J entails a strictly higher value of V_J at any given value of p.

(b) To show that $[p_S, p_C]$ forms a connected set satisfying the condition that $V_J(p; p_J) \geq V_L(p) \quad \forall p \in (0, p_J),\ \ p_J \in [p_S, p_C]$ it is sufficient to show that $V_J(p; p_J)$ is increasing in both of its arguments for all $p_J \in (0, p_C]$. Since $\partial V_J / \partial p_J$ evaluated at p_C equals zero, we can write

$$\frac{\partial V_J}{\partial p_J} \geq 0 \text{ for } p_J \in (0, p_C]. \tag{A5.3}$$

It can easily be seen that

$$\frac{\partial V_J}{\partial p} = B_J(p_J) b_0 p^{b_0 - 1} > 0 \ \ \forall p > 0. \tag{A5.4}$$

Q.E.D.

The designated leader's investment problem

The designated leader's investment point, p_D, is defined implicitly by

$$(b_1 - b_0) B_L p_D^{b_1} + (b_0 - 1)\frac{hp_D}{r + h - m} - b_0 K = 0$$

where B_L is as defined following expression (25). Having solved numerically for the value of the trigger point p_D, the option value constant B_D is then defined by

$$B_D = \frac{p_D^{-b_0}}{b_0}\left\{ \frac{hp_D}{r + h - m} - b_1 B_L p_D^{b_1} \right\}.$$

Lemma 6. $p_L < p_1$ for $h > 0$.

Proof. From rent equalisation at p_L we know that $V_L(p_L) = V_F(p_L)$. Using (21) and (25) to substitute for the respective value functions at this point, and their derivatives, we can write

$$V_F'(p_L) = V_L'(p_L) - \frac{b_1 K}{p_L} + \frac{(b_1 - 1)h}{(r + h - m)}. \tag{A6.1}$$

From lemma 4 we know that V_L crosses V_F at p_L from below and must therefore have the steeper slope, thus $V_L'(p_L) > V_F'(p_L)$. Thus, the following upper bound for p_L can be derived

$$p_L < \frac{b_1}{(b_1 - 1)} \frac{(r + h - m)}{h} K \equiv p_M . \qquad \text{(A6.2)}$$

Given the shape of the function $Y(p)$ (defined by (A1.1) above) of which p_1 is the root, and since $p_M < p_2$, it is sufficient to show that $Y(p_M) < 0$. It can readily be shown that

$$Y(p_M) = \frac{K}{(b_1 - 1)} \left\{ b_0 - b_1 - (b_1 - b_0) \left[\frac{(r - m)}{(r + 2h - m)} \right]^{b_1} \right\} < 0 \text{ for } h > 0. \qquad \text{(A6.3)}$$

Thus, $p_L < p_M < p_1$. Q. E. D.

Figure 1: Preemptive leader-follower equilibrium

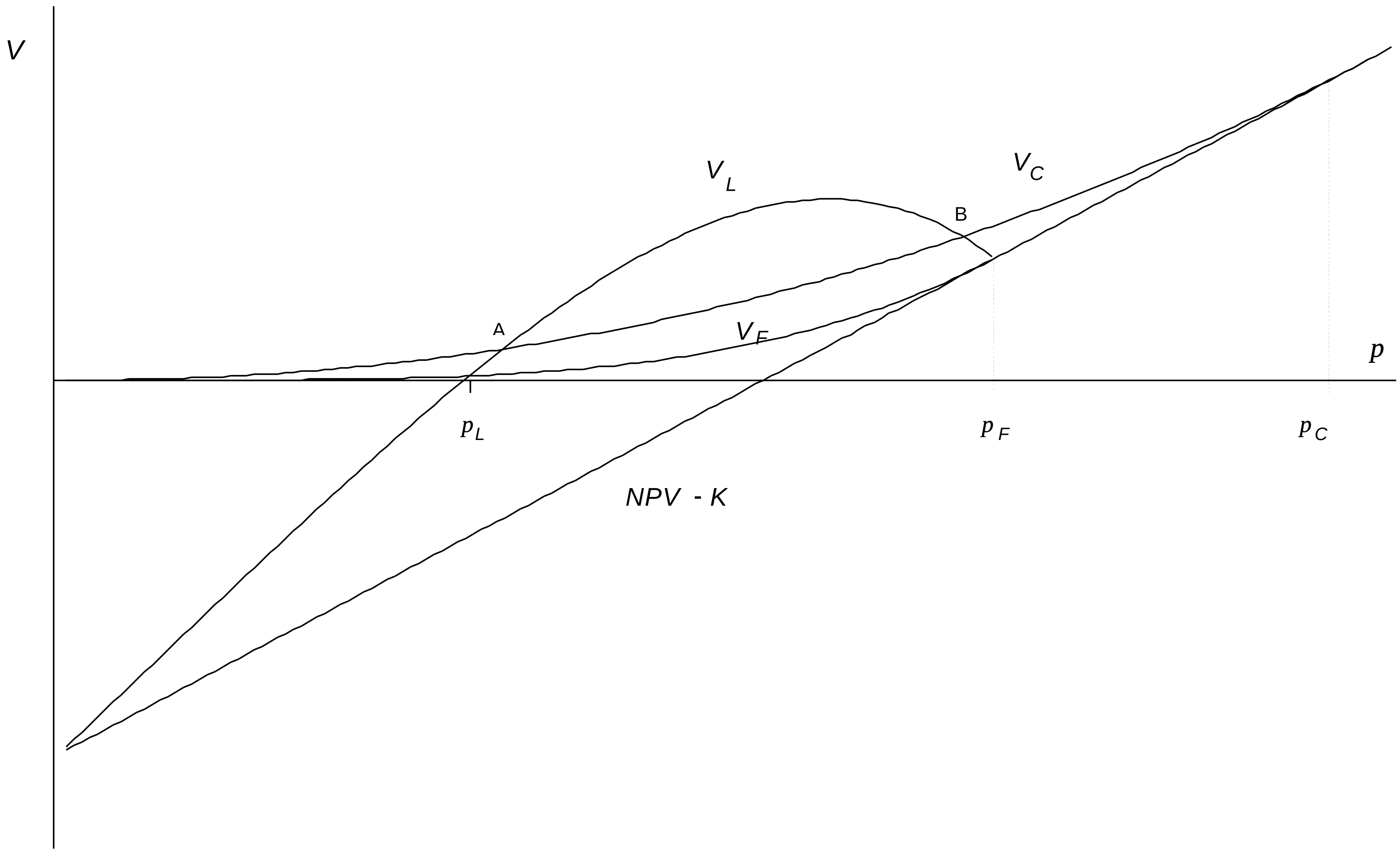
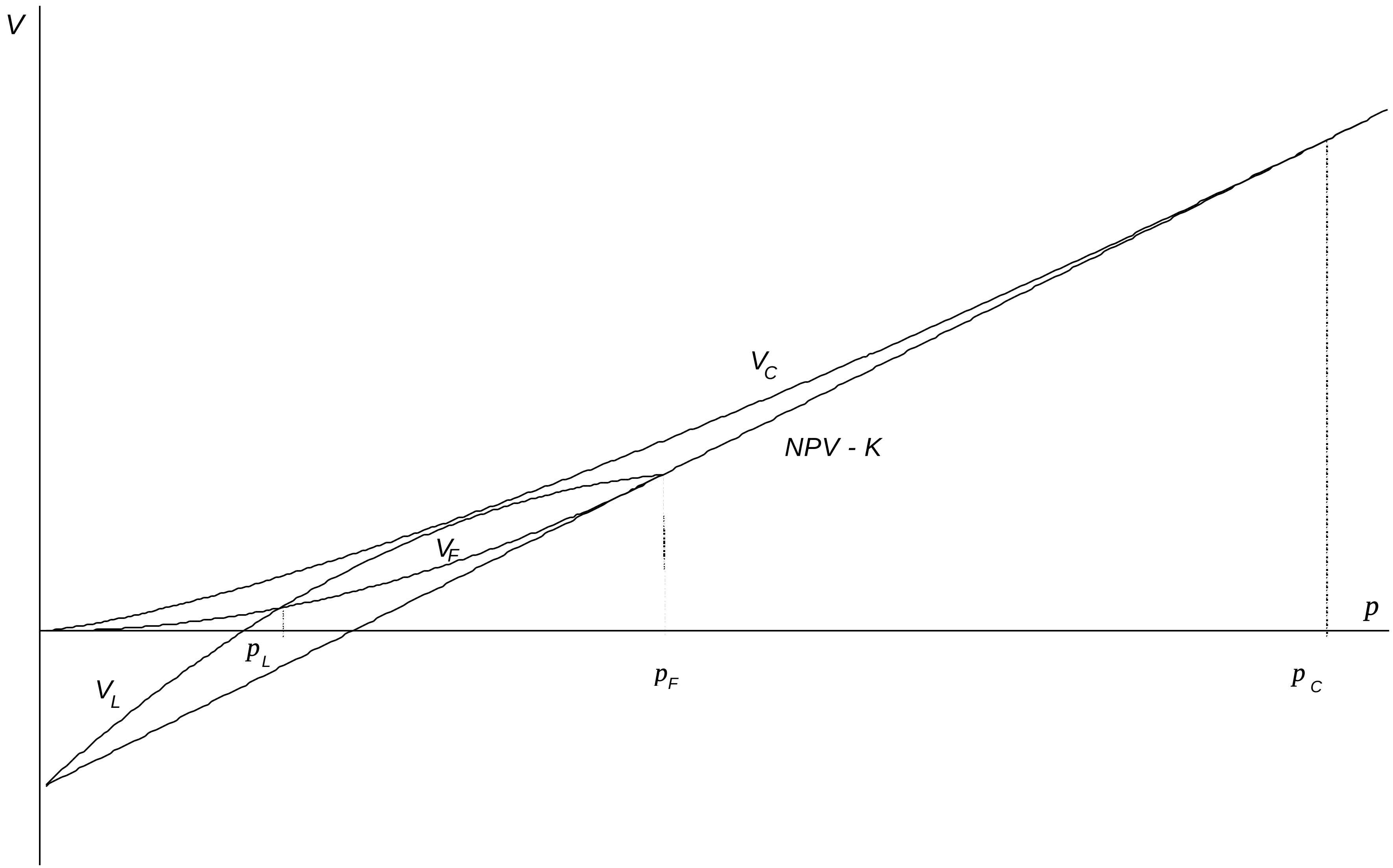


Figure 2: Joint investment equilibrium



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1 This value could be interpreted as the expected NPV of profits in the relevant product market or, if further sunk costs are required, might itself be the value of the option to invest in this market, making investment in the research stage a compound option problem.

2 The restriction that $m < r$, commonly found in real options models, is necessary to ensure that there is a strictly positive opportunity cost to holding the option, so that it will not be held indefinitely. A large negative drift term would, *ceteris paribus*, encourage earlier investment to raise the probability of winning the prize before its value declines significantly, counteracting the option effects in the model. To avoid such an outcome we make the assumption that m is non-negative. Since the model is concerned with the effects of uncertainty, not expected trends, the conclusions from the analysis are unaffected by this assumption.

3 Thus the cost of R&D is fixed, or contractual in the terminology of Kamien and Schwartz (1982).

4 To be precise, the statement that a firm invests at a trigger point p^* means that the firm invests at the time when the stochastic process p first hits the value p^*, approaching this level from below.

5 For further details see Fudenberg and Tirole (1985), section 4.

6 If smooth-pasting were violated and instead a kink arose at p_U, a deviation from the supposedly optimal policy would raise the firm's expected payoff. By delaying for a small interval of time after the stochastic process first reached p_U, the next step dp could be observed. If the kink were convex, the firm would obtain a higher expected payoff by entering if and only if p has moved (strictly) above p_U, since an average of points on either side of the kink give it a higher expected value than the kink itself. If the kink were concave, on the other hand, second order conditions would be violated. Continuation along the initial value function would yield a higher payoff than switching to the alternative function and switching at p_U could not be optimal. More detailed explanation of this condition can be found in appendix C of chapter four in Dixit and Pindyck (1994). Note that this condition applies for all diffusion processes, not just a geometric Brownian motion such as (1).

7 It is implicitly assumed that side payments may be used to ensure that neither firm has an incentive to deviate; alternatively, the two firms may be separate research units controlled by an integrated firm.

8 Smooth-pasting ensures that the first derivative of V_{L+F} is continuous at p_1.

9 An analogous effect is found in the duopoly models of Smets (1991) and Grenadier (1996). The second, option value effect of rival investment is absent from these models, however.

10 With K adjusted appropriately so that the project's expected value remains constant.

11 With m adjusted in line so that the opportunity cost $d = r - m$ remains constant.

12 Given that this proportion is determined largely by the duration of the patent and the degree of monopoly power conveyed by the patent grant, this would not seem to be an unreasonable assumption.

13 All values and trigger points are scaled up by the same proportion, leaving the optimal timing of investment unchanged.