Temporary Equilibrium, Expectations and Notional Spillovers*

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1. Introduction

In most temporary equilibrium models the market operates as follows. At the start of the market period the relative price vector is announced. Agents (consumers and producers) then compute and announce their initial market offers based upon Walrasian supply and demand curves. If the relative price vector is not the Walrasian constellation then some market offers go unsatisfied and markets clear on the "short side". Agents who face a quantity constraint on one market adjust their behaviour upon others in an attempt to achieve levels of transactions consistent with the solution to their constrained utility maximisation problems. However this approac assumes sufficient flexibility within each market period to allow behaviour in each market to adjust completely to the quantity constraints actually experienced in other markets in the same period. It can be argued that such flexibility is less than perfect, particularly in the upward direction. In that case, agents will have to base their initial offers not merely on the fixed prices, but also upon their expectations of quantity constraints in other markets. It is these offers that are confronted in each market within a market period. They can be revised downwards, but not upwards, if the actual quantity constraints in other markets turn out to be different from those expected. With this mechanism, each period's markets clear by the familiar quantity adjustment. This will be shown to generate three new types of temporary equilibria termed "expectational" Keynesian, Classical, and Repressed Inflation. These new temporary equilibria will be shown to have interested intra period adjustment properties.

2. The Microeconomic Model.

(a) Consumers

The consumer maximises a single period Cobb-Douglas utility function.[1/]

$$U = X^{\alpha} (T-L)^{\beta} (M_1/p)^{\gamma} \qquad \alpha + \beta + \gamma = 1$$

subject to a budget constraint

$$M_0 + wL = p.X + M_1$$

where X is consumption good, T total time, L labour, M_0 initial money balance, w wage rate, M_1 end of period money balance and p is the price level about which consumers hold unit elastic expectations.

Solution of this problem without further quantity constraints yields the Walrasian supplies and demands familiar in the literature[2/]

$$X = \alpha\left(\frac{M_0 + wT}{p}\right) \tag{1}$$

$$T-L = \beta\left(\frac{M_0 + T}{w}\right) \tag{2}$$

$$M_1 = \gamma(M_0 + wT) \tag{3}$$

Next consider two auxiliary problems with equation constraints. By standard Cobb-Douglas properties the constrained supply and demand functions are

For choice of X and M_1 when $L = \bar{L}$

$$X = \frac{\alpha}{\alpha+\gamma}\left(\frac{w\bar{L} + M_0}{p}\right) \tag{4}$$

$$M_1 = \frac{\alpha}{\alpha+\gamma}(w\bar{L} + M_0) \tag{5}$$

1/ The Cobb-Douglas form is not crucial to the results but does allow a simple exposition.

2/ Identical type functions can be found in Malinvaud (1977).

For choice of L and M_1 when $X = \bar{X}$

$$L = \left(\frac{\gamma}{\beta+\gamma}\right) T - \left(\frac{\beta}{\beta+\gamma}\right)\frac{M_0}{w} + \left(\frac{\beta}{\beta+\gamma}\right)\frac{p.\bar{X}}{w} \tag{6}$$

$$M_1 = \frac{\gamma}{\beta+\gamma}\ (M_0 + wT - p\bar{X}) \tag{7}$$

Now the actual constraint a consumer expects will be of the form $L \leq \bar{L}$ (or $X \leq \bar{X}$). It will be shown that the optimal choice of X (resp L) is then the smaller of the amounts given by (1) and (4) (resp (2) and (6)).

From expressions (1)-(7) it is evident that initial supplies and demands, on the labour and goods markets, expected by the consumer may take one of two forms constrained or Walrasian.

The next step is to determine under what conditions consumers' initial supplies and demands are based on the Walrasian functions and when on the expectations constrained functions.

Consider first the consumer's behaviour on the goods market given an expectation of a quantity constraint in the form of an upper bound on labour sales. His maximisation programme is as follows:

$$\text{Max} \quad U = X^{\alpha}(T-L)^{\beta}(M_1/p)^{\gamma}$$

$$\text{S.T.} \quad M_0 + wL \geq p.X + M_1$$

$$L \leq \bar{L}$$

$$\Lambda \geq 0 \ \mu \geq 0 \quad \text{complementary slackness}$$

Setting up the Lagrangian

$$Z = \alpha \log(X) + \beta \log(T-L) + \gamma \log(M_1) + \Lambda(M_0 + wL - pL - M_1) + \mu(\bar{L}-L)$$

(i) $$\frac{\partial Z}{\partial X} = \frac{\alpha}{X} - \Lambda p = 0$$

(ii) $$\frac{\partial Z}{\partial L} = \frac{\beta}{T-L} + \Lambda w - \mu = 0$$

(iii) $$\frac{\partial Z}{\partial M_1} = \frac{\gamma}{M_1} - \Lambda = 0$$

Examination of the Kuhn-Tucker conditions demonstrates that the solution to the programme implies that the consumer always bases his initial goods purchase offers on the minimum of the two demand curves (1) and (4).

Case (1) $L < \bar{L}$, $\mu = 0$

From (2) (i) and (ii) it can be shown that this case is valid if

$$(1-\beta)T - \beta\left(\frac{M_0}{w}\right) < \bar{L}$$

and[3/] $$\bar{L} < (1-\beta)T$$

which reduces to the condition

$$w < \frac{\beta M_0}{(1-\beta)T-\bar{L}} \qquad (8)$$

Thus case (1) is the solution to the programme when w satisfies the inequality (8), which implies that the expected labour constraint does not bind and goods purchase offers by consumers are determined by the Walrasian goods demand curve.

3/ This condition is required for $w > 0$ to be ensured.

Case (2) $L = \bar{L}$, $\mu > 0$

From (4) (i) and (ii) it can be shown that this case is valid if

$$\mu = \frac{(1-\beta)w}{w\bar{L}+M_0} - \frac{\beta}{T-\bar{L}} > 0$$

which reduces to the condition

$$w > \frac{\beta M_0}{(1-\beta)T-\bar{L}} \qquad (9)$$

Thus case (2) is the solution to the problem when w satisfied inequality (9), the implication being that consumers initial goods purchase offers are determined by the constrained demand function.

Case (3) $L = \bar{L}$, $\mu = 0$

From (2), (4), (i) and (ii) it can be shown that this case is valid where the following condition holds

$$w = \frac{\beta M_0}{(1-\beta)T-\bar{L}} \qquad (10)$$

This is the special case where both Walrasian and expected constrained demand function yield the same initial goods purchase offer.

Expressing (1) and (4) together in $(p/w,X)$ space holding P fixed clarifie the proceeding argument.

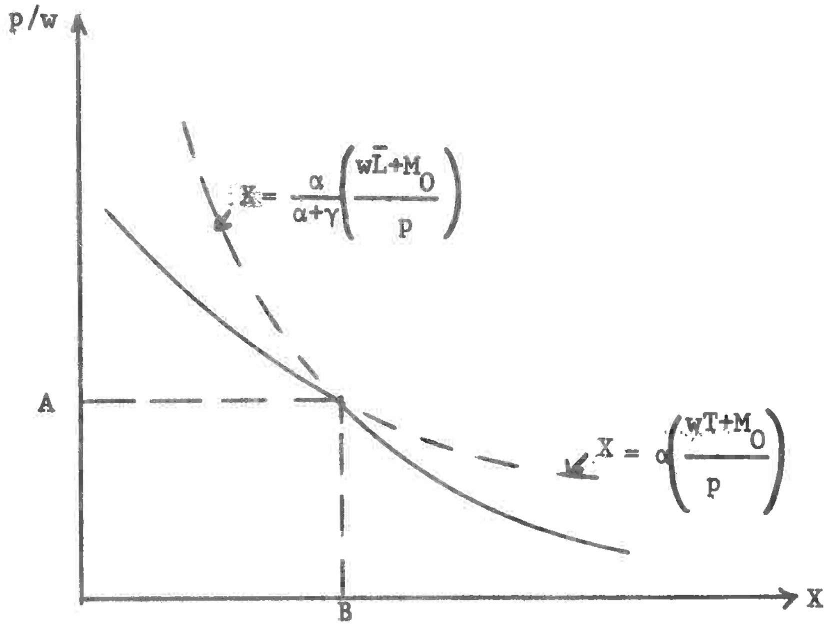

Figure 1

In Figure 1 at the point (A,B) both the Walrasian and expectations constrained curves yield the same initial goods purchase offers and condition (10) holds. Above (A,B) offers are made according to the Walrasian curve as prescribed by condition (8), below (A,B) offers are made according to the expectations constrained curve as prescribed by condition (9). Hence the heavy black line represents the consumer's initial goods demand curve. This is termed the "expectational" goods demand function [4/] and may be written

$$X = F_1(p, w, M_0, \bar{L}) \tag{11}$$

The consumer's behaviour on the labour market given an expectation of a good ration can be analysed in a similar manner. The programme being

$$\text{Max} \quad U = X^{\alpha}(T-L)^{\beta}(M_1/p)^{\gamma}$$
$$\text{S.T.} \quad M_0 + wL \geq p.X + M_1$$
$$X \leq \bar{X}$$
$$\Lambda \geq 0 \quad \rho \geq 0 \quad \text{Complementary slackness}$$

4/ Proof that the demand curves have properties as drawn can be found in Appendix 1.

For brevity the details are omitted, the conditions where the cases are valid, as prescribed by the Kuhn-Tucker conditions are summarised by the following:

$$w \gtreqless \frac{p.X - \alpha M_0}{\alpha T} \quad \text{as} \quad X \lesseqgtr \bar{X} \quad \text{and} \quad \rho \geqq 0 \tag{12}$$

The treatment of the consumer's labour market behaviour is very similar to that of the goods market, with (12) being a summary to the conditions equivalent to (9), (10) and (11). This implies that as in the goods market case, the consumer's labour supply offers are the minimum of their notional and constrained functions. This point is more intuitively obvious if the two functions (2) and (6) are considered together in $(w/p, L)$ space, again represented with fixed p, viz. Figure 2.

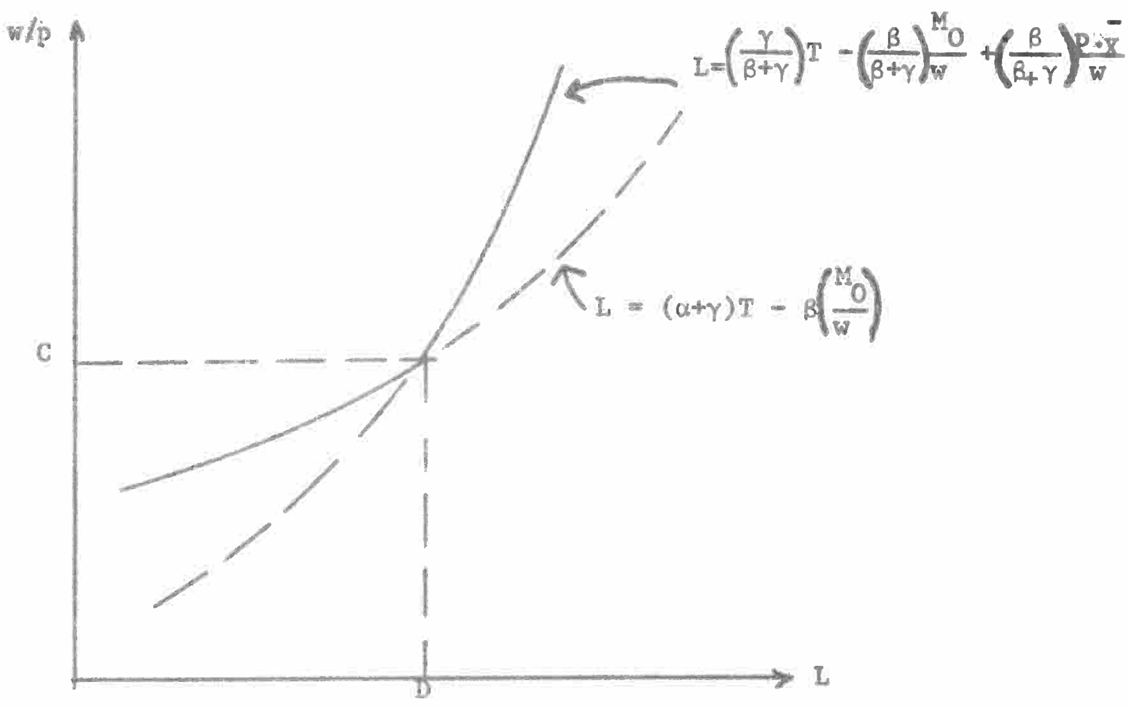

Figure 2

In Figure 2 at the point (C,D) both the Walrasian and expectations constrained curves yield the same initial labour supply offers and condition (12) holds with equalities. Above (C,D) offers are made according to the expectations constrained curve with the constraint binding, below (C,D) offers are made according to the Walrasian curve with the complementary slackness in (12). Hence the heavy black line represents the consumer's initial labour supply curve. This is termed the "expectational" labour supply function[5/] and may be written

$$L = F_2(p,w,M_0,\bar{X}) \tag{13}$$

Notice that this partially cures the Malinvaud money illusion problem. Expressions (11) and (13) imply that the expectational demand for end of period money balances will take the form

$$M_1 = F_3(M_0,w,p,\bar{L},\bar{X}) \tag{14}$$

Following the method of the dual decisions hypothesis, the next question is how will the use of expectational rather than Walrasian curves in making initial market offers affect the final market outcome. The potential consequences for the goods market are illustrated in Figure 3. From the diagram it can be seen that the adoption of the "expectational" curve changes which set of agents are on the short side of the market for relative price vectors falling in region A, and reduces goods purchase offers for relative prices in region B. In the region denoted $\tilde{X}$ consumers face an effective good supply constraint and are on the long side of the market. A symmetric case obtains for the labour market but will be omitted for brevity. Note that the important point here is that the expectation of a constraint on one market causes a spillover effect onto the other market, changing initial market offers upon the market

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5/ Proof that the supply curves have the properties as drawn can be found in Appendix 2.

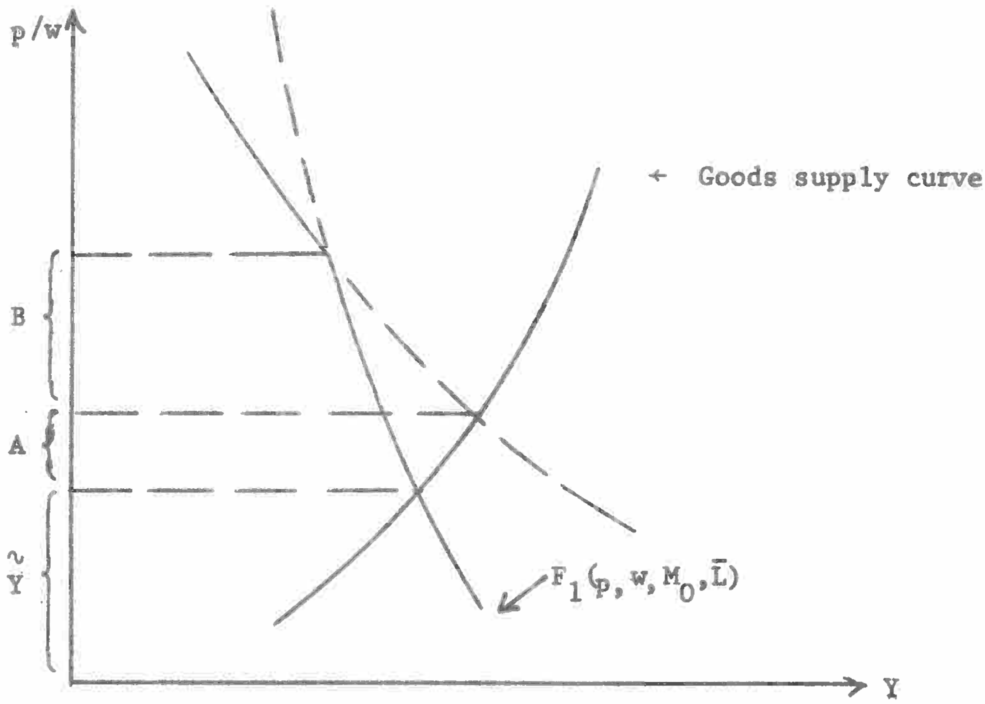


Figure 3

in which a binding constraint is not anticipated, but not on the market where the consumer expects to be rationed.[6/]

(b) Producers

The producer maximises profit subject to a short run production function

$$\text{Max } \Pi = p.Y - wL$$

$$\text{S.T. } Y = kL^{\delta} \qquad 0 < \delta < 1$$

where Y = output

k = capital

$$\partial Y/\partial L = \delta k L^{\delta-1} > 0 \quad \text{and} \quad \delta^2 Y/\partial L^2 < 0$$

There is a positive but diminishing marginal product of labour, and the profit

6/ The introduction of transactions costs would make a constraint expectation influence offers on both markets, this is a case I wish to consider at some future date.

maximising condition is

$$w/p = \delta k L^{\delta-1} \tag{15}$$

Manipulating the production function and (15) yields the producer's unconstrained labour demand and goods supply functions.

$$L = \left(\frac{w}{p} \cdot \frac{1}{\delta k}\right)^{-\frac{1}{1-\delta}} \tag{16}$$

$$Y = kL^{\delta} = k\left(\frac{w}{p} \cdot \frac{1}{\delta k}\right)^{-\frac{\delta}{1-\delta}} \tag{17}$$

Assuming that producers hold constraint expectations, $(\bar{L},\bar{Y})$ and have no means of inter-period income transfer, then the constrained labour demand and goods supply functions are the following

$$L = \left(\frac{\bar{Y}}{k}\right)^{1/\delta} \tag{18}$$

$$Y = k\bar{L}^{\delta} \tag{19}$$

Expressions (18) and (19) follow directly from the production function, notice that if the producer expects a binding constraint on one market, as he has no degrees of freedom[7/] (no third good as in the consumer case), then he simply responds by feeding this constraint through the production function as in (18) and (19). Thus his goods supply is

$$Y = \min\,(k\bar{L}^{\delta}, k(w/p.1/\delta)^{-\frac{\delta}{1-\delta}}) \tag{20}$$

7/ The case here is clear cut and investigation of the Kuhn-Tucker condition is not necessary.

representing this condition in (p/w,Y) space gives

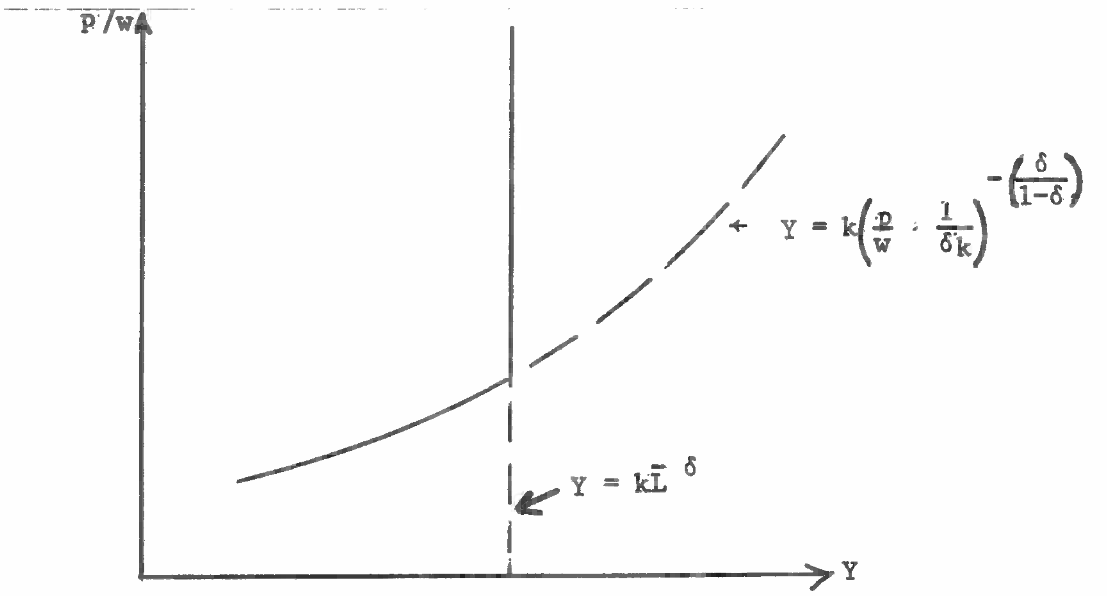


Figure 4

The heavy black line is the producer's "expectational" commodity supply curve.

$$Y = F_4(p,w,k,\bar{L}) \tag{21}$$

as in the consumers case it follows by symmetry that the producer's "expectational" labour demand curve can be written

$$L = F_5(p,w,k,\bar{Y}) \tag{22}$$

Notice that the producer cannot expect to meet binding constraints on both markets, unless the expected constraints are mutually consistent. But then he has no need to hold both expectations.

3. Characterisation of Microeconomic Outcomes

Having examined both agents problems all that is required to characterise market outcomes is a definition of the market mechanism. Here agents base initial market offers upon the expectational curves. Once made market offers can only be varied downwards, because agents on the other side of each market adjust to the initial short-side offers. Consequently two types of market outcome are possible, either both sets of agents achieve a mutually consistent outcome or, one set achieves a mutually consistent outcome and the other wishes to revise upwards. Assuming that $x = y$ all output is available for consumption, then the market outcome can be characterised by the following minimum coditions:

$$X^d = \begin{cases} F_1(p_1w, M_0, \bar{L}) & \text{if} \quad \bar{L} < L^d \\ F_1^*(p_1w, M^0, L^d) & \text{if} \quad L^d \leq \bar{L} \end{cases} \tag{23}$$

$$X^s = \begin{cases} F_4(p_1w, k, \bar{L}) & \text{if} \quad \bar{L} < L^s \\ F_4^*(p_1w, k, L^s) & \text{if} \quad L^s \leq \bar{L} \end{cases} \tag{24}$$

$$X = \min(X^d, X^s) \tag{25}$$

$$L^s = \begin{cases} F_2(p_1w, M_0, \bar{X}) & \text{if} \quad \bar{X} < X^s \\ F_2^*(p_1w, M_0, X^s) & \text{if} \quad X^s \leq \bar{X} \end{cases} \tag{26}$$

$$L^d = \begin{cases} F_5(p_1w, k, \bar{X}) & \text{if} \quad \bar{X} < X^d \\ F_5^*(p_1w, k, X^d) & \text{if} \quad X^d \leq \bar{X} \end{cases} \tag{27}$$

$$L = \min(L^s, L^d) \tag{28}$$

There are two points of note here, firstly the pairs of expressions (23), (24), (26) and (27) have the same functional form, and differ only in that the first case of each pair the constraint is expected whilst in the second it is actually generated in (25) and (28). Secondly it is assumed that the producer cannot be off his production function.

4. Expectations and Macroeconomic Outcomes

To examine the macroeconomic implications of the preceeding analysis some accounting identities are required to define the structure of the economy.

$$X_t = X_t + g_t \qquad (I1)$$

$$S_t = w_t L_t - p_t X_t + d_t \qquad (I2)$$

$$\Pi_t = p_t y_t - w_t L_t \qquad (I3)$$

$$\Delta M_t^H = S_t + M_t^A \qquad (I4)$$

$$\Delta M_t^F = \Pi_t - d_t = p_t Y_t - w_t L_t - d_t \qquad (I5)$$

$$p_t g_t + M_t^A = p_t Y_t - p_t X_t + M_t^A = \Delta M_t^H + \Delta M_t^F = \Delta M_t \qquad (I6)$$

where S_t = flow of household savings

d_t = dividend payment to households at start of period t.

g_t = Government expenditure : which has a prior claim on output

M_t^A = Exogenous change in the money stock.

Superscripts F and H indicate firms and household respectively and Δ is a one period difference operator.

To express the macroeconomic potentialities of this model, the familiar wedge diagram developed by Muellbauer and Portes (1978) is adopted, this allows consumers constrained labour supply and commodity demand and the production function to be expressed together in (X,L) space. The interpretation of the wedge here however is different in one significant respect, for the consumers two constrained curves represent initial offer curves if the constraint is an expected constraint, and represent loci of temporary equilibria if the constraint is effective. It will later be shown that the expected constraint ($\bar{L}$ or $\bar{X}$) truncates the wedge at the level of the initial market offer it generates. Notice that the production function assumption rules out the possibility of an underconsumption case, and we are thus only interested in points on or inside the consumers wedge.

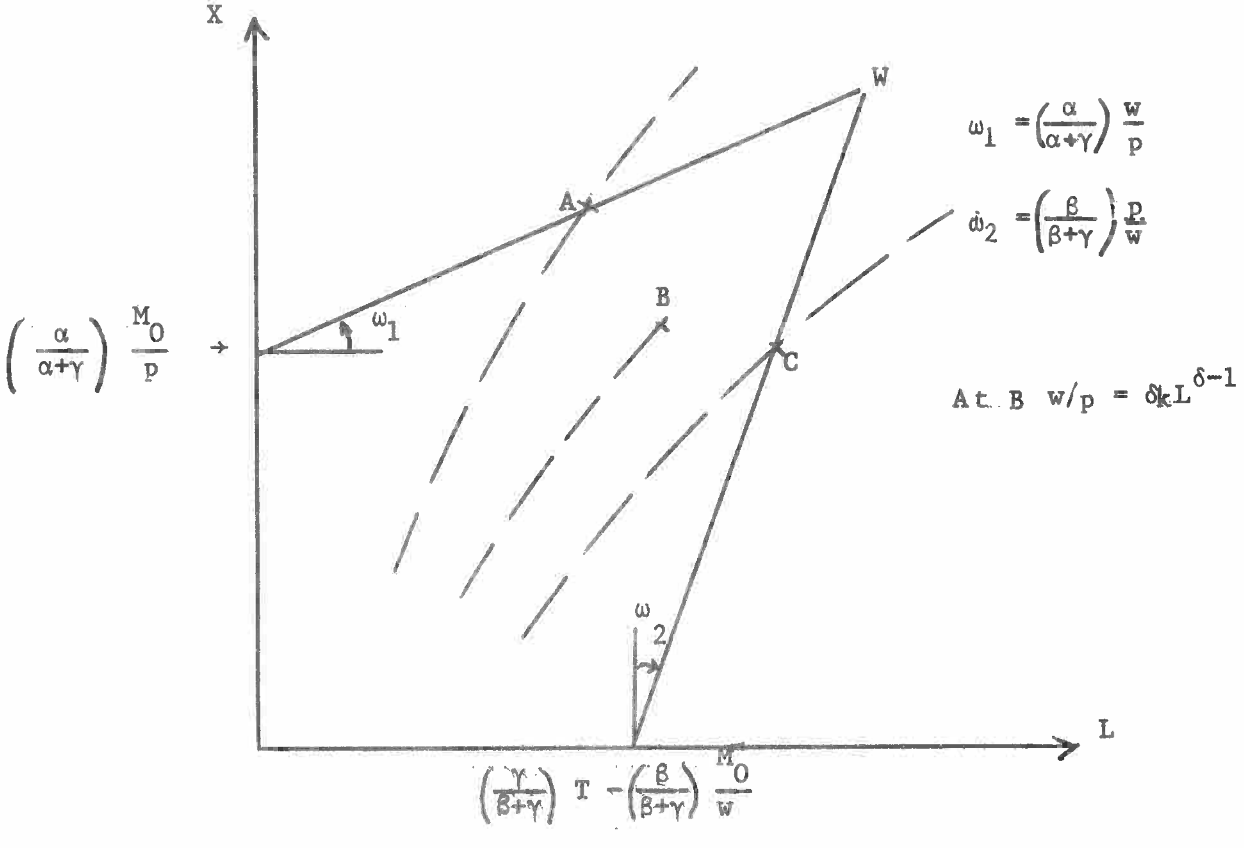


Figure 5

The broken lines represent possible production functions, points A, B and C are standard Keynesian, Classical and Repressed inflation temporary equilibria.

To examine the effects of expectations, first assume only firms hold constraint expectations, then, provided the firms expected constrained input/output combination lies within the wedge and below the unconstrained choice point, an expecational classical temporary equilibrium will result as indicated in Figure 6.

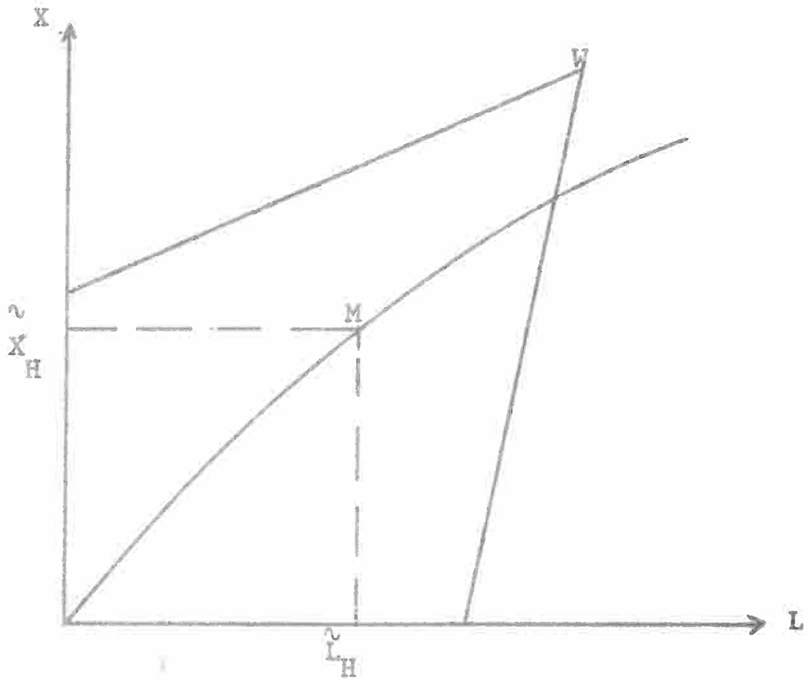


Figure 6

where $M = (\bar{L}, k\bar{L}^{\delta})$ if $k\bar{L}^{\delta} < \bar{X}$

$= ((\bar{X}/p)^{1/\delta}, \bar{X})$ if $k\bar{L}^{\delta} > \bar{X}$

At point M the firm expecting to encounter a constraint on trade in one market adjusts offers to be mutually consistent, the household wishes to trade more on both markets. Hence the economy has achieved an expectational classical outcome.

The next step is the introduction of household expectations, assuming that firms hold no constraint expectations, but enforcing the rule that they have to be on their production functions. The case where households expect a constraint on their labour sales is described by Figure 7.

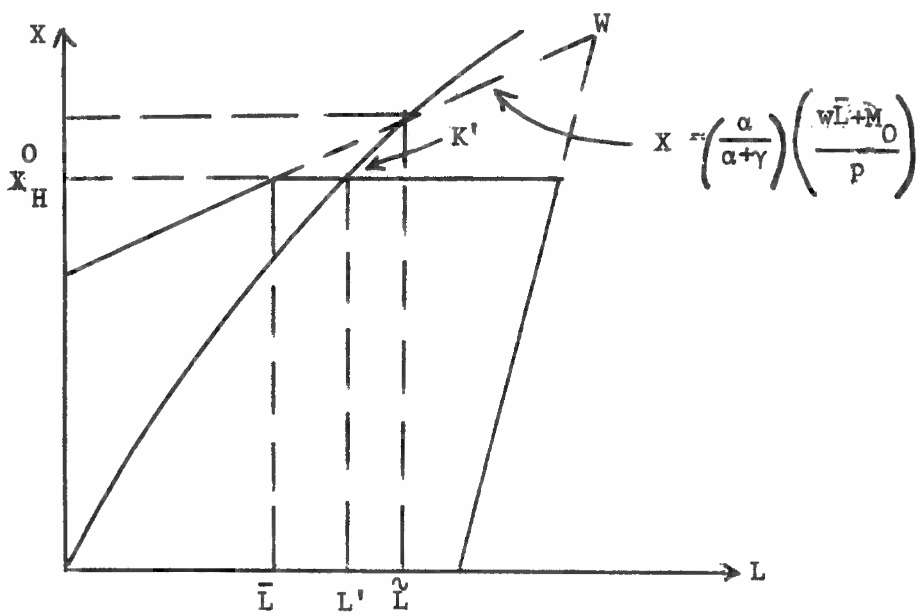


Figure 7

In Figure 7 households solve a constrained utility maximisation problem subject to $\bar{L}$, and hence offer X_H^0 on the good market. Once the offer X_H^0 is made this truncates the consumers wedge as no upward adjustment is allowed, the area above X_H^0 is now unobtainable, and the truncated wedge now shows actual levels of X as a function of actual labour sales. Firms are constrained by X_H^0 and employ L'. At the point K', households achieve

L' on the labour market and their expected labour constrained good demand, giving unexpected saving $w(L'-\bar{L})$ This market outcome is essentially Keynesian in character, expectations impinge on effective demand which in turn constrain firms and affects their labour demand.

If the household expects to be constrained on the goods market then Figure 8 is relevant.

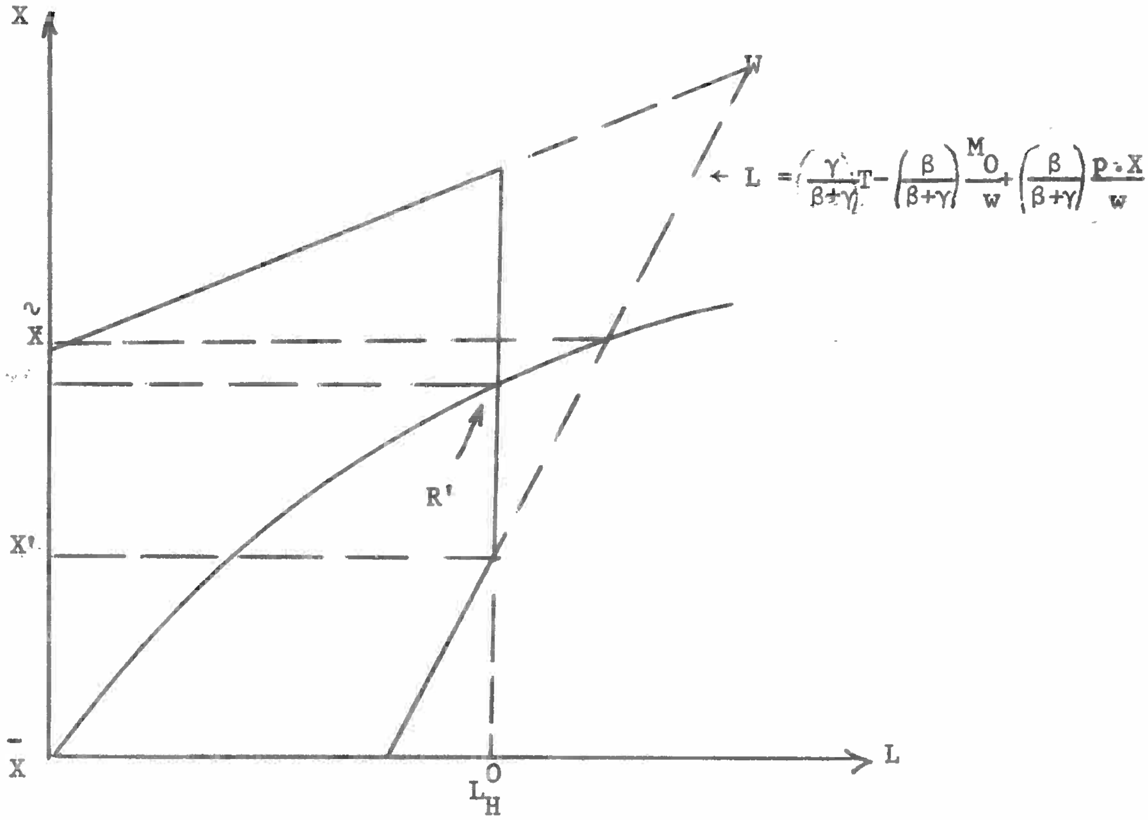


Figure 8

Here households expecting X offer L_H^0 on the labour market, thus truncating the consumers wedge at L_H^0. Firms constrained by the households labour offer provide X to the goods market. At the point R' households achieve their expected goods constrained labour sales, and purchase X', giving unanticipated dissaving $p(X'-\bar{X})$. The market outcome has repressed inflation characteristics, expectations influence consumers labour offers which in turn constrain the firms output.

Diagrams 7 and 8 refer to situations where consumers only expect to be constrained on one market, there is no reason why they should not anticipate an upper bound upon their trade in both markets simultaneously, this is not to say that they will actually be bound by the expected constraints on both markets at the same time. Figure 9 indicates how the consumers wedge would be modified in this case.

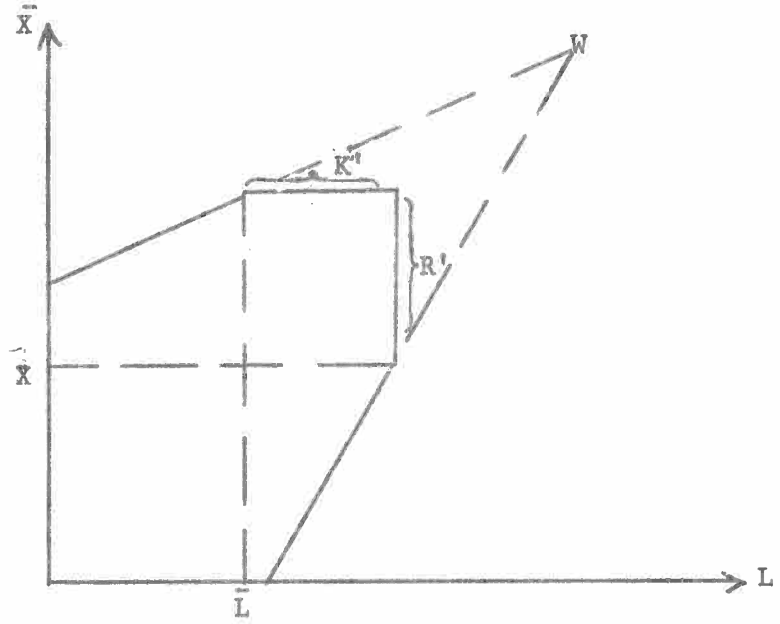


Figure 9

If Figure 9 interesection of the production function with the boundary of the truncated wedge along the K' line will yield expectational Keynesian temporary equilibria, intersection with the K' line yields repressed inflation temporary equilibria. The mechanisms being as in Figure 7 and 8.

Notice that although termed Keynesian and Repressed inflation the outcomes K' and R' are not the same as A and C in Figure 5, for in Figure 5 the market outcomes are mutually consistent for both sets of agents, whereas the points K' and R' are only mutually consistent for the producer. For the consumer the level of money balances changes as a residual. The consumer is however achieving transactions consistent with his expectations, even though these expectations may not fulfilled on the market they were formed about. It is therefore argued that consumers are in a semi-classical situation. Notice also that the relative price vector does not uniquely determine the market outcome, the (X,L) combination which is achieved in any market period is dependent upon w, p, $\bar{L}$, $\bar{X}$ and the shape of the production function.[8/] Finally these equilibria are not "bootstrap", expectations themselves will only be correct in the unlikely circumstances of $\bar{L} = \tilde{L}$ or $\bar{X} = \tilde{X}$, rather it is the market offers generated by expectations which are always achieved. I shall thus refer to these market outcomes, R' and K', as expectional equilibria.

5. Expectations adjustment and Market Outcomes over Several Periods

In this section two basic questions will be considered. Firstly if relative prices are stable over several periods will the system automatically adjust to higher levels of employment and output? Secondly, what effects will relative price changes have? From this juncture onwards the Keynesian cases will be examined in detail, symmetric cases can be constructed for repressed inflation outcomes the results of which will be stated but not analysed here.

Consider the case where only consumers hold expectations. Let relative prices be fixed for several periods, the monetary environment be static and expectations be formed as follows[9/]

$$\bar{L}_t = \bar{L}_{t-1} + \sigma_1(L'_{t-1} - \bar{L}_{t-1}) \qquad (29)$$

8/ Hence the standard Malinvaud market characterisation diagram in (p,w) space is not useful here.

9/ For the repressed inflation case read L as X.

For simplicity let $\sigma_1 = 1$. This will give the behaviour described by Figure 10.

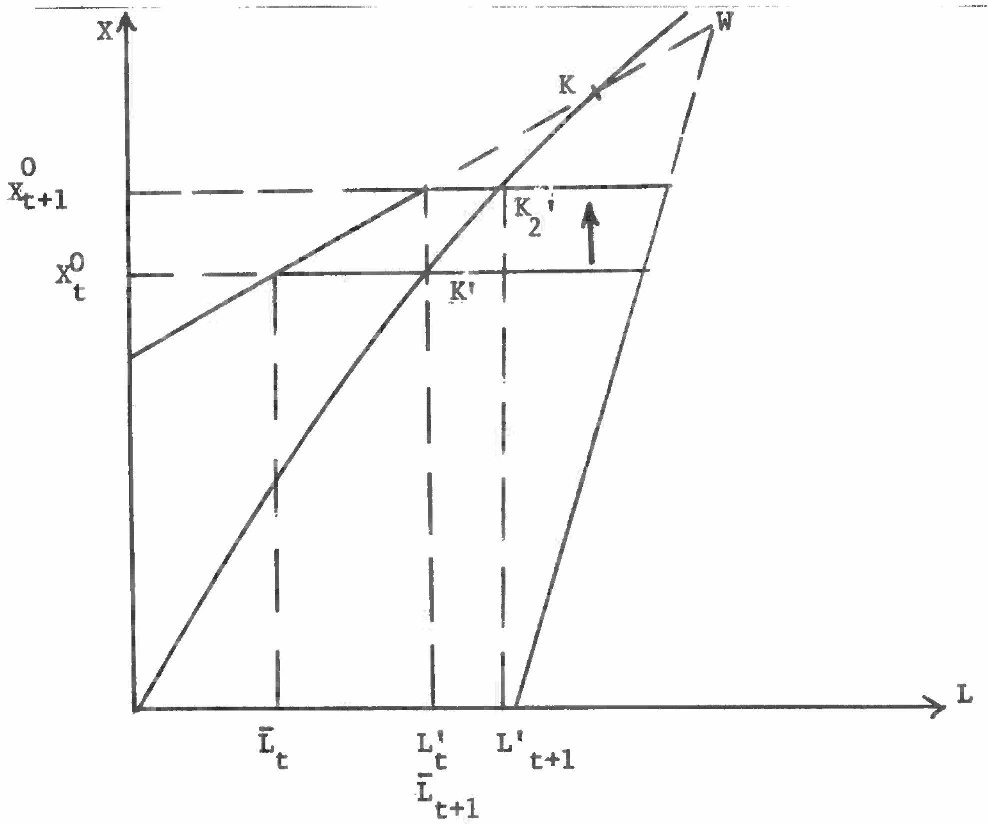


Figure 10

In Figure 10 as expectations adjust consumers initial goods market purchase offers rise from X^0_t to X^0_{t+1} over time period t. This has the effect of pushing up the ceiling of the truncated wedge as indicated by the arrow. Thus over successive time periods the system will move out along the production function through a series of expectational Keynesian equilibria until the point K is achieved, here expectations are correct and there is no tendency for further movement. A similar result holds for R' points tending to a R point on the other regime line.

This result is significant in that it demonstrates that the system does have a self-equilibrating tendency, but this tendency is towards one of the traditional temporary equilibrium regimes, not to Walrasian equilibrium.

Relaxing the assumption of a static monetary environment yields some interesting results. As the level of money balances varies the intercept terms of the consumers wedge shift, this will trace out a locus of consumers unconstrained choice points[10/] which is described by the following expression.

$$L = \left(\frac{\gamma}{\beta+\gamma}\right)T + \frac{p}{w}\left(\frac{\beta}{\beta+\gamma}\right)\left(\bar{X} - X\left(\frac{\alpha+\gamma}{\alpha}\right)\right) - \left(\frac{\beta}{\beta+\gamma}\right)\bar{L} \tag{30}$$

which has slope

$$\frac{\partial X}{\partial L} = \frac{\alpha}{\beta}\left(\frac{\beta+\gamma}{\alpha+\gamma}\right) \cdot \frac{w}{p} < 0 \tag{31}$$

which is a negatively shaped straight line as drawn in Figure 11.

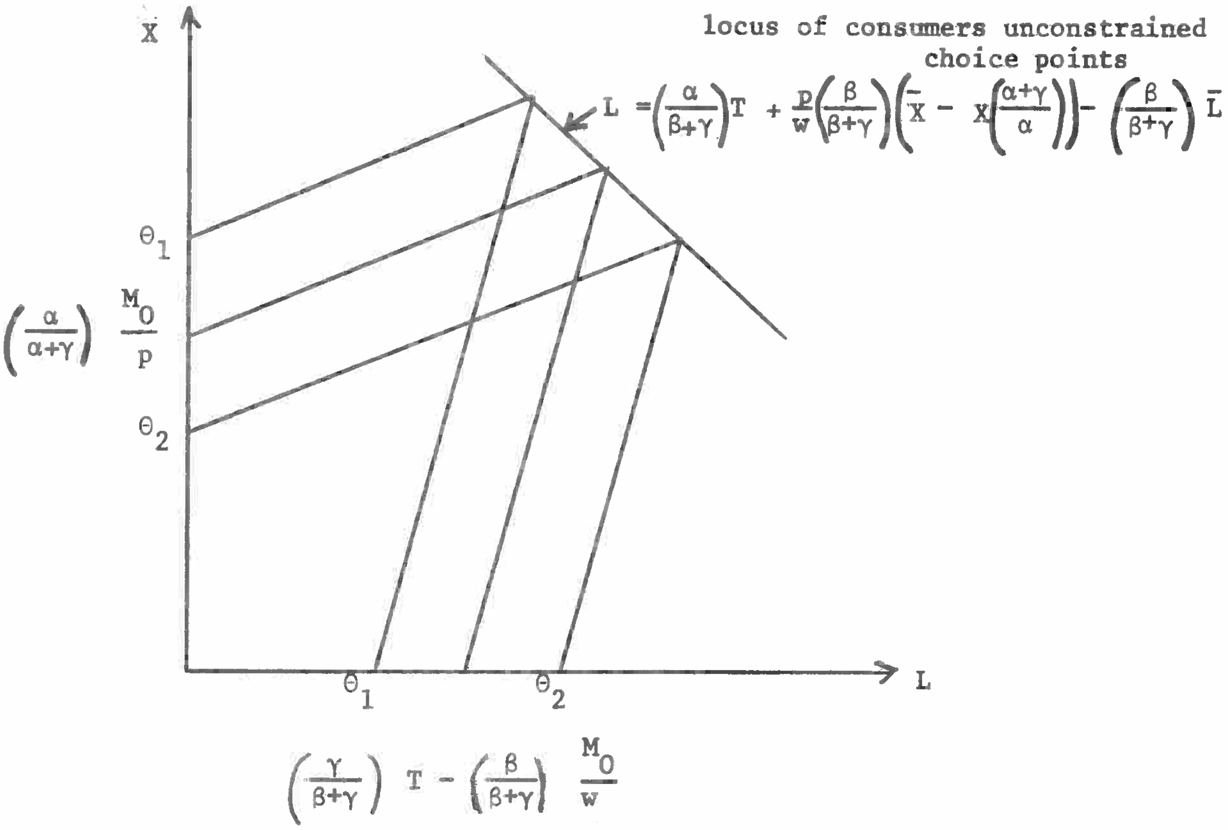


Figure 11

10/ Deviations of (30) and (31) can be found in Appendix 3.

The points marked θ_1 on the diagram represent the intercept given a rise in the money stock, the θ_2's represent a fall. As the intercepts vary the locus of consumers choice points is traced out as indicated.

If the production function and consumers expectations are reintroduced it can be shown that in certain monetary environments consumers expectations push the system towards the locus of consumers points.

If the relative price vector is fixed in successive periods, the money stock increasing (i.e. $\hat{M}_t^A > 0$) and the system initially in an expectational Keynesian temporary equilibrium then Figure 12 describes the market behaviour.

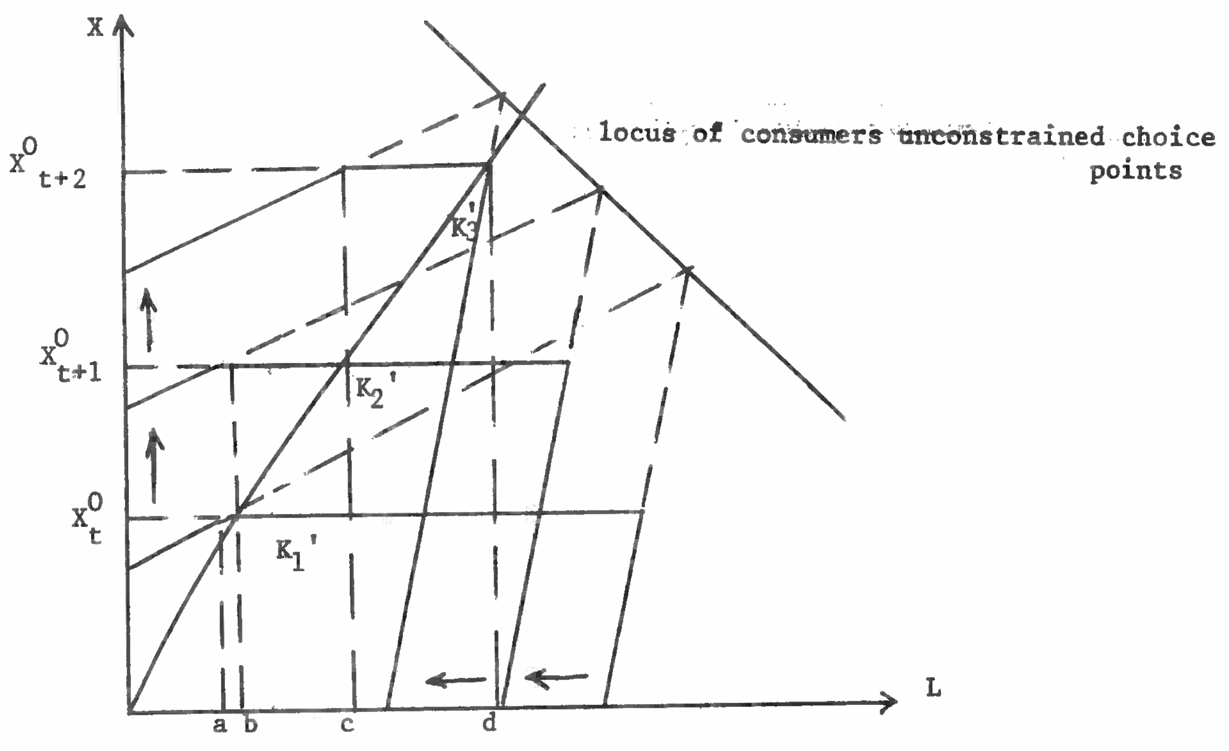


Figure 12

$a = \bar{L}_t \quad b = L'_t, \ \bar{L}_{t+1} \quad C = L'_{t+1}, \ \bar{L}_{t+2} \quad d = L'_{t+2}$

Here as the money stock grows the consumers wedge shifts upwards and to the left. Expectations adjust via the partial adjustment mechanism and goods purchase offers correspondingly rise, thus expectations push the system out along the production function towards the locus of consumers choice points. If on this regime the money supply were to be contracting then the wedge would shift in the opposite direction to that indicated, tracing out a series of orthodox Keynesian temporary equilibrium points. On the repressed inflation regime the same results hold but in each case the direction of the money supply change is reversed, i.e. a cut in the money supply is required to shift the system towards the locus of consumers choice points. Should expectations actually manage to push the system to the locus of consumers choice points, then output and employment will be defined by the production function, consumers choice point locus intersection. Giving levels of employment and output as defined below

$$L + L^{\delta} = \left(\frac{\gamma}{\beta+\gamma}\right)\left(\frac{\alpha+\gamma}{\alpha}\right)\left(\frac{\beta}{\beta+\gamma}\right)^{kT} - \left(\frac{\alpha}{\alpha+\gamma}\right)\frac{\bar{L}}{k} + \left(\frac{\alpha}{\alpha+\gamma}\right)\frac{p}{w}\,\frac{\bar{X}}{k}$$

$$X + X^{1/\delta} = \left[\left(\frac{\gamma}{\beta+\gamma}\right)\left(\frac{\alpha\,\gamma}{\alpha}\right)^{2}\left(\frac{\beta}{\beta+\gamma}\right)^{2}\right]\frac{p^{2}T}{w^{2}k^{1/\delta}} - \frac{1}{\beta(\beta+\gamma)k^{1/\delta}}\left[\bar{L} + \frac{\bar{X}}{w}\right]$$

If producers expectations are also reintroduced, and are also based on a partial adjustment mechanism, then there will come a point at which their expectations will place them on the short side of the markets. Consequently the consumers adjustment process will be halted by the occurrance of an expectational classical outcome.[11/] The market outcomes over several periods will thus depend upon three rates of change, consumers expectations, producers expectations and exogenous monetary changes. Also highly significant will be the "original"

11/ Here if the adjustment were infinitely fast as assumed in the consumers case the economy would always return to a classical outcome after one period.

state of agents expectations. There are of course many permutations of rates of change and original expectations which will produce quantitatively different outcomes, but the likelihood of these outcomes may well be a question for empirical study.

The preceeding argument examines expectations adjustment with a static relative price vector, this aspect is important but only part of the story. Consequently next consider the effect of changes in the price vector. Hence consider Figure 13.

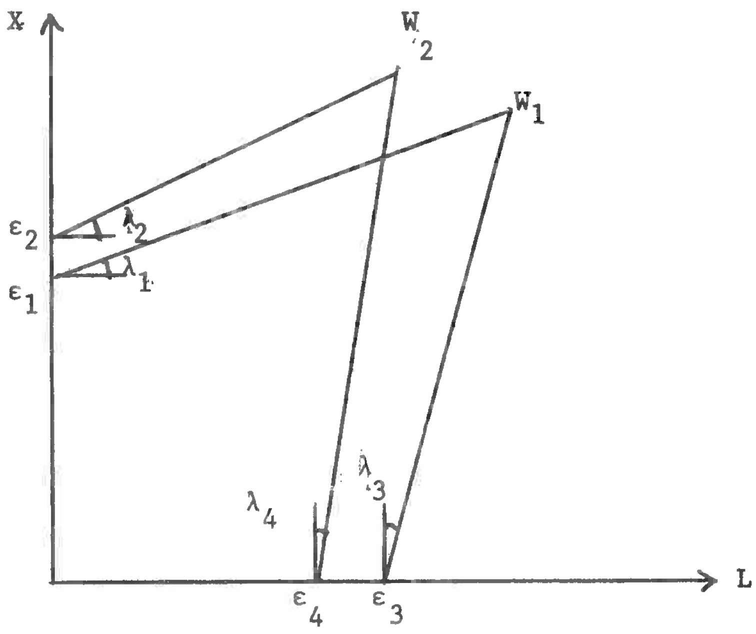


Figure 13

where $\lambda_1 = \left(\frac{\alpha}{\alpha+\gamma}\right)\frac{w_0}{p_0}$ $\lambda_2 = \left(\frac{\alpha}{\alpha+\gamma}\right)\frac{w_1}{p_1}$ $\lambda_3 = \left(\frac{\beta}{\beta+\gamma}\right)\frac{p_0}{w_0}$ $\lambda_4 = \left(\frac{\beta}{\beta+\gamma}\right)\frac{p_1}{w_1}$

$$\varepsilon_1 = \left(\frac{\gamma}{\alpha+\gamma}\right)\frac{M_0}{p_0} \quad \varepsilon_2 = \left(\frac{\alpha}{\alpha+\gamma}\right)\left(\frac{M_0+M_1^A}{p_1}\right) \quad \varepsilon_3 = \left(\frac{\gamma}{\beta+\gamma}\right) T - \left(\frac{\beta}{\beta+\gamma}\right)\frac{M_0}{w_0}$$

$$\varepsilon_4 = \left(\frac{\gamma}{\beta+\gamma}\right) T - \left(\frac{\beta}{\beta+\gamma}\right)\left(\frac{M_0+M_1^A}{w_1}\right)$$

In Figure 13 $w_1 > w_0$, $p_1 > p_0$ and $\frac{M_0}{w_0} < \frac{M_0+M_1^A}{w_1}$

For different parameter values of p_0, p_1, w_0, w_1, M_0, M_1^A a whole spectrum of regime line shifts can be generated. If one adds the production function to these, the potential outcomes are numerous, and I do not wish to examine them all here. To illustrate the model's properties consider what some may claim to be a familiar Keynesian contention, that a rise in the real wage will cure unemployment. Here it is demonstrated that a rise in the real wage will alleviate an expectational Keynesian unemployment situation. Consider Figure 14.

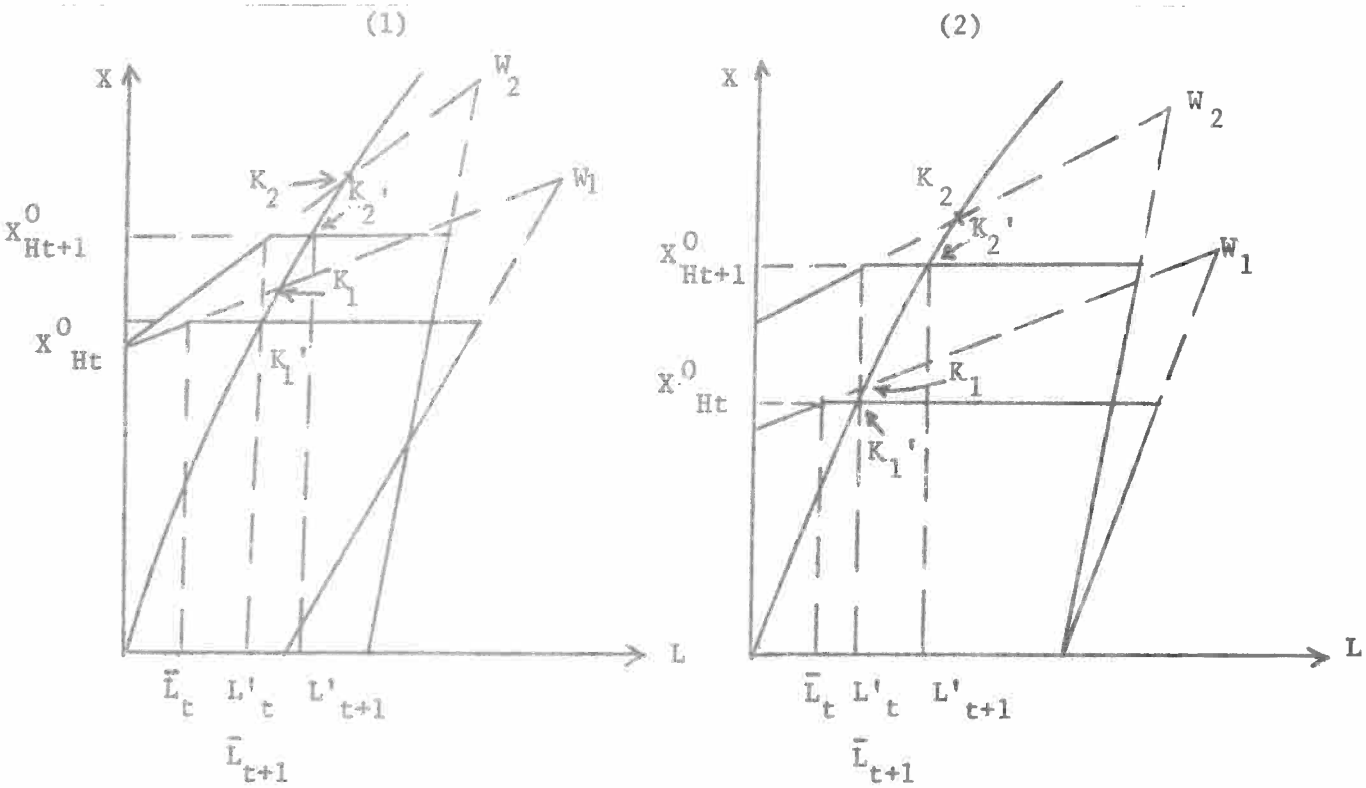


Figure 14

In Figure 14 (1) the rise in the real wage is achieved by increasing the nominal wage, this rotates the consumers wedge anti-clockwise and the system tends to K_1 rather than K_2, consequently both output and employment will rise. In (2) the price level falls hence raising the real wage, this again rotates the wedge anti-clockwise, but the effect is reinforced by an upward movement of the intercept on the goods axis due to an increase in real money balances. Hence here there is both a wedge rotating and real balance effect serving to increase both output and employment.[12/] Similar standard contentions can be made for real wage cuts on the repressed inflation regime.

(b) Government Policy

This section examines the effectiveness of government policy. Fiscal policy takes the form of changes in government expenditure (g), monetary policy involves the government choosing the exogenous monetary variable (M^A_t).

Con sider first the single period effect of a change in government expenditure on an expectational Keynesian outcome.

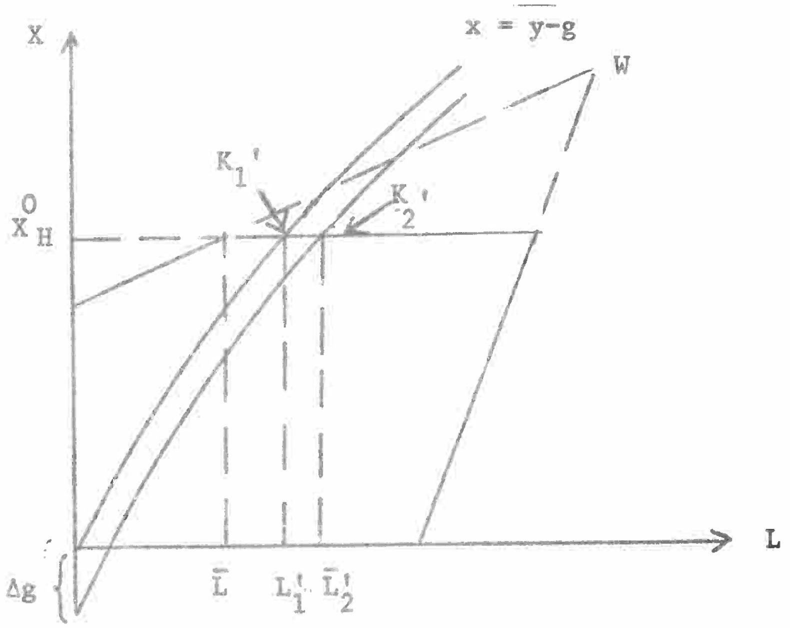


Figure 15

12/ Both figures are drawn for a static nominal money supply.

The increase in government expenditure pushes the production function down hence increases employment and income, but cannot affect consumption. The consumers are constrained on the goods market to their original offer. All extra output accrues to the government. This can be viewed as crowding out as none of the extra output is available for private consumption. If however the process is considered over two periods then both consumption and employment increase as indicated in Figure 16. Here the extra labour sold in the first period affects consumers second period constraint expectations and hence their goods purchase offers, which are achieved.

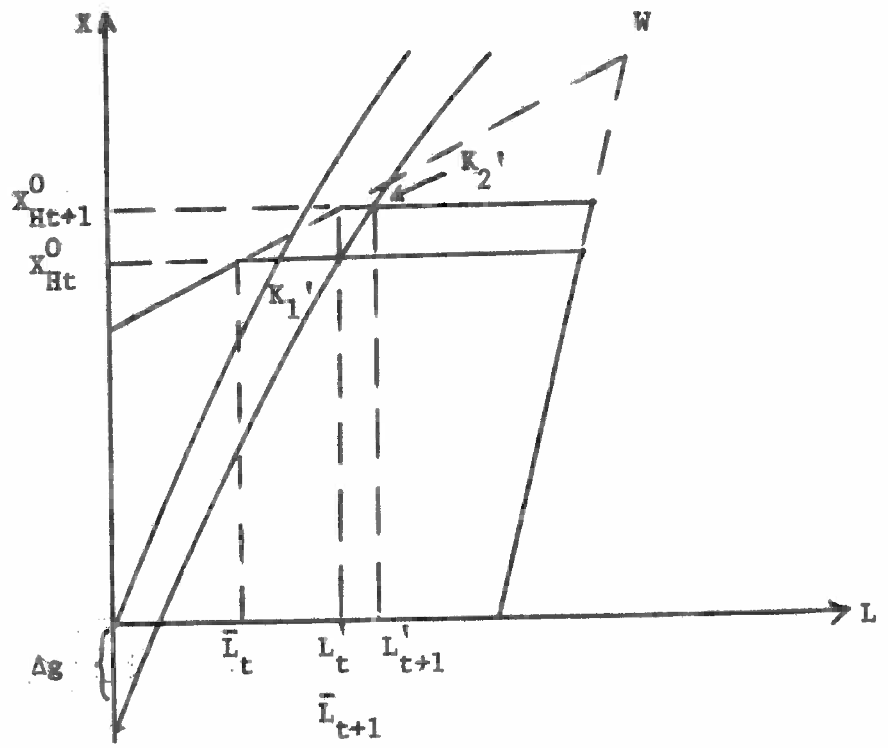


Figure 16

Figures 15 and 16 ignore the implications of assumption (I6), which suggests that the government purchases goods by printing money. Hence consider Figure 17.

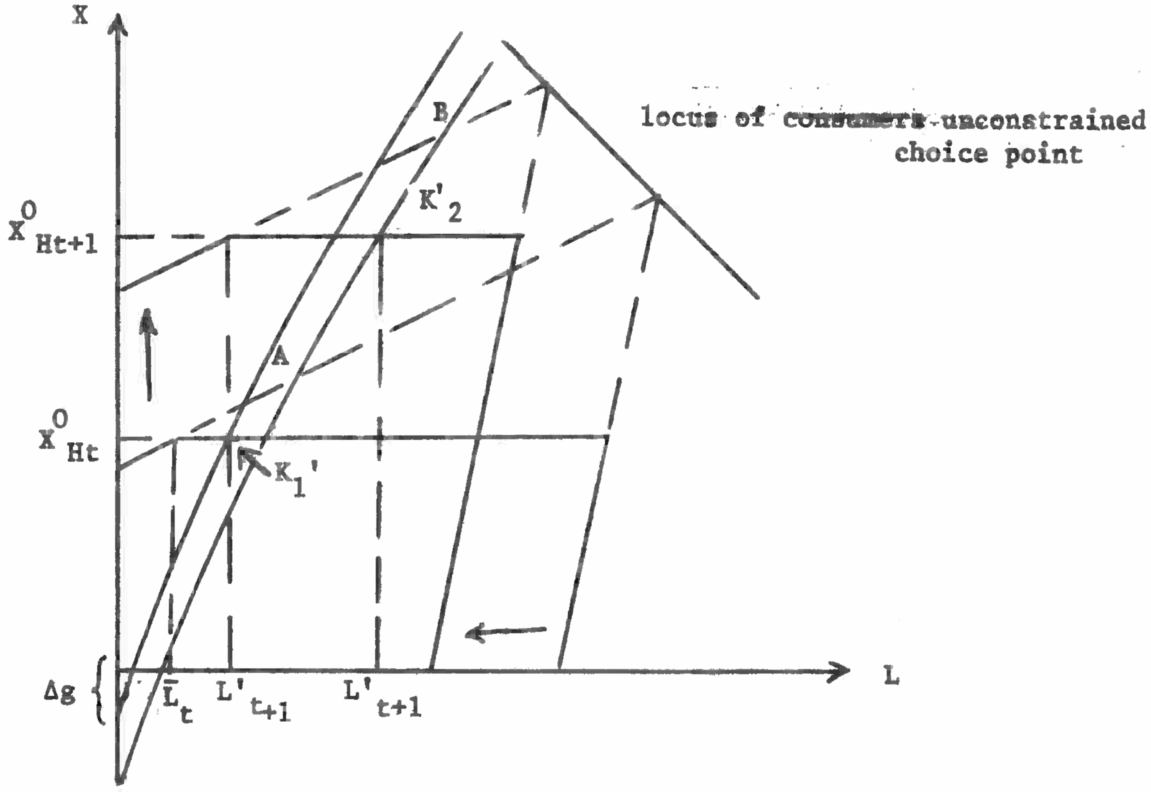


Figure 17

In Figure 17 the increased money balances generated by the government shift the consumers wedge upwards and to the left, thus reinforcing the effects of the production function shift. The system will now tend to point B rather than A. In this instance government policy is very effective in pushing the system towards the locus of consumers choice points. Again similar results hold for the repressed inflation regime but with the signs of the government expenditure and money supply changes reversed.

In the above it is implicitly assumed that producers expectations do not enter the problem. If however the system is in an expectational classical equilibrium then changes in government expenditure can produce a number of interesting outcomes. The two most obvious are illustrated in Figure 18.

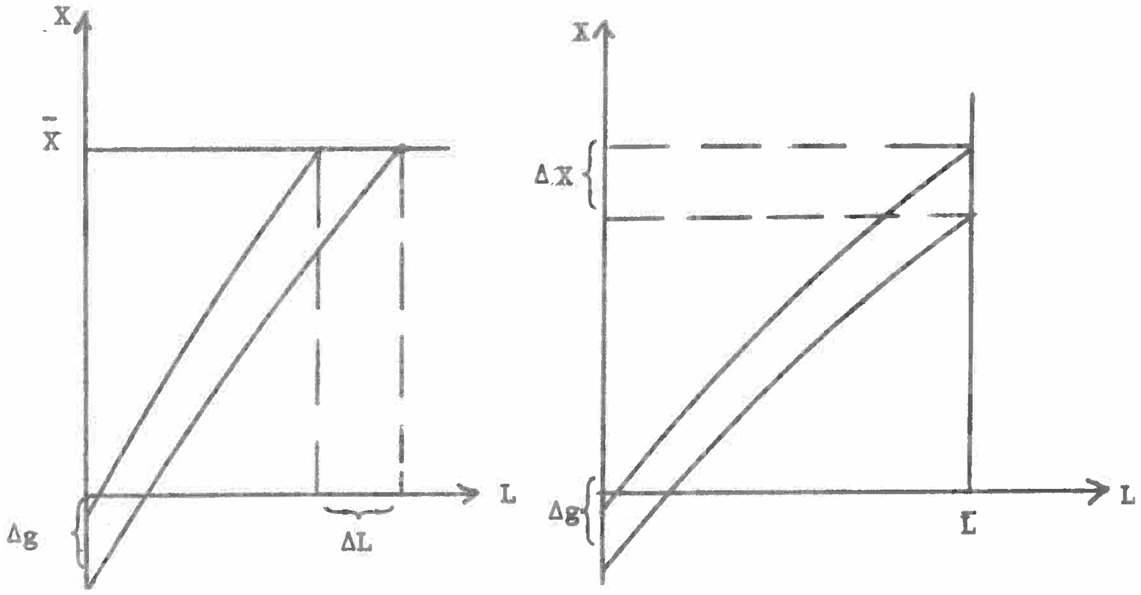


Figure 18

In (1) changes in government expenditure only change employment and not consumption, and vice versa for (2). These movements may cause the production function to cut one of the regime lines and switch to one of the other outcomes. However a more interesting case is generated when the producers unconstrained choice point is examined. As the production function shifts it traces out a locus of points where the real wage equals the marginal product of labour. The locus of producers choice points is defined by the expression

$$X = \frac{wL}{p\delta}^{2\delta-1} \qquad (32)$$

has slope $\frac{\partial X}{\partial L} = 2\delta-1 \left(\frac{wL}{p\delta}^{2\delta-2}\right) < 0$ iff $\delta < \frac{1}{2}$

Thus changes in government expenditure may cause a switch from an expectational to a true classical equilibrium as indicated in Figure 19.

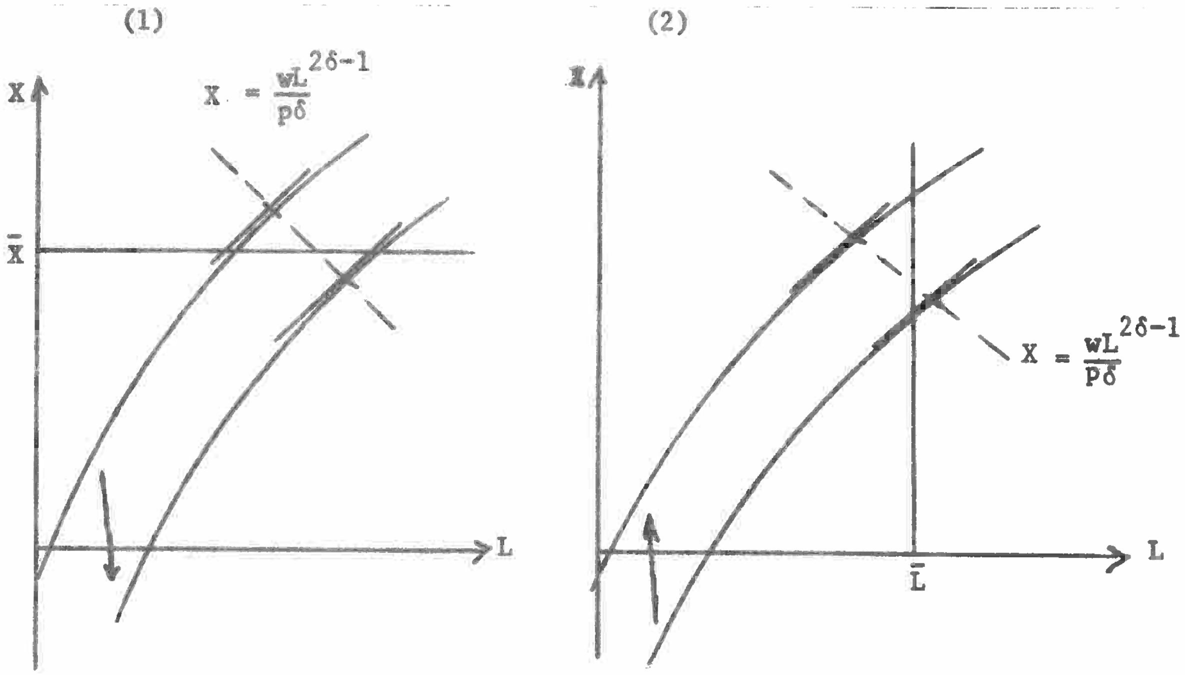


Figure 19

The system switches in (1) where

$$\bar{X} = \frac{wL^{2\delta-1}}{p\delta} \quad \text{in (2) where} \quad \bar{L} = \left(\frac{Xp\delta}{w}\right)^{\left(\frac{1}{2\delta-1}\right)}$$

Notice that if either of the two classical outcomes are encountered any movement up the production function, as in Figures 15, 16 or 17, will be choked off.

Government monetary policy takes the form of manipulating M_t^A, effectively discretionary lump-sum changes in consumers money balances. Changes in the money supply as a government instrument will produce the same outcome as in Figure 12, and is subject to the same provisos. Over the range in which regimes do not switch the money multipliers can be seen to be signed as follows.[13/]

13/ A derivation of these results can be found in Appendix 4.

Expectational Keynesian $\frac{\partial L}{\partial M_0} < 0$ $\frac{\partial X_H^0}{\partial M_0} > 0$

Expectational Repressed Inflation $\frac{\partial L_H^0}{\partial} < 0$ $\frac{\partial X}{\partial M_0} < 0$

In a world of scarce information as this model represents, agents may well take into account government policy when forming their expectations.

Let expectations be formed as follows:

(1) $\bar{L}_i = L'_{t-1} + f_j(\Delta g)$ $i = H, F$ $j = 1,2$

(2) $\bar{X}_i = X'_{t-1} + f_k(\Delta g)$ $k = 3,4$

Where the f's are positive functions.

How changes in government expenditure will affect expectations will depend upon the sign and magnitude of $\partial\bar{X}/\partial f$ and $\partial\bar{L}/\partial f$, this will not be independent of the regime obtaining. For the consumers Keynesian case the expected labour constraint is relevant, and the two possible effects of a government expenditure increase are analysed in Figure 20.

in Figure 20 (1) $\partial\bar{L}/\partial f > 0$

(2) $\partial\bar{L}/\partial f < 0$

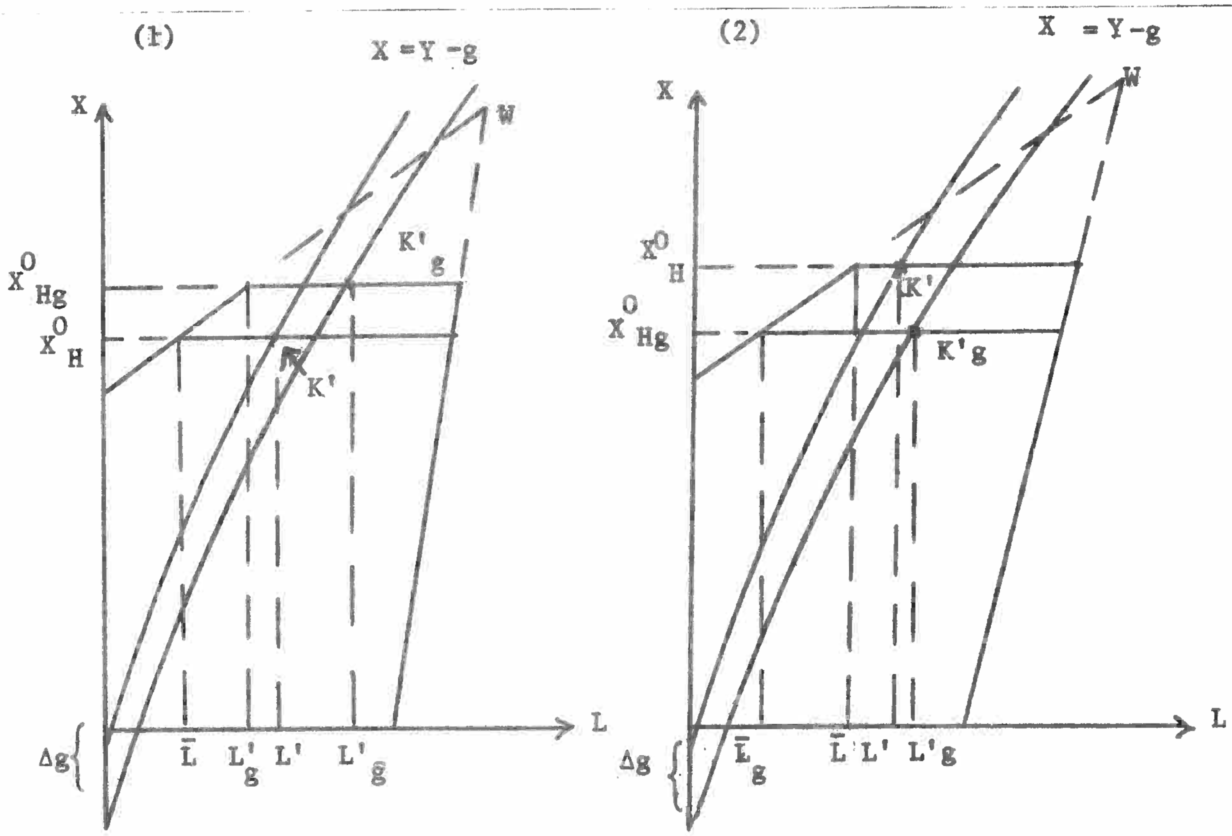


Figure 20

In (1) consumers optimistic expectations lead to an increase both in employment and consumption. In (2) consumption definitely decreases and employment may rise or fall dependent upon the relative magnitudes of the changes in consumer and government demand.[14/] In the expectational Keynesian case a positive correlation between changes in government expenditure and ration level expectations is most probable. A similar case holds for the repressed inflation regime, except optimist expectations will be associated with a government expenditure cut.

If the initial position is an expectional classical unemployment equilibrium, then how expectations adjustment in reaction to government expenditure changes affects the system depends upon which constraint binds the producer. Consider Figure 21.

14/ The timing of the government expenditure change may be quite crucial here.

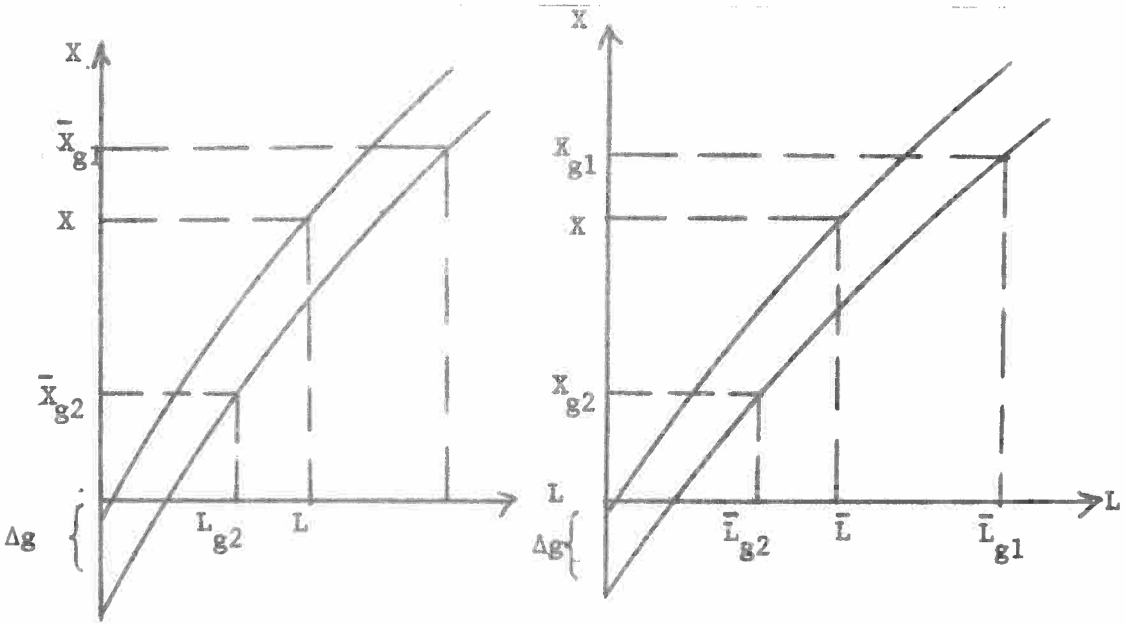


Figure 21

The effect of government expenditure increases in Figure 21 can easiest be summarised by a little algebra:

$$\partial\bar{X}/\partial f = \bar{X}_{g1} - \bar{X} > 0 \Rightarrow \Delta L > 0 \text{ and } \Delta_X > 0$$

$$\partial\bar{X}/\partial f = \bar{X}_{g2} - \bar{X} < 0 \Rightarrow \Delta L \gtreqless 0 \text{ and } \Delta X < 0$$

$$\partial\bar{L}/\partial f = \bar{L}_{g1} - \bar{L} > 0 \Rightarrow \Delta L > 0 \text{ and } \Delta X \gtreqless 0$$

$$\partial\bar{L}/\partial f = \bar{L}_{g2} - \bar{L} < 0 \Rightarrow \Delta L < 0 \text{ and } \Delta X < 0$$

where g1 indicates expectations revised in the same direction as government expenditure, g2 indicates expectations revised in the opposite direction.

Two results are clear here, if producers expect a goods ration and respond optimistically to an expenditure increase then both output and employment rise. If producers expect a labour ration and respond pessimistically to an expenditure increase then both output and employment fall.[15/]

15/ Here I have abstracted from any possibility of regime switch, and from consumers expectations.

(1) $\bar{L}_i = L'_{t-1} + R_j(\Delta M)$ $\quad i = H,F \quad j = 1,2$

(2) $\bar{X}_i = X'_t + R_k(\Delta M)$ $\quad k = 3,4$

where the R's are positive functions.

How expectations adjustments will modify the effects of money supply changes will depend upon the sign and magnitude of $\partial\bar{X}/\partial R$ and $\partial\bar{L}/\partial R$. For the consumers expectational Keynesian case, the expectational labour constraint is relevant and the two possible effects of the government increasing the money supply are analysed in Figure 22.

In 22 (1) $\partial\bar{L}/\partial R > 0$ (2) $\partial\bar{L}/\partial R < 0$

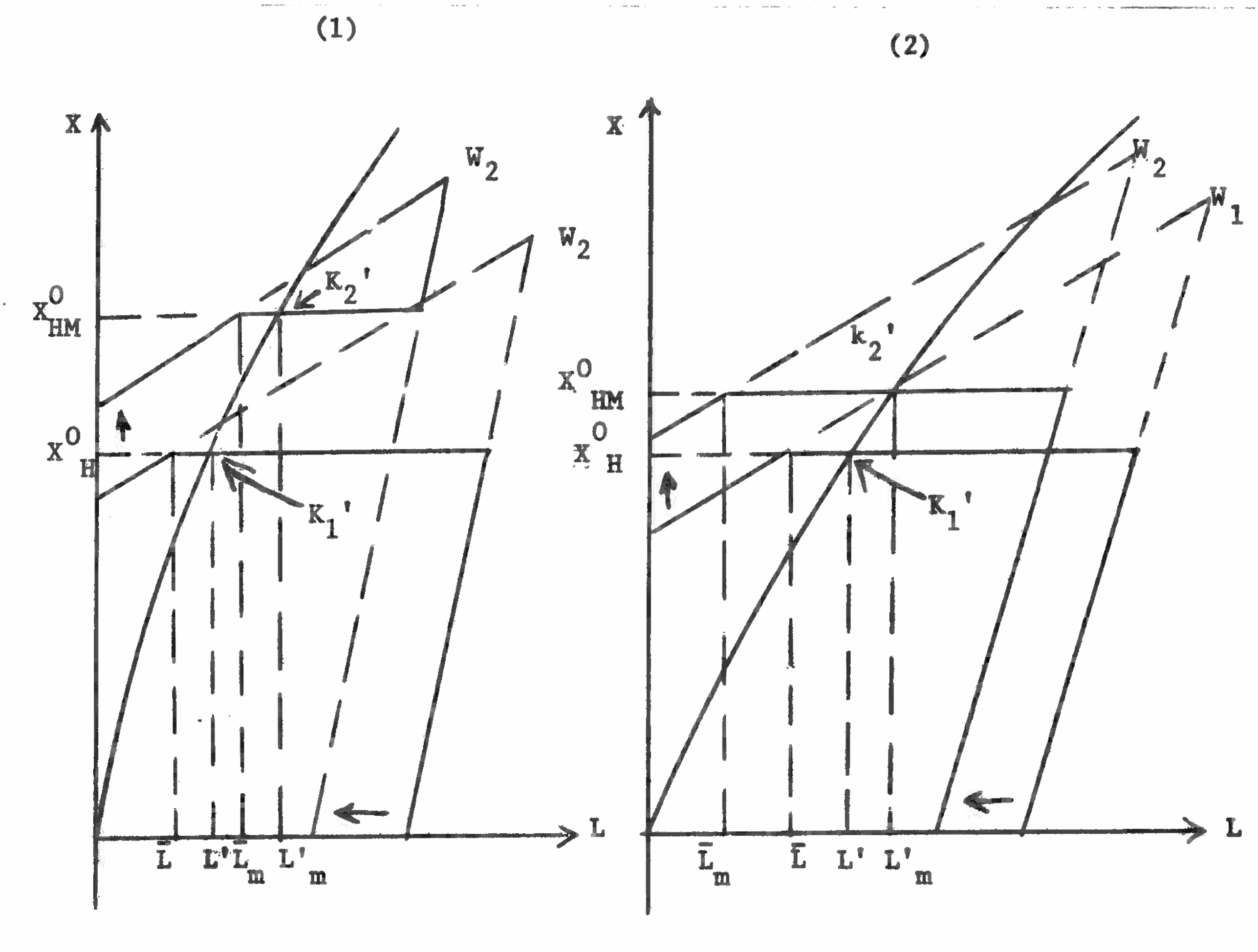


Figure 22

As the diagram demonstrates if an increase in the money supply generates optimism then output and employment will rise. If the change creates pessimism then the effects are ambiguous dependent upon the relative magnitudes of the monetary expansion and the expectations caused contraction. Here the case encapsulating optimism may be more reasonable. As in several prior instances the repressed inflation results are symmetric except that here a decrease in the money stock will unambiguously increase output and employment, provided the monetary contraction causes consumer optimism.

If the system is initially in an expectational classical unemployment equilibrium, then Figure 23 is relevant.

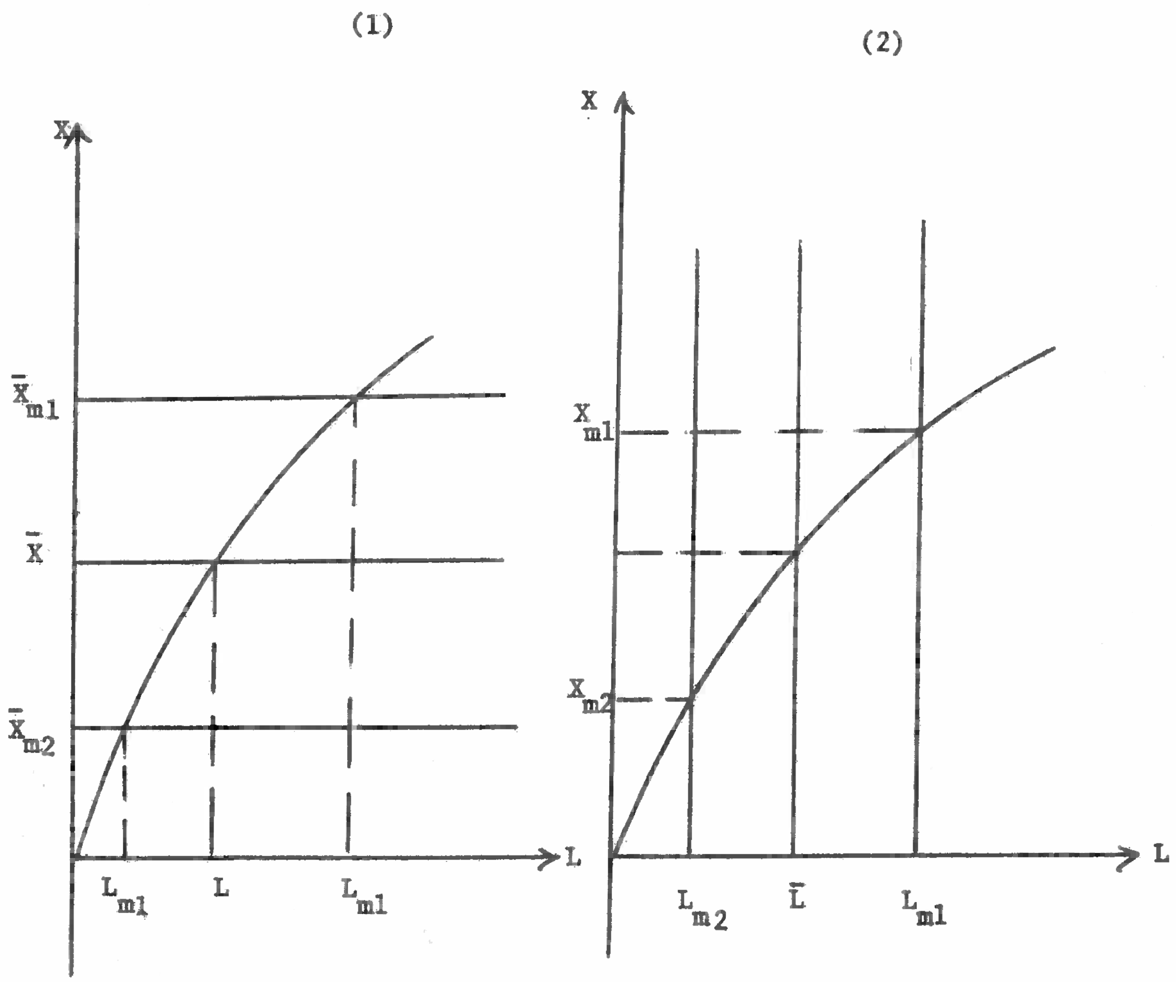


Figure 23

Hence if expectations are optimistally revised with a rise in the money supply then both output and employment rise, if they are pessimistically revised then the opposite happens.

Finally and most importantly it is interesting to consider whether government policy has a differential effect if it is announced or not. The "Rational Expectations" literature contends that prices clear markets and that government policy can only affect the economy if agents are somehow fooled in the short-run. The results obtained here are diametrically opposed to the rational expectations conclusions. Here announced policy measures will, if constraint expectations are rational, be enhanced by agents behavioural adjustments. For example compare Figures 15 and 20 (1), 15 represents an unnanounced and 20 (1) an announced fiscal expansion. The rational expectation philosophy would suggest that only the unannounced policy could be effective, yet here both policies are effective the announced policy more so.

7. Conclusions

This paper has emphasised the import of expectations in a non-Walrasian world. It is argued that expectations of rationing impinge on agents notional maximisation problems and hence are implicit in initial market offers. It is shown that agents will base these market offers on the "expectational" curves representing the minimum of the rationed and Walrasian curves. The point of note here being that the rationed portion of the expectational curve is the consequence of a spillover effect, the expectation of a constraint on one market spilling over to affect offers on the other.

Having established the expectational curves their implications were considered in a temporary equilibrium context, and were shown to generate three new temporary equilibria, termed expectational Keynesian, Classical and Repressed Inflation. It was also demonstrated that these expectational equilibria adjust over several market periods as expectations are revised in the light of experience. In a static monetary environment the expectational Keynesian and Repressed Inflation equilibria adjusted towards their standard namesakes, in dynamic monetary environment the expectational equilibria did under certain circumstances display self-adjustment towards the locus of unconstrained consumers choice points.

Finally it was shown that both fiscal and monetary policy are effective in the model, and that expectations and agents behavioural adjustments will tend to enhance rather than hinder government policy measures. This suggests that rather than keeping out of the economy as rational expectations proponents would suggest, the government should manipulate its policy instruments and make its behaviour perfectly clear to economic agents.

The model presented here is clearly limited in certain respects. However it is constructed to emphasise a particular point and any loss of generality that may have been incurred in doing so I feel is justified.

APPENDIX 1

Proof that the expectational goods demand function has the properties as shown in Figure 1.

Two properties are required:

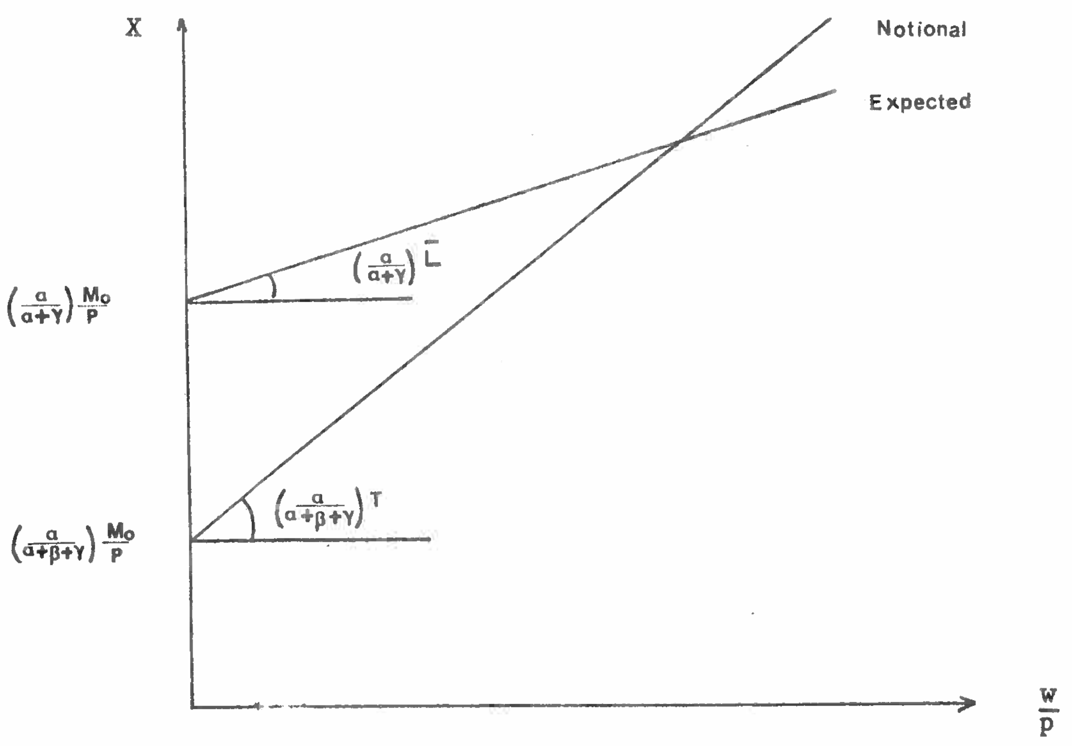
(1) Two curves, notional and expected, which form the expectational, interesect in the positive quadrant.

(2) The expected curve is steeper than the notional at the point of interesection.

notional $$X = \frac{\alpha}{\alpha+\beta+\gamma}\left(\frac{wT+M_0}{p}\right)$$

expected $$X = \frac{\alpha}{\alpha+\gamma}\left(\frac{w\bar{L}+M_0}{p}\right)$$

That these curves possess the appropriate properties is clear if they are examined in (X,w/p) space.



The intercept of the expected is greater than that of the notional

$$\left(\frac{\alpha}{\alpha+\beta}\right) > \left(\frac{\alpha}{\alpha+\beta+\gamma}\right)$$

Hence the condition for a positive intersection is that the slope of the notional is steeper than the slope of the expected.

$$\left(\frac{\alpha}{\alpha+\gamma}\right)^{\bar{L}} < \left(\frac{\alpha+\gamma}{\alpha+\beta+\gamma}\right)^{T}$$

OR $\quad \frac{\bar{L}}{T} < \frac{\alpha+\gamma}{\alpha+\beta+\gamma}$ which is true by equation (2)'

APPENDIX 2

Proof that the expectational labour supply function has the properties as shown in Figure 2.

Here three properties are required:

(1) The two curves, notional and expected which form the expectational, intersect in the positive quadrant.

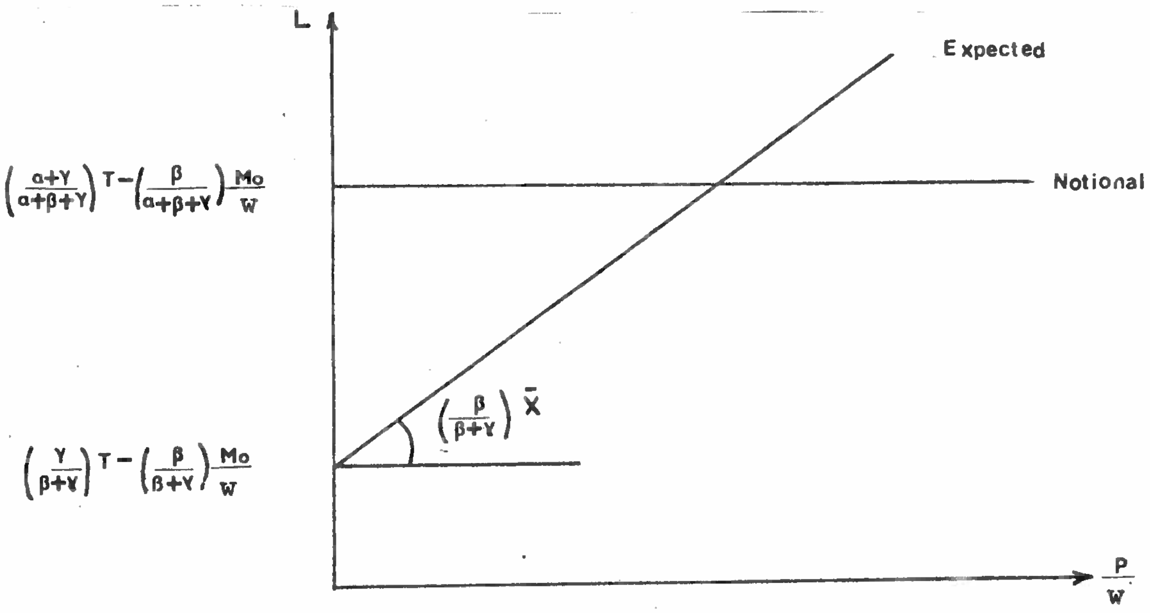
(2) The expected curve is steeper than the notional at the point of intersection.

(3) The point of interesection of the two curves occurs where labour is less than total time, i.e.: that the intersection is meaningful.

notional $\quad L = \left(\frac{\alpha+\gamma}{\alpha+\beta+\gamma}\right) T - \left(\frac{\beta}{\alpha+\beta+\gamma}\right)\frac{M_0}{w}$

expected $$L = \left(\frac{\gamma}{\beta+\gamma}\right) T - \left(\frac{\gamma}{\beta+\gamma}\right)\frac{M_0}{w} + \left(\frac{\beta}{\beta+\gamma}\right)\frac{p.\bar{X}}{w}$$

Examining these curves in (L,p/w) space clarifies matters.



We know that the slope of the expected is steeper than the slope of the notional.

If $$\left(\frac{\beta}{\beta+\gamma}\right)\bar{X} > 0$$

To demonstrate a positive interesection it is required that the notional curve has a positive intersection with the L axis.

Let $$\left(\frac{\alpha+\gamma}{\alpha+\beta+\gamma}\right) = E \qquad \left(\frac{\beta}{\alpha+\beta+\gamma}\right) = 1 - E$$

$$\left(\frac{\gamma}{\beta+\gamma}\right) = C \qquad \left(\frac{\beta}{\beta+\gamma}\right) = 1 - C$$

Hence (1-C) > $\Rightarrow$ E > C.

We require

$$ET - (1-E)\frac{M_0}{w} > 0$$

$$ET > (1-E)\frac{M_0}{w}$$

It is also required that the notional intersection with the L axis be greater than the expected.

$$ET - (1-E)\frac{M_0}{w} > CT - (1-C)\frac{M_0}{w}$$

$$ET - CT > (1-E)\frac{M_0}{w} - (1-C)\frac{M_0}{w}$$

$$T(E-C) > (C-E)\frac{M_0}{w} \quad \text{which is obviously satisfied.}$$

The condition for a meaningful intersection comes easily out of the notional expression

$$L = ET - (1-E)\frac{M_0}{w}$$

$$\frac{1}{E}L = T - \frac{(1-E)}{E}\frac{M_0}{w}$$

$$\frac{L}{E} + \frac{(1-E)}{E}\frac{M_0}{w} = T$$

$$\text{As } E > 1 \Rightarrow \left(\frac{1-E}{E}\right) > 0 = T > L$$

Thus the intersection is always meaningful.

APPENDIX 3

Derivation of the locus of consumers choice points, and proof that it is negatively sloped.

Rearranging the goods demand function (4)

$$X = \left(\frac{\alpha}{\alpha+\gamma}\right)\left(\frac{w\bar{L}+M_0}{p}\right)$$

$$p.X = \left(\frac{\alpha}{\alpha+\gamma}\right)(w\bar{L} + M_0)$$

$$\left(\frac{\alpha+\gamma}{\alpha}\right) p.X = w\bar{L} + M_0$$

$$\left(\frac{\alpha+\gamma}{\alpha}\right) p.X - w\bar{L} = M_0$$

Substituting for M_0 into the labour supply function (7) we get

$$L = \left(\frac{\gamma}{\beta+\gamma}\right) T - \left(\frac{\frac{\beta}{\beta+\gamma}}{w}\right)\left[\left(\frac{\alpha+\gamma}{\alpha}\right) p.X - w\bar{L}\right] + \left(\frac{\beta}{\beta+\gamma}\right)\frac{p}{w}\,.\,\bar{X}$$

$$L = \left(\frac{\gamma}{\beta+\gamma}\right) T - \left(\frac{\alpha+\gamma}{\alpha}\right)\left(\frac{\beta}{\beta+\gamma}\right) X.\frac{p}{w} - \left(\frac{\beta}{\beta+\gamma}\right)\bar{L} + \left(\frac{\beta}{\beta+\gamma}\right)\frac{p}{w}\,.\,\bar{X}$$

$$L = \left(\frac{\gamma}{\beta+\gamma}\right) T = \frac{p}{w}\left(\frac{\beta}{\beta+\gamma}\right)\left(\bar{X} - X\left(\frac{\alpha+\gamma}{\alpha}\right)\right) - \left(\frac{\beta}{\beta+\gamma}\right)\bar{L}$$

As the slope of the locus of consumers choice points depends on the relative responses of the two curves, good demands and labour supply, to changes in households money balances, then it can be derived as follows.

Consider

$$X = \left(\frac{\alpha}{\alpha+\gamma}\right)\left(\frac{w\bar{L} + M_0}{p}\right)$$

$$L = \left(\frac{\gamma}{\beta+\gamma}\right) T - \left(\frac{\gamma}{\beta+\gamma}\right)\frac{M_0}{w} + \left(\frac{\beta}{\beta+\gamma}\right)\frac{p}{w} \cdot \bar{X} \qquad (7)$$

From (4)

$$\frac{\partial X}{\partial M_0} = \left(\frac{\alpha}{\alpha+\gamma}\right)\frac{1}{p}$$

$$\partial X = \left(\frac{\alpha}{\alpha+\gamma}\right)\frac{\partial M_0}{p}$$

From (7)

$$\frac{\partial L}{\partial M_0} = \left(\frac{\beta}{\beta+\gamma}\right)\frac{1}{w}$$

$$\partial L = \left(\frac{\beta}{\beta+\gamma}\right)\frac{\partial M_0}{w}$$

Hence

$$\frac{\partial X}{\partial L} = -\frac{\frac{\alpha}{\alpha+\gamma}}{\frac{\beta}{\beta+\gamma}} \cdot \frac{\frac{\partial M_0}{p}}{\frac{\partial M_0}{w}} = \frac{\alpha(\beta+\gamma)}{p(\alpha+\gamma)} \cdot \frac{w}{p}$$

$$\therefore \quad \frac{\partial X}{\partial L} < 0$$

Hence the locus is negatively sloped.

APPENDIX 4

Derivation of money multipliers:

(1) Expectational Keynesian case

$$X_H^0 = \left(\frac{\alpha}{\alpha+\gamma}\right)\left(\frac{w\bar{L}+M_0}{p}\right)$$

$$L = \left(\frac{X_H^0}{k}\right)^{1/\delta}$$

$$L = \left(\frac{\left(\frac{\alpha}{\alpha+\gamma}\right)\left(\frac{w\bar{L}+M_0}{p}\right)}{k}\right)^{1/\delta}$$

$$\frac{\partial X_H^0}{\partial M_0} = \left(\frac{\alpha}{\alpha+\gamma}\right)\frac{1}{p} > 0$$

$$\frac{\partial L}{\partial M_0} = \frac{1}{\delta}\left(\left(\frac{\alpha}{\alpha+\gamma}\right)\left(\frac{w\bar{L}+M_0}{pk}\right)\right)^{\frac{1-\delta}{\delta}} \left(\frac{\alpha}{\alpha+\gamma}\right)\frac{1}{pk}$$

$$\frac{\partial L}{\partial M_0} > 0$$

(2) Expectational Repressed Inflation Case

$$L_H^0 = \left(\frac{\gamma}{\beta+\gamma}\right) T - \left(\frac{\beta}{\beta+\gamma}\right)\frac{M_0}{w} + \left(\frac{\beta}{\beta+\gamma}\right)\frac{p}{w} \cdot \bar{X}$$

$$X = kL_H^{0^{\delta}}$$

$$X = k\left(\left(\frac{\gamma}{\beta+\gamma}\right) T - \left(\frac{\beta}{\beta+\gamma}\right)\frac{M_0}{w} \left(\frac{\beta}{\beta+\gamma}\right)\frac{p}{x} \cdot \bar{X}\right)^{\delta}$$

Hence $$\frac{\partial L_H^0}{\partial M_0} = -\left(\frac{\beta}{\beta+\gamma}\right)\frac{1}{w} < 0$$

$$\frac{\partial X}{\partial M_0} = -\delta\, k\left[\left(\frac{\gamma}{\beta+\gamma}\right) T - \left(\frac{\beta}{\beta+\gamma}\right)\frac{M_0}{w} + \left(\frac{\beta}{\beta+\gamma}\right)\frac{p}{w}\,.\,\bar{X}\right]^{\delta-1}\left(\frac{\beta}{\beta+\gamma}\right)\frac{1}{w}$$

$$\frac{\partial X}{\partial M_0} < 0$$

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