Investment Subsidies and Time-Consistent Environmental Policy*

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ABSTRACT

We describe a model of dynamic pollution abatement choices with heterogeneous agents, where, due to the presence of a distributional objective and to the absence of incentive-compatible compensation mechanisms, the choice of a second-best level of emission taxation is time-inconsistent. In this model, we investigate whether investment subsidies can act as a substitute for policy commitment.

KEY WORDS: Pollution Abatement, Emission Taxes, Investment Subsidies.

JEL CLASSIFICATION: H2, Q3.

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1 Introduction

This paper investigates how subsidies to environment improving investment can be used to overcome dynamic consistency problems in environmental policy choices, in the presence of dynamic pollution abatement, heterogeneous agents, and governments that pursue both efficiency and distributional objectives.

There is ample evidence that pollution abatement is closely linked to investment and innovation. Firms are committed to certain modes of production in the short run, and changing production methods typically involves some investment in R&D and new equipment (Carraro and Siniscalco, 1994; Popp, 1998). At the same time, environmental taxes can generate unwanted distributional effects (Johnson *et al.*, 1990; Poterba, 1991; Jorgenson *et al.*, 1992),[1] which, due to information-related constraints, cannot easily be undone through compensation.[2]

When policymakers care about distribution, the presence of dynamic abatement decisions can give rise to a policy commitment problem. This is because, although emission taxes are required to generate incentives for environmental innovation, once innovation has taken place, a policymaker may find it optimal *ex post* to lower them in order to minimize distributional impacts; as private agents recognize the *ex-post* incentives the policymaker faces, the promise of high future emission taxes will not be credible. This commitment problem will, in turn, force policymakers to achieve their objectives by relying more heavily on investment subsidies, which are paid immediately and therefore do not suffer from the same dynamic inconsistency problem that affects emission taxes.[3]

While time inconsistency problems in tax policy choices have been examined in some detail, especially with reference to capital income taxation (see, e.g., Fischer, 1980; Chari and Kehoe, 1990; Xiaodong, 1995), much of the existing literature on policy commitment has focused on efficiency considerations only. A recent exception is Pearce and Stacchetti (1997), who analyze time-consistent taxation in a context where a government is interested both in efficiency and equity. Dynamic inconsistency problems in environmental policies have also so far received relatively little attention in the literature. Biglaiser, Horowitz, and Quiggin (1995) examined dynamic permit regulation when firms can behave strategically against the regulator; in their structure, emission permits are time inconsistent, but the inconsistency problem can be solved by the use of emission taxes. Marsiliani and Renström (1998) have analyzed the role of tax earmarking of environmental taxes to overcome dynamic inconsistency in environmental taxes. More recently, Gersbach and Glazer (1999) have examined the "investment hold-up" problem when output reductions are socially undesirable and regulators cannot commit to a certain level of stringency in environmental regulation. But, to the best of our knowledge, the implications of distribution-related time-consistency constraints for the choice between taxes and subsidies have not been explored before.

In the next section, we describe a two-period model of pollution abatement with heterogeneous agents having different consumption requirements of a polluting good. Pollution abatement takes the form of an alternative production method which requires a special, additional investment in the first period, with the associated rents being dispersed unequally between the two agents' types. In the model, environmental policies affect distribution both through the consumption side of the economy—via their impact on the price of the polluting good—and through the production side—via their effects on profits from abatement activities.

Section 3 examines the optimal choice of emission tax or abatement subsidy. Both generate incentives for pollution abatement, and both have an adverse distributional effect, due to the government's inability to disperse revenues so as to compensate losers. Because of these distributional effects, the second-best optimal level of emission taxation—even when the government can commit to future policies—will lie below the efficient level. But in the absence of a commitment mechanism, the presence of a first-period, abatement-related, private investment choice gives rise to a dynamic inconsistency problem in policy choices, which results in the time-consistent choice of emission taxes lying below the second-best choice.

Section 4 analyses the policymaker's problem when abatement incentives can also be affected by a subsidy to abatement-related investment (rather than to abatement itself). This is a less efficient instrument in comparison with an emission tax or abatement subsidy—because it distorts input choices—but it may be superior on distributional grounds; consequently, even when commitment is possible, a second-best policy will involve a mix of emission taxes and investment subsidies. If, however, commitment is not possible, the time-consistent choice will involve a level of investment subsidization which departs from the second-best choice. Nevertheless, we find that, if abatement technologies exhibit constant elasticity and profit shares are identical across the two agent types, the consistent and inconsistent optimal policies will coincide; otherwise, the comparison between the consistent and inconsistent subsidy is generally ambiguous. It is only when distributional effects stem uniquely from the distribution of abatement related profits, and emission taxes and abatement subsidies are substitute instruments at the margin from the point of view of the policymaker, that the consistent subsidy, as conjectured above, will unambiguously lie above the corresponding inconsistent choice.

2 A Model of Dynamic Abatement Choices

This section describes a stylized model of dynamic abatement choices with heterogeneous agents. There are two time periods, 1 and 2. Two goods are produced in the second period, a clean good and a pollution generating good. Each unit of the latter generates one unit of emissions, and can be produced at a constant marginal cost of unity. Thus, if the government levies an emission tax of t per unit of emissions, its gross-of-tax price is

$$p = 1 + t. \tag{1}$$

2.1 Investment and Abatement

There exists an alternative method for producing a perfect substitute of the polluting good without generating emissions, but this involves a marginal cost in excess of unity and requires an additional investment, N, in period 1. Let V be the amount of the good produced using this clean technology, and suppose that its production in period 2 requires one unit of income (as its "dirty" counterpart does), plus an additional cost which depends positively on V and on the unit (opportunity) cost of N, denoted with q.[4] Thus, the long-run cost of producing an amount V can be written as

$$\widehat{c}(V, q) = V + H(V, q). \tag{2}$$

In order to develop our argument, we will assume technologies to be homothetic—i.e., such that the cost minimizing optimal combination of N and other inputs for given prices is independent of the level of abatement V—and costs to be isoelastic (i.e., the output elasticity of marginal abatement costs, $\eta \equiv H_{VV}V/H_V$, is constant) and convex in V ($H_{VV} > 0$). This implies the following representation:

$$H(V, q) \equiv h(q)V^{1+\eta}, \tag{3}$$

where $\eta > 0$, $h'(q) > 0$, $h''(q) < 0$. Note that $H_q = h'(q)V^{1+\eta} > 0$ represents compensated demand; thus concavity of $h(\cdot)$ corresponds to the standard requirement that the compensated own-price effect, H_{qq}, be negative. Homotheticity also implies $H_{qV} > 0$. Finally, we shall also assume $h(\cdot)$ to be isoelastic, implying that $\omega = h''(q)q/h'(q)$ is constant.

Using Shephard's Lemma, and employing subscripts to denote derivatives, the indirect demand for N is given by

$$\widetilde{N}(V, q) = H_q. \tag{4}$$

Since each unit of the "clean" good can sell at a price of p, the revenue from producing and selling V units is pV, and the associated profits are

$$\Pi(V, p) = tV - H(V, q). \tag{5}$$

The first-order condition for an interior profit-maximizing choice of V is

$$t - H_V = 0; \tag{6}$$

Convexity of $H(V, q)$ in V guarantees that the second-order conditions for an optimum are satisfied.

Condition (6) simply states that pollution abatement will take place up to the point where marginal abatement costs equal the marginal benefit from abatement (the tax). Notice that if $t = 0$, we have $V = 0$, meaning that no abatement will take place. Condition (6) defines V and indirectly N—via (4)—as functions $\widehat{V}(t, q)$ and $\widehat{N}(t, q) \equiv \widetilde{N}[\widehat{V}(t, q), q]$ of t and q. Comparative statics effects are:

$$\frac{\partial \widehat{V}}{\partial t} = \frac{1}{H_{VV}} > 0; \tag{7}$$

i.e. the amount of pollution abatement increases with the tax;

$$\frac{\partial \widehat{V}}{\partial q} = -\frac{\partial \widehat{N}}{\partial t} = -\frac{H_{qV}}{H_{VV}} < 0; \tag{8}$$

i.e. the amount of pollution abatement decreases with the price of investment, and the amount of investment increases with the tax;

$$\frac{\partial \widehat{N}}{\partial q} = H_{qq} - \frac{(H_{qV})^2}{H_{VV}} < 0; \tag{9}$$

i.e. investment is negatively related to its price.

The above analysis describes the "long-run" choice by producers. If we focus, instead, on the "short-run" choice of V—made in the second period after a certain level of N has been installed in the first period—then the short-run cost of producing an amount V becomes

$$\widehat{c}(V, q, N) = V + H[V, q^*(V, N)] - [q^*(V, N) - q]N, \tag{10}$$

where $q^*(V, N)$ is the shadow price of N, which is the value that solves

$$\widehat{N}[V, q^*(V, N)] = N. \tag{11}$$

The second-period optimal choice of V is then characterized by the interior first-order condition

$$t - H_V + (N - H_q)\frac{\partial q^*}{\partial V} = 0. \tag{12}$$

Condition (12) defines V as an implicit function $\hat{\hat{V}}(t, N)$ of t and N. Note that

$$\frac{\partial q^*}{\partial V} = -\frac{H_{qV}}{H_{qq}}; \tag{13}$$

i.e. the shadow price of investment increases with the level of abatement. The other comparative statics effects are as follows:

$$\frac{\partial \hat{\hat{V}}}{\partial t} = \frac{1}{H_{VV} + \Gamma}; \tag{14}$$

where

$$\Gamma \equiv \frac{\partial\left[(N - H_q)H_{qV}/H_{qq}\right]}{\partial V} = -\frac{(H_{qV})^2}{H_{qq}} > 0; \tag{15}$$

and

$$\frac{\partial \hat{\hat{V}}}{\partial N} = \frac{1}{H_{qV}} > 0. \tag{16}$$

It is straightforward to establish the following result (all proofs are given in the Appendix):

Lemma 1: *The short-run abatement response to a marginal increase in the emission tax, $\partial\hat{\hat{V}}/\partial t$, is less than the long-run response, $\partial\hat{V}/\partial t$.*

The above result follows from basic principles: abatement choices are more inflexible in the short-run, when investment cannot adjust. This is the mechanism at the heart of the policy inconsistency problem that we describe in the next section.

2.2 Consumption and Damage

There are equal numbers of two consumer types, A and B, living in the second period. Consumers of each type are endowed with exogenous income levels respectively equal to Y^A, Y^B. We assume that in the second period individuals must consume fixed given amounts, $X^A = \delta^A X$, $X^B = \delta^B X$ ($\delta^A + \delta^B = 1$), of the pollution generating commodity (or of its clean substitute), with the rest of their disposable income being available for consumption of the other good.[5] Each of the two consumer groups receives a share θ^i ($i = A, B$; $\theta^A + \theta^B = 1$) of the profits from abatement activities. Although stylized, the above specification captures the two main channels through which environmental policies affect distribution, namely, differences in consumption patterns and differences in income patterns.

Environmental emissions are equal to

$$E = X - V; \tag{17}$$

and tax revenues from emission taxes are

$$R = tE. \tag{18}$$

We assume that these are returned to the two consumer groups in lump-sum fashion and in equal shares. This assumption reflects the idea that tax policies must be anonymous and that there exists no feasible, incentive-compatible means of identifying the two consumer types.[6] In this specification there are no other taxes and no public spending. Thus, our analysis abstracts from any "double-dividend" considerations—whereby, in the presence of a revenue requirement financed by distortionary taxes, the social marginal value of environmental tax revenues exceeds unity.

Consumers are also affected directly by environmental emissions. Because we wish to focus on the distributional effects of abatement activities—leaving aside any direct distributional impacts associated with different preferences for environmental quality across consumers—we assume that the valuation of damage is the same for all individuals, and equal to

$$D(E)/2, \tag{19}$$

with $D' > 0$ and $D'' > 0$. We also adopt an additive formulation for the impact of damage from emissions, where utility can be written as effective consumption of goods other than the polluting good—which is equal to income, gross of profits and tax revenues received and net of the cost of purchasing the required amounts of the polluting good, X^A and X^B—minus environmental damage:

$$U^i = Y^i + \theta^i \Pi - p\delta^i X + [R - D(E)]/2, \qquad i = A, B. \tag{20}$$

In the next two sections, this simple model structure is used to investigate the role of alternative tax-based incentive mechanisms, and their implications for policy choices when policy commitment is infeasible.

3 Consistent and Inconsistent Policy Choices: Emission Taxes

3.1 The Policymaker's Problem

Suppose that the only instrument available to the policymaker is an emission tax, and that, without loss of generality, the cost of investment is unity, i.e. $q = 1$.

We shall assume that policymaker's objective is the maximization of a symmetric, strictly concave social welfare function:[7]

$$W = W(U^A, U^B). \tag{21}$$

Maximization of W by choice of t yields

$$\left[\left(\frac{1}{2} - \delta^A\right) X - \left(\frac{1}{2} - \theta^A\right) V + \frac{D'(X-V) - t}{2} \frac{\partial \widehat{V}}{\partial t}\right] \frac{\partial W}{\partial U^A}$$
$$+ \left[\left(\frac{1}{2} - \delta^B\right) X - \left(\frac{1}{2} - \theta^B\right) V + \frac{D'(X-V) - t}{2} \frac{\partial \widehat{V}}{\partial t}\right] \frac{\partial W}{\partial U^B} = 0, \tag{22}$$

where $V = \widehat{V}(t, 1)$. The first two terms in the square brackets of each of the two terms on the left-hand side of (22) reflect distributional effects stemming from nonuniform consumption and ownership patterns. If $\delta^i = \theta^i = 1/2$, $i = A, B$, these two terms disappear and the remaining terms imply $t = D'(E)$, the efficient choice.

Throughout the rest of our analysis, we shall also maintain the following assumption:

$$Y^A - \delta^A X = Y^B - \delta^B X. \tag{23}$$

This is a normalization condition, whose role is to ensure that in the absence of environmental emissions there is no independent redistributive role to play for environmental taxes; formally, (23) implies that, if $D'(E) = 0$, a choice of $t = D'(E) = 0$ results in $U^A = U^B$, and is thus optimal according to (22); i.e., in the absence of damage, the optimal tax would be zero.

In the second period, once investment decisions have been made and N is fixed, social welfare maximization yields

$$\left[\left(\frac{1}{2} - \delta^A\right) X - \left(\frac{1}{2} - \theta^A\right) V + \frac{D'(X-V) - t}{2} \frac{\partial \widehat{\widehat{V}}}{\partial t}\right] \frac{\partial W}{\partial U^A}$$
$$+ \left[\left(\frac{1}{2} - \delta^B\right) X - \left(\frac{1}{2} - \theta^B\right) V + \frac{D'(X-V) - t}{2} \frac{\partial \widehat{\widehat{V}}}{\partial t}\right] \frac{\partial W}{\partial U^B} = 0, \tag{24}$$

where $V = \widehat{\widehat{V}}(t, N)$. In an equilibrium where investors anticipate policy choices, N will also satisfy the "rational-expectations" condition

$$N = \widehat{N}(t, 1). \tag{25}$$

This says that N will be chosen in the first period on the basis of the anticipated tax rate t, resulting in a choice of abatement level which coincides with the optimal "long-run" choice for the given t, i.e., $V = \widehat{V}(t, 1)$.

Thus, the only difference between (22) and (24) is in the expressions $\partial \widehat{V}/\partial t$ and $\partial \widehat{\widehat{V}}/\partial t$, reflecting the difference between short- and long-run responses of pollution abatement to tax changes: condition (24) identifies the *consistent* (i.e., subgame perfect) optimal choice of tax, $\widehat{\widehat{t}}$, while (22) characterizes the *inconsistent* choice, $\widehat{t}$, which can only be an equilibrium outcome if the policymaker can credibly commit to it in period 1. If $\delta^i = \theta^i = 1/2$ $(i = A, B)$, the first-best choice of $t = D'(E)$ will also satisfy (24), and thus will be time-consistent, but if there are distributional impacts from the tax, the consistent and inconsistent optimal rates will diverge.[8]

3.2 Comparison of Consistent and Inconsistent Policy Choices

In order to compare the two outcomes, we can express δ^B as $1 - \delta^A$ and θ^B as $1 - \theta^A$, and rewrite (22) and (24) as

$$\frac{D'(X - V) - t}{2} \frac{\partial \widehat{V}}{\partial t} \left(\frac{\partial W}{\partial U^A} + \frac{\partial W}{\partial U^B} \right)$$
$$= \left[\left(\delta^A - \frac{1}{2} \right) X - \left(\theta^A - \frac{1}{2} \right) V \right] \left(\frac{\partial W}{\partial U^A} - \frac{\partial W}{\partial U^B} \right), \tag{26}$$

and

$$\frac{D'(X - V) - t}{2} \frac{\partial \widehat{\widehat{V}}}{\partial t} \left(\frac{\partial W}{\partial U^A} + \frac{\partial W}{\partial U^B} \right)$$
$$= \left[\left(\delta^A - \frac{1}{2} \right) X - \left(\theta^A - \frac{1}{2} \right) V \right] \left(\frac{\partial W}{\partial U^A} - \frac{\partial W}{\partial U^B} \right). \tag{27}$$

Let us first focus on the case $\theta^A = \theta^B = 1/2$, $\delta^A \neq \delta^B$. First notice that $\partial \widehat{V}/\partial t > 0$ and $\partial \widehat{\widehat{V}}/\partial t > 0$. If $\delta^A > 1/2$, because of (23) a choice of $t > 0$ implies $U^A < U^B$, and so $\partial W/\partial U^A > \partial W/\partial U^B$; if, on the other hand, $\delta^A < 1/2$, we have $U^A > U^B$, implying $\partial W/\partial U^A < \partial W/\partial U^B$; either way, the right-hand side of both (26) and (27) will be positive, implying that, due to the presence of a distributional objective, both the consistent and the inconsistent optimal tax rates lie below the social marginal damage $D'(E)$. The same is true if $\theta^A \neq \theta^B$, and $\delta^A = \delta^B = 1/2$. If both $\theta^A \neq \theta^B$, and $\delta^A \neq \delta^B$, however, we cannot exclude that both the consistent and the inconsistent optimal tax rates could lie above $D'(E)$.

When $D'(E) > t$, however, it can be shown that the inconsistent choice unambiguously lies below the consistent choice:

Proposition 1: *If only demand shares or if only profit shares are unequal across individuals, the consistent choice of emission tax lies below the inconsistent choice.*

Thus, when the distributional impacts of emission taxation arise exclusively from either the production side or the consumption side of the economy, they will not only cause the welfare maximizing tax to lie below the efficient level, but also cause a policy commitment problem, resulting in a time-consistent choice of tax lying below the corresponding second-best level.

If distributional effects on income and consumption are both simultaneously present, no general conclusion is possible. This is because, when both $\delta^A \neq \delta^B$ and $\theta^A \neq \theta^B$, an increase in the tax above the efficient level $t = D'(E)$ could improve income distribution (if, for example, $\delta^A > \delta^B$ and $\theta^A > \theta^B$) at the expense of efficiency.[9] Consequently, the post-investment optimal policy could involve a higher tax than the second-best consistent policy.

Note that in this model, where the demand for the pollution generating good is fixed, an emission tax with $\delta^A = \delta^B = 1/2$ is equivalent to a scenario where $\delta^A \neq \delta^B$ and where an abatement subsidy (a subsidy to V) is used in place of an emission tax: this is because an abatement subsidy is distributionally neutral with respect to differences in consumption patterns, although it will still generate distributional effects if abatement profits shares are unequal. Thus, the above analysis also implies that, if only demand shares are unequal across consumers, both the consistent and inconsistent choice will coincide with the efficient abatement subsidy. But if profit shares are unequal, the consistent choice of abatement subsidization will be less than the inconsistent choice.

4 Subsidies to Environmental Investment

In principle, emission taxes, if feasible, are a perfectly adequate means of generating appropriate incentives to reduce environmental emissions, whether abatement is achieved through technical innovation or otherwise (Baumol and Oates, 1988). Yet, we observe many countries providing other inducements, typically in the form of direct tax incentives for environment-related investment.[10] It is well understood that such policies are not first-best as they distort input choices. Furthermore, tax incentives tend to be imperfectly targeted, due to the impossibility of distinguishing between true environment-related investment and other forms of investment, and differentiating tax preferences according to the specific environmental impacts of different types of investment (typically a single rate of subsidy is used for all qualifying forms of investment).

In some of the environmental literature, the use of investment incentives for innovation, either in isolation or in combination with emission taxes, has been associated with the existence of non-competitive environments. Ferrante (1996), for instance, has developed a model with environmental externalities, technical change, and Cournot competition. His main finding is that a subsidy to research and development either alone or together with an emission tax would be superior to an emission tax only. A similar argument is developed by Kim and Chang (1993). In contrast, in this paper we characterize the use of investment subsidies as reflecting distributional concerns.

4.1 The Policymaker's Problem

Suppose that abatement choices can also be influenced by a subsidy to environmental investment, N, paid in the first period (when investment occurs) at a rate s. Then the net-of-subsidy price of investment becomes

$$q = 1 - s. \tag{28}$$

Accordingly, the profit-maximizing choice of N and V will depend on s as well as on t. Net tax revenues become

$$R = tE - sN. \tag{29}$$

Given (28), the functions for V and N defined by condition (6) now also involve s.

The relevant comparative statics effects are as follows:

$$\frac{\partial \widehat{V}}{\partial s} = -\frac{\partial \widehat{V}}{\partial q} = \frac{H_{qV}}{H_{VV}} > 0; \tag{30}$$

subsidization;

$$\frac{\partial \widehat{N}}{\partial s} = -\frac{\partial \widehat{N}}{\partial q} = -H_{qq} + \frac{(H_{qV})^2}{H_{VV}} > 0; \tag{31}$$

i.e. clean production and investment both increase with the level of investment subsidization.

The first-order conditions for an interior welfare-maximizing choice of t and s are

$$\widehat{-}_t \equiv \left[\left(\frac{1}{2} - \delta^A\right) X - \left(\frac{1}{2} - \theta^A\right) V + \frac{D'(X-V) - t}{2}\frac{\partial \widehat{V}}{\partial t} - \frac{s}{2}\frac{\partial \widehat{N}}{\partial t}\right]\frac{\partial W}{\partial U^A}$$

$$+ \left[\left(\frac{1}{2} - \delta^B\right) X - \left(\frac{1}{2} - \theta^B\right) V + \frac{D'(X-V) - t}{2}\frac{\partial \widehat{V}}{\partial t} - \frac{s}{2}\frac{\partial \widehat{N}}{\partial t}\right]\frac{\partial W}{\partial U^B} = 0, \quad (32)$$

and

$$\widehat{-}_s \equiv \left[-\left(\frac{1}{2} - \theta^A\right) N + \frac{D'(X-V) - t}{2}\frac{\partial \widehat{V}}{\partial s} - \frac{s}{2}\frac{\partial \widehat{N}}{\partial s}\right]\frac{\partial W}{\partial U^A}$$

$$+ \left[-\left(\frac{1}{2} - \theta^B\right) N + \frac{D'(X-V) - t}{2}\frac{\partial \widehat{V}}{\partial s} - \frac{s}{2}\frac{\partial \widehat{N}}{\partial s}\right]\frac{\partial W}{\partial U^B} = 0. \quad (33)$$

The subsidy is a second-best instrument as it distorts input choices in abatement activities: with $\delta^i = \theta^i = 1/2$ $(i = A, B)$, a first-best choice will involve $t = D'(X - V)$ and $s = 0$. If, however, there are distributional effects from taxes, a solution to (32)-(33) will generally involve $s \neq 0$, and so tax incentives to investment will have a role to play in a second-best environmental policy mix.

As for the case where subsidies are not available, little specific can be said when $\theta^A \neq \theta^B$, $\delta^A \neq \delta^B$. When $\theta^A \neq \theta^B$, $\delta^A = \delta^B$ or $\theta^A = \theta^B$, $\delta^A \neq \delta^B$, one can verify that, when $s = 0$ and for the level of t that satisfies $\widehat{-}_t = 0$, the expression $\widehat{-}_s$ is positive,[11] implying that the consistent choice of s will rise above zero. For $s > 0$, however, the sign of $\widehat{-}_{ts} \equiv \partial\widehat{-}_t/\partial s$ is ambiguous, implying that the consistent tax can fall or increase relative to a scenario where $s = 0$ (and it is indeed possible to find examples where this occurs). Thus, the presumption that, when either profit shares or consumption shares are unequal, the consistent optimal policy choice will involve substitution of emission taxes with investment subsidies is not generally valid.

The reason for this ambiguity is as follows. An increase in the level of subsidy directly encourages abatement and thus reduces the need for emission taxes, which should then result in lower taxes. But from (7), we have $\partial^2\hat{V}/(\partial t \partial s) > 0$, implying that an increase in the subsidy also raises the responsiveness of abatement choices to marginal tax increases, making them relatively more attractive to the policymaker. Although, this is only a "second-order" effect (appearing in the expression for $\widehat{-}_{ts}$), in principle it can more than offset the negative "first-order" effect on the optimal level of tax, making the tax and subsidy complementary rather than substitute instruments from the perspective of the policymaker.

If we now focus on a time-consistent policy choice sequence, then, given s and N, a second-period optimal choice of t identified by the necessary first-order condition

$$\begin{aligned}\hat{\hat{-}}_t \equiv & \left[\left(\frac{1}{2}-\delta^A\right)X-\left(\frac{1}{2}-\theta^A\right)V+\frac{D'(X-V)-t}{2}\frac{\partial\hat{\hat{V}}}{\partial t}\right]\frac{\partial W}{\partial U^A} \\ & +\left[\left(\frac{1}{2}-\delta^B\right)X-\left(\frac{1}{2}-\theta^B\right)V+\frac{D'(X-V)-t}{2}\frac{\partial\hat{\hat{V}}}{\partial t}\right]\frac{\partial W}{\partial U^B}=0. \end{aligned}\tag{34}$$

The optimal choice of subsidy prior to the choice of N taking place is then found as the solution to the problem of maximizing W subject to (34) and to the forward-looking condition (25); this yields the necessary condition

$$\hat{\hat{-}}_s \equiv \hat{-}_s + \hat{-}_t\frac{\hat{\hat{-}}_{ts}}{\hat{\hat{-}}_{tt}} = 0; \tag{35}$$

where $\hat{\hat{-}}_{tt} \equiv \partial\hat{\hat{-}}_t/\partial t$. In conjunction with (34) and (25), the above identifies an optimal, time-consistent choice of emission tax and investment subsidy.[12]

Note that, if we totally differentiate (34) with respect to t and s, we obtain

$$\frac{d\hat{\hat{t}}}{ds} = -\frac{\hat{\hat{-}}_{ts}}{\hat{\hat{-}}_{tt}}, \tag{36}$$

which is the negative of the ratio that appears on the right-hand side of (35). Thus, the consistent optimal choice of investment subsidization depends on how the subsidy affects the second-period consistent choice of t at the margin. In turn, since the denominator is negative (from the second-order conditions for an optimum), the sign of (36) agrees with the sign of $\hat{\hat{-}}_{ts}$. When $\hat{\hat{-}}_{ts} < 0$, the tax and the subsidy are marginal substitutes, i.e., a marginal increase in the tax induces a decrease in the consistent optimal level of emission taxation; otherwise, the opposite will be true.

4.2 Comparison of Consistent and Inconsistent Policy Choices

When $\theta^A = \theta^B$, a second-best policy choice will generally involve a non-zero investment subsidy.[13] However, it can be shown that in this case the two sets of first-order conditions become equivalent, implying that the consistent and inconsistent choices coincide:

Proposition 2: *When the profits from abatement activities are uniformly distributed across consumer groups, the optimal time-consistent mix of emission taxes and investment subsidies coincides with the inconsistent choice.*

Thus, when profits shares are equal across consumers (meaning that distributional effects arise only from the consumption side of the economy), not only does an investment subsidy have a distributional role to play, but it effectively eliminates the need for policy commitment. The reason for this result can be more easily understood if one compares the expressions for (32) and (34). The term $(s/2)\partial\hat{N}/\partial t$ in (32) represents a marginal efficiency cost associated with tax-induced changes in investment, which is due to the subsidy driving a wedge between the social and private cost of investment. This term, however, is absent from the short-run optimality condition (34)—under which N is constant—making tax increases relatively more attractive *ex post* (i.e., after N has been installed) on efficiency grounds. In the constant elasticity case with $\theta^A = \theta^B$, for the second-best level of subsidy that satisfies (33), this positive effect exactly offsets the *ex-post* incentive to reduce emission taxes because of the lower tax responsiveness of short-run abatement choices. As a result, the consistent and inconsistent choices are the same. Effectively, the presence of the subsidy makes marginal increases in emission taxes distortionary ex ante but not *ex post*, which can be exploited to bring credibility to the long-run second-best policy choice.

If profit shares are unequal, on the other hand, the above equivalence between the consistent and inconsistent optimal policy mix will not hold even in the isoelastic case. Furthermore, the comparison between the two solutions is generally ambiguous. The only case for which it is possible to obtain an unambiguous prediction is when $\theta^A \neq \theta^B$ and $\delta^A = \delta^B$,[14] and when the tax and subsidy are substitutes:

Proposition 3: *If the consistent optimal policy choice involves positive subsidization of investment, and if the consumption of the polluting good is uniformly distributed across consumers and the tax and subsidy are policy substitutes at the margin, then, were commitment feasible, the policymaker would find it optimal to raise the tax and to lower the subsidy in comparison with the consistent choice.*

The wording used in the above statement should make it clear that this is only a local result, characterizing the policymakers' incentives "around" the inconsistent choice. Under mild monotonicity conditions, this result also applies to the comparison between the consistent and inconsistent choice, i.e., the inability to commit to second-period taxes will result in a higher subsidy and a lower tax relative to the second-best choice.

This outcome can be illustrated with the help of a simple parameterized example. Let $X = 1/2$, $Y^B = 1$, $Y^A = 3/2$, $D(E) \equiv E$, $h(q) \equiv 2q^{1/2}$, $\eta = 1$, $W(U^A, U^B) = U^A U^B$. The first-best, efficient choice is $t = D'(E) = 1$, $s = 0$. Suppose that $\theta^A = \theta^B = 1/2$ and $\delta^A = 1$ (with $\delta^B = 0$); then, the inconsistent and consistent optimal choices are both $\widehat{t} = \widehat{\widehat{t}} \approx 0.46$ and $\widehat{s} = \widehat{\widehat{s}} \approx 0.61$.[15] With $Y^B = Y^A = 1$, $\delta^A = \delta^B = 1/2$ and $\theta^A = 1$ ($\theta^B = 0$), the inconsistent optimal choice is $\widehat{t} \approx 0.89$ and $\widehat{s} \approx 0.11$; while the consistent choice is $\widehat{\widehat{t}} \approx 0.85 < \widehat{t}$ and $\widehat{\widehat{s}} \approx 0.17 > \widehat{s}$.[16]

Intuitively, the difference between this latter case and the case with equal profit shares lies in the fact that subsidies have here a direct effect on distribution (since they directly affect profits and hence the distribution of income), whereas with $\theta^A = \theta^B$ the distributional effect of subsidies is only indirect (through their impact on tax choices). Formally, when $\theta^A \neq \theta^B$, (33) involves an additional negative term ($-(1/2 - \theta^A)N(\partial W/\partial U^A) - (1/2 - \theta^A)N(\partial W/\partial U^B) = -(1/2 - \theta^A)N(\partial W/\partial U^A - \partial W/\partial U^B) < 0$[17]), reflecting a direct distributional cost of marginal subsidy increases. This, in turn, leads to a lower second-best subsidy, and, hence, to ex-ante tax increases having a lower marginal efficiency cost (the term $(s/2)\partial \hat{N}/\partial t$ in (32)) in comparison with *ex-post* marginal tax changes. As a result, the second-best subsidy is insufficient to eliminate incentives to lower taxes *ex post*, and a commitment problem remains.

Finally, if both distributional effects are present, i.e. $\theta^A \neq \theta^B$ and $\delta^A \neq \delta^B$, or if taxes and subsidies are viewed as complementary instruments by the policymaker at the margin, the nature of the consistent solution relative to the first-best efficient policy cannot be characterized in general. Furthermore, the relationship between the consistent and inconsistent choices becomes ambiguous, and it is thus possible for the consistent subsidy to lie below the inconsistent one, with the reverse applying to the tax. This possibility can again be illustrated using our previous parameterized example. If we make $\delta^A = 1$ and $\theta^A = 1$ (with $Y^B = 1$, $Y^A = 3/2$, $\delta^B = 0$ and $\theta^B = 0$), we obtain $\widehat{t} \approx 0.58$, $\widehat{s} \approx 0.52$; and $\widehat{\widehat{t}} \approx 0.62 > \widehat{t}$, $\widehat{\widehat{s}} \approx 0.47 < \widehat{s}$, i.e. the consistent subsidy lies below the corresponding inconsistent level.

To summarize, if investment subsidies are used, and the distributional effects of taxes are restricted to the demand side, consistent and inconsistent policies choices will coincide. Otherwise, consistent and inconsistent choices will diverge, but the conditions under which the consistent choice of subsidy is unambiguously higher than the corresponding inconsistent level—with the reverse being true for the tax—are quite restrictive, even in a model which, admittedly, is already quite restrictive. A more general model, incorporating, for example, endogenous demand choices, income effects, or general substitution patterns between environmental quality and private consumption, would introduce additional dimensions of choice and, potentially, additional sources of ambiguity.

5 Concluding Remarks

This paper has examined how investment subsidies could be used to alleviate distribution-related commitment problems in environmental policies, when pollution abatement has a dynamic dimension.

Investment subsidies may be used in conjunction with emission taxes or abatement subsidies in order to offset the distributional effects of first-best policies. Furthermore, when distributional impacts only involve the consumption side of the economy, our analysis suggests that investment subsidies, thanks to their distortionary effects on long-run investment choices, may be able to fully eliminate the need for policy commitment. In contrast, when environmental taxes and subsidies affect the distribution of income, a dynamic consistency problem in environmental policy choices remains, and the attainment of a second-best policy mix is hindered by a government's inability to commit to future taxes.

Our simple model structure could be extended in several directions. As we have already mentioned, the demand for the polluting good could be made endogenous, and the implications of budgetary constraints in the presence of other distortionary taxes could be considered.[18] Our model could also be augmented by an explicit formalization of incentive-constrained compensation schemes and political choice mechanisms, and our analysis extended to an infinite-horizon setting.[19] Finally, a government's inability or unwillingness to commit could be given a formal foundation as an optimal response to uncertainty about the damage associated with environmental emissions, whereby there exists a positive "option value" in delaying commitment until new information becomes available.

Before concluding, a few remarks are in order with respect to a key premise of our analysis, namely that commitment to future policies is not feasible. Is there indeed a commitment problem in environmental policy making? Experience in both the US and elsewhere—with a tough environmental policy stance by political candidates and incumbents often being followed by a softer line *ex post*—seems to suggest that credibility is indeed a problem for environmental regulators. One could argue, however, that mechanisms for committing to future taxes are available. A possible approach, which is now widely adopted in matters of monetary policy, is to appoint an independent body, relatively detached from the short-run fluctuations of the political process, and transfer decision-making authority to it. Other institutional mechanisms for achieving commitment may relate to the budgeting process, e.g., if revenues from environmental taxes are pre-committed or earmarked (Marsiliani and Renström, 1998).

International agreements may also be instrumental in achieving commitment. There has been considerable debate on the need for international coordination of environmental policies in the presence of transboundary effects or "eco-dumping" through trade and investment. The European Carbon Tax proposal which was tabled in the early 1990s (Agostini *et al.*, 1992; Carraro and Siniscalco, 1993) was a reflection of this debate. Plans for a unilateral but coordinated European response to global warming were subsequently shelved in favour of a global treaty approach, which has resulted in the December 1997 Kyoto agreement, and whereby individual countries agree to country-specific emission cuts to be achieved by independent national policies. Given that such a coordinating agreement has now been reached, is there any scope left for European countries to delegate competence on environmental policy to the center? Our analysis suggests that the answer may be yes. A European carbon tax may still be needed as a means of overcoming a policy commitment problem faced by national governments: without delegation to a centralized and independent institution, emission taxes as a means to support the Kyoto agreement could be fragile.

But even if means of commitment are available, there may be other reasons why they are not used. First, in the presence of technology shocks or other forms of uncertainty (e.g., about the costs and benefits of environmental protection), it may be desirable for environmental regulation to remain flexible. Furthermore, even if commitment is desirable and institutionally feasible, it may not be politically feasible. Commitment to certain policies effectively involves their removal from the political process; if there is disagreement among voters about the desirability of environmental policies, their removal from the electoral debate could damage the very political parties that have been elected on a relatively more environmentally focused platform, by weakening their chances for re-election. Thus, due to the still limited degree of political consensus on environmental issues, this separation may be difficult to achieve at this point.

Notes

[1]Distributional effects are particularly significant in the case of greenhouse emissions, less so with other types of emissions.

[2]For example, achieving compensation through lump-sum transfers (so as not to interfere

with abatement incentives) would require full information about individuals' characteristics. Pirttila (1997) rationalizes the practical difficulties of implementing compensation mechanisms as stemming from asymmetric information and adverse selection. Brett and Keen (1997) view earmarking of environmental taxes—which is practiced both in the United States and in some European countries—as a means of getting around the difficulties associated with compensation.

[3]Parry (1998) examines the efficiency implications of introducing environmental subsidies when tax distortions are present.

[4]As will be discussed later, q can vary depending on the presence of investment subsidies; for the time being, we shall take q as exogenous.

[5]Assuming a fixed level of demand for the polluting good will suffice for establishing our results. As we note later, the endogenization of demand choices would introduce further dimensions of choice that would combine with those described in our analysis.

[6]In a model with explicit information-related constraints, this would correspond to the extreme scenario where the optimum mechanism is one that supports a pooling equilibrium; in less extreme cases, separation may be feasible but costly, implying that incentive-compatible optimal schemes may involve less than full compensation. We should note that in the absence of any restrictions on the form of compensation (i.e., under an arbitrary non-linear mechanism), a second-best solution may not call for the disruption of production efficiency—which in this context means selecting the Pigouvian level of taxation (Cremer and Gahvari, 1999)—but this will not generally be the case under more restrictive schemes.

[7]This objective can alternatively be interpreted as reflecting political support in a probabilistic voting framework (see, for example, Coughlin and Nitzan, 1981).

[8]Formally, the consistent choice is a subgame perfect equilibrium strategy for a three-stage game where first investment decisions are made and then the policymaker selects a tax rate, after which firms reduce emissions; whereas the inconsistent choice is an equilibrium strategy for a game structure in which the order of actions for the first two stages of the game is reversed.

[9]This implies that the distribution of welfare is not necessarily monotonic in t (since, by construction, with $t = 0$ utility levels are identical across consumers, and, in a neighborhood of $t = 0$, utility levels are becoming more unequal as t increases).

[10]Industrialized countries that offer tax incentives for pollution control investments include Japan, Korea, Taiwan, France, Germany, Netherlands, and Canada. Such incentives consist mainly of accelerated depreciation, investment credits, partial expensing, exemptions and deferrals. See, for example, Jenkins and Lamech (1992) and OECD (1994).

[11]This can be shown by solving for $D'(X - V) - t$ from (32) and substituting the resulting expression into (33).

[12]The consistent choice is a subgame perfect equilibrium strategy for a four-stage game where the subsidy is selected before firms invest (in the first period), and the tax rate is selected before firms abate (in the second period); the inconsistent choice is an equilibrium strategy for a scenario where both the subsidy and the tax are selected first.

[13]When profit shares are identical, an *abatement subsidy* can achieve a first-best outcome. As we discussed in the previous section, such a subsidy is equivalent to an emission tax with $\delta^A = \delta^B$; hence, if we also have $\theta^A = \theta^B$, an abatement subsidy generates no distributional

effects and there is no need for an investment subsidy (when $\delta^A = \delta^B$ and $\theta^A = \theta^B$ a choice of $t = D'(X - V)$ and $s = 0$ solves both sets of first-order conditions).

[14]As mentioned above, this case is also equivalent to a scenario where abatement subsidies are used; in this scenario, as long as $\theta^A \neq \theta^B$, the direct subsidy will be supplemented by an investment subsidy.

[15]All numerical values were found using numerical optimization techniques.

[16]Although we find that the sign of the expression $\partial\widehat{\widehat{t}}/\partial s$ at an optimum with $\theta^A = \theta^B$ is ambiguous, we have performed systematic sensitivity analysis with a constant-elasticity-of-substitution social welfare function and did not encounter any case where the expression is positive.

[17]If $(1/2 - \theta^A) > 0$ then $U^A < U^B$, implying $(\partial W/\partial U^A - \partial W/\partial U^B) > 0$; similarly, $(1/2 - \theta^A) < 0$ implies $(\partial W/\partial U^A - \partial W/\partial U^B) < 0$.

[18]In practice, revenue consideration are likely to be important for the choice of policy instruments: since subsidies generate a negative revenue, the presence of a premium on public funds would make them less attractive.

[19]The distributional impacts of emission taxes stem in part from the existence of short-term adjustment costs (e.g., the displacement of workers from adversely affected sectors), and are therefore less severe over the long run.

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Appendix

Proof of Lemma 1: Since $\Gamma > 0$, we have

$$\frac{\partial \widehat{\widehat{V}}}{\partial t} = \frac{1}{H_{VV} + \Gamma} < \frac{1}{H_{VV}} = \frac{\partial \widehat{V}}{\partial t}. \tag{37}$$

Proof of Proposition 1: For a given fixed level of N, social welfare can be written as a function of N and the tax rate t, i.e. $\widehat{\widehat{W}}(t, N)$. If we allow the choice of N to respond to changes in t, we can write $\widehat{\widehat{W}}[t, \widehat{N}(t, 1)] \equiv \widehat{W}(t)$, and

$$\widehat{W}_t = \widehat{\widehat{W}}_t + \widehat{\widehat{W}}_N \widehat{N}_t. \tag{38}$$

The second-order conditions for (22) and (24) to identify an optimum are

$$\widehat{\widehat{W}}_{tt} < 0; \tag{39}$$

$$\widehat{W}_{tt} = \widehat{\widehat{W}}_{tt} + 2\widehat{\widehat{W}}_{Nt}\widehat{N}_t + \widehat{\widehat{W}}_N \widehat{N}_{tt} < 0. \tag{40}$$

Consider a convex combination of $\widehat{W}(t)$ and $\widehat{\widehat{W}}(t, N)$:

$$\alpha \widehat{W}(t) + (1 - \alpha)\widehat{\widehat{W}}(t, N); \tag{41}$$

with $0 < \alpha < 1$. Suppose we maximize the above by choice of t. The first-order condition for an interior optimum is

$$\alpha \widehat{W}_t(t) + (1 - \alpha)\widehat{\widehat{W}}_t(t, N) = 0. \tag{42}$$

Combining (42) with the forwarding-looking condition $N = \widehat{N}(t, q)$, and totally differentiating (42) and re-arranging terms yields

$$\frac{\partial t}{\partial \alpha} = \frac{\widehat{\widehat{W}}_t - \widehat{W}_t}{\alpha \widehat{W}_{tt} + (1 - \alpha)\widehat{\widehat{G}}_{tt}}, \tag{43}$$

where

$$\widehat{\widehat{G}}_{tt} = \widehat{\widehat{W}}_{tt} + \widehat{\widehat{W}}_{tN}\widehat{N}_t = \widehat{W}_{tt} - (\widehat{\widehat{W}}_{tN}\widehat{N}_t + \widehat{\widehat{W}}_N \widehat{N}_{tt}). \tag{44}$$

Using (22) and (24), the numerator of (43) can be written as

$$\widehat{\widehat{W}}_t - \widehat{W}_t = -\widehat{\widehat{W}}_N \widehat{N}_t = \frac{D'(X-V)-t}{2}\left(\frac{\partial W}{\partial U^A} + \frac{\partial W}{\partial U^B}\right)\left(\frac{\partial \widehat{\widehat{V}}}{\partial t} - \frac{\partial \widehat{V}}{\partial t}\right). \tag{45}$$

By Lemma 1, $\partial\widehat{V}/\partial t > \partial\widehat{\widehat{V}}/\partial t$. This implies that, if $D'(X-V)-t > 0$, (45) is negative, and, since $\widehat{N}_t$ is positive (from (8)), that $\widehat{\widehat{W}}_N$ is also positive. Differentiation of (22) with respect to N gives $\widehat{\widehat{W}}_{tN} > 0$, while differentiating (8) it can be readily seen that $N_{tt} > 0$. Since $\widehat{N}_t$ and $\widehat{\widehat{W}}_N$ are also both positive, we can conclude that $\widehat{\widehat{G}}_{tt}$ is negative, and so is the denominator of (43). Hence $\partial t/\partial\alpha > 0$. Condition (42) says that the optimal tax is given by $\widehat{\widehat{t}}$ when $\alpha = 0$, or $\widehat{t}$ when $\alpha = 1$. Since $\partial t/\partial\alpha > 0$ for $0 < \alpha < 1$, we can conclude that $\widehat{t} > \widehat{\widehat{t}}$.

Proof of Proposition 2: For $\widehat{t}$ to be equal to $\widehat{\widehat{t}}$, the first-order conditions (32) and (34) must be equivalent. With $\theta^A = \theta^B$, from (33), we find the optimal subsidy to be

$$s = \frac{[D'(X-V)-t]\ \partial\widehat{V}/\partial s}{\partial\widehat{N}/\partial s}. \tag{46}$$

Substituting this into (32), and using (34), one finds that equivalence between conditions (32) and (34) implies

$$\frac{\partial\widehat{V}}{\partial t} - \frac{\partial\widehat{\widehat{V}}}{\partial t} = \frac{\partial\widehat{V}}{\partial s}\,\frac{\partial\widehat{N}/\partial t}{\partial\widehat{N}/\partial s}. \tag{47}$$

Using (7), (14), (30) and (31), it can be readily seen that the above equivalence condition is indeed satisfied in the constant-elasticity case.

Proof of Proposition 3: Subtracting (34) from (32) and rearranging yields

$$\widehat{-}_t \equiv \frac{1}{2}\left(\frac{\partial W}{\partial U^A} + \frac{\partial W}{\partial U^B}\right)\left\{[D'(X-V)-t]\left(\frac{\partial\widehat{V}}{\partial t} - \frac{\partial\widehat{\widehat{V}}}{\partial t}\right) + s\frac{\partial\widehat{N}}{\partial t}\right\}. \tag{48}$$

If $s > 0$, given that $\frac{\partial\widehat{N}}{\partial t} > 0$ and $D'(X-V) > t$ (from (32)), and given Lemma 1, this expression is always positive. From (35), the optimal consistent choice of subsidy implies $\widehat{\widehat{-}}_s = 0$, and hence

$$\widehat{-}_s = -\widehat{-}_t \frac{\widehat{\widehat{-}}_{ts}}{\widehat{\widehat{-}}_{tt}}. \tag{49}$$

Since $\widehat{-}_t > 0$, a positive $\widehat{\widehat{-}}_{ts}$ implies $\widehat{-}_s > 0$.