# Time, Self-Selection and User Charges for Public Goods

by

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## Abstract

Many public goods generate utility only when combined with a time-input. Important examples include road networks and publicly provided leisure facilities. If it is possible to charge for the time spent using the public good it is generally a second-best Pareto optimal policy to do so even in the absence of congestion. An optimal linear user charge is analysed within a standard optimum income-tax framework. Second-best public good provision in the presence of a user charge is also characterized and factors that in‡uence the direction of optimal distortion of the public good supply are identi..ed.



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# 1 Introduction

Why are there fees for entering most publicly run museums? Marginal costs are essentially zero, and crowding rarely seems to be a problem. Moreover, it seems that the distributional effects of such fees are, at best, doubtful. Similarly, charges for entering national parks and municipal swimming pools seem hard to justify on efficiency grounds using received economic theory. Granted, there is an argument that each activity should 'bear its own cost', but this notion has little support in economic theory when dealing with public goods. Relatedly, the 'benefit principle', which states that public goods should be paid for by people enjoying their benefits, does not call for introducing charges at the margin.

In this paper, we suggest a rationale for such user charges. This rationale stems from the fact that 'consumption' or 'use' of public goods often requires that individuals devote scarce time to it. A user charge based on the chosen time inputs then alters the trade-offs faced by individuals when dividing their time endowment among various activities. User charges in effect amount to taxing the time spent using the public goods; since time spent working is taxed indirectly through the income tax system, there is an obvious second-best argument for taxing this time use as well. In more precise terms, it turns out that in an optimum-tax framework, taxing the time spent using public goods relaxes the incentive constraint that highly productive workers not shirk in order to obtain the more attractive tax-benefit package intended for less productive workers.

The mechanism introduced may be important in a wide variety of applications. One example is charges for the use of public roads, be that either directly through tolls or indirectly by means of fuel taxes. In the case of roads there is a congestion argument for charges, but the mechanism alluded to provides an additional and independent rationale for user charges.[1] Another set of examples—appearing in current policy discussion—include public pay-TV channels and charges for use of the internet, cases in which charges are naturally time-related: Why should, e.g., the BBC charge for its satellite channels when the service is not congested?

We will explore a model with a public good that requires time for its use. The public good is excludable—so that the time devoted to it can be charged for—but not necessarily congestionable. Our results will thus not rely on congestion externalities. We ask under what conditions a charge for the use of the good is welfare-improving in the presence of a general

[1] Note that fuel taxes which are commonly viewed as the most important form of charge for road use is a very poor anti-congestion measure since it is not specific about time and place.

income tax whose optimal design is subject to self-selection constraints. A charge turns out to be desirable under a strikingly simple and mild condition. We also characterize the optimal charge and discuss its links with some well-known results on commodity taxation. From this perspective, this paper can be seen as a contribution to the literature on user charges for publicly provided goods. Previous attempts at identifying the economic rationale behind user charges for publicly provided excludable goods include e.g. Besley (1991)? and Munro (1992) ?. These papers focus on optimal linear income tax systems and suggest that user charges are socially desirable as the social planner can employ them for improving the income distribution.

A second question addressed is whether the presence of a charge a¤ects the optimal provision of public goods. Boadway and Keen (1993) argued that public-good provision should in general deviate from the Samuelson rule in the presence of a general income tax, this in order to account for self-selection e¤ects. We demonstrate an extended version of that modi…cation, and discuss an example which gives some insight into the direction in which the provision of the public good is optimally distorted. The paper is thus also intended as a contribution to the literature on optimal public good provision in second-best environments. This is an issue that has recently been studied by e.g. Wilson (1991),?, Arnott (1994),? Mirrlees (1994)? and Kaplow (1996)?. Following the seminal contribution by Atkinson and Stern (1974), the focus in this literature is on the question whether public goods should under- or over-provided compared to the …rst-best.

The paper is structured as follows. Section 2 sets forth the model and presents the …rst-best scenario. Section 3 illustrates second-best policy: the instruments include the general income tax, the level of provision of the public good, and, provided the use of the latter is anonymously observable, a linear charge. Section 4 discusses the condition under which a charge is welfare-improving and also derives the optimal charge. Section 5 is concerned with the optimal public good provision. Section 6 presents several extensions of the model. Finally, section 7 sums up.

## 2 A Simple Model

Each household has $T$ hours available which are devoted to labour, $l$; pure leisure, $L$, and to the consumption of a public good, $t$. An amount $g$ of the public good is made available by the government. The '‡ow of services' that a household obtains from the public good depends on $g$ and the time input $t$; and is denoted $h(t; g)$. In addition there is a private good $x$ which for simplicity does not require a time input.[2] The households have identical preferences over the private good, the services from the public good, and over leisure $U(x, h, L)$. The public good and the private good are produced with labour as only input and with a simple linear technology. Units are chosen so that $g$ and $x$ both have producer prices equal to unity. A household's wage rate $w$ is the effective labour supplied per hour of labour. There are two types of households, a household's type being defined by its wage rate $w$. The wage rates are $w_i$, $i = 1, 2$ with $w_1 < w_2$, for low- and high-ability households. There are $n_i$ households of type $i$.

## First-best

Throughout we study properties of Pareto optimal arrangements. As a point of reference we first describe first-best. A first-best Pareto optimal allocation is a solution to

$$\begin{aligned} \max \quad & U(x_1, h(t_1, g), T - l_1 - t_1) \\ \text{s.t.} \quad & U(x_2, h(t_2, g), T - l_2 - t_2) \geq \overline{U} \qquad (\mu) \\ & n_1 x_1 + n_2 x_2 + g = n_1 w_1 l_1 + n_2 w_2 l_2 \quad (\lambda) \end{aligned} \tag{1}$$

where $l_1, l_2, t_1, t_2, x_1, x_2$ and $g$ are the objects of choice. From the first order conditions one obtains the 'Samuelson rule' (where subscripts denote derivatives),

$$n_1 \frac{U_h^1 h_g^1}{U_x^1} + n_2 \frac{U_h^2 h_g^2}{U_x^2} = 1, \tag{2}$$

which states that the sum of the marginal rates of substitution between the public good and the private good should equal the marginal rate of transformation. We also obtain the first-best rules for the choice of working hours and the allocation of non-working hours

$$U_L^i / U_x^i = w_i, \quad \text{and} \quad U_h^i h_t^i = U_L^i, \quad \text{for } i = 1, 2. \tag{3}$$

This allocation could, if type was observable, be implemented by using type-specific income transfers for redistribution and for financing the public good. Time spent using the public good does not generate any externality (there being no congestion). Hence there should be no user charge. Neither should there be a tax on the private good or on labour income.

[2] The setup draws heavily on Ebrill and Slutsky (1982).? However, to focus the analysis we assume that private consumption is not time demanding, and we assume that there is no congestion. Both these extensions are discussed in the section 6.

## 3 Second-Best Policy

Which policies are feasible depends on what the government can observe. We will consider two scenarios. The first scenario is a benchmark case where use of the public good is not observed, whence no charge can be levied. The second and main case it that where the government observes 'anonymous' purchases of $x$ and 'anonymous' use of the public good: That the transactions are 'anonymous' means that the identity of the consumer is not observed, which makes the private good and use of the public good suitable for linear taxation/subsidization..

The government thus determines a consumer price $p$ for the private good and a user charge $q$ for the public good. The government also observes pre-tax income, $y$, and sets a tax function that maps pre-tax income to disposable income, $b$. Focusing on self-selection the government can be seen as determining two pairs of pre-tax- and disposable incomes $(y_i, b_i)$, $i = 1, 2$. The government also determines the level of the public good, $g$.

### Consumer Optimization

It is convenient to divide the consumer's optimization problem into two steps. The first consists of choosing labour supply. Given the income tax schedule ($T(\cdot)$ below), this also yields a value for the disposable income $b$.[3] The second step consists of allocating the disposable income between private consumption and use of the public good. In the latter step the consumer faces a budget constraint $px + qt = b$. Since $p, q$ as well as $b$ will be policy variables for the government this indicates that a normalization will be necessary. It is convenient to assume that the private good is untaxed, i.e. we set $p \equiv 1$. Solving the second step of the consumer's problem then gives the following partially indirect utility

$$u(q, g, b, l) \equiv \max_{x,t} \{U(x, h(t, g), T - l - t) \mid x + qt = b\}. \tag{4}$$

This problem also defines (conditional) demands $x(q, g, b, l)$ and $t(q, g, b, l)$. As usual, it is convenient to write these functions with the observable variables as argument. Noting that $y = wl$ and slightly abusing the notation we can write the partially indirect utility

$$u(q, g, b, y, w) = u\left(q, g, b, \frac{y}{w}\right). \tag{5}$$

The same rewriting will be used for the demands. Some properties will be useful:

$$u_q = -\alpha t; \qquad u_b = \alpha; \qquad u_g = U_h h_g; \quad \text{and} \quad u_y = -U_L/w. \tag{6}$$

[3] As the government's problem is set up, the consumer is essentially choosing only between two levels of labour supply; the solution can, however, be implemented by a tax schedule, albeit not necessarily a differentiable one.

where $\alpha$ is the multiplier associated with the budget constraint. This shows that 'Roy's identity' holds for the time input, $u_q = -u_b t$.

For future reference we will at this stage also introduce the dual of problem (4). Define

$$e(q; g; \bar{u}; y; w) \equiv \min_{x;t} \{x + qt \mid U(x; h(t; g); T - y/w - t) \geq \bar{u}\}; \tag{7}$$

This problem also defines conditional compensated demands $x^c(q; g; \bar{u}; y; w)$ and $t^c(q; g; \bar{u}; y; w)$. There are two relations between the uncompensated and the compensated demands that we will use frequently. The first is the usual Slutsky equation, while the second relation is a bit more non-standard, but straightforward to demonstrate; in terms of the time-input:

$$\frac{\partial t^c}{\partial z} = \frac{\partial t}{\partial z} - \frac{u_z}{u_b}\frac{\partial t}{\partial b}; \quad z = g; y. \tag{8}$$

Intuitively, the r.h.s. is the effect on $t$ of a change in $z$ when accompanied by a change in disposable income that just keeps the consumer's utility constant.

### The Government's Problem

Since the indirect utility function $u(q; g; b; y; w)$ gives a consumer's utility in terms of the government's policy variables we are now ready to consider the government's problem of finding an optimal policy. We focus on what Stiglitz (1982) ?termed the 'normal case'; i.e., the case where the planner wishes to redistribute income to low-ability households. This is clearly the main case although one can construct reasonable circumstances under which the opposite is true; while many of our results would reverse in that case, the method of analysis of this paper is valid for the other case as well.[4] Further we assume that a single-crossing condition is satisfied by the indifference curves of $u(\cdot)$ in $(y; b)$-space; $|u_y/u_b|$ is assumed to be decreasing in type, $w$. This ensures that at most one self-selection constraint binds at any solution. Note that—in contrast to the simplest optimum-tax model—this condition is imposed on the indirect utility function. It is straightforward, albeit tedious, to show that the condition can be phrased in terms of $U(\cdot)$; doing so, the assumption seems perfectly reasonable, although sufficient conditions are rather strong.

[4] The case where the opposite may hold is one where the high-ability type works more, and where this calls for monetary compensation. Even in the simplest case (one private good, only labour/leisure) with a utilitarian social objective, a very strong condition (additively separable utility) is needed to rule it out; as the model is enriched by making time use a choice variable, new interactions are introduced, and even stronger conditions are required.

A high-ability household that 'misrepresents' its type is referred to as a 'mimicker'—variables pertaining to mimickers are distinguished by the accent $\hat{}$. The government's problem is to

$$\begin{array}{lll} \max & u(q,g,b_1,y_1,w_1) & \\ \text{s.t.} & u(q,g,b_2,y_2,w_2) - \bar{u} \geq 0, & (\mu) \\ & u(q,g,b_2,y_2,w_2) - u(q,g,b_1,y_1,w_2) \geq 0, & (\phi) \\ \text{and} & \sum n_i (y_i - b_i + q t_i) - g \geq 0. & (\lambda) \end{array} \tag{9}$$

by choice of $q, g, b_i, y_i$. We will perform the analysis in two steps—first we describe the optimal policy conditional on $q$, and then we consider the usefulness of a user charge.

Differentiating w.r.t. $y_i, b_i$, and $g$ gives the first order conditions,

$$u_y^1 - \phi \hat{u}_y^2 + \lambda n_1 \left( 1 + q \frac{\partial t_1}{\partial y_1} \right) = 0, \tag{10}$$

$$u_b^1 - \phi \hat{u}_b^2 + \lambda n_1 \left( -1 + q \frac{\partial t_1}{\partial b_1} \right) = 0, \tag{11}$$

$$(\mu + \phi) u_y^2 + \lambda n_2 \left( 1 + q \frac{\partial t_2}{\partial y_2} \right) = 0, \tag{12}$$

$$(\mu + \phi) u_b^2 + \lambda n_2 \left( -1 + q \frac{\partial t_2}{\partial b_2} \right) = 0, \tag{13}$$

$$u_g^1 + (\mu + \phi) u_g^2 - \phi \hat{u}_g^2 + \lambda \left( q \sum_i n_i \frac{\partial t_i}{\partial g} - 1 \right) = 0. \tag{14}$$

The usual caveat applies; the problem is generally not well–behaved. Hence we can only suppose the existence of a solution in which $\lambda$ and $\phi$ are both positive.

### The Income Tax Schedule

At this stage we can give a brief characterization of the income tax schedule. Consider first the high-ability households. Let $y - b = \Theta(y)$ be the income tax, and recall that $\Theta'(y) = u_y/u_b + 1$ is the implicit marginal income tax rate (see Stiglitz (1982)). The total tax liability is

$$\tau(y) = \Theta(y) + q\, t(q, g, y - \Theta(y), y, w). \tag{15}$$

Differentiating and using the definition of the marginal income tax rate, we obtain the following expression for the marginal effective tax rate,

$$\tau'(y) = 1 + \frac{u_y}{u_b} + q \left( \frac{\partial t}{\partial y} - \frac{u_y}{u_b} \frac{\partial t}{\partial b} \right). \tag{16}$$

Note that from (8) this means that $T'(y) = \tau'(y) - q(\partial t^c/\partial y)$. Starting with the high-ability households, combining (12) and (13) immediately yields that, at the second-best optimum, $\tau'(y_2) = 0$; i.e. these households should face a zero marginal effective tax rate (this is in accordance with Edward et al. (1994)). Proceeding then to the low-ability households, manipulating (10) and (11) in the way similar to Edwards et al. (1994) gives that $\tau'(y_1) = (\phi^*/n_1)\left(\hat{u}_y^2/\hat{u}_b^2 - u_y^1/u_b^2\right)$; where $\phi^* = \phi\hat{u}_b^2/\lambda$. We can summarize these results in terms of the marginal income tax rates: given a user charge $q$, Pareto efficient taxation requires that

$$T'(y_1) = -q\frac{\partial t_1^c}{\partial y} + \left(\frac{\phi^*}{n_1}\right)\left(\frac{\hat{u}_y^2}{\hat{u}_b^2} - \frac{u_y^1}{u_b^2}\right); \tag{17}$$

and

$$T'(y_2) = -q\frac{\partial t_2^c}{\partial y}. \tag{18}$$

By the single-crossing assumption the second term on the r.h.s. of (17) is positive. Hence, when $q = 0$, we obtain the standard result that the high-ability households should face a zero marginal income tax rate, while the low-ability households should face a positive marginal income tax rate.

When $q$ is different from zero, a second component appears. This term captures the revenue generated by the charge. In particular, if $q$ is positive we see that both types will face positive marginal income tax rates if $\partial t^c/\partial y < 0$. This is indeed likely to be the case; a consumer faces two constraints, a budget constraint and time constraint. A compensated increase in $y$ is simply an increase in working hours, i.e. a tightening of the time constraint, accompanied by an increase in disposable income such that the consumer's utility is unaffected. The direct effect of longer working hours on $t$ is negative if, plausibly, $t$ is normal in available non-working hours. The compensation on the other hand has a positive indirect effect on $t$ if $t$ is normal in disposable income. As long as the hours of work effect dominates the disposable income effect, $\partial t^c/\partial y$ will indeed be negative. Intuitively, when this condition holds, restricting the labour supplies while adjusting the disposable incomes increases the time spent using the public good. With $q > 0$ this generates revenue which justifies a downward distortion of the labour supplies.

### Non-observability of the Time Input

An important reference case is that where, due to non-observability of the time input $t$, no charge can be levied, i.e. $q = 0$. It then only remains for the government to determine the level of public good supply. In this case the public good should be supplied according to Boadway and Keen's (1993)? modification of the Samuelson rule. To see this set $q = 0$ in the first order conditions. By extending (14) and substituting from (11) and (13) one obtains that at the optimum,

$$n_1 \frac{u_g^1}{u_b^1} + n_2 \frac{u_g^2}{u_b^2} = 1 + \frac{\varphi \hat{u}_b^2}{\lambda} \left[ \frac{\hat{u}_g^2}{\hat{u}_b^2} - \frac{u_g^1}{u_b^1} \right]. \tag{19}$$

This is the modification derived by Boadway and Keen (1993) (to compare with (2) recall that $u_g = U_h h_g$ and $u_b = U_x$). It shows that whether the sum of the households' marginal rates of substitution should exceed or fall short of the marginal rate of transformation depends on who has the largest marginal willingness to pay for $g$, a mimicker or a low-ability household. Boadway and Keen provide an intuition for this result which is equally valid in this case.

That (19) holds indicates a certain robustness of the Boadway and Keen modification—while it was originally derived with the public good being a final consumption good it extends to the current case where the public good is an 'intermediate' good in the consumption process.

# 4 Observability of Time Input and a User Charge

Turning now to the case where use of the public good can be charged for we start by identifying a simple condition which guarantees that the introduction of a user charge allows a Pareto improvement.

### The Introduction of a User Charge

To check when a user charge is Pareto improving we apply the envelope theorem on the government's problem (9). Differentiating the associated Lagrangian w.r.t. $q$ we obtain

$$W'(q) = u_q^1 + (\mu + \varphi)\, u_q^2 - \varphi \hat{u}_q^2 + \lambda \sum_i n_i \left( t_i + q \frac{\partial t_i}{\partial q} \right). \tag{20}$$

Using Roy's identity to replace the derivatives of the indirect utilities we have

$$W'(q) = -u_b^1 t_1 - (\mu + \varphi)\, u_b^2 t_2 + \varphi \hat{u}_b^2 \hat{t}_2 + \lambda \sum_i n_i \left( t_i + q \frac{\partial t_i}{\partial q} \right). \tag{21}$$

When evaluating at $q = 0$, using (11) and (13), and simplifying, we find that

$$W'(0) = \varphi \hat{u}_b^2 \left( \hat{t}_2 - t_1 \right). \tag{22}$$

Since $\varphi$ and $\hat{u}_b^2$ are both strictly positive we thus have the following strikingly simple result:

Claim 1. The introduction of a positive user charge is Pareto improving if $\hat{t}_2 > t_1$.

The condition in the claim is very likely to hold. A mimicker differs from a low-ability household only by having more non-working hours to allocate. In particular, since there is initially no charge they both consume the same amount of the private good. If in terms of the consumer's problem (4), $t$ is increasing in $T - l$; i.e. if the chosen time input $t$ is normal in available non-working hours, then $\hat{t}_2$ will exceed $t_1$ and the introduction of a user charge allows a Pareto improvement.

The intuition for the result is straightforward. When a charge is introduced it generates revenue. Suppose that the revenue from both types, 1 and 2, are returned in a lump-sum fashion through corresponding increases in $b_1$ and $b_2$. Since there is initially no charge this leaves both types, as well as the government's budget constraint, unaffected. However, if $\hat{t}_2$ exceeds $t_1$ a mimicker will pay larger total charges than type 1. In particular, the adjustment of $b_1$ (which corresponds to the total charges paid by type 1) will not be sufficient to compensate the mimicker for the imposition of the charge. Hence the mimicker will be made worse off and the self-selection constraint will be relaxed. This enables a Pareto improvement through a restructuring of the income tax schedule.

Note that in deriving this result we did not make use of the optimality of the public good provision. Indeed, this result would go through even if the level of the public good was exogenously fixed. In particular, achieving a Pareto improvement does not require that the charge is accompanied by a modification of the supplied level of the public good.

## An Optimal User Charge

There is now a developed theory for how commodity taxes can supplement a non-linear income tax. In this section we demonstrate that the optimal charge is isomorphic in structure to such optimal supplementing commodity taxes.

A necessary condition that an optimal user charge must satisfy is that $W'(q)$ expressed in (21) is equal to zero. After substituting for $u_b^1$ and $(\mu + \phi)\, u_b^2$ using (11) and (13), collecting terms we find that $W'(q) = 0$ requires that

$$\lambda q \sum_i n_i \left( \frac{\partial t_i}{\partial q} + \frac{\partial t_i}{\partial b_i} t_i \right) + \phi u_b^2 \left( \hat{t}_2 - t_1 \right) = 0. \tag{23}$$

Finally, using the Slutsky relation we can state the following claim:

**Claim 2.** A Pareto efficient user charge, supplementing an optimal income tax, satisfies

$$\sum_i n_i\, q \frac{\partial t_i^c}{\partial q} = \frac{\phi u_b^2}{\lambda} \left( t_1 - \hat{t}_2 \right). \tag{24}$$

This rule is familiar from the optimal taxation literature and it corresponds directly to the rule for optimal commodity taxation in the presence of an optimal income tax. Indeed, the derivation closely follows the derivation of the commodity tax rules reported by Edwards et al. (1994) ? and Nava et al. (1996)?; it is essentially a corollary of their results. Moreover, it is well-known that the left-hand side of (24) can be interpreted as a first-order approximation to the change in aggregate compensated use of the public good due to the charge.

Noting that the expenditure function $e$ is concave in $q$ for the usual reasons it follows that the own price effect for the compensated time input $\partial t^c/\partial q$ is negative. The optimal charge is therefore positive precisely if $\hat{t}_2 > t_1$; a sufficient condition for this is that, in terms of problem (4), $t$ is strictly increasing in $T - l$. Thus we have:

**Corollary 3.** If the chosen time input $t$ is normal in available non-working hours $T - l$, then a Pareto optimal user charge is strictly positive.

This shows that the local result on the desirability of a charge carries over as a global result; this is somewhat remarkable since there is an almost complete lack of global results concerning optimal provision level of public goods (this is discussed in the next section).

# 5 Public Good Provision Revisited

So far we have explored the usefulness of a user charge. It remains to consider how a charge feeds back on the provision of the public good. We saw above that when no charge can be levied, then the second-best provision of the public good generally differs from the first-best Samuelson rule (2). Indeed, the Boadway-Keen modification (19) shows how such a departure is justified inasmuch as the public good offers 'screening power'. We now ask whether a user charge affects the provision rule and, if so, in what direction. Of course, we apply to our argument the usual proviso: the analysis does not permit us to make conclusive statements concerning the relative levels of the public good, because, like in most optimal policy problems, it is based on a manipulation of the first-order conditions without any guarantee that the standard concavity properties hold.[5]

Repeating the steps of the derivation of (19) gives an expression for the optimal public good

[5] Level comparisons can be made if specific functional forms are assumed; see for instance, Atkinson and Stern (1974) and Wilson (1991).??

provision which applies with a user charge and which incorporates revenue effects,

$$n_1 \frac{u_g^1}{u_b^1} + n_2 \frac{u_g^2}{u_b^2} = 1 + \frac{\phi \hat{u}_b^2}{\lambda}\left(\frac{\hat{u}_g^2}{\hat{u}_b^2} - \frac{u_g^1}{u_b^1}\right) - q\sum_i n_i \left(\frac{\partial t_i}{\partial g} - \frac{u_g^i}{u_b^i}\frac{\partial t_i}{\partial b_i}\right). \tag{25}$$

By (8) the last parenthesis is just the derivative of the conditional compensated time input w.r.t. $g$. Thus we arrive at the following characterization:

Claim 4. In the presence of an optimal income tax and a user charge $q$ for use of the public good, Pareto efficient provision of the public good requires that:

$$n_1 \frac{u_g^1}{u_b^1} + n_2 \frac{u_g^2}{u_b^2} = 1 + \frac{\phi \hat{u}_b^2}{\lambda}\left(\frac{\hat{u}_g^2}{\hat{u}_b^2} - \frac{u_g^1}{u_b^1}\right) - q\sum_i n_i \frac{\partial t_i^c}{\partial g}. \tag{26}$$

To understand this result, start from a situation where the Samuelson rule applies and assume that initially there is no user charge. Consider then changing the public good supply by a small amount $dg$. Simultaneously change the disposable incomes according to the marginal willingness to pay, $db_i = -\left(u_g^i/u_b^i\right) dg$, $i = 1, 2$. This leaves the utilities of type 1 and 2 as well as the government's budget unaffected. However, when the mimicker's marginal willingness to pay differs from that of type 1, the mimicker's utility will be affected. By deviating from the Samuelson rule, the self-selection constraint can then be relaxed. This motivates the second term on the right hand side (see Boadway and Keen (1993)). If in addition there is a charge, the same mechanism applies, only the compensated change in $g$ will in general not be revenue neutral. Thus the revenue effect has to be taken into account; this is the last term of (26).

We want to compare this provision rule with the first-best rule (2) and the modification (19) due to Boadway and Keen. To do this we introduce some additional terminology. If, at the optimum, the sum of the marginal rates of substitution exceed (falls short of) 1, then we say that the public good is 'S-underprovided' ('S-overprovided'), as short for 'underprovided relative to the Samuelson rule'. If, at the optimum, the sum of the marginal rates of substitution exceed (falls short of) the first two terms on the r.h.s. of the (26), then we say that the public good is 'BK-underprovided' ('BK-overprovided'), as short for 'underprovided relative to the Boadway-Keen rule'. To gain some insight into which direction the public good provision is optimally distorted, we first discuss weakly separable preferences and then turn to a more specific case with an additively separable utility function.

### Weak Separability and Public Good Provision

Weak separability is a property of utility functions which plays an important role in the theory of optimal public policy.[6] Boadway and Keen (1993) showed that with a weakly separable utility function of the form $U(Z(g; x); L)$, the Samuelson rule applies at the second-best optimum. Intuitively, in that case, the di¤erence in leisure enjoyed by the mimicker and the low-ability individual does not a¤ect the marginal rate of substitution between $g$ and $x$, so the public good has no screening power. This result is not robust to the generalization of the environment considered in this paper.

Just as in the Boadway-Keen model, a mimicker has in the current model more non-working hours available than a low-ability household. However, when decomposed into two uses—use of the public good and pure leisure—non-working hours will generally not be separable from the public good. In particular, even if leisure enters separably in the utility function there is generally scope for two forms of redistributive policy to supplement the non-linear income tax:

1. if the time-input $t$ is not observable the planner can distort the provision of $g$— following the Boadway-Keen modi…cation (19)—exploiting the di¤erences in preferences for the public good that results from the consumers' control over the allocation of non-working hours;
2. if the time input $t$ is observable, it can be viewed as a 'signal' of the true ability type and the planner can devise a charge which distorts the consumers' allocation of non-working hours. The planner can also distort the public good provision, now following (26).

### Separability and Public Good Provision with Non-observable Time Inputs

Having said this, we proceed to develop an example with a separable utility function so as to gain some insights into which direction the public good provision is optimally distorted. Thus assume that the consumers' preferences can be written in the form:

$$U(x; h(t; g); L) = v(x; h(t; g)) + f(L) \tag{27}$$

[6] The importance of weak separability was …rst noted in the direct versus indirect taxation controversy by Atkinson and Stiglitz (1976)? and Mirrlees (1976).?

where $f$ is increasing and concave, $v_x > 0$; $v_h > 0$ and, importantly, $v_{hh} = 0$.[7] Leisure is $L = T - t - l$ as before. The specification excludes complementaries with leisure which typically are the focus in the literature. Instead, as the specification indicates, we focus on two potential sources of complementarities; that between private consumption $x$ and services from the public good $h$; and that between the two inputs in the production of $h$, i.e. the time input $t$ and the public good $g$.

We consider first the case where, due to non-observability of $t$, no charge can be imposed. In that case $q = 0$ and $x = b$. It is then straightforward to show that the chosen time input $t$ is normal in available non-working hours. This implies that $\hat{t}_2 > t_1$. From the envelope theorem $u_b = v_x(b;h)$ and $u_g = v_h(b;h)\, h_g(t;g)$. Comparing a mimicker and a low ability household, two effects can now be traced.

First, looking at the components of $v(b;h)$ we see that $\hat{t}_2 > t_1$ implies that $h\left(\hat{t}_2; g\right) > h(t_1; g)$; i.e. the mimicker obtains a larger flow of services from the public good than the low-ability household. On the other hand, both consume the same amount of the private good. This means that if $x$ and $h$ are Edgeworth complements, $v_{xh} > 0$, the mimicker will have a largest marginal utility of disposable income, i.e. $\hat{u}_b^2 > u_b^1$. Conversely, if $x$ and $h$ are Edgeworth substitutes, $v_{xh} < 0$, then $\hat{u}_b^2 > u_b^1$. In words, since a mimicker has more non-working hours available he obtains a larger flow of services from the public good. If the services from the public good and the private good are complements, this works in favour of the mimicker having a high marginal valuation of disposable income.

Second, since $\hat{t}_2 > t_1$ while everyone enjoys the same level of $g$, complementarity (substitutability) of $t$ and $g$ in generating public good services (i.e. in $h$) works in favour of the mimicker having a larger (smaller) marginal valuation of the public good than the low-ability household. In particular, (using that $v_{hh} = 0$) it is easy to see that $h_{tg} > 0$ implies that $\hat{u}_g^2 > u_g^1$ while $h_{tg} < 0$ implies $\hat{u}_g^2 < u_g^1$. In words, if $t$ and $g$ are complementary inputs, then a mimicker who has a relative abundance of non-working hours values an increase in the provision of the public good more than a low-ability household.

To conclude, when a public good is combined with a time input, separability does not guarantee that the Samuelson rule applies at the second best optimum. Indeed, we see that:

[7] This is not very restrictive; in many cases it is possible to choose $v$ and $h$ so that $v_{hh} = 0$. More specifically $v_{hh} = 0$ holds when $v(x;h) = \gamma(x)\, h(t;g) + \delta(x)$ for some functions $\gamma$ and $\delta$. The restriction thus implies that consumer's preferences over lotteries in $t$ and $g$ are independent of the level of $x$. Weak separability, $U = U(v(x; h(t;g)); L)$ would do equally well as long as the chosen $t$ is normal in non-working hours.

1. 'complementarity' of $x$ and $h$ works in favour of the mimicker having a relatively low marginal valuation of the public good and thus in favour of 'S-overprovision'.

2. 'complementarity' of $t$ and $g$ works in favour of the mimicker having a relatively high marginal valuation of the public good and thus in favour of 'S-underprovision'.

### Separability and Public Good Provision with Observable Time Input

When the time-input can be observed and a charge can be levied, the optimal public good provision follows (26). Note that with separable preferences as in (27) we know that the mimicker always devotes more time to using the public good. Hence an optimal charge is positive.

We can now focus on the sign of the revenue correction, i.e. the last term of (26). As in the case with non-observable time input, we can gain some insight by looking at the 'complementarity' ('substitutability') of $x$ and $h$ in the utility function $v$ and of $t$ and $g$ as inputs in the consumption process, i.e. in $h$. A compensated increase in the public good provision is accompanied by a decrease in disposable income; thus, holding $t$ constant, this would imply an increase in the services from the public good and a decrease in the consumption of the private good. Therefore, if $h$ and $x$ are 'complements', we would expect that a household reacts by decreasing the time input $t$: given a positive user charge, this reaction would generate a decrease in $h$ as well as an increase in the consumption of the private good. Instead, if $t$ and $g$ are 'complementary' inputs, we would have the opposite effect: i.e. we would expect an increase in $g$ to be matched by an increase in $t$ since the increase in $g$ would increase the marginal utility of using the public good. Thus we can say the following:

1. 'complementarity' of $x$ and $h$ works in favour of $\partial t_i^c/\partial g$ being negative and thus in favour of 'BK-underprovision';

2. 'complementarity' of $t$ and $g$ (in $h$) works in favour of $\partial t_i^c/\partial g$ being positive and thus in favour of 'BK-overprovision'.

If we combine these observations with those of the preceding subsection, we see that (26) augments the original Samuelson rule by two terms which will often have opposite signs. When $x$ and $h$ are 'complements' the mimicker tends to have a relatively smaller marginal willingness to pay than the low-ability household: thus, the adjustment emanating from the self-selection constraint tends to be positive. Instead, with a positive charge, the adjustment emanating from the revenue constraint tends to be negative. When $t$ and $g$ are 'complements', the opposite holds:

the self-selection effect tends to be negative, while the revenue effect tends to be positive. In both cases, the net effect is indeterminate.

## 6 Extensions

In this section we will mention a number of extensions. First, to focus the analysis on the self-selection rationale for a user charge we have assumed that there is no congestion. Congestion could be included in the analysis e.g. by assuming that the flow of services from the public good depends on aggregate use in addition to the level of the public good and the private time input, i.e. one could have $h = h(t; g; \bar{t})$ where $\bar{t} = \sum n_i t_i$: A second-best Pareto efficient charge will in such an extension consist of two positive parts. The first part emanates from the effect of a charge on the incentive constraint and involves exactly the terms in (24). The second part is an externality correction. As long as the government has only an indirect instrument (the linear charge) for controlling the externality this part will be rather complicated. However, following Pirtillä and Tuomala (1997)? it can be decomposed into a feedback effect, a direct Pigouvian part, a self-selection effect and a revenue effect.

Secondly, we have assumed that consumption of the private good, $x$, does not require a time input. Generalizing this, one may assume that a consumer allocates non-working hours to three uses—private consumption, consumption of the public good, and pure leisure. The plausible case is that where the time input in consumption of the private good cannot be observed at all (and hence cannot be taxed). It turns out that the analysis is essentially unaffected by such an enrichment.[8] Even though the high-ability type is likely to face a tighter time constraint due to his larger goods private consumption, thereby ceteris paribus having a higher marginal utility of income, this tendency is very unlikely to reverse the planners desire to redistribute resources to the low-ability type.

Thirdly, one could consider more than two types. As long as only 'downward' incentive constraints bind at the second-best optimum (i.e. no type wants to mimic a higher ability type) then the case for a user charge is intact. Indeed, if only downward self-selection constraints bind, then every mimicker has more non-working hours than the type that he mimics. If time spent using the public good is increasing in the number of available non-working hours every mimicker spends more time using the public good than the type he mimics. In that case a user

[8] Granted, the consumer's maximizing over a space of higher dimension in such a case may render some of the assumptions about the behaviour of the problem more demanding.

charge helps to relax every binding self-selection constraint. This justifies its existence.

Fourthly, one could consider a non-linear user charge. In that case it is easy to show that high-ability households should be completely undistorted—they should face a zero marginal income tax as well as a zero (marginal) user charge. The low-ability households should typically face a positive marginal income tax as well as a positive (marginal) user charge. Moreover, it can be shown that the public good should be supplied according to Boadway and Keen's modification (19).

# 7 Conclusions

Adopting the household production approach of Ebrill and Slutsky (1982), a public good is viewed as a stock from which consumers obtain a flow of services when devoting scarce time to it. The modelling of the households' time allocations provides a notion of use of the public good. When combined with a sufficient degree of excludability it is possible to levy a charge on the use of the public good, i.e. on the time devoted to it.

The purpose of the paper is to provide a rationale for user charges that does not rely on congestion effects or arbitrary restrictions on the instruments available to the government.

Assuming that the public good is not congested we investigate whether it is a second-best optimal policy to levy a positive linear charge, given that the government has at its disposal a general income tax. We demonstrate that the imposition of a user charge allows a Pareto improvement under the very weak condition that the time input to the public good is normal in available non-working hours. An optimal user charge is characterized and is shown to be isomorphic in structure to optimal commodity taxes supplementing a non-linear income tax (Edwards et al., 1994). These results do not rely on the supply of the public good being variable.

We do, however, also characterize second-best provision of the public good in the current setting, allowing for a charge. The existence of competing uses for non-working hours implies that no simple separability condition guarantees that the second-best provision follows the first-best Samuelson rule. Indeed, assuming separable preferences we outline some properties of the preferences that influence the direction of the optimal distortion of the public good provision.

The model can be extended to allow for congestion effects whereby a optimal charge will consist of the self-selection component demonstrated in the current paper and an externality correction component.