External Effects : An Alternative Formulation

by

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I Introduction

Economists have long recognised that the actions of individual agents may affect the decisions or the well being of others in important ways without necessarily being mediated through the market. This recognition has spawned a large and evergrowing literature.

When modelling the behaviour of consumers, this literature has generally used the direct utility function as the basic tool of analysis. In this paper, we discuss the use of alternative, dual formulations of consumer behaviour. In Section III we discuss the use of the minimum expenditure function and of the indirect utility function in modelling the behaviour of an individual consumer who acts as a price taker in markets for tradeable commodities and as a quantity taker in his consumption of certain environmental commodities. Subsequently, we look at a simple model involving reciprocal externalities which has recently been the subject of discussion by Diamond and Mirrlees (1973), Sandmo (1978), Sadka (1978) and Sheshinski (1978). Part of this literature deliberately abstracts from real income effects, and in discussing this part we find the minimum expenditure function and its associated compensated demand functions particularly fruitful. It is our belief that the dual approach serves to clarify a number of issues in this area.

II Modelling the Individual : The Utility Function

The consumer faces a set of prices $p \equiv (p_1 \ldots p_n)$ for private commodities, quantities of which are denoted by $x \equiv (x_1 \ldots x_n)$. In addition, he faces a set of quantities $q \equiv (q_{n+1} \ldots q_r)$ of "environmental" commodities. Such commodities may be thought of as being consumed by him in quantities which are not wholly under his control. In particular, we may interpret them as externalities which influence the environment within which the consumer allocates his resources, but which are themselves determined wholly by the actions of others. Possible reasons for the persistent failure of the market system to eliminate or internalise such externalities have been extensively discussed elsewhere and lie beyond the scope of this paper, which takes their existence as given. The absence of certain markets denies the individual an opportunity to trade up to the point where the marginal utilities of all commodities are proportional to the prices which he faces. Throughout our analysis, we assume that the consumer is a price-taker for all private commodities, and a quantity-taker for all environmental commodities. While this ignores a number of interesting problems, and certainly cannot be justified as a literal description of behaviour when small numbers of agents are involved, it is an abstraction which has proved useful for certain limited purposes.

A common formulation of the consumer's problem models the consumer as solving the problem

$$\underset{x}{\mathrm{Max}} \{U(x,q) \mid px = y \text{ , } q \text{ preassigned}\}. \tag{1}$$

This defines a "restricted" indirect utility function, $I(p,q,y)$, from which "restricted" Marshallian demand functions for private commodities may be generated using Roy's identity:

$$m_i(p,q,y) = \frac{\partial I}{\partial p_i} / (\sum_j \frac{\partial I}{\partial p_j} \frac{p_i}{y})$$

Since I(p,q,y) is homogeneous of degree zero in p and y,

$$\sum \frac{\partial I}{\partial p_j} p_j + \frac{\partial I}{\partial y} y = 0,$$

$$\therefore \qquad m_i(p,q,y) = -\frac{\partial I}{\partial p_i} / \frac{\partial I}{\partial y} . \tag{2}$$

Also, the partial derivative $\partial I/\partial q_k$ is the marginal utility of q_k. Dividing this by the marginal utility of income yields the amount of money which the individual would be prepared to pay for an extra unit of q_k. In short, we may define the marginal valuation functions for environmental commodities,

$$v_k(p,q,y) = \frac{\partial I}{\partial q_k} / \frac{\partial I}{\partial y} . \tag{3}$$

$\partial I/\partial y$, the marginal utility of income, is assumed strictly positive throughout our analysis. $\partial I/\partial q_k$, and hence $v_k(\ )$, may be of either sign. If it is positive, an increase in q_k represents a beneficial externality, while a negative or detrimental externality is associated with negative $\partial I/\partial q_k$.

The amount of income required to provide exact compensation for a change dq_k is given by

$$dy = -(\frac{\partial I}{\partial q_k} / \frac{\partial I}{\partial y}) \, dq_k . \tag{4}$$

III An Alternative Formulation : the Expenditure Function

For certain purposes, it is more useful to formulate the consumer's problem in terms of the "restricted" expenditure function. The consumer is modelled as solving the problem

$$\min_x \{px \mid U(x,q) = u, \quad q \text{ preassigned}\}.$$

This defines the restricted expenditure function $E(p,q,u)$. We assume that for any vector q the structure of consumer preferences over the private commodities is such that $U(x\ q)$ and $E(p\ q\ u)$ have their usual properties. In particular we assume that $E(\ )$ is differentiable as many times as is convenient. The following properties of $E(\ )$ will be used subsequently:

(i) $E(\ )$ is an increasing function of u. That is, $E_u \equiv \partial E/\partial u > 0$. Its reciprocal is the marginal utility of income.

(ii) $E(\ )$ is concave in all prices. This implies that $E_{pp} \equiv \{\partial^2 E/\partial p_i\ \partial p_j\}$ is negative semidefinite.

(iii) $E(\ )$ is homogeneous of degree 1 in all prices. Therefore $p.E_p(\ ) = \sum_{i=1}^{n} p_i\ \partial E/\partial p_i = E(\ )$. $E_{pp}(\ )$ is therefore singular.

(iv) Differentiation of $E(\ )$ with respect to private commodity prices yields compensated demand functions. In vector notation,

$$c(p,q,u) = E_p(p,q,u)$$

Various properties of $c(\ )$ follow from this. In particular the matrix of compensated substitution effects is $\{\partial c_i/\partial p_j\} = \{\partial^2 E/\partial p_i\ \partial p_j\}$ which is symmetric and, as we have already noted, negative semidefinite and singular.

It remains to consider the relationship between q on the one hand and E( ) and c( ) on theother. For simplicity, suppose q is a scalar. Then $E_q \equiv \partial E/\partial q$ is the compensating variation associated with dq. It is the amount by which y must change to provide exact compensation for dq. This is clearly so, since from ( )

$$E_q \, dq + E_u \, du = dy$$

If exact compensation is provided, du = 0, and the necessary change in income is given by

$$dy = e_q \, dq \tag{5}$$

Comparing this with (4), $E_q = -I_q/I_y$, or $E_q/E_u = -I_q$, so that beneficial (detrimental) externalities dq are associated with negative (positive) E_q.

Turning to the compensated demand function, the parameters $\partial c_i/\partial q$ are the demand responses to changes in q at constant prices and utility. These responses satisfy the relationship

$$p.c_q = \sum p_i \, \partial c_i/\partial q = E_q$$

Apart from this, there are no restrictions on the terms $\partial c_i/\partial q$. An increase in the incidence of pollutants in tap water may, at constant utility, increase the demand for purifying tablets and for gin, while reducing that for concentrated fruit juice.

Changes in q have two conceptually distinct implications for the recipient. First, there is an effect on his welfare:

$$\partial I/\partial q \equiv I_q = -E_q/E_u \; .$$

This may be termed the welfare effect. Second, there is what may be termed the "compensated behaviour" effect, summed up by the vector of compensated responses

$$\partial c/\partial q \equiv c_q.$$

The total response by the recipient to a change in q is summed up by the Marshallian response

$$\partial m/\partial q \equiv m_q = c_q + I_q c_u \tag{6}$$

It is the welfare effect, I_q, together with its implications for behaviour, that is the usual focus of interest in the literature on externalities. Typically, externalities are modelled by inserting an additional argument into the recipient's utility function - see, for example, Heller and Starrett (1976) or Diamond and Mirrlees (1973). This tends to confound the welfare and behavioural aspects of external effects. As Sandmo has pointed out, if our interest is in relative price and quantity movements in externality models, demand functions are the appropriate starting point. The present formulation has the great advantage of deriving behavioural functions in a way which keeps the "pure welfare" and "compensated behaviour" effects separate. (6) shows very clearly that, even if $I_q=0$, so that there is no pure welfare effect, either detrimental or beneficial, substitution effects may exist which determine the comparative static behaviour of the system. In any given context, both effects can be expected to be present, but it may be illuminating to consider polar cases in which one or other is absent.[1] Section IV will take up the issue of reciprocal interdependence between consumers in a model in which welfare effects are suppressed, so that pure substitution responses to price and quantity changes determine outcomes.

For the purposes of this section, the precise manner in which q is determined does not matter. Its important characteristic is that, though variable, it is unaffected by any action of the recipient, who acts as a quantity taker. However, we indicate the possible applications of the analysis by listing some of the common mechanisms by which q may be determined.

(i) Consumption externality: $q = f(x_i^h)$. The environmental commodity depends on the consumption of i by individual h.

(ii) Production externality: $q = f(s_i^f)$. Here s_i^f represents supply of commodity i by firm f. s_i^f may be negative.

(iii) Public goods: $q = f(S_i)$ where $S_i = \sum_f s_i^f$. If the i^{th} commodity is a nonoptional public good, or if it is optional but its total supply is less that that which the individual would most prefer at zero price, the present formulation is appropriate. Public "bads" are, of course, dealt with by changing signs where appropriate.

(iv) Non-meddlesome altruism or envy:[2] $q = f(u^h)$. q depends on the utility level of individual h.

(v) Congestion: $q = f(\sum_h x_i^h, S_i)$: this formulation seems to capture the notion of congestion as discussed by Oakland. The amount of congestion, q, is a decreasing function of the level of provision of a service S_i and an increasing function of the aggregate consumption of that service, $\sum_h x_i^h$.

IV Reciprocal Interdependence

Two consumers, a and b, each choose quantities of commodities 1 and 2. Each individual's consumption of commodity 2 generates externalities. and each takes the other's consumption of commodity 2 as given. Using the "restricted" minimum expenditure function as the basis for analysis, the i^{th} individual's problem is:

$$E^i(p_1,p_2,q^i,u^i) \equiv \min_{x_1^i,x_2^i} \{p_1 x_1^i + p_2 x_2^i \mid u^i(x_1^i,x_2^i,q^i) = u^i,\ q^i = x_2^j\} \tag{6}$$

where i=a, b ; j=a, b ; i≠j.

The level of utility which appears as an argument in $E^i(\ )$ is determined by introducing the budget constraint:

$$E^i(p_1,p_2,q^i,u^i) = y^i .$$

In fact, much of the literature to which we have referred has concentrated on the possibility of anomaly even in the absence of real income effects. In much of what follows, we will put $du^i = 0$. The present formulation then makes the analysis very straightforward.

Using (6) we can readily define the restricted compensated demand functions:

$$c_k^i(p_1,p_2,q^i,u^i) = \partial E^i(p_1,p_2,q^i,u^i)/\partial p_k \qquad \begin{matrix} i = a,b \\ k = 1,2 \end{matrix} \tag{7}$$

The comparative static properties of (7) have already been discussed. The only new feature in the present exercise is that, while q^a and q^b are exogenous from the point of view of the recipient agent, they are endogenous to the whole system.

We now pose the following question. What will be the response of total demand for commodity 2, $X_2 = x_2^a + x_2^b$ to changes in p_2 keeping p_1 and u^i fixed? In particular, can dX_2/dp_2 be positive as a result of the interdependence in the model? Such a possibility is termed a price anomaly.

Consider individual a. Putting i = a and k = 2 in equation (7), and recalling that du^a is zero by assumption,

$$dc_2^a/dp_2 = c_{22}^a + c_{2q}^a(dq^a/dp_2) \tag{8}$$

The total response is the sum of the conventional compensated price response and the response by individual a to any change which may come about in the level q^a, which he regards as exogenous.

But $q^a = c_2^b(p_1, p_2, q^b, u^b)$, so that

$$dq^a/dp_2 = c_{22}^b + c_{2q}^b(dq^b/dp_2) \tag{9}$$

Substituting (8) into (9),

$$dc_2^a/dp_2 = (c_{22}^a + c_{2q}^a c_{22}^b)/(1 - c_{2q}^b c_{2q}^b) \tag{10}$$

A similar expression holds for individual b's response, with the superscripts reversed. Summing the two responses yields the change in total demand:

$$dX_2/dp_2 = (c_{22}^a + c_{22}^b + c_{2q}^a c_{22}^b + c_{2q}^b c_{22}^a)/(1 - c_{2q}^a c_{2q}^b) \tag{11}$$

Without further analysis, there appear to be two possible sources of anomaly. The second term in brackets in the numerator of (11) could conceivably be positive and dominate the conventional negative responses in the first term. Also the denominator has yet to be signed.

As it turns out, stability considerations require the denominator to be positive, leaving the numerator to determine whether behaviour is anomalous or not. We must now discuss stability.[3]

Suppose there is an equilibrium. Let consumer prices be fixed, and let real income be constant for both individuals. Then the demand function becomes:

$$x_2^a = c_2^a(q^a) \quad , \quad x_2^b = c_2^b(q^b) \; .$$

The possibility of instability arises in the following way: suppose a small perturbation pushes the system away from the equilibrium, $\bar{x}_2^a$, $\bar{x}_2^b$. Too high a level of x_2^a, say, provokes a reaction in x_2^b. This feeds back to x^a, pushing it even further away from the equilibrium. You turn your hifi set a little too high, so I turn up the volume on my television, so you turn up your hifi a bit more, and so on, with possibly explosive results.

It is shown in the appendix that stability with respect to quantity adjustments of the linearised system around $\bar{x}_2^a$, $\bar{x}_2^b$ requires that

$$c_{2q}^a \, c_{2q}^b \quad < \quad 1.$$

Thus if we assume stability with respect to the specified process, the denominator in (11) must be positive. Figure 1 shows some possibilities.

Restricting attention to locally stable equilibria, therefore we can say that the comparative static response depends upon the sign of the numerator in (11).

Equation (11) allows us to consider some special cases:

(i) Completely symmetric behaviour: if $c_{22}^a = c_{22}^b \equiv c_{22}$, then (11) becomes

$$dX_2/dp_2 = 2c_{22}(1 + c_{2q})/|1 - (c_{2q})^2| .$$

Stability implies $|c_{2q}| < 1$, so that we have sign dX_2/dp_2 = sign c_{22}, which is nonpositive.

(ii) Partially symmetric behaviour: indeed, a moments' reflection shows that we may dispense with equality between c_{22}^a and c_{22}^b, and still have the result given for case (i).

(iii) $c_{2q}^a \geqq 0$, $c_{2q}^b \geqq 0$.
The numerator of (11) must be positive, so that any stable equilibrium exhibits a normal price response.

(iv) Unilateral dependence: let $c_{2q}^b = 0$. Then (11) becomes

$$dX_2/dp_2 = c_{22}^a + c_{22}^b + c_{2q}^a c_{22}^b$$

Stability is assured, and a normal response requires that

$$c_{2q}^a > -(c_{22}^a + c_{22}^b)/c_{22}^b .$$

An analogous result holds for c_{2q}^b when $c_{2q}^a = 0$. Figure II, which graphs the elasticities $\eta_{2q}^i \equiv (q^i c_{2q}^i/c_2^i)$, summarises the results so far. In the northeast quadrant, any stable equilibrium evinces normal behaviour for the total compensated price response dX_2/dp_2. Similarly, for pairs of values on QR, the response is normal. Finally, there are points S and T such that along the η_{2q}^a axis to the right of S, and along the η_{2q}^b axis above T, behaviour is normal, while to the left of S and below T perverse behaviour arises.

This analysis suggests that a curve may be drawn through S, Q and T - the dashed curve in Figure II - such that below and to the left of it stable equilibria exhibit perverse behavior. One general statement which may be made is that the possibility of anomaly depends upon a certain amount of asymmetry in the system, in the sense that $\eta_{2q}^a \neq \eta_{2q}^b$. More this cannot be said, without making further assumptions.

The analysis of this section demonstrates the crucial role of the parameters $\partial c_k{}^i/\partial q^i$ and $\partial c_k{}^i/\partial p_k$ in determining stability and comparative static properties of equilibrium. We have assumed that, at least locally, external effects are neutral in the sense that the recipient is made neither worse nor better off by changes in q. There is simply a "compensated behaviour" effect, and in the resulting equilibrium behaviour is determined by pure substitution terms. We have also ignored the real income effects of price changes. These are easily incorporated, as we now show.

Suppose that each individual has a fixed money income, y^i, so that:

$$E^i(p_1, p_2, q^i, u^i) = y_i$$

Then $$E_2{}^i\, dp_2 + E_q{}^i\, dq + E_u{}^i\, du = 0.$$

We retain the assumption that changes in q^i have no welfare effect - we are simply incorporating the effect of price changes on utility. Then it may be confirmed that the total response is

$$\frac{dX_2}{dp_2} = \frac{1}{1-c_{2q}{}^a c_{2q}{}^b}\Bigg[(c_{2p}{}^a + c_{2q}{}^a c_{2p}{}^b) + (c_{2p}{}^b + c_{2q}{}^b c_{2p}{}^a) - \frac{E_p{}^a}{E_u{}^a}(c_{2u}{}^a + c_{2q}{}^b c_{2u}{}^a) - \frac{E_p{}^b}{E_u{}^b}(c_{2u}{}^b + c_{2q}{}^a c_{2u}{}^b)\Bigg]. \tag{12}$$

The patterns in this expression are clear. The first two expressions within the square brackets are "pure substitution" terms, while the last two are real income terms. In each case there is a direct and an indirect response. The consequences of adding in the real income terms depend on such considerations as whether the commodity is an inferior good. Detailed analysis does not seem worthwhile, but it is worth emphasising again that this result has been obtained in a model in which the "pure externality", or welfare effect of dq^i is zero. Appendix B provides a treatment of the most general case, in which dp_2 and dq^i generate real income effects.

V Concluding Comments

Using the minimum expenditure function, we have modelled the behaviour of consumers who are price-takers in markets for private commodities, and quantity-takers with respect to environmental commodities, or externalities. We have used this formulation to investigate a well-known problem in the literature on consumption externalities. We find that in the absence of real income, or welfare, effects aggregate demand may vary positively with price. This is because, in addition to the conventional compensated price responses there are compensated responses to quantity changes. The possibility of unstable equilibria, and of stable equilibria with upward-sloping demand schedules exists when externalities are locally "neutral" - that is, neither detrimental nor beneficial.

Adding in real income responses complicates the picture, but the point remains that there is nothing surprising about some of the responses once thought to be perverse or anomalous, once the role of compensated quantity responses is isolated and clearly understood.

APPENDIX

A. The Stability Condition

(i) Constant Supply Price

$$dx_2^a/dt = \mu(c_2^a(q^a) - x_2^a(t))$$

$$dx_2^b/dt = v(c_2^b(q^b) - x_2^b(t))$$

where μ, $v > 0$. Linearise $c_2^a(\ )$ and $c_2^b(\ )$ in the neighbourhood of equilibrium $(\bar{c}_2^a, \bar{c}_2^b)$. Then, writing $\tilde{c}_2^a \equiv c_2^a - \bar{c}_2^a$ and $\tilde{c}_2^b \equiv c_2^b - \bar{c}_2^b$,

$$\begin{pmatrix} dx_2^a/dt \\ dx_2^b/dt \end{pmatrix} = \begin{pmatrix} -\mu & \mu c_{2q}^a \\ v c_{2q}^b & -v \end{pmatrix} \begin{pmatrix} \tilde{c}_2^a \\ \tilde{c}_2^b \end{pmatrix}$$

whence stability requires that $\mu v(1 - c_{2q}^a\, c_{2q}^b) > 0$. Since μ and v are positive, the stability condition is

$$1 - c_{2q}^a\, c_{2q}^b > 0.$$

(ii) Variable Supply Price

Let there be a supply function $S_2(p_2)$, $dS_2/dp_2 \equiv S_{22} > 0$. Consider the system,

$$dx_2^a/dt = \mu(c_2^2(p_2, q_2^a) - x_2^a(t))$$

$$dx_2^b/dt = v(c_2^b(p_2, q_2^b) - x_2^b(t))$$

$$dp_2/dt = \beta(x_2^a + x_2^b - s_2)$$

Again, we linearise:

$$\begin{pmatrix} dx_2^{a}/dt \\ dx_2^{b}/dt \\ dp_2/dt \end{pmatrix} = \begin{pmatrix} -\mu & \mu c_{2q}^{a} & \mu c_{22}^{a} \\ \nu c_{2q}^{b} & -\nu & \nu c_{22}^{b} \\ \beta & \beta & -\beta s_{22} \end{pmatrix} \begin{pmatrix} \tilde{x}_2^{a} \\ \tilde{x}_2^{b} \\ \tilde{p}_2 \end{pmatrix}$$

A necessary condition for stability is that the determinant of coefficients be negative. This implies that

$$1 - c_{2q}^{a} c_{2q}^{b} - \left\{ (1 + c_{2q}^{b}) c_{22}^{a} + (1 + c_{2q}^{a}) c_{22}^{b} \right\} / s_{22} > 0 \qquad \text{(A.1)}$$

Since c_{22}^{a} and c_{22}^{b} are negative, this condition certainly holds if both c_{2q}^{a} and c_{2q}^{b} lie between -1 and $+1$. Hence, a necessary condition is still that $c_{2q}^{a}\ c_{2q}^{b} < 1$. However, this is certainly not sufficient. For example, if $c_{2q}^{b} = 0$, equilibrium may still be unstable if c_{2q}^{a} is negative and sufficiently large.

It is instructive to consider a special case of the present system. Suppose p_2 adjusts slowly, while x_2^{a} and x_2^{b} adjust rapidly. We can think of a succession of short-run equilibria, in each of which p_2 is regarded as exogenous. The condition $c_{2q}^{a}\ c_{2q}^{b} < 1$ is the condition of stability of the quantity adjustment processes, given p_2. Over time, p_2 itself adjusts, and the stability of the price adjustment process involves the familiar consideration of the downward-sloping excess demand function. It turns out that this requires the expression in curly brackets in (A.1) to be negative.

B. dX_2/dp_2 in the General Case

For convenience, we drop p_1, which is constant, and denote the price of commodity 2 simply by p. Then

$$q^a = c_2{}^b(p, q^b, u^b) \quad , \quad q^b = c_2{}^a(p, q^a, u^a).$$

Now $dX_2/dp_2 = dq^a/dp + dq^b/dp.$

Consider dq^a/dp:

$$dq^a = c_{2p}{}^b\, dp + c_{2q}{}^b\, dq^b + c_{2u}{}^b\, du^b \tag{B.1}$$

To solve for u^b, note that

$$E^b(p, q^b, u^b) = y^b.$$

If y^b is fixed,

$$du^b = -(E_p{}^b\, dp + E_q{}^b\, dq^b)/E_u{}^b \tag{B.2}$$

Substituting into (B.1)

$$dq^a + (c_{2u}{}^b\, E_q{}^b/E_u{}^b - c_{2q}{}^b)dq^b = (c_{2p}{}^b - c_{2u}{}^b\, E_p{}^b/E_u{}^b)dp.$$

By similar reasoning, starting from $q^b = c_2{}^a(p, q^a, u^a)$,

$$(c_{2u}{}^a\, E_q{}^a/E_u{}^a - c_{2q}{}^a)dq^a + dq^b = (c_{2p}{}^a - c_{2u}{}^a\, E_p{}^a/e_u{}^a)dp.$$

Using Cramer's rule to solve for dq^a/dp, the numerator is

$$(c_{2p}{}^b - c_{2u}{}^b\, E_p{}^b/{}_u{}^b) - (c_{2u}{}^b\, E_q{}^b/E_u{}^b - c_{2q}{}^b)(c_{2p}{}^a - c_{2u}{}^a\, E_p{}^a/E_u{}^a). \tag{B.3}$$

The denominator is

$$1 - (c_{2u}{}^a\, E_q{}^a/E_u{}^a - c_{2q}{}^a)(c_{2u}{}^b\, E_q{}^b/E_u{}^b - c_{2q}{}^b). \tag{B.4}$$

(B.3) though rather complicated, shows a clear pattern. Within the first bracket are the conventional pure substitution ($c_{2p}{}^{b}$) and income ($-c_{2u}{}^{b}\, E_p{}^{b}/E_u{}^{b}$) terms associated with the price change. The product of the two other brackets captures the interaction which arises from the dependence of b's welfare and behaviour on a's consumption, $c_2{}^{a}$. Again, responses are decomposed into compensated terms and income terms.

Summing the solutions for dq^a/dp and dq^b/dp gives the required result. The special cases dealt with in the text are:

(i) $du^a = du^b = 0.$

Whether by compensatory policies, or fortuitously, utility levels are unchanged. Then (B.1) becomes

$$dq^a = c_{2p}{}^{b}\, dp + c_{2q}{}^{b}\, dq^b,$$

(B.2) is not needed, and equation (11) quickly follows.

(ii) $E_q{}^{a} = E_q{}^{b} = 0$

Here, (A.3) becomes

$$(c_{2p}{}^{b} - c_{2u}{}^{b}\, E_p{}^{b}/E_u{}^{b}) + c_{2q}{}^{b}(c_{2p}{}^{a} - c_{2u}{}^{a}\, E_p{}^{a}/E_u{}^{a})$$

and (B.4) becomes

$$(1 - c_{2q}{}^{a}\, c_{2q}{}^{b}).$$

from which (12) is easily obtained.

References

1. Brennan, H.G. (1973) "Pareto desirable redistribution: the case of malice and envy", Journal of Public Economics, 2, 173-83

2. Collard, D. (1978) Altruism and Economy (Martin Robertson).

3. Diamond, P.A. and J. Mirrlees (1973) "Aggregate production with consumption externalities", Quarterly Journal of Economics, 87, 1-24.

4. Diewert, W.E. (1974) "Applications of duality theory" in Intriligator, M.D. and D.A. Kendrick (eds), Frontiers of Quantitative Economics, Vol. III (North-Holland), 107-71.

5. Heller, W.P. and D. Starrett (1976) "On the nature of externalities", in Lin, S.A.Y. (ed), Theory and Measurement of Economic Externalities (Academic Press), 9-22.

6. Katzner, D. (1970) Static Demand Theory (Macmillan).

7. Oakland, W.H. (1972) "Congestion, public goods and welfare", Journal of Public Economics, 1, 339-57.

8. Sadka, E. (1978) "A note on aggregate production with consumption externalities", Journal of Public Economics, 9, 101-5.

9. Sandmo, A. (1978) "Anomaly and stability in the theory of externalities", unpublished paper.

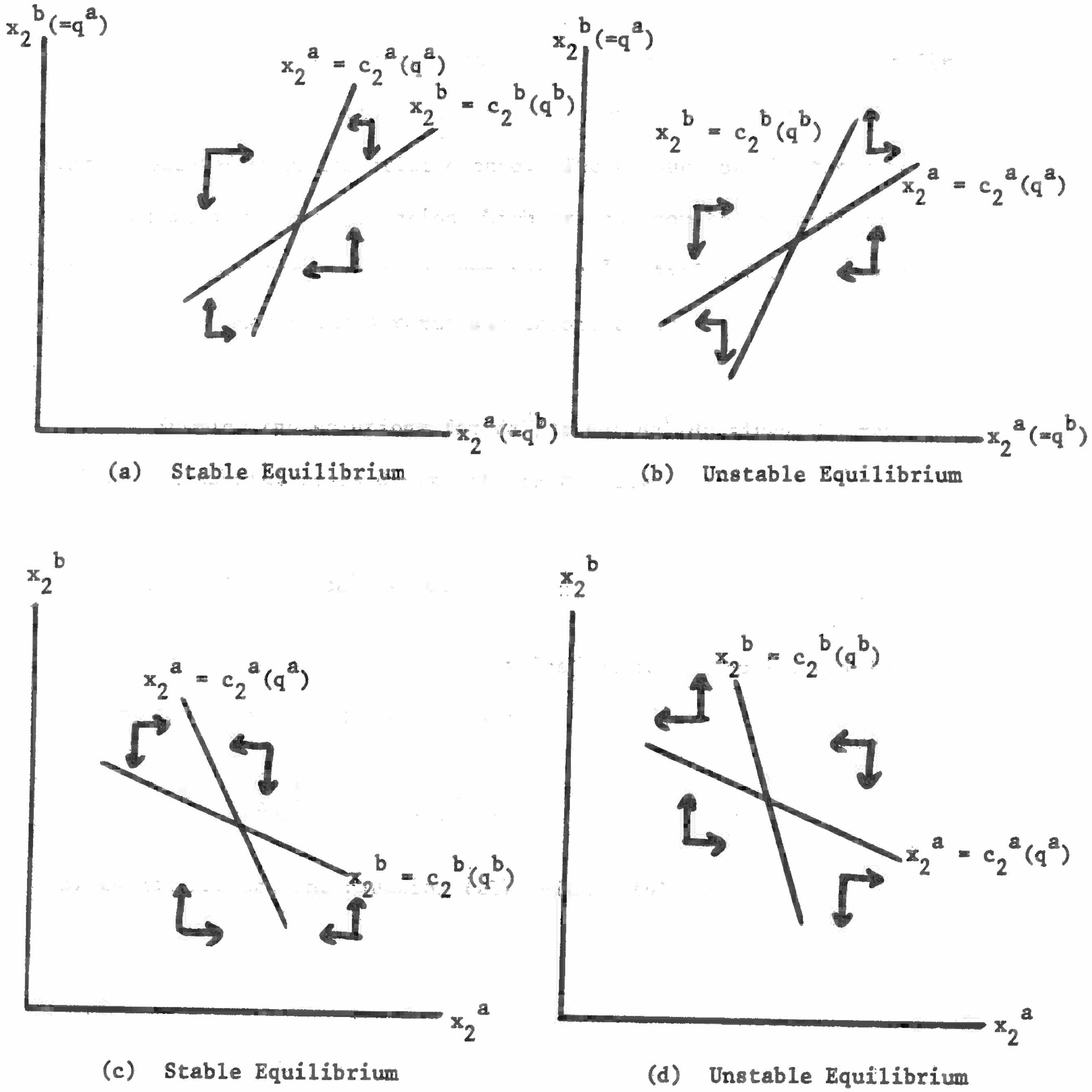
10. Sheshinski, E. (1978) "Note on the relation between quantity and price anomalies under externalities", Economics Letters, 1, 111-5.

Footnotes

1. Throughout the paper, we restrict attention to the local analysis of comparative static properties.

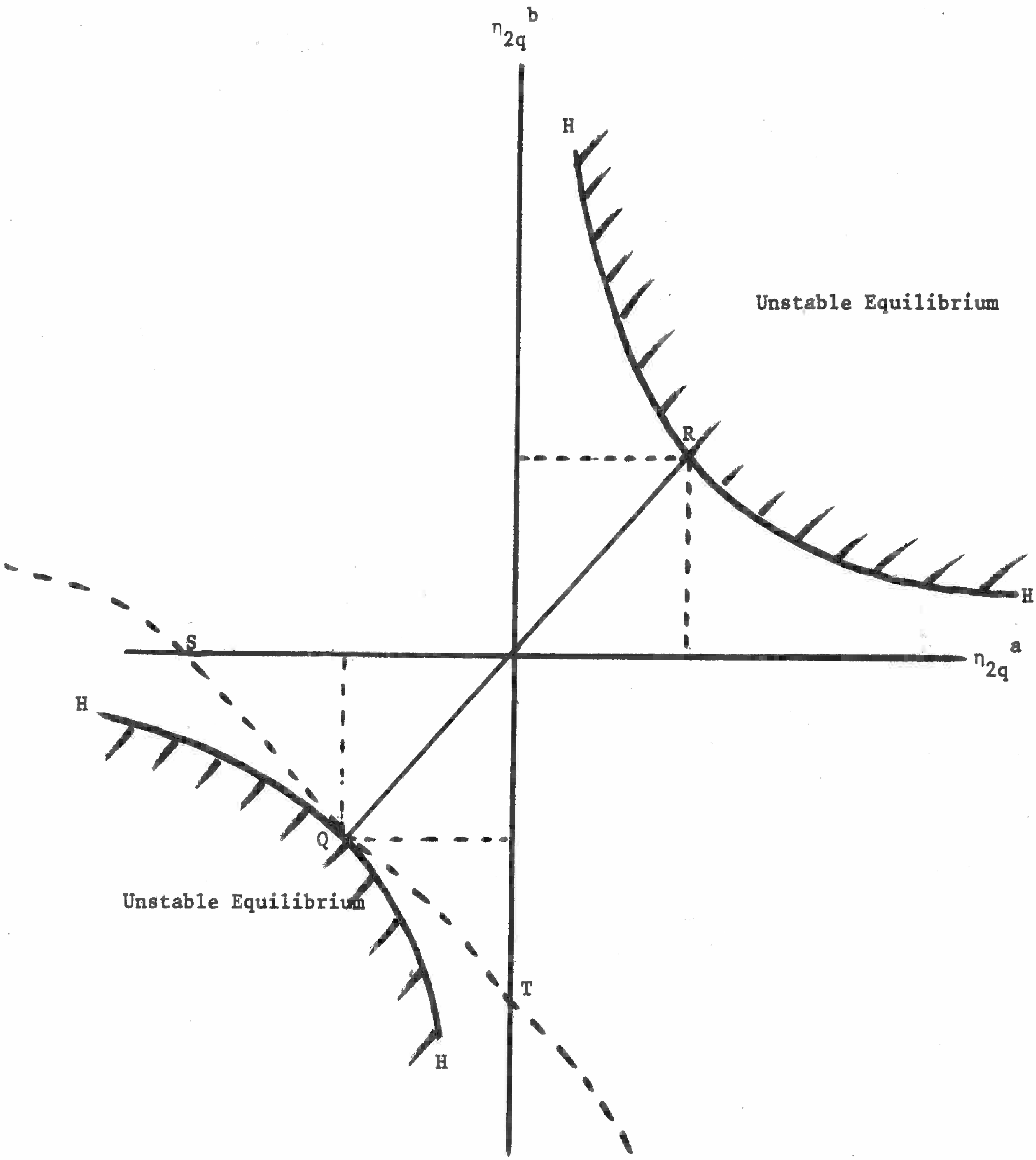
2. See, for example, Brennan (1973) or Collard (1978).

3. It should be noted that there are two stability issues. First, there is stability with respect to quantity adjustments. It is this which is dealt with here. Second, as we note later, there is the problem of whether the slope of dX_2/dp_2 is such as to produce an excess supply curve with positive slope. The latter may be termed stability with respect to price.



$x_2^b (=q^a)$
$x_2^a = c_2^a(q^a)$
$x_2^b = c_2^b(q^b)$
$x_2^a (=q^b)$
(a) Stable Equilibrium
$x_2^b (=q^a)$
$x_2^b = c_2^b(q^b)$
$x_2^a = c_2^a(q^a)$
$x_2^a (=q^b)$
(b) Unstable Equilibrium
x_2^b
$x_2^a = c_2^a(q^a)$
$x_2^b = c_2^b(q^b)$
x_2^a
(c) Stable Equilibrium
x_2^b
$x_2^b = c_2^b(q^b)$
$x_2^a = c_2^a(q^a)$
x_2^a
(d) Unstable Equilibrium

FIGURE I



η_{2q}^{b}
H
Unstable Equilibrium
R
H
S
η_{2q}^{a}
H
Q
Unstable Equilibrium
T
H

FIGURE II