A FELDMAN-TYPE MODEL

OF WAR ECONOMY

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Introduction

In trying to sort out some of the economic issues raised by Soviet experience before and during the last war, I thought that some clarification might be obtained from the Feldman model of expanded reproduction. The Feldman model has often been used to address the issues of rapid Soviet industrialisation and of priority to heavy industry. It can be developed to illustrate the choice of priorities in resource mobilisation for war. In my view the significance of the results is historical and, for reasons given below, I do not attribute to them any contemporary or future significance whatsoever.

The Feldman model

The basic Feldman model has been summarised by Domar (1957, pp. 223-258) from Feldman's articles of 1928 and 1929, and in the main I follow Domar's approach.

Starting from Marx's analysis of expanded reproduction Feldman proposed that the economy can be divided between activities which sustain production at its previously existing level, and activities which expand production to higher levels. To give this idea operative content he had to displace its focus away from different kinds of activities onto different kinds of products. In the Feldman model as it is known to us today the economy is divided into two productive sectors. Sector 1 produces the means of production employed in both sectors. Sector 2 produces the means of consumption of the employees of both sectors and of the population at large. The initial capital stock is divided between the two sectors and this division is assumed to be inflexible, i.e. capital goods once invested in one line of production cannot be shifted to another. However the allocation of new capital goods between the two sectors is completely flexible, and attention is focused upon their distribution.

In each line of production there is a given marginal capital-output ratio and, with unlimited supplies of labour, the capital stock is always fully employed.

Initial levels of investment and consumption are constrained by the initial size and division of the capital stock. The subsequent time-paths of investment and consumption are constrained by the output of new capital goods from sector 1, and by their division between sector 1 (investment in capital goods capital) and section 2 (investment in consumer goods capital). If the share of new capital goods invested back in sector 1 is designated by the letter α, then a high value of α will result in rapid growth of sector 1 while sector 2, starved of new capital goods, will at first stagnate. Over time, however, sector 2's relatively small share of the exponentially growing output of new capital goods will mean a growing increment to consumer goods capital, both absolutely and in proportion to the sector's existing capital stock. Eventually the growth of consumer goods production will accelerate to match that of the capital goods sector, yielding a high rate of steady-state growth. Given a long enough time horizon this will be the course chosen. But if short-term considerations predominate then α will be given a low value and the bulk of new capital goods will be allocated to the immediate production of consumer goods. The production of consumer goods will grow rapidly at first. But this gain to immediate consumption will interrupt the expansion of sector 1, the capital goods sector, and sustained growth will not be achieved.

The long-run steady state is achieved when the growth rate of sector 2 has decayed to match the stagnation of sector 1.

Hence the Feldman model illustrates a trade-off between immediate and future consumption, and shows the extent to which future production and consumption are lowered when immediate consumption interrupts the process of expanded reproduction based in the capital goods sector. Demand factors play no endogenous role and are reflected solely in the externally determined parameters which allocate new capital goods. Production possibilities are constrained only on the supply side, not from the demand side. Production takes place independently of consumption and of consumer demand. Savings propensities adjust passively to the accumulation process. Hence the Feldman model is different in kind from other macroeconomic models which have been seen as relevant to the operation of war economies, which focus on problems of realisation, inflation and war finance.

The basic Feldman model can be extended to incorporate capital depreciation (Domar, 1957), a raw materials sector (Raj and Sen, 1961) as a complement to the Mahalanobis version underlying early Indian planning, and foreign trade (ibid.). Each of these extensions is potentially relevant to the problems of war economy, and some of their implications are considered below.

What kind of war economy?

The kind of war economy which can be modelled after Feldman is a particular kind of economy engaged in a particular kind of war. In my view the Feldman model is relevant to some, though not all problems of the Soviet economy before and during the last world war. This is because the Feldman model can be limited in scope and application to an economy in an early or intermediate stage of industrialisation, centralised and mobilised for an unlimited war based on productive effort, such as the second world war became. The Feldman model will throw some light on other kinds of economy and other kinds of war, but mainly by way of contrast. These points are now considered in more detail.

Firstly, why is the Feldman model particularly relevant to an economy in an early or intermediate stage of industrialisation? An essential feature of the model is the clear differentiability of consumer goods and capital goods production. Experience suggests that this assumption is particularly operative in the early stages of industrialisation when the emerging industrial economy is still relatively simple, and when the main drift of production development is towards innovation of machine technology. In the course of this development the metallurgical, engineering and machine-building sectors clearly emerge as a distinct heavy industrial core, surrounded by less strongly defined extractive, energy, transport and construction industries (less strongly defined because they already produce significantly for household consumption as well as for production; in addition agriculture, generally counted as producing only for consumption, continues to produce many of its own means of production).

For the Feldman model to have explanatory power it is necessary both to be able to distinguish products by final use in production or consumption, and to be able to distinguish them in terms of their conditions of production. Beyond the intermediate phases of industrialisation, however, we commonly find that the sharp division between heavy and light industries decays. The economy's intersectoral linkages become more complex and roundabout, while the focus of innovation shifts from machine technology to automation technology. These new production interrelationships are complemented by other new links, just as damaging to the power of the Feldman model, between production and consumption which degrade the distinction between products by final use. The ability of an economy to accumulate new means of production and to employ them effectively is increasingly influenced by its ability to develop new products incorporating new technologies in response to new social wants. As a result the pattern of accumulation in both socialist and capitalist economies today is far more complex than the rigid framework of the Feldman model allows.

The development of an economy beyond the stage typified by Feldman can be defined not only by the changing character of products and accumulated means of production, but also by reference to the aspect of the Feldman model which requires economic growth to be sustained by absorbing new supplies of labour-power in unlimited quantity. This can be understood in terms of a 'primary' accumulation process in which the economy grows by appropriating and restructuring external resources - here the labour-power - of other systems of economy. Beyond the primary phase attention shifts from absorbing an external population to the internal reproduction of the existing labour force, and to raising the productiveness of labour rather than the sheer number of working hands. The Feldman model shows the production of subsistence and the reproduction of the capital stock, but has very little to say about the reproduction processes and productiveness of human labour-power which may become increasingly important behind the scenes.

Secondly, why is the Feldman model relevant to an economy centralised and mobilised for unlimited warfare based in productive effort? The Feldman model shows the economy responding coherently to a limited number of priorities ordered from above. This is appropriate to the situation of some nations in some wars, but not every style of warfare requires such an economy. Milward distinguishes two kinds of war economy among the combatants of the second world war. Some states were committed to limited war, based upon limited resource mobilisation. These states were mainly the aggressors - Germany, Italy and Japan. Other states, mainly the victims or opponents of aggression, found themselves committed to a total war based upon a single-minded productive effort.

The German Nazis, for example, followed a strategy of limited campaigns enabling Germany to expand at the expense of weaker neighbours among whom Hitler included Austria, Czechoslovakia, Poland, France and the Low Countries, the countries of the Balkans and Scandinavia and, mistakenly, the USSR. This choice was influenced by perceptions of Germany's economic weaknesses, especially shortages of raw materials which would serious inhibit the sustained productive effort required for a successful war of unlimited character under conditions of blockade and aerial bombardment. In turn Germany's expansion was partly directed towards sources of materials which would alleviate these weaknesses. The choice was also influenced by the Nazi style of political organisation which favoured shock methods and ad hoc administration towards limited objectives but which was unable to generate detailed, systematic co-ordination or to achieve social and economic coherence (Milward, 1977, pp. 26-28).

Therefore while the Nazi strategy worked the Germany economy was able to avoid full mobilisation. War production concentrated on stock-piling particular materials and equipment for particular, limited campaigns which were not intended to incur heavy losses or to generate prolonged demands. War production demonstrated flexibility and rapid changes in emphasis rather than massive, sustained flows of output. The economy as a whole was not subordinated to a single set of dominant priorities, but responded to a plurality of objectives reflecting the corporate interests and competing personal bureaucracies of the Third Reich.

By contrast the other major combatants rapidly evolved a clear commitment to building up war production and its basis of productive capacity and technology. Their purpose was to equip themselves to fight a long war which would progressively blunt the German assault, disrupt its productive base and ultimately convert the war into a war for the annihilation of the Third Reich. During 1942 the guarantee of this aim was effectively secured, following the stalemate of the German offensive against Moscow. From this time onward Germany too became committed, involuntarily, to a war of productive effort.

> In retrospect the enormous volume of production attained is one of the most remarkable aspects of the war. Once the strategy of the aggressors had failed the strategies of all the major combatants became aligned, and the most important element in each was production . . . The abandonment of the Blitzkreig strategy marked the start for Germany of a battle against hopeless economic odds, the more so as she was giving several years' start in long-term investment for war production to a combination of powers which was, in any case, in terms of such a strategy, economically more powerful (Milward, 1977, p. 56).

The notion of war of productive effort is particularly applicable to the Soviet case, where the military build-up which started in the late 1920s clearly reflected general apprehensions of a coming war in which some coalition of forces would seek the destruction of the Soviet state. Soviet rearmament was not directed towards some specific war or particular aggressor, although stimulated by frictions with Japan; at that time the Soviet Union was friendly with Germany but feared the Western Europeans. Of necessity such preparations concentrated on the productive basis for warfare, to be translated into military might only as the imagined dangers became real and perceptible.

The centralised priorities appropriate to a fully war-mobilised productive system can be simply incorporated into a Feldman-type model. The remaining problems of the model's definition concern the economic role of war production itself.

The economic role of war production

How should war production be integrated into the two-sector model? Kravchenko reports two basic views among Soviet scholars. The first view retains the two-sector structure of the economy. Those who adhere to it argue that war production should be assigned to the capital goods sector on the grounds that war products are clearly not means of consumption; or to the consumer goods sector because some war products are clearly not means of production either; or to both sectors because some war products resemble means of consumption (e.g. uniforms and rations) while others resemble means of production (e.g. vehicles and aircraft). Not unreasonably Kravchenko (1970, p. 6) rejects all these variations of the first view, basing his argument on the strict distinction maintained by Engels between the production of useful objects and that of objects serving the purpose of destruction. He particularly singles out for criticism the assignment of war production to sector 1, the capital goods sector, because of the resemblance of products:

> Production is not classified by resemblances, but by essence and role in the process of reproduction. War production (literally everything from rockets and tanks to uniforms and ammunition), once assigned to war purposes for fulfilment of a military role, can no longer enter into productive usage, except perhaps as scrap (Kravchenko, 1970, p.16).

The alternative view which Kravchenko adopts defines a third productive sector of the economy the capital stock of which is produced in sector 1, whose employees consume the products of sector 2, and which produces the means of destruction.

In my view Kravchenko does not clearly distinguish between the classification of the product and the classification of the production technology. I agree with Kravchenko that in accounting for final products tractors and tanks should not be aggregated under the same heading. However the production technologies required to produce tanks and tractors have much in common. Much Soviet fixed capacity in heavy industries laid down in the 1930s for civilian production was deliberately designed for rapid conversion to production of related war products (Cooper, 1976, pp. 26-27). The economies of all the combatants in the second world war displayed considerable interpenetration of civil and military production technologies.

This can be illustrated by considering the main trends of development of military technology in the war, comprising enhanced mobility, electronic aids and firepower. Enhanced mobility involved the motorisation of land forces, the rapid advance of aviation and marine engineering. Mobility in turn required new electronic methods of communication, guidance and detection. Forces so equipped were additionally endowed with greatly increased firepower (artillery, mortars, rocketry and explosives) against which opposing forces deployed enhanced defensive armour, threatened in turn by new armour-piercing shells and blockbuster bombs - a cycle of development culminating at the end of the war with the atomic bomb.

Of these three components of wartime technology development - mobility, electronics and firepower - the first two had major links with civilian production. The automotive, aviation and shipbuilding industries experienced rapid conversion from peaceful to wartime needs. So did the prewar electronics industries. Only the third category of arms and armour required a productive apparatus fully independent of civilian production and yielded no spin-off to civilian technology, apart from nuclear technology which today
is seen by many as having a militarising and destructive significance even in its peaceful applications.

The solution adopted in this paper is to count all war products in a separate category from products of civilian usage (including capital goods). However from the point of view of production we initially treat war products as if they all belong to the first two groups listed above, and as if they are all produced within the capital goods sector (from now on we shall refer to this sector as the 'heavy industries' producing both capital goods and war products). For the time being we ignore those aspects of military technology which require a distinct productive base, and the economy continues to comprise just two sectors of production. Sector 1 (the 'heavy industries') produces both the means of production and the means of destruction in a balance determined by the authorities. In the next two sections of this paper I develop a model within this framework and demonstrate its properties. In a subsequent section I introduce strategic materials which are required for war production and which require a distinct productive base outside sector 1.

Basic features of the war economy model

The economy produces three products: means of consumption, of production and of destruction. The production of capital goods is interchangeable with the production of war products, so the economy consists of two production sectors. Sector 1 produces objects which can be invested in sector 1, invested in sector 2 or assigned to the military as war products. Sector 2 produces means of consumption for the employees of both sectors, the armed forces and their dependents.

As in the basic Feldman model the capital stock is initially divided between the two sectors and, once invested, cannot be shifted across the division. However the allocation of new products of sector 1 between capital goods of sector 1-type, of sector 2-type and war products is completely flexible. The capital stock of each sector is added to by the products of sector 1. In addition the capital stock of each sector depreciates as a result of wear and tear proportional to the rate of production. Depreciation is introduced into the model mainly to allow the authorities the opportunity to stockpile war products at the expense of not replacing worn-out capital goods in the civilian economy. The treatment of depreciation is therefore simpler than that of Domar (1957, pp. 240-242) in intent and execution.

As in the basic Feldman model we assume a given marginal capital-output ratio in each sector, unlimited supplies of labour and continuous full employment of capital. These assumptions are inherent in the Feldman-type model and can seriously restrict its relevance. Most war economies encountered labour scarcities which could only be resolved by technical change, social reorganisation and sacrifice, meaning longer hours of harder work combined with increased participation from a declining civilian population. The only exceptions were the United States which for much of the war were able to draw on previously existing reserves of unemployed labour and fixed capacity, and Germany which was able to import slave labour from her occupied territories. Because the model assumes that labour supplies are always available, it also removes the need to consider a common wartime option, that of shutting down non-essential industries in order to concentrate scarce inputs upon priority sectors. We accept this loss of scope in order to focus attention on the quantitative circulation of products and means of production. The Feldman-type model will also have nothing to say about the drive to qualitative improvement of war products which characterises an arms race.

In this economy the authorities have to make a series of choices. It probably makes best sense to imagine an objective function with three arguments, to be maximised within a given time horizon. I prefer the time-horizon approach to that of discounting future values, firstly because of well-known inherent difficulties in establishing the social discount rate, secondly because wars are discrete events which can be anticipated over a given period of time and the outcome of which, once begun, cannot easily be postponed. Within the given time horizon the authorities will set some value, firstly upon the production of war products, secondly upon the production of consumer goods, and thirdly upon the terminal capital stock of sector 1. Other things being equal the authorities will wish to maximise, at a given point in time, not the rate of war production but the cumulative stockpile of war products. Of two time-paths yielding equal levels of war production at the end of the period the authorities will prefer the one with higher levels of production in the intervening years. In the same way the authorities will prefer to maximise the integral of consumer goods production over a period of time, not the rate of production at a point in time. However the utility of these two objectives will be maximised only when considered together with that of a third, the terminal capital stock of sector 1. The importance of this is that, while the authorities cannot see beyond their chosen time-horizon, they know that the uncertain future will eventually materialise. The capital stock of sector 1 guarantees the possibility of then continuing with the existing balance of production and accumulation or of revising it. This guarantee of continuity and flexibility is especially important because the structure of the Feldman model requires the authorities to fix the values of their policy parameters at the outset and not to vary them within the operative time horizon. If one believes that it would be desirable to alter these parameters frequently, one should operate with a short time horizon. The larger the capital stock of sector 1 at the end of each period, the wider becomes the authorities' field of choice in the next period.

In formulating the model the following symbols are used:

I	gross product of sector 1 (heavy industries)
C	gross product of sector 2 (light industries)
I_1	gross investment in sector 1
I_2	gross investment in sector 2
W	war production

$$I = I_1 + I_2 + W \qquad (1)$$

α share of I allocated to I_1

β share of I allocated to I_2

$$I_1 = \alpha I \; ; \; I_2 = \beta I \; ; \; \alpha + \beta \leq 1 \tag{2}$$

K_1 capital stock of sector 1

K_2 capital stock of sector 2

v_1 marginal capital-output ratio of sector 1

v_2 marginal capital-output ratio of sector 2

$$\frac{dI}{dt} = \frac{dK_1}{dt} \frac{1}{v_1} \; ; \; \frac{dC}{dt} = \frac{dK_2}{dt} \frac{1}{v_2} \tag{3}$$

D_1 depreciation in sector 1

D_2 depreciation in sector 2

$$\frac{dK_1}{dt} = I_1 - D_1 \; ; \; \frac{dK_2}{dt} = I_2 - D_2 \tag{4}$$

r_1 depreciation-output ratio in sector 1

r_2 depreciation-output ratio in sector 2

$$D_1 = r_1 I \; ; \; D_2 = r_2 C \tag{5}$$

In this system the rate of war production at a point in time is given by equations 1 and 2 as:

$$W = (1 - \alpha - \beta) I \tag{6}$$

that is, the rate of war production is given by the rate of output of sector 1 not assigned to investment either in sector 1 (heavy industries) or in sector 2 (light industries). The rate of output of heavy industries, I, is given from equations 3, 4 and 5 as follows :

$$\frac{dI}{dt} = \frac{dK_1}{dt}\frac{1}{v_1}$$

and

$$\frac{dK_1}{dt} = I_1 - D_1$$

$$= (\alpha - r_1)I$$

therefore

$$\frac{dI}{dt} = \left(\frac{\alpha - r_1}{v_1}\right) I$$

and

$$I = I_o e^{\left(\frac{\alpha - r_1}{v_1}\right) t} \qquad (7)$$

that is, heavy industrial output at time t is given by its initial level I_o expanded over t periods at the continuously compounded rate of $(\alpha - r_1)/v_1$. This growth rate varies negatively with the rate of depreciation and the marginal capital-output ratio of sector 1, and in positive proportion to α , the share of heavy industry products allocated as capital goods to sector 1.

If we substitute equation 7 back into equation 6, we find the time-path of war production defined as:

$$W = (1 - \alpha - \beta) I_o e^{\left(\frac{\alpha - r_1}{v_1}\right) t} \qquad (8)$$

Here we see that the higher are α and β , the lower will be initial values of W . However the higher is α the greater will be the subsequent growth rate of war production. Equation 8 thus illustrates the trade-off between allocating new resources generated in heavy industries to civilian production in general (represented by α and β), to the productive basis of the economy in general and of war production in particular (represented by α) , and to immediate war use.

At this stage it is convenient also to derive the determinants of the capital stock of sector 1 which, it was argued above, would be found among the authorities' three objectives. This is derived simultaneously with equation 7, since increases in sector 1 output are in fixed proportion to increases in the sector's capital stock, and is given as:

$$K_1 = v_1 I_o e^{\left(\frac{\alpha - r_1}{v_1}\right) t} \qquad (9)$$

Thus a higher level of I at a given point in time will always imply a higher level of K_1 , and this will also be implied by a higher level of W .

The time paths of war production and heavy industry capital stock can now be compared with that of consumer goods production. The level of output of consumer goods, C , is derived from equations 3, 4 and 5 as follows :

$$\frac{dC}{dt} = \frac{dK_2}{dt} \frac{1}{v_2}$$

and

$$\frac{dK_2}{dt} = I_2 - D_2$$

$$= \beta I - r_2 C$$

therefore

$$\frac{dC}{dt} = \frac{\beta I - r_2 C}{v_2}$$

substituting equation 7 into this,

$$\frac{dC}{dt} = \frac{\beta I_o e^{\left(\frac{\alpha - r_1}{v_1}\right)t} - r_2 C}{v_2}$$

The solution to this is more complicated than in the case of heavy industry output and for nonzero values of α is given by the following:

$$C = \left[C_o - \frac{\beta}{\alpha - (r_1 - r_2)\frac{v_1}{v_2}} \frac{v_1}{v_2} I_o\right] e^{\left(\frac{-r_2}{v_2}\right)t}$$

$$+ \frac{\beta}{\alpha - (r_1 - r_2)\frac{v_1}{v_2}} \frac{v_1}{v_2} I_o e^{\left(\frac{\alpha - r_1}{v_1}\right)t} \qquad (10)$$

Here the time path of consumer goods production is given by two terms, one of them declining at the continuous exponential rate of $-r_2/v_2$, the other growing at the rate of $(\alpha - r_1)/v_1$ (of course if $\alpha < r_1$ then the second term will decline as well, but more slowly). Because the second exponent is larger than the first, over time the second term will come to dominate in determining C , and the growth rate of C will tend to that of I and W . In the short term the growth of C may differ, and may even have opposite sign. For example if β is small the relative weight of the first term will be large; new capital goods are allocated away from consumer goods production which may even decline in the short term if worn-out capital is not replaced. There will be the same effect if I_0 is small relative to C_0 . However as long as β exceeds zero the long-run performance of the light industries will depend faithfully upon the heavy industries. If the capital goods not being allocated to consumer goods production are being allocated to capital goods production at a rate exceeding capital depreciation in that sector, then α will be large enough to enable the heavy industries to grow, and the consumer goods sector's allotment of new capital goods will eventually enable it to resume growth as well. However if the heavy industrial products not allocated to consumer goods production are being assigned as war products, then the second term will grow slowly or not at all. War production not only interrupts immediate consumption but also interrupts the process of accumulation for future production of all kinds of products.

This illustrates the role of the policy parameters α and β . An important role in equations 8, 9 and 10 is also played by the technical parameters r and v . These can be understood as affecting the opportunity cost of different policies. In general a high rate of depreciation, r , necessitates a high rate of commitment of new capital goods just to keep production up to its initial level. A high marginal capital-output ratio, v, necessitates a high rate of commitment of new capital goods to achieve even small increases in output. So resources can be mobilised more rapidly and cheaply for war production, the lower are r_1 and v_1, while the consumption goods sector will display greater resilience, the lower are r_2 and v_2.

In order to focus attention on the policy parameters I have chosen to make life simpler and easier by assuming that $r_1 = r_2$, and that $v_1 = v_2$. Under these conditions equations 8, 9 and 10 reduce to the following:

$$W = (1 - \alpha - \beta) I_0 \, e^{\left(\frac{\alpha - r}{v}\right) t} \tag{11}$$

$$K_1 = v \, I_0 \, e^{\left(\frac{\alpha - r}{v}\right) t} \tag{12}$$

when α exceeds zero,

$$C = \left(C_0 - \frac{\beta}{\alpha} I_0\right) e^{\left(\frac{-r}{v}\right) t} + \frac{\beta}{\alpha} I_0 \, e^{\left(\frac{\alpha - r}{v}\right) t} \tag{13}$$

or when α equals zero,

$$C = \left(C_0 + \frac{\beta t}{v} I_0\right) e^{\left(\frac{-r}{v}\right) t} \tag{14}$$

It can be noted, lastly, that this economy behaves as if war production had a distinct productive apparatus of its own outside heavy industry, in the special case when the relative shares of each sector in production happen to equal their shares in the allocation of new capital goods, that is, in the steady state.

Results of simulation

I have chosen to illustrate further the influence of policy by simulating the behaviour of the economy over time for given values of the model's parameters. This reveals three main aspects to the model. Firstly we compare the expansion of C and W through time for given values of α and β. We consider what is the consumption cost of achieving a particular level or rate of growth of war production. The second aspect of the model is revealed by examining the trade-offs between accumulated war products and accumulated consumption goods output over 5, 10, 15 and 20 periods. The third aspect emerges from considering the trade-offs between accumulated war products and accumulated heavy industrial capital stock over a similar time-scale. We find that some choices are highly sensitive to small changes in the time horizon considered, or in the relative priority of consumption.

The values chosen for simulation are given as follows. We take $I_0 = 1$, and $C_0 = 2$, giving an initial gross savings-investment ratio of one third - a high value but not unrealistic for the USSR in the 1930s. The value of r is set at 0.1 and that of v at 2.5. This combination is not varied below, but it could be varied in future work. With this combination we could potentially examine all the combinations of α and β which satisfy the condition that $0 \leqslant \alpha + \beta \leqslant 1$. In fact there are two critical values which help us to restrict the search. These are:

$$\alpha = r \; ; \quad \frac{\beta}{\alpha} = \frac{C_0}{I_0} \tag{15}$$

The first condition $(\alpha = 0.1)$ is the condition for sector 1 exactly to reproduce its capital stock without growing or contracting. If we substitute this into the second condition we obtain $\alpha = 0.1$, $\beta = 0.2$ which is the condition for consumption goods capital also to be exactly reproduced at a constant level (sector 1 just replaces the worn-out capital in both sectors). Therefore I use below a set of values of α and β which straddle these critical values ($\alpha, \beta = 0, 0.1, 0.2, 0.3$) and which should therefore reveal all interesting variations.

Charts 1-4 show possible expansion paths through time of C and W measured on a logarithmic scale (the advantage of the logarithmic scale is that the curve of a variable which expands at a given constant exponential rate will exhibit a given constant slope). Each chart shows the effect of varying α, while β is held constant, on the time-paths of C and W. The initial level of production of consumption goods is the same under all conditions. The initial level of war production depends inversely upon the sum of α and β. The subsequent growth of consumer goods production depends on how α and β are combined. A given level of β means a given light industry claim on heavy industrial output for new capital goods, but the growth of heavy industrial output through time is determined by α. (A special case arises in Chart 1 when β is set equal to zero, with the result that consumption goods production declines at a constant rate reflecting the depreciation of its capital stock, independently of heavy industrial growth.) The subsequent growth of war production is varied exclusively by changes in α.

Four important rules can be taken from these charts. For a given value of β, varying α from 0 to 0.3 has three effects. Firstly the initial level of war production is reduced but its rate of growth is increased. Secondly the accumulation of capital in sector 1 is increased. Thirdly the steady-state growth rate of consumption is increased, at least

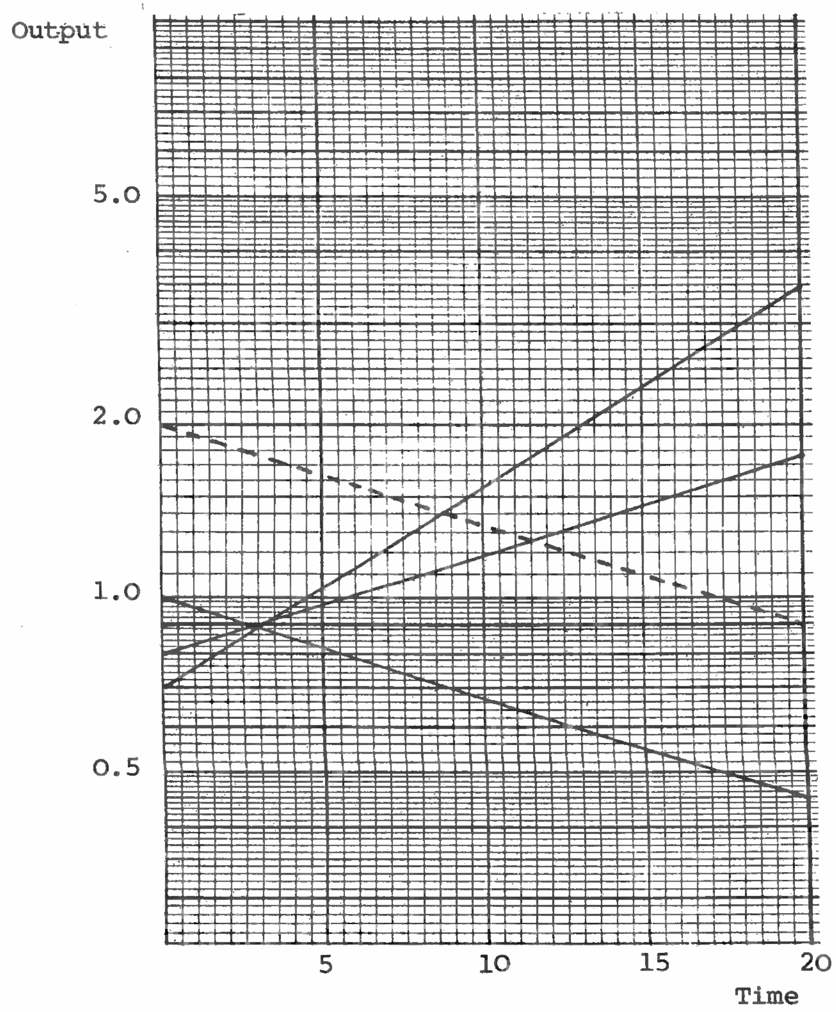

Chart 1 ($\beta = 0$, $\alpha = 0, 0.1, 0.2, 0.3$)

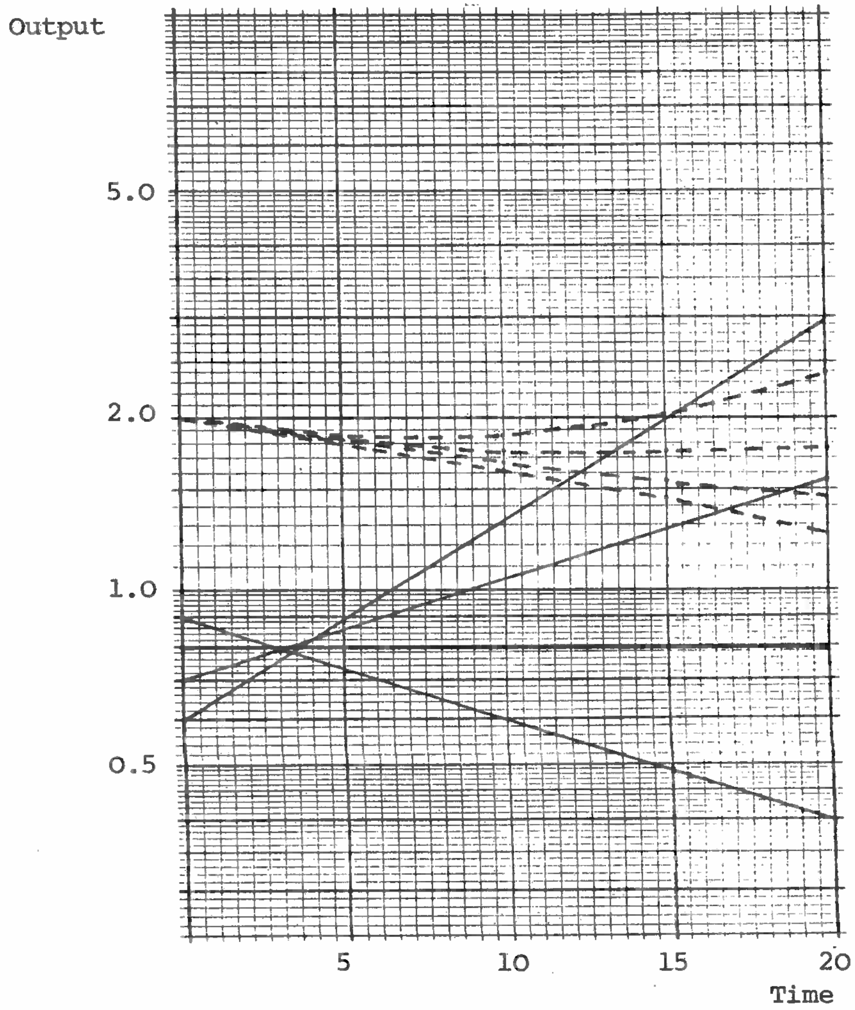

Chart 2 ($\beta = 0.1$, $\alpha = 0, 0.1, 0.2, 0.3$)

Note: the level of war production, W, is indicated by a solid line and the level of consumption goods production, C, is indicated by a broken line. Each higher rate of growth of W and of C indicates a higher level of α.

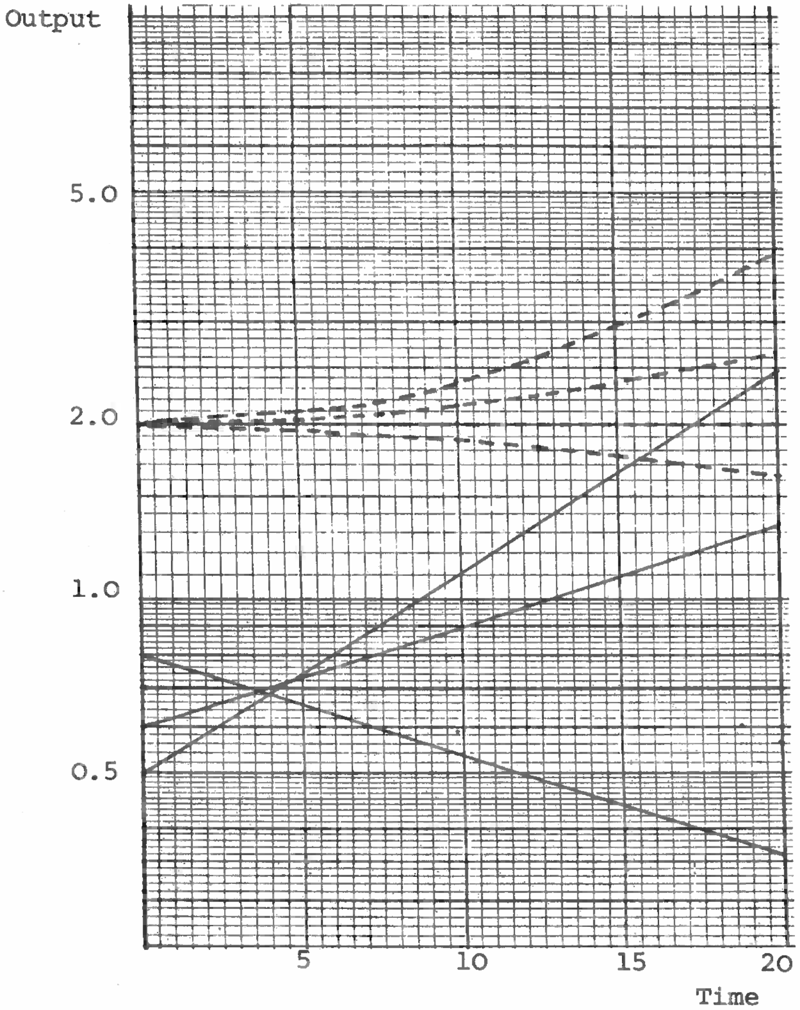

Chart 3 (β = 0.2, α = 0, 0.1, 0.2, 0.3)

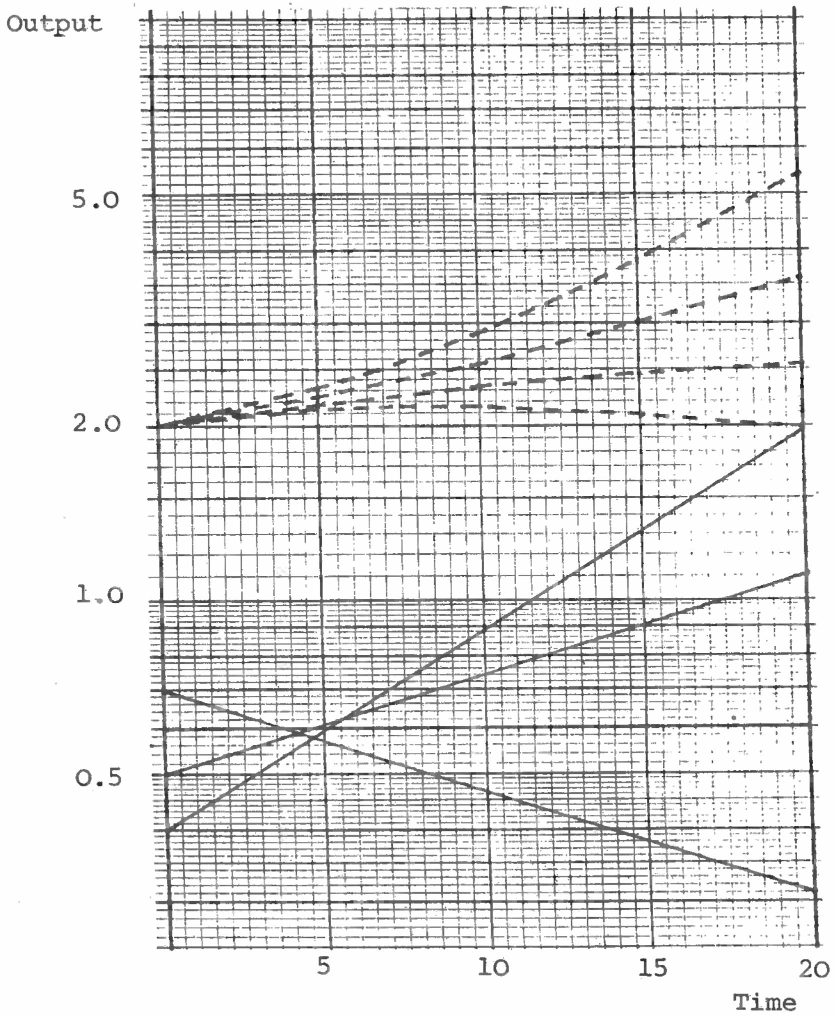

Chart 4 (β = 0.3, α = 0, 0.1, 0.2, 0.3)

Note: the level of war production, W, is indicated by a solid line and the level of consumption goods production, C, is indicated by a broken line. Each higher rate of growth of W and of C indicates a higher level of α.

for nonzero values of β . By comparing each chart in order, we can also generate a fourth rule, that the effect of varying β from 0 to 0.3 while holding α constant is in each case to postpone the attainment of a given level of war production (and also of K_1) while bringing forward the attainment of a given level of consumption goods production; however changing the value of β does not affect the growth rate of war production, nor does it affect either the initial level of consumption or the steady-state growth rate to which it converges.

Charts 1-4 also reveal a phenomenon noted by Domar in relation to the basic Feldman model. Other things being equal, small changes in the time horizon can have disproportionate effects on the values of α which must be chosen in order to maximise the level of war production at a point in time. For example when $\beta = 0$, the value of α which yields a maximum level of war production in period 2.5 is shown in Chart 1 as $\alpha = 0$, but for period 3.5 it is shown as $\alpha = 0.3$. Each of the following charts shows a similar interval of high sensitivity to changes in the time horizon. However as the priority attached to consumption increases in each successive chart the critical interval is moved steadily into the future while the degree of sensitivity is increasingly attenuated.

Looking at production levels at points in time, as in Charts 1-4, gives us a good idea of the movement of the economy in familiar terms. However I suggested above that the authorities will be less interested in levels of output at any point in time than in cumulative values of war production, of consumption and of sector 1's capital stock. The implications of Charts 1-4 for these cumulative values are spelt out in Chart 5 and Table 1. In Chart 5 cumulative war production is traded against cumulative consumption goods output over four different time horizons (5, 10, 15 and 20 periods) by continuously varying β while α is held constant. We do this for two values of α , 0 and 0.3. We find a straightforward transformation frontier which is bounded by maxima of both cumulative war production (when $\beta = 0$) and of cumulative consumption (when $\alpha + \beta = 1$), and which marches steadily outwards with time. The effect on the frontier of an increase in α confirms what we could judge from Charts 1-4. If the time horizon is 5 periods, an increase in α has a minimal effect on the availability of war products. A society orientated towards immediate consumption might just as well immediately turn over its heavy industrial products to the military without bothering about renewing or expanding the industrial base of war production. However as the time horizon expands the advantage shifts more and more towards accumulation in heavy industries over immediate use (either for consumption or for war). Moreover as the time horizon expands the advantage shifts more and more towards accumulation in heavy industries because an increase in α steepens the frontier so that war products become implicitly cheaper in terms of consumer goods.

The last piece in the jigsaw is filled by Table 1, which shows how cumulative war production is traded against the accumulation of capital in heavy industry. For each given time horizon and for each level of β reflecting the priority accorded to consumption we can find the level of α which will maximise the accumulation of war products. This will fall below the level of α which maximises the accumulation of capital in sector 1 by an amount given in the table as $\theta (\alpha + \beta + \theta = 1)$, and of course θ is also the proportion of heavy industrial products assigned to the military in each period.

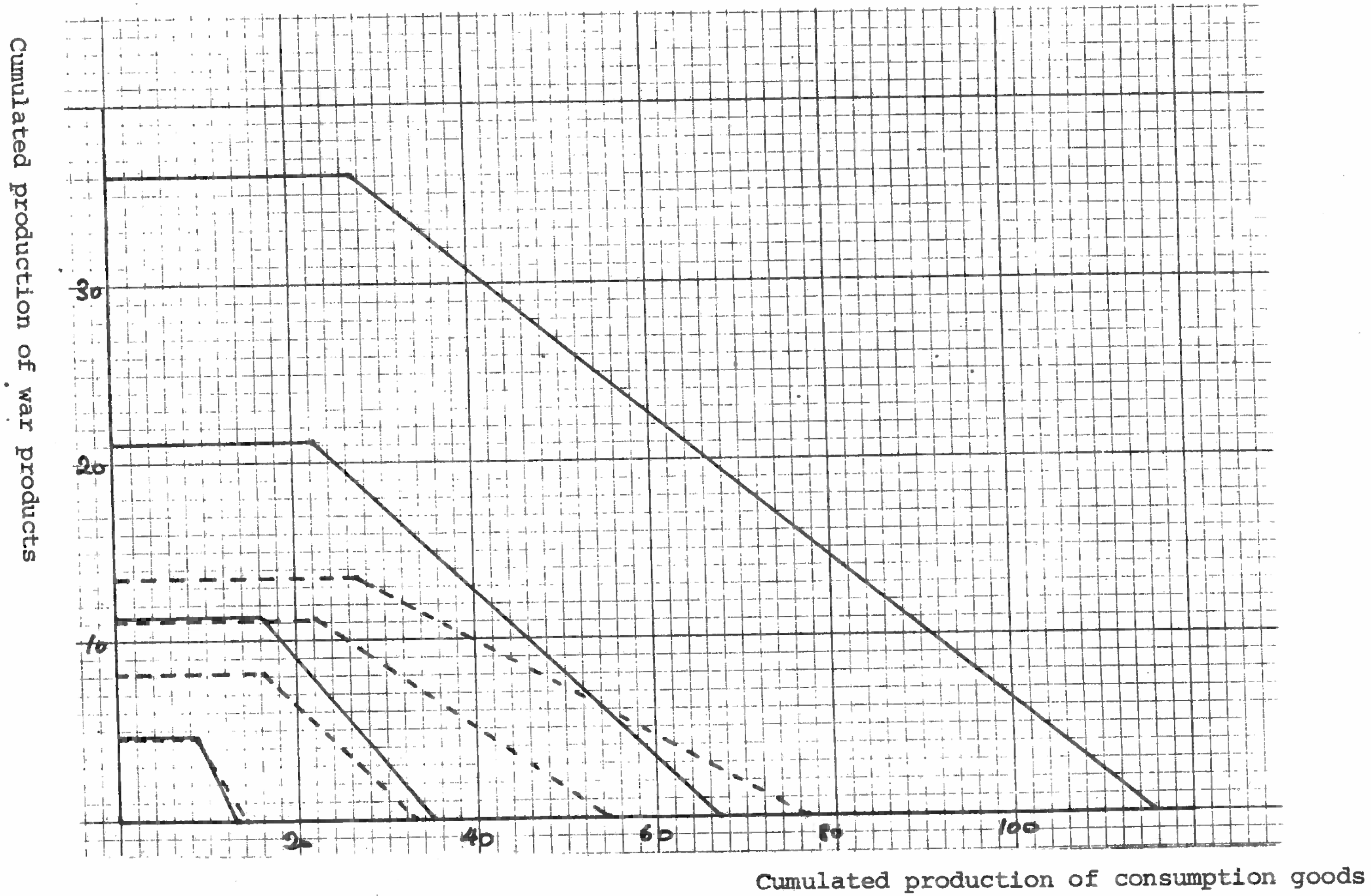

Chart 5. Possible combinations of cumulated production of war products and of consumption goods over 5, 10, 15 and 20 periods.

Note: the combinations are derived by varying β while α is held constant. The broken line indicates $\alpha = 0$ and the solid line indicates $\alpha = 0.3$. Each shift upwards to the right indicates a 5 period interval.

Table 1. Values of α and θ which maximise the cumulative stockpile of war products within a given time horizon for given values of β .

Value of β	Periods over which war products are accumulated: 5		10		15		20	
	α	θ	α	θ	α	θ	α	θ
0	0.19	0.81	0.65	0.35	0.79	0.21	0.85	0.15
0.1	0.06	0.84	0.54	0.36	0.69	0.21	0.75	0.15
0.2	0.00	0.80	0.42	0.38	0.58	0.22	0.65	0.15
0.3	0.00	0.70	0.29	0.41	0.46	0.24	0.54	0.16

As the time horizon expands we find that the value of α which maximises the stockpile of war products rises from an initially low value, tending asymptotically towards its hypothetical maximum. Even when $t = 20$, however, there is still considerable divergence between the α which maximises cumulative war production and that which maximises the accumulation of capital in sector 1. The table shows how maximising cumulative war production operates to the detriment of expanded reproduction, especially in the short run, but the detriment cannot be ignored even when a lengthy time horizon is considered. Moreover if the priority accorded to consumption is increased, the trade-off becomes more acute since the quota of heavy industrial products available for both heavy industrial accumulation and military use is correspondingly reduced; increases in β are reflected mainly in reductions in α rather than θ , unless the time horizon is so short and production possibilities so restricted that α is reduced to zero and θ encounters its maximum limit.

The economic role of strategic materials

So far we have addressed the problems of a relatively uncomplicated economy. How does the behaviour of the Feldman-type model change when the economy becomes more complex? We can answer this question usefully by introducing the fresh problem of strategic materials, which is also interesting in its own right.

We can introduce strategic materials by defining another branch of the economy (sector 3) devoted to the production of materials required in fixed proportion to the flow of war products from heavy industry (sector 1). These correspond to the special chemicals, alloys and fuels required for the arming, protection and mobility of conventional military equipment. Some of them are inputs into war production, while others are really products complementary to war production (generally I find it convenient to refer to them as inputs). Other materials required for civilian production can be regarded, as before, as produced and consumed within heavy industry. Strategic materials, on the other hand, are produced with capital goods which are produced in heavy industry but which constitute a distinct, nonshiftable productive apparatus. This approach is inspired by (but different from) that of Raj and Sen (1961) whose focus was on raw materials which enter into all stages of production of all goods in final demand. Below we draw up a material balance for the new sector in order to assess its implications for the expansion of both military and civilian products.

The same notation is used as before, with the following additional symbols:

M gross product of sector 3

γ share of I allocated to I_3

$$I = I_1 + I_2 + I_3 + W \tag{16}$$

$$I_3 = \gamma I \ ; \quad \alpha + \beta + \gamma \leq 1 \tag{17}$$

$\bar{M}$ demand for products of sector 3

m materials-output ratio in war production

$$\bar{M} = mW \tag{18}$$

Given the assumptions about technology which we used before, the production of materials will expand through time in the same kind of pattern as that derived for the production of consumer goods in equations 13 and 14 above:

$$M = \left(M_O - \frac{\gamma}{\alpha} I_O\right) e^{\left(\frac{-r}{v}\right) t} + \frac{\gamma}{\alpha} I_O \, e^{\left(\frac{\alpha - r}{v}\right) t} \qquad (\alpha > 0) \tag{19}$$

or:

$$M = \left(M_O + \frac{\gamma t}{v} I_O\right) e^{\left(\frac{-r}{v}\right) t} \qquad (\alpha = 0) \tag{20}$$

That is, for nonzero values of α the production of materials will converge towards the steady-state growth path of the economy from its initial level M_O , the convergence being from above if $\gamma/\alpha < M_O/I_O$ and from below in the opposite case. When $\alpha = 0$ the production of materials will decline continuously at a constant rate.

The demand for materials, however, will start at a level determined initially by the allocation of heavy industrial products to military use, and will grow at the same constant rate as war production and heavy industry as a whole. This can be shown by modifying equation 11 to allow for the new parameter γ and substituting it into equation 18:

$$\bar{M} = m(1 - \alpha - \beta - \gamma) I_O \, e^{\left(\frac{\alpha - r}{v}\right) t} \qquad (21)$$

Now dynamic balance between war production and the supply of strategic materials is achieved under two conditions. Firstly the initial level of war production is constrained by the initial level of materials supply, and I assume this constraint to be binding. If $t = 0$ and $\bar{M}_O = M_O$, then equation 21 will read :

$$M_O = m(1 - \alpha - \beta - \gamma) I_O$$

Using $\theta(\alpha + \beta + \gamma + \theta = 1)$ again, we can rewrite this condition:

$$\theta = \frac{M_O}{mI_O} \qquad (22)$$

This first condition states that θ is determined independently of the preferences of the authorities by the economy's initial proportions I_O and M_O, and by the technical variable m. The second condition for dynamic balance is that thereafter materials supply must grow at the same constant rate $(\alpha - r)/v$ as war production. Equations 19 and 20 suggest that this will only be achieved when:

$$\frac{\gamma}{\alpha} = \frac{M_O}{I_O} \qquad (\alpha > 0) \qquad (23)$$

or:

$$\gamma = 0 \qquad (\alpha = 0) \qquad (24)$$

Thus the second condition further states that the relative values of γ and α are also determined by the economy's initial proportions independently of the authorities' preferences. And it should not go unnoticed that these two conditions can also be expressed as a third condition which determines the relative value of β, namely :

$$\beta = 1 - \alpha - \frac{M_O}{I_O}\left(1 + \frac{1}{m\alpha}\right) \qquad (\alpha > 0) \qquad (25)$$

or:

$$\beta = 1 - \frac{M_O}{mI_O} \qquad (\alpha = 0) \qquad (26)$$

In this economy the authorities can choose between alternative futures, but the alternatives are more tightly bounded than before, especially in the short term. Let us take the production capacities I_O, C_O and M_O as inherited from the previous phase of development, and m as dictated by military needs. Equation 22 states that as a result θ and W_O are also determined thereby. The authorities may choose α subject to equations 23 and 24, bearing in mind that their choice of α will now also determine γ, that it will further determine β through equations 25 and 26, and that the sum of α and γ together must not exceed $1 - \theta$. Thus a given choice of α will uniquely determine the economy's pattern and the levels of each of its outputs in all future periods, as well as the steady-state path to which these will converge. Put more simply still, in the two-sector economy the authorities could previously choose the values of any two out of (α, β, θ). In the three-sector economy a new policy parameter has been added, but as a result one of the old ones has become constrained, and the authorities can now choose the value of only one out of (α, β, γ). In the long run war production and consumption can still be traded against each other, but in the short run war production is relatively constrained, no matter what other choices are made.

Two lessons can be drawn from this discussion. Firstly the effects on the economy's performance of introducing a strategic materials requirement can be assessed, and will not necessarily be quantitatively great, even in the short run. To show this we can calculate the maximum possible drain of new resources into strategic materials development under varying technical assumptions. Let us rewrite equation 25 to show how desired combinations of α and β imply a given value of γ, and equation 23 to show the value of M_O which must previously have been achieved to match the resulting combinations of α and γ:

$$\gamma = \frac{m(1 - \alpha - \beta)}{m + \frac{1}{\alpha}} \; ; \; M_O = \frac{\gamma}{\alpha} I_O \tag{27}$$

We assume that the authorities happen to desire the combination of α and β which will maximise the implied value of γ, i.e.:

$$\alpha = \frac{1}{1 + \sqrt{1 + m}} \; ; \; \beta = 0 \tag{28}$$

Table 2 sets out the values of γ and θ $(\theta = 1 - \alpha - \beta - \gamma)$ which will then obtain at different levels of materials input-intensity of war production.

Table 2. Combinations of policy parameters which maximise γ subject to varying war production technologies

m	α	β	γ	θ	γ/α
0.1	0.49	0	0.02	0.49	0.05
0.5	0.45	0	0.10	0.45	0.22
1.0	0.41	0	0.17	0.41	0.41

In the table we see that even if the materials content of war production approaches unity the strategic materials sector can absorb a maximum of 17 per cent of heavy industrial output ($\gamma = 0.17$). This amounts to about 30 per cent of what would otherwise be available as war products. At lower levels of materials input intensity the maximum possible value of γ becomes relatively small.

In addition, however, the table also reminds us that these paths are only feasible given the correct level of previous development of strategic materials production; for example in the worst case shown ($m = 1.0$) the materials sector must already have been built up to over 40 per cent of heavy industrial production ($\gamma/\alpha = 0.41$). Otherwise the desired value of γ will not be a feasible choice (γ must be lowered and β given a positive value).

The second lesson, therefore, is that although the quantitative impact of the strategic materials requirement may not be large under all circumstances, it is qualitatively of the first importance. A small increase in the complexity of the economy being modelled has resulted in a large increase in the model's rigidity and a narrowing in the range of choice of possible futures which it can generate. There are two reasons for this. The first is that this is what the world is really like. Historical accounts of Soviet rearmament in the 1930s stress the urgency of demands placed upon strategic materials exploitation, the difficulties encountered in the search for substitutes and the narrow margins of balance imposed upon planning agencies (Cooper, 1976, pp. 28-30). The Feldman-type model does not allow any easy resort to inferior substitutes or temporary deficits, but neither does the real world. The Feldman-type model particularly highlights the role of initial production capacities in constraining the choice between future growth-paths. At the same time there is also a second reason why small increases in complexity tend so rapidly to overdetermine the model. This is the basic design of the Feldman-type model, with the constancy of planning parameters and convergence towards steady-state equilibrium growth which it requires. These requirements are intended to be convenient abstractions, not typical of reality, but as the model becomes more complex they constrain the possible choices within it more and more actively.

One way of relaxing such inconvenient restrictions is to allow the possibility of external trade. For example, if in our model the initial conditions M_0 and I_0 did not allow the pattern of investment allocation which would take us to our desired target values, we could compensate for this by temporarily exporting consumption goods and importing machinery, strategic materials or even armaments. The effects would be to alter the initial levels of availability of these goods and to expand the range of their possible growth paths. Trade requirements would be transitory and would normally decay over time, because trade is not based in lasting comparative cost advantages but is required for the external substitution of products which can always be obtained through domestic transformation in the long term. Trade would only be revived if, at the expiry of the authorities' time horizon, it was necessary to alter the parameters of investment allocation for the next phase of growth. In fact multisectoral Feldman-type models frequently use trade to provide an extra degree of freedom (Das, 1974, pp. 32-33).

In the real world trade does, of course, relax constraints. The view of trade just outlined is similar to that embodied in most import-substitution models, whether or not of the Feldman type, and can be applied to some aspects of Soviet trade history. Introduced into the Feldman-type model it helps to ameliorate but not to avoid the consequences of the model's basic design.

Conclusions

Firstly, the structure of the Feldman model is relevant to some features of the Soviet economy before and during the last World War, and can be adapted to show some of the choicesof priority between war production, expanded reproduction and immediate consumption. It is not relevant to all economies in all wars, and its relevance to the Soviet economy in the last one is limited by its silence on the labour front. It is capable of some further development, and it is possible to illustrate the implications of including capital depreciation, strategic materials and external trade.

Secondly, the Feldman-type model indicates how expanded reproduction processes within heavy industry become more important, the longer the time horizon and the lower the priority attached to immediate increases in consumption. In addition it indicates critical areas where small alterations in time horizon, or in the priority of consumption, will imply large changes in policy parameters, in the patterns of war production and capital goods output, and in the steady-state equilibrium. It also indicates how increases in the degree of complementarity in the war economy progressively constrain the freedom of action of policy-makers in allocating new resources, and how these constraints can be relaxed through external trade.

Thirdly, in what sense does the Feldman-type model simulate historical reality? It is tempting to visualise the Soviet government in 1931 with 10 years to prepare for war (say, $\theta = 0.4$) , or in mid 1941 with a few months in which to secure the conditions of military survival. The Feldman-type model should help to illustrate such predicaments. However it does not attempt to simulate history; its parameters once set are inflexible, it cannot respond to discrete events and it cannot pass critical judgement upon what was actually decided. In these senses it is different in kind from the multisectoral linear programming model of the interwar Soviet economy which has been the basis of Hunter (1973) and subsequent work.

Fourthly, the advantage of the Feldman-type model is its simplicity, which makes it easier to see what is happening within it and why it generates given results, even when these were not intuitively predicted. Abstraction lends clarity precisely when reality is complex and therefore opaque. In this paper the Feldman-type model is used primarily to abstract and to illustrate. Attempts to make it more realistic have the effect of raising its complexity more rapidly than its realism. The foregoing analysis marks out a rough limit beyond which the Feldman-type model loses its utility and should give way to the greater realism combined with flexibility of the linear programming approach to multi-sectoral balance.

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