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TESTING NORMALITY IN ECONOMETRIC MODELS

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Abstract

A specification test based on an Edgeworth expansion is proposed and some of its useful properties are noted. In particular the test has an important additivity property, in that a test for higher-order alternatives simply adds additional, asymptotically independent χ^2 variates to tests against lower order alternatives.

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1. Introduction

Asymptotic expansions are a common device for approximating the possibly unknown frequency function of a random variable. For instance, Edgeworth or Gram-Charlier series may be used to expand an arbitrary density f(x), in a series based on the derivatives of the standard normal density $\phi(x)$

$$f(x) = \sum_{j=0}^{\infty} c_j H_j(x)\phi(x) \qquad (1)$$

where the $H_j(x)$ are Hermite polynomials of order j in the random variable x so that $(-D)^r\phi(x) = H_r(x)\phi(x)$ and the coefficients c_j are determined by the particular choice of expansion. (See Kendall and Stuart (1969), p. 156-159). The first few terms of an Edgeworth expansion in terms of the cumulants κ_j may be written as

$$f(x) = \phi(x) \{1 + \frac{\kappa_3}{6} H_3 + \frac{\kappa_4}{24} H_4 + \frac{\kappa_5}{120} H_5 + \frac{\kappa_6 + 10\kappa_3^2}{720} H_6 + \dots\},$$

or when reexpressed in terms of central moments μ_j and truncated, as

$$f(x) = \phi(x) \{1 + \frac{1}{6} \mu_3 H_3 + \frac{1}{24} (\mu_4 - 3) H_4 + \dots\} \qquad (2)$$

where $H_3 = x^3 - 3x$ and $H_4 = x^4 - 6x^2 + 3$.

The leading term returns the normal density and the succeeding terms essentially then correct the normal approximation for skewness and kurtosis respectively. Higher order terms would correct for the effects of higher order moments.

We propose a simple score test for normality of errors in a regression equation. This application illustrates some convenient properties of our procedure. A further application to the more complicated, but very important, problem of testing the normality assumption in selectivity models is given by Kiefer and Salmon (1982).

2. The Test

We assume an econometric model of the form

$$y_t = g(x_t,\beta) + u_t \qquad t=1, \ldots, T$$

where the regressors x_t are assumed to be strictly exogenous and the error process follows a density $f(u)$ with finite moments. Since the errors are unobserved the test must be based on the residuals derived from consistent estimates of the regression parameters β. We deal for simplicity with the standardised residuals so that an expansion in standard measure may be employed.

The likelihood function for a set of N independent drawings on the residual process may then be written

$$L = \prod_t f(u_t)$$

$$= \prod_t \sigma^{-1}\phi(u_t)\ \{1 + \frac{\mu_3}{6} H_3(u_t) + \frac{\mu_4 - 3}{24} H_4(u_t)\}$$

and

$$\log L = \sum_t \log(\sigma^{-1}\phi(u_t)) + \sum_t \log\{1 + \frac{\mu_3}{6} H_3(u_t) + \frac{\mu_4 - 3}{24} H_4(u_t)\}$$

$$= \sum_t \log(\sigma^{-1}\phi(u_t)) + \sum_t \log A_t \,. \qquad (3)$$

The null hypothesis of normality is $H_o: \mu_3 = 0,\ \mu_4 = 3$, and hence denoting $\theta = (\mu_3, \mu_4)$ the score test of H_o may be based on

$$S = \left(\frac{\partial \ln L}{\partial \theta}\right) E\left(-\frac{\partial^2 \ln L}{\partial\theta\partial\theta'}\right)^{-1} \left(\frac{\partial \ln L}{\partial \theta}\right)'$$

evaluated at the restricted estimates.

This simple form arises since $E(\partial^2 \ln L/\partial\theta\partial\beta) = E(\partial^2 \ln L/\partial\theta\partial\sigma^2) = 0$ under the null hypothesis and hence the following tests are independent of the regression parameters (β,σ^2) and subsequently we shall regard them as known.

The computed statistic should be compared with the χ^2_p distribution where p represents the number of restrictions implied by the null. For the present problem we have

$$\frac{\partial \log L}{\partial \mu_3} = \Sigma \frac{1}{6} \frac{H_3(u_t)}{A_t} \qquad \frac{\partial \log L}{\partial \mu_4} = \Sigma \frac{1}{24} \frac{H_4(u_t)}{A_t}$$

$$\frac{\partial^2 \log L}{\partial \mu_3^2} = -\Sigma \frac{H_3^2(u_t)}{36 A_t^2} \qquad \frac{\partial^2 \log L}{\partial \mu_4^2} = -\Sigma \frac{H_4^2(u_t)}{(24)^2 A_t^2}$$

and
$$\frac{\partial^2 \log L}{\partial \mu_3 \partial \mu_4} = -\Sigma \frac{H_3(u_t) H_4(u_t)}{6(24) A_t^2}$$

One of the properties of Hermite polynomials is that they are orthogonal under a normal weighting function, i.e.

$$\int_{-\infty}^{\infty} H_m(x) H_n(x) \phi(x)\, dx = 0 \qquad m \neq n$$
$$= n! \qquad m = n$$

This implies that the sub information matrix relating to (μ_3,μ_4) is diagonal. The score test for $\mu_3=0$, $\mu_4=3$ then becomes

$$S = \frac{\{\Sigma H_3(u_t)\}^2}{6N} + \frac{\{\Sigma H_4(u_t)\}^2}{24N}$$

which when rewritten in terms of the sample moments becomes

$$S = \frac{N}{6}(\hat{\mu}_3 - 3\hat{\mu}_1)^2 + \frac{N}{24}(\hat{\mu}_4 - 6\hat{\mu}_2 + 3)^2 \qquad (5)$$

Note that the test statistic is made up from the sum of two asymptotically independent chi square variates each with one degree of freedom; the resulting test is therefore χ_2^2. This score test can be shown to be asymptotically equivalent under the null to the test suggested by Bowman and Shenton (1975) based on the skewness and curtosis coefficients

$$\sqrt{\beta_1} = \mu_3\mu_2^{-3/2} \quad \text{and} \quad \beta_2 = \mu_4/\mu_2^2 .$$

However a critical advantage of the procedure given above is that the null hypothesis may effectively be partitioned so that attention may first be directed to skewness and only the first expression in the score test used with a χ_1^2 critical value. Alternatively if kurtosis alone is of interest then only the second term in score test should be used and again compared with the χ_1^2 distribution. This ability to determine more finely the direction of deviation from normality will, it is suggested, provide greater diagnostic information than the combined test.

Moreover this ability to explore closely the nature of the alternative distribution also extends to the effects of higher order moments, a possibility that does not appear to be available in procedures based, for instance, on Pearson family alternatives (see Ord (1972), p. 29) or the Bowman Shenton procedure. In fact the test of $H_o: \mu_5 = 0$ is not independent of the test for $H_o: \mu_3 = 0$ but if attention is directed at the cumulants rather than the moments directly the sequence of tests remain independent up to and including the fifth cumulant. The score test for the normality null hypothesis $H_o: \kappa_r = 0,\ r > 2$ will then be

$$S = \frac{N}{6}(\hat{\mu}_3 - 3\hat{\mu}_1)^2 + \frac{N}{24}(\hat{\mu}_4 - 6\hat{\mu}_2 + 3)^2 + \frac{N}{120}(\hat{\mu}_5 - 10\hat{\mu}_3)^2$$

and compared with the χ^2_3 critical value.

The asymptotic independence of the successive chi squared variates follows from the diagonality of the information matrix which in turn follows from the orthogonality of the Hermite polynomials under normal measure. In consequence it can be seen that higher order tests against the null of normality could be based on the coefficients of the original expansion (1).

3. Conclusion

We have proposed a simple test for the null hypothesis that the errors in an econometric relation follow a normal distribution. Since the procedure is based on the score test principle only estimation under the null hypothesis of normality is required. An important property of the test is the additivity of successive chi square variates so that the effects of higher moments and cumulants may be examined individually.

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