INCOME DISTRIBUTION AND INPUT-OUTPUT :   AN ANALYSIS

FOR SRI LANKA

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Graham Pyatt

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NUMBER 102

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1. Introduction

In a forthcoming volume relating to the economy of Sri Lanka,[1] a case is made for developing particular types of macro data framework together with a demonstration of the methodology needed in practice to do so. The outcome is a very detailed empirical picture of the Sri Lankan economy for 1970. According to our terminology the statistical base is a Social Accounting Matrix (SAM).

The case for carrying out such an extensive empirical task is incomplete, however, without at least some foray into analysis or some consideration of models that are compatible with the social accounting base thereby created. While it is true that underlying every model there is an implicit data system, it is also true that every data system implies a _class_ of economic models. Thus, by compiling a particular SAM we have inevitably restricted ourselves to the class of model implied by it. For Sri Lanka, classification schemes had to be chosen so as to be consistent with data which currently exists; without doing so, the demonstration would have been incomplete. However, this has restricted the class of models that can possibly be considered to those which make use of these classification schemes.

In this paper, as a preliminary to any future consideration of models consistent with this SAM for Sri Lanka, some very elementary fixed coefficient models are considered. Though we are obviously aware of the deficiences of the fixed coefficient approach, we would defend

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[1] Pyatt and Roe, with Lindley, Round _et al_. (forthcoming).

its use on this occasion, on the grounds that we are still at the stage of trying to elicit information about directions of effect and broad orders of magnitude rather than to aspire to any great precision. In fact, greater precision generally implies non-linear models, and will usually require data additional to that which is included in our SAM framework, such as data on elasticities of substitution in production and consumption. Non-linear models therefore detract attention from SAM data to a degree. By avoiding such a diversion we much be content to work with fixed coefficient, linear models in the knowledge that they are well-tried and a proper means of obtaining a first approximation to orders of magnitude for many problems. [1]/

This paper is, therefore, an attempt at some analysis of structure and interdependence, using the SAM that has been developed for Sri Lanka. Our concern is to emphasize distributional considerations and as such, the factoral and institutional classifications assume prime significance. We shall show in due course that a generalized inverse can be defined in such a way that the activity by activity Leontief inverse matrix, which is traditionally considered, is augmented by rows and columns relating to factor and institutional accounts. A decomposition of this inverse leads to some interesting insights into interdependence between various classes of account. The inverse (or multiplier) decomposition separates out 'own account' multipliers from those which arise out of interactions with other accounts. All of this leads, in a final section to considerations of how changes in activity levels may affect the distribution of income across households of different types.

1/ Detailed techniques for working with non-linear models in a SAM frame-work (and some of the pros and cons of doing so) are discussed in Pyatt and Thorbecke (forthcoming).

2. Input-Output Structure of the Accounts

A useful way to analyze income distribution within a macro-economic setting is to disaggregate households analogous to the disaggregation of production structure which underlies input-output analysis. In this section, empirical results are presented on these aspects of the accounts and it is shown how they permit a generalization of input-output to cover not only structure of production but also the factoral distribution of income and its distribution across household types.

There are two main reasons for choosing this initial illustration of the accounting framework and the data therein. The first is that the exercise serves to illustrate the structure of the system and the circular flow of income. Not least it becomes clear that the framework is ideally suited to multiplier analysis of the effects of exogenous injections into the economy, such as increased export demand, or outputs, employment and incomes, with each of these being disaggregated according to the classification system embodied in the social accounts.

A second reason for choosing this exercise as an initial illustration is that it is a natural extension of previous work on two fronts. One of these refers to earlier modelling work in Iran.[1] In this study an attempt was made to capture the two-way linkages whereby (i) income distribution affects final demand, and hence the structure of production; and (ii) the structure of production influences factor demands and hence the structure of income distribution. In the Iran study there were no explicit accounts for the factors of production so that the routing of this second linkage via these accounts had to be

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[1] See Pyatt et al. (1972) or Blitzer, Clark and Taylor (1975) Chapter 5.

an implicit one. One consequence of this is that employment effects
could not be fully integrated into the analysis. Since factor accounts
do appear explicitly in the present framework, it is natural to re-
examine this earlier work in the light of this innovation.

While our own earlier work lacked factor accounts, previous
studies in Sri Lanka had fallen short on the treatment of income dis-
tribution. This should not be taken as a criticism, however. Foundations
laid in data collection and reconciliation[1] were built on by S. Narapala-
singham, who constructed a dynamic input-output model of the Sri Lankan
economy.[2] His work was based on that of the Cambridge Growth Project
which in turn pioneered social accounting in the form which is now
established as the UN SNA.[3] Narapalasingham's was a pioneering effort
which shared with its antecedents a treatment of institutions which did
not disaggregate the household sector. However, within this restriction,
Narapalasingham was perhaps first in the field to explore empirically
how a change in income distribution might effect production structure, that
is, the first of the two links discussed above. Not least then, in
view of the interest in these questions in Sri Lanka, the initial
illustration of the SAM relates to the interdependence of production
structure and income distribution.

Table 1 sets out the basic structure of the Sri Lanka social
accounting matrix used in this study.[4] Four classes of accounts are
distinguished: factors, institutions (current accounts), production

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1/ By Abdul Meguid and Lal Jayawardena as part of a UNDP Planning Project.

2/ S. Narapalasingham (1970).

3/ See Cambridge, Department of Applied Economics, (1962-   ).

4/ The full SAM framework for Sri Lanka is broader to the extent that
it permits a disaggregation of activities, institutions (and households
in particular) as well as of factors of production.

Table 1. A Social Accounting Matrix for Sri Lanka 1970.

EXPENDITURES → / RECEIPTS ↓

Column groups: **1–3 Factors: Labor** (1 Urban, 2 Rural, 3 Estate) · **4–6 Factors: Capital** (4 Housing, Other; 5 Private; 6 Public) · **7–9 Institutions: Households** (7 Urban, 8 Rural, 9 Estate) · **10–11 Institutions: Companies** (10 Private, 11 Public) · **12 Government** · **13–24 Activities** (13 Tea, 14 Rubber, 15 Coconut, 16 Rice, 17 Other Agriculture, 18 Food, Drink & Tobacco, 19 Manufacturing, 20 Mining-Construction, 21 Power, Trade, Transport & Communication, 22 Services, 23 Dwellings, 24 Government Services) · **25–27 Other Accounts** (25 Commodity Tax, 26 Consolidated Capital, 27 Rest of World)

#	Label	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	Σ
1	Labor Urban													-6	1	9	10	37	39	202	183	520	126		540				1673
2	Labor Rural													57	88	213	148	276	74	268	527	678	129		727				3185
3	Labor Estate													470	98	55	4	26	5	1	16	14	14		8				711
4	Capital Housing, Other																							633					633
5	Capital Private													54	133	307	850	1248	182	376	343	1282	322					-113	4984
6	Capital Public															-3	1	-12	12	80	-5	29	72						174
7	Households Urban	1673			137	662					434	6	91																3003
8	Households Rural		3185		330	3026					203	6	151																6901
9	Households Estate			711	31	30					7	6	6																791
10	Companies Private				135	1266							57																1458
11	Companies Public						174						237																411
12	Government							305	141	2	272	74														1404	43		2241
13	Tea							14	56	6			2														-55	839	864
14	Rubber																										25	341	374
15	Coconut							74	261	40			4		1	433											29	217	1124
16	Rice							154	760	106			18				1082										105		2242
17	Other Agriculture							350	985	141			16														106		1846
18	Food, Drink & Tobacco							223	589	64			8																1005
19	Manufacturing							216	544	66			38	91	23	15	44	50	15	463	367	167	31				37	11	2292
20	Mining-Construction							2	6	1			92						32	23	57	10	6				1595	90	1853
21	Power, Trade, Transport & Communication							409	1096	144			117	61	12	64	49	23	65	242	218	162	63	42			154	19	3089
22	Services							208	460	35			7	6	2	6	13	1	1	1		1					154	210	1057
23	Dwellings							258	400	32																	281		690
24	Government Services												1275																1275
25	Commodity Tax							63	54	2	29			33	4	14	7	19	291	212	68	132					130		1404
26	Consolidated Capital							519	807	11	526	308	43														270	425	2639
27	Rest of World							208	772		16		79	75	12	11	32	53	204	363	68	78	14				364		2574
	Σ	1673	3185	711	633	4984	174	3003	6901	791	1458	411	2241	864	374	1124	2242	1846	1005	2292	1853	3089	1087	690	1275	1404	2639	2574	50492

activities and all other accounts. A brief sketch of the cell entries contained within the intersection of these blocks of accounts is as follows. Factoral income is received by six types of factor (accounts 1 to 6), both from domestic production activities[1] (accounts 13 to 26) as well as from the rest of the world (account 27). This income is distributed to a set of six institutions, which comprise three household types (accounts 7 to 9), two kinds of corporate enterprise (accounts 10 and 11), plus government (account 12). Inter-institution transfers are shown in the second diagonal submatrix comprising the intersection of accounts 7 through 12, while transfers from abroad are shown as receipts by domestic institutions from the rest of the world, account 27. Government (account 12) shows in its row a receipt of 1404 in commodity taxes, plus 43 from the consolidated capital account. Receipts by domestic production activities (activities 13 to 24) arise from the expenditures of institutions (accounts 7 to 12), inter-activity purchases of commodities (accounts 13 to 24), as well as receipts from 'other' accounts (accounts 25 to 27) which comprise exports and fixed capital formation. The inter-activity matrix therefore comprises the third diagonal submatrix of the SAM. Commodity and other indirect taxes are collected as receipts of account 25, and are paid into the government current account as the only element (1404) of the column for account 25. Receipts of the consolidated capital account essentially comprise institution savings plus the deficit on the balance of payments current account. Expenditures of account 26 include gross fixed capital formation and capital transfers. The final account (account 27) shows imports

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[1] Government services are included as an activity of production: this merely routes government expenditure in wages and salaries through to the factor accounts. Household expenditures on domestic services are routed through the activity: Services.

along the row, and exports plus net transfers from abroad down the column.

This then is the aggregative social accounting matrix for Sri Lanka which will form the basis for the analysis and related empirical work in this paper. However, although the matix is specific to Sri Lanka for the year 1970, the analysis which follows is quite general. To facilitate this analysis, Table 2 sets out the basic structure of the social accounting matrix we have shown in Table 1. The notation $T_{i.j}$ is used to represent the submatrix of transactions which are receipts of the i'th set of accounts resulting from expenditures by the j'th set. Thus $T_{1.3}$ is the 6 x 12 matrix showing how each of the six factors of production derives income from each of the 12 production activities. The null matrices in Table 2 are significant; they represent flows between accounts which are necessarily zero. For example, there are no receipts by institutions direct from activities, since all institutional receipts are either from factors or transfers between domestic and foreign institutions. Accounting balance in Table 2 necessitates the row and column totals for each block of accounts to be equal. The row rums represented by the vectors t_1 to t_4 also appear in transposed form as column sums t_1' to t_4'. The normalization of a transaction matrix $T_{i.j}$ by the vector t_j will be denoted by $A_{i.j}$ where

$$A_{i.j} = T_{i.j} \; \hat{t}_j^{-1} \tag{1}$$

Thus $A_{i.j}$ is obtained from $T_{i.j}$ by dividing elements of the latter by the sum of the column in which they appear. This treatment will be adopted for all the $T_{i.j}$'s in each of the first three columns of Table 2. Doing so implies that the accounting constraints across the rows can be expressed as

Table 2:

SCHEMATIC SOCIAL ACCOUNTING MATRIX

	Expenditures				Totals
	Factors	Institutions	Activities	Other Accounts	
Factors	0	0	$T_{1.3}$	$T_{1.4}$	t_1
Institutions	$T_{2.1}$	$T_{2.2}$	0	$T_{2.4}$	t_2
Activities	0	$T_{3.2}$	$T_{3.3}$	$T_{3.4}$	t_3
Other Accounts	0	$T_{4.2}$	$T_{4.3}$	$T_{4.4}$	t_4
Totals	t_1'	t_2'	t_3'	t_4'	

$$
\begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & A_{1.3} \\ A_{2.1} & A_{2.2} & 0 \\ 0 & A_{3.2} & A_{3.3} \\ 0 & A_{4.2} & A_{4.3} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{2}
$$

where x_i is the row sum of the submatrix $T_{i.4}$ for each $i = 1,2,3,4$.

Subsequent analysis assums that each of the x_i's is an exogenous set of numbers, and that each of the $A_{i.j}$ matrices in equation (2) has constant elements. Combining these two sets of assumptions implies that values of t_1 to t_4 can always be obtained from any assumed values of x_1 to x_4. And the mathematics of this can be expressed as

$$
\begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & A_{1.3} \\ A_{2.1} & A_{2.2} & 0 \\ 0 & A_{3.2} & A_{3.3} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{3}
$$

and

$$
t_4 = A_{4.2} \, t_2 + A_{4.3} \, t_3 + x_4 \tag{4}
$$

Equation (4) above shows how the balance of the fourth set of accounts can be derived once t_2 and t_3 are known, that is, once the first three sets of accounts are balanced.[1] This residual balance equation is of no further interest to us. We can therefore concentrate on equation (3) which can now be written as

[1] The fourth set includes an account for commodity taxes. Clearly, it is anomalous to treat commodity taxes exogenously in this way while all other government revenue and expenditure are endogenous. Moreover, the structure of the system requires the only entry in the outgoings of account 25 to equal all incomings. As we have it, this may not occur. Endogenizing commodity taxes creates structure difficulties in the system which we have not yet overcome.

$$t = At + x \tag{5}$$

so that

$$t = (I - A)^{-1} x \tag{6}$$

These last two results show how t (i.e. t_1, t_2 and t_3) can be derived from x (i.e. x_1, x_2 and x_3), through a generalized inverse $(I-A)^{-1}$. This is a strictly analygous procedure to that followed in conventional input-output analysis which is concerned with the determination of t_3 only. From equation (3) it is apparent that

$$t_3 = A_{3.3} \, t_3 + (A_{3.2} \, t_2 + x_3) \tag{7}$$

so that

$$t_3 = (I - A_{3.3})^{-1} \, (A_{3.2} \, t_2 + x_3) \tag{8}$$

Equation (8) is a part of our system and is therefore completely consistent with it. It is also the end of the story in conventional static input-output analysis since $A_{3.2} \, t_2$ is assumed to be exogenous. Thus, in the latter approach t_3 (the level and structure of output) is derived through the inverse $(I-A_{3.3})^{-1}$ of direct and indirect commodity requirements on the basis of assumed demands or activities from other accounts. Our extension of input-output is to decompose these assumed demands and to allow for a part of them, viz. $A_{3.2}t_2$, to be determined simultaneously with t_3. This part depends on t_2, i.e. on the level and distribution of income across institutions. Accordingly, we are considering a system in which output structure and income distribution are determined simultaneously, as opposed to one which treats them as separable.

3. Decomposition of the generalized inverse

The preceding discussion, and in particular equation (6), shows that the simultaneous determination of factor and institution incomes and production levels can simply be regarded as a generalized problem in input-output analysis, given appropriate definition of the matrix A. However, it also follows from the subsequent discussion of how this links to conventional input-output that it may be useful to decompose the generalized inverse $(I-A)^{-1}$ into contributory parts which reflect the different mechanisms at work within it, resulting from the interconnections within the system. This is the approach pursued in this section. From it we conclude that it is useful to write $(I-A)^{-1}$ as

$$(I - A)^{-1} = M = M_3 \, M_2 \, M_1 \tag{9}$$

where the notation M is used to indicate a multiplier matrix in the sense that all its elements are greater than (or equal to) the corresponding elements of an identity matrix.

The aggregate multiplier matrix, M, shows how an increase in any element of x will increase the corresponding element of t by at least the same amount, and may also have indirect effects on other elements of t. The right-hand side of equation (9) states that these aggregate multipliers can be decomposed into three separate multipliers. M_1 and M_2 are referred to as being 'own effects' multipliers, as opposed to M_3 which collects what we shall term 'cross effects'. The distinction between M_1 and M_2 is as follows. A change in an element x_i of x will influence t_i for two sets of reasons. One is that there may be transactions or transfers within the i'th set of accounts so that, for example, an increase in demand on a production sector will cause it to increase its

demand on another production sector. Another example is that increased income for companies from an exogenous source will result in increased income for government through profits taxation. These and other multiplier processes which operate within a set of accounts can be referred to as 'own direct effects' or 'own transfer effects'. These are captured by the multiplier matrix M_1. They contrast with M_2 which collects together 'own indirect effects' which arise if an increase in x_i affects t_i after feeding through other accounts. For example, an increase in demand on a production activity will cause it to hire more factors. This will raise incomes in the factor accounts which will, in turn, raise incomes in the institution accounts. These latter accounts will spend some of the increased income, and by so doing will raise demand on the production accounts beyond the level of the initial increase which came from an exogenous source. Finally, the multipliers M_3 record cross effects, that is, the impact of an increase in x_i on t_j for j not equal to i.

To establish the existence and properties of these multiplier effects requires manipulation of equation (3). This can start most easily with the own direct effects, M_1, which depend on transfers within a particular set of accounts. Some slight rearrangement of equation (3) allows us to write it as

$$
\begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_{2.2} & 0 \\ 0 & 0 & A_{3.3} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & A_{1.3} \\ A_{2.1} & 0 & 0 \\ 0 & A_{3.2} & 0 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (10)
$$

The basic operation involved here is to separate out the diagonal elements of A (i.e. $A_{2.2}$ and $A_{3.3}$) from the other non-zero elements. Having done so, the equation can be written as

$$
t = M_1 \begin{bmatrix} 0 & 0 & A_{1.3} \\ A_{2.1} & 0 & 0 \\ 0 & A_{3.2} & 0 \end{bmatrix} t + M_1 x \quad (11)
$$

where

$$
M_1 = \begin{bmatrix} I & 0 & 0 \\ 0 & (I - A_{2.2})^{-1} & 0 \\ 0 & 0 & (I - A_{3.3})^{-1} \end{bmatrix} \quad (12)
$$

Equation (12) defines the multiplier matrix M_1 introduced in equation (9). This definition shows that it captures the impact of interindustry dependence in the conventional input-output sense via the inverse $(I - A_{3.3})^{-1}$. Indeed equation (8) is contained in equation (11). This is clearly seen when the latter is written extensively as

$$
\begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & (I - A_{2.2})^{-1} & 0 \\ 0 & 0 & (I - A_{3.3})^{-1} \end{bmatrix} \left\{ \begin{bmatrix} 0 & 0 & A_{1.3} \\ A_{2.1} & 0 & 0 \\ 0 & A_{3.2} & 0 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right\} \quad (13)
$$

Equation (8) occurs as the expression for t_3 in equation (13). At the same time it can be noted that a symmetric treatment of the institutions accounts leads to the novelty of having the multiplier matrix $(I-A_{2.2})^{-1}$ in the system.

Result (11) can be written as

$$
t = A^* t + M_1 x \quad (14)
$$

and the point to emphasize is that A^* has zeros on its diagonals: the importance of diagonal elements of A is taken care of by the multiplier matrix M_1. This means that apart from M_1 all other multiplier effects must be due to the connectedness of different accounts in the whole system. This will be discussed shortly. Meanwhile, it should be noted that the procedures assume that M_1 exists. From (12) this requires

the existence of the Leontief inverse $(I - A_{3.3})^{-1}$ and of $(I - A_{2.2})^{-1}$. The necessary conditions needed are weaker than with respect to $(I - A)^{-1}$ and present no problems in practice. Assuming they are satisfied, $(I - A_{i.i})^{-1}$ can be written as

$$(I - A_{i.i})^{-1} = I + A_{i.i} + A_{i.i}^2 + A_{i.i}^3 + \text{ad infinitum} \qquad (15)$$

which must be greater than I since all elements of $A_{i.i}$ are positive. Hence M_1 exists and is a multiplier matrix.

Turning now to the effects which depend on connections between the accounts it follows from equation (14) that

$$t = (I - A^*)^{-1} M_1 x \qquad (16)$$

assuming that the inverse $(I - A^*)^{-1}$ exists. Again this assumption is quite reasonable, and we can write

$$(I - A^*)^{-1} = I + A^* + A^{*2} + A^{*3} + \dots \text{ad inf.} \qquad (17)$$

$$= (I + A^* + A^{*2})(I + A^{*3} + A^{*6} + \dots \text{ad inf.}) \qquad (18)$$

$$= (I + A^* + A^{*2})(I - A^{*3})^{-1} \qquad (19)$$

The second of the two matrices on the right hand side of this equation is our definition of M_2, and the first of the pair is M_3. From (18) it is apparent that both these matrices are indeed multiplier matrices.

The decomposition (19) may seem to be somewhat arbitrary since it is generally true that

$$(I - A^*)^{-1} = (I + A^* + \ldots + A^{*k-1})(I - A^{*k})^{-1} \qquad (20)$$

However, the choice of $k = 3$ in (19) is appropriate in the present case because we are dealing with a system of three accounts; factors, institutions and activities. If the system were to be extended, for example by adding commodity accounts, then $k = 4$ would be the appropriate choice.

The significance of $k = 3$ in the present case is seen most easily by considering the successive terms A^*, A^{*2} and A^{*3} which appear in (19). Writing A^* as

$$
A^* = \begin{bmatrix} 0 & 0 & A^*_{1.3} \\[2em] A^*_{2.1} & 0 & 0 \\[2em] 0 & A^*_{3.2} & 0 \end{bmatrix} \tag{21}
$$

it follows that

$$
A^{*2} = \begin{bmatrix} 0 & A^*_{1.3}A^*_{3.2} & 0 \\[2em] 0 & 0 & A^*_{2.1}A^*_{1.3} \\[2em] A^*_{3.2}A^*_{2.1} & 0 & 0 \end{bmatrix} \tag{22}
$$

and

$$
A^{*3} = \begin{bmatrix} A^*_{1.3}A^*_{3.2}A^*_{2.1} & 0 & 0 \\[2em] 0 & A^*_{2.1}A^*_{1.3}A^*_{3.2} & 0 \\[2em] 0 & 0 & A^*_{3.2}A^*_{2.1}A^*_{1.3} \end{bmatrix} \tag{23}
$$

Thus A^{*3} is a block diagonal matrix and therefore so is M_2. The elements of this matrix show the multipliers that result from tracing an initial impact from its source through the system and back to the account from which it started. Since we have a system of three accounts here, getting back to source requires three steps. [1]

----

[1] The accounts would be spurious if it was impossible to return to source. In fact the non-zero entries of A must correspond to a permutation effect.

The fact that M_2 is block diagonal implies that it captures only the effects of x_1 on t_1. Cross effects can be derived from M_3 which, from (21) and (22), is given by

$$M_3 = \begin{bmatrix} I & A^*_{1.3} \, A^*_{3.2} & A^*_{1.3} \\ A^*_{2.1} & I & A^*_{2.1} \, A^*_{1.3} \\ A^*_{3.2} \, A^*_{2.1} & A^*_{3.2} & I \end{bmatrix} \tag{24}$$

M_3 clearly captures only cross effects, since its diagonal contains only identity matrices.

The structure of the decomposed multiplier matrices can be summarized by rewriting equation (9) using the observed structure of the multiplier matrices M_1, M_2 and M_3 as shown in equations (12), (23) and (24). With some obvious notation, M can therefore be expressed as

$$\begin{bmatrix} M(1,1) & M(1,2) & M(1,3) \\ M(2,1) & M(2,2) & M(2,3) \\ M(3,1) & M(3.2) & M(3,3) \end{bmatrix}$$

$$= \begin{bmatrix} I & M_3(1,2) & M_3(1,3) \\ M_3(2,1) & I & M_3(2,3) \\ M_3(3,1) & M_3(3,2) & I \end{bmatrix} \begin{bmatrix} M_2(1,1) & 0 & 0 \\ 0 & M_2(2,2) & 0 \\ 0 & 0 & M_2(3,3) \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & M_1(2,2) & 0 \\ 0 & 0 & M_1(3,3) \end{bmatrix} \tag{25}$$

Each of the submatrices shown in equation (25) can be expressed in terms of the component submatrices of A. For example,

$$M_1(2,2) = (I - A_{2.2})^{-1} \tag{26}$$

is the 'own direct' effect multiplier for the institution accounts.
The 'own indirect' effect multipliers for the institution accounts, $M_2(2,2)$,
augment these 'own direct' effects, through equation (25), making the
total 'own' multiplier effects for these accounts equal to

$$M(2,2) = M_2(2,2) \, M_1(2,2)$$

$$= \left[I - A^{*}_{2.1} \, A^{*}_{1.3} \, A^{*}_{3.2}\right]^{-1} \left[I - A_{2.2}\right]^{-1} \tag{27}$$

Some empirical results obtained from applying the above analysis
to the Sri Lanka data are shown in Tables 3 and 4. Only the matrices M_1
M are presented. In the light of equation (25) these contain some of the
more interesting submatrices for present discussion. A presentation of
all four matrices would be somewhat overwhelming.

Table 4 shows the matrix M. From equation (25) it follows
that the first of the submatrices forming the diagonal, $M(1,1)$, is such that

$$M(1,1) = M_2(1,1) \tag{28}$$

since the leading submatrices of M_1 and M_3 are both identity matrices.
This shows the full multiplier effect of a unit exogenous increase in the
income of a particular factor on that factor and all others. Using
equation (28) this means that the total multiplier effects are in fact own
indirect effects since there are no own direct effects within the factor
accounts. To be specific on the interpretation of this matrix we may note
that, for example, a unit exogenous increase in income for rural labor
results in an overall increase of 1.36 units in income for that factor
after allowing for the own indirect effects via other accounts. However,
other factors also receive increased income, as a result of the one unit
exogenous increase, notably 'other private capital' which benefits to the

TABLE 3

MULTIPLIER MATRIX M_1

Note: the original is a large hand-drawn 24 × 24 social-accounting multiplier matrix, with RECEIPTS labelling the rows and EXPENDITURES labelling the columns. Many entries are faint or marked with an asterisk (*). The transcription below preserves the row/column labels and the legible values; blank cells indicate values that could not be read reliably.

Row/column categories (1–24):

No.		Category
	FACTORS	
1	LABOUR	URBAN
2		RURAL
3		ESTATE
4	CAPITAL	PUBLIC
5		OTHER PRIVATE
6		HOUSING
	INSTITUTIONS	
7	HOUSEHOLDS	URBAN
8		RURAL
9		ESTATE
10	COMPANIES	PRIVATE
11		PUBLIC
12	GOVERNMENT	
	ACTIVITIES	
13		Tea
14		Rubber
15		Coconut
16		Rice
17		Other agric.
18		Food, P. & T.
19		Manufacturing
20		Mining + Const".
21		Power, Trade + T.
22		Services
23		Dwellings
24		Govt. services

Matrix entries (RECEIPTS rows × EXPENDITURES columns):

Factor identity block (self-receipts):

Row	Col	Value
1	1	1.00
2	2	1.00
3	3	1.00
4	4	1.00
5	5	1.00
6	6	1.00

Institutions block (cols 7–13, legible readings):

Expenditure col	Legible values (by receipt row)
7	1.01 ; … ; .10
8	1.00 ; .02
9	1.00
10	.01 ; .16 ; .01 ; 1.01 ; .02 ; .23
11	1.00 ; .01 ; .01 ; 1.02 ; .19
12	.31 ; .16 ; .01 ; .03 ; .11 ; 1.03
13	.01 / .05 / .07 / .03 ; 1.00

Activities block (cols 13–24) — principal diagonal multipliers (legible):

Activity (col)	Diagonal value
13 Tea	1.00
14 Rubber	1.63
15 Coconut	1.93
16 Rice	1.06
17 Other agric.	1.03
18 Food, P. & T.	1.04
19 Manufacturing	1.27
20 Mining + Const.	1.06
21 Power, Trade + T.	1.02
22 Services	1.06
23 Dwellings	1.01
24 Govt. services	1.00

Sample legible off-diagonal activity coefficients (receipt row 19, Manufacturing, across activity columns): .14, .08, .03, .05, .04, .03, .27, .07, .05, .02

Sample legible activity coefficients (receipt row 20, Mining + Const.): .09, .04, .10, .05, .02, .08, .14, .16, .07, .01

Other scattered legible values: .01, .03, .04, 1.04, 1.06, 1.07

TABLE 4: MULTIPLIER MATRIX M

RECEIPTS (rows, down the left) × EXPENDITURES (columns 1–24)

					FACTORS					INSTITUTIONS					ACTIVITIES												
			LABOUR			CAPITAL			HOUSEHOLDS			COMPANIES		Gov.													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
		URBAN	RURAL	ESTATE	PUBLIC	o.priv	Housing	URBAN	RURAL	ESTATE	PRIVATE	PUBLIC	Govt	Tea	Rubber	Coconut	Rice	O.Agric	Food,D.&T.	Manuf.	Mining&c	Pow.Tr&Tr	Services	Dwellings	Govt.serv
1	URBAN	1.20	.19	.19	.19	.19	.20	.19	.19	.16	.08	.42	.19	.19	.31	.20	.21	.14	.26	.31	.36	.28	.19	.63	.62
2	RURAL	.35	1.36	.34	.34	.35	.35	.36	.38	.28	.67	.41	.59	.67	.47	.66	.56	.44	.66	.56	.44	.35	.35	.19	.93
3	ESTATE	.03	.03	1.03	.03	.03	.03	.03	.03	.13	.02	.03	.11	.03	.12	.12	.03	.02	.02	.03	.04	.02	.02	.04	—
4	PUBLIC	.74	.86	.93	1.13	1.74	.76	.86	.93	.48	.73	.86	1.17	1.36	1.48	1.47	.65	.79	.98	1.16	.95	.75	.80	—	—
5	o.private	-.02	.02	.02	.02	1.00	.02	.02	.02	.13	*	.02	.03	.02	.13	.12	.06	.12	.12	.12	.10	.11	.15	—	—
6	Housing	.15	.14	.13	.13	.02	.15	.14	.13	.09	.02	.01	.86	.11	.12	.13	.12	.06	.09	.12	.12	.05	.02	.04	—
7	URBAN	.23	.35	.44	.48	.12	.33	.35	.35	.61	.67	.24	.25	.58	.52	.56	.30	.46	.56	.64	.51	.65	.84	—	—
8	RURAL	.95	1.99	1.05	1.46	1.59	.93	1.99	2.06	.81	1.28	1.17	1.05	1.42	1.56	1.50	1.51	.73	1.37	1.39	1.39	1.42	1.54	—	—
9	ESTATE	.04	.05	1.05	.05	.01	.04	.05	.05	.02	.04	.04	.14	.09	.31	.12	.05	.03	.05	.07	.04	.09	.05	—	—
10	PRIVATE	.35	.27	.44	.48	.04	.33	.27	.27	.04	.05	.05	.09	.25	.33	.35	.41	.18	.22	.38	.32	.42	.24	—	—
11	PUBLIC	.14	.19	.19	.19	*	.14	.19	.14	.30	1.47	.02	.02	.15	.16	.17	.09	.14	.16	.14	.18	.17	—	—	—
12	Government	.02	.02	.02	.02	.02	.02	.02	.01	*	.02	*	1.02	1.02	.15	.17	.09	.12	.14	.14	.18	.17	—	—	—
13	Tea	*	*	*	*	*	*	*	*	*	*	*	.02	1.00	*	*	*	*	.01	*	*	*	*	—	—
14	Rubber	*	*	*	*	*	*	*	*	*	*	*	.02	1.00	*	*	*	*	*	*	*	*	—	—	—
15	Coconut	.14	.15	.15	.14	.02	.13	.19	.09	.02	.13	.15	.14	.15	1.77	.14	.16	.11	.13	.12	.14	.15	—	—	—
16	Rice	.37	.49	.55	.41	.07	.37	.55	.55	.38	.42	.44	.43	2.33	.41	.30	.38	.20	.37	.44	.44	—	—	—	—
17	Oth Agric	.36	.39	.44	.35	.06	.36	.37	.44	.32	.36	.35	.38	1.39	.23	.24	.31	.32	.30	.33	.39	—	—	—	—
18	Food,D.&T.	.21	.22	.23	.20	.03	.21	.22	.23	.13	.18	.20	.18	.20	.19	.13	.18	.18	.19	.22	—	—	—	—	—
19	Manufacturing	.31	.32	.35	.30	.05	.31	.33	.35	.19	.41	.37	.33	.32	.41	.37	.52	.34	.26	.30	.33	.39	—	—	—
20	Mining&constr	.03	.02	.02	.02	.04	.03	.02	.02	.19	.01	.06	.02	.02	.01	.02	.03	.17	.02	.09	.02	—	—	—	—
21	Pow,Trans&Tr	.47	.51	.55	.45	.08	.47	.51	.55	.30	.47	.52	.50	.56	.48	.46	.47	.56	.41	.45	.49	—	—	—	—
22	Services	.17	.17	.16	.15	.03	.17	.17	.14	.10	.03	.14	.13	.16	.14	.15	.16	.07	.10	.13	.15	.16	—	—	—
23	Dwellings	.18	.15	.14	.15	.12	.18	.15	.14	.02	.14	.15	.16	.16	.12	.14	.10	.10	.07	.08	.10	—	—	—	—
24	Govt service	.12	.08	.07	.11	.12	.12	.08	.07	.17	.12	.07	.08	.07	.09	.10	.10	.05	.07	.08	.10	1.10	—	—	—

extent of 0.85. Table 4 has a number of interesting features. Particularly worthy of note are the low multipliers for state labor and public capital; these factors benefit relatively little from increased demand for other factors. Even own-multipliers are low for these factors; 1.03 for state labor and as low as unity for public capital.

For the institution accounts, own indirect effects are not the only contribution to the overall multipliers. In addition there are 'own direct' (or transfer) effects and these are represented by

$$M_1(2,2) = (I - A_{2.2})^{-1} \tag{29}$$

There are shown in Table 3 as the second diagonal submatrix. Generally, these transfer effects are quite small: the largest elements refer to the impact of increased income for private companies on the incomes of urban households (0.31) and government (0.23). The reason for this structure is very easily explained with reference to the transfers between institutions shown in Table 1. All of the own-indirect effect linkage arises from government receipts and transfers, plus distributed profits of private companies.

When own indirect and direct effects are combined, the full income multipliers for institutions are obtained as M(2,2), where

$$M(2,2) = M_2(2,2) \, M_1(2,2) \tag{30}$$

This is shown as the second diagonal submatrix of Table 4. The elements in M(2,2) are clearly substantially greater in almost every cell than the corresponding element in $M_1(2,2)$. Such a result, indicating a large own-indirect effect compared with the own-direct effect, is to be expected. Institutions' major own-linkage is via the activity and factor accounts,

although it is significant that some multiplier effect arises out of the transfers between institutions.

The redistributive effects en households in particular, and institutions in general, that arise from the multipliers shown in Tables 3 and 4, will be considered more fully in Section 5. It is interesting to note, however, that rural households appear to gain more than estate households from an exogenous increase in income for the latter. The magnitude of the redistributive effects are, of course, in absolute terms, so the measure of redistribution requires more careful consideration.

The third diagonal submatrices in Tables 3 and 4 relate to the own-direct effects, $M_1(3,3)$, and total 'own' effects, $M(3,3)$, of the production activities, where the relationship between them is given by

$$M(3,3) = M_2(3,3) \ M_1(3,3) \tag{31}$$

These correspond to the open and closed Leontief inverse matrices, respectively, for the twelve activity accounts.[1]

A summary of the interdependence between the domestic production activities can be seen in Table 5, which sets out statistics of off-diagonal elements of the table in frequency distribution form. It is clear from the large number of zero and negligible numbers in the Leontief inverse that the direct interdependence of industries in Sri Lanka is limited. However, once indirect effects are taken into account, 93 percent of the elements are recorded as 0.01 or larger. This then gives a sense of magnitude to the general expectation that allowing for indirect effects enhances interdependence within the economy.

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1/ Versions of these matrices for a more disaggregated production structure are shown in Pyatt and Roe with Lindley, Round et al.(forthcoming), Table 4.5.

Table 5: Frequency Distributions for the size of off-diagonal elements of the matrices $M_1(3,3)$ and $M(3,3)$

Element Size	$M_1(3,3)$ (Leontief inverse)	$M(3,3)$
0	22	−
* = < .005	69	10
.01 to .05	30	32
.06 to .10	7	14
.11 to .15	2	12
.16 to .20	1	14
.21 to .25	−	12
.26 to .30	1	3
.31 +	−	35
Total	132	132

Perhaps the most interesting results of all are contained in some of the off-diagonal submatrices of Table 4. In particular, M(1,3) and M(2,3) show the full effects of an exogenous unit change in the outputs of activities upon factor and institution incomes, respectively. Being off-diagonal matrices of M, they result from an interesting combination of own direct and indirect activity effects together with some cross effects between factors, institutions and activities. The relevant expressions that derive from equation (25)

$$M(1,3) = M_3(1,3) \ M_2(3,3) \ M_1(3,3) \tag{32}$$

and
$$M(2,3) = M_3(2,3) \ M_2(3,3) \ M_1(3,3)$$

The matrix M(1,3) shows that the effects on capital are larger than those on labor, while M(2,3) shows that it is households which stand to benefit most once the distribution of profits along with other interactions are taken into account.

4. Normalized Multipliers

The results discussed briefly in the preceding section serve first
and foremost to illustrate the interdependence of social accounts and
the structure of the particular set of accounts which we have implemented.
This emerges most clearly from the decomposition of the general multiplier,
M, and in particular, its component, M_2. This last shows how an exogenous
injection into the system will circulate within it, thus amplifying the
initial impact. More generally, the full matrix M captures the repercussions
of any impact on all the different accounts, thus extending multiplier
analysis beyond the simple Leontief inverse in the same way that SAM extends
the basic Leontief information system. The empirical results show that the
difference between M and M_1 is quite marked, so the extension is important.

The multipliers, so determined, set out the effects on incomings
(or receipts) of factors, institutions and activities that arise from a unit
exogenous change in the level of outgoings of the various accounts. For some
purposes it is useful to know, for example, the effect on incomes received by
different types of households which arise out of a unit change in output of a
certain activity. This is captured by the multipliers we have so far defined.
We have already noted that a unit increase in outgoings of most of the accounts
give rise to a large increase in incomes of rural households, relative to
the corresponding increases of other institutions. However, when interpreting
these results it should be remembered that a multiplier increase for a rural
household should be divided by the total income of rural households to obtain
the proportionate change in rural incomes. Redistribution will then be in
favor of rural households as opposed to, say, urban households, only if the
proportionate change is greater for the former than the latter.

In much the same way, the multipliers can be normalized so as to show the effect of a unit percentage exogenous change in the levels of income and output on the whole system. To normalize both the exogenous increase and the multiplier increase by the level of income or output of each account, first express equation (6) in absolute changes. We have

$$\Delta t = (I - A)^{-1} \Delta x \tag{34}$$

and normalizing with respect to levels this becomes

$$\hat{t}^{-1} \Delta t = \hat{t}^{-1} (I - A)^{-1} t (\hat{t}^{-1}\Delta x) \tag{35}$$

Writing percentage changes of t and x with respect to income and output levels, t, as ∇t and ∇x, equation (35) can be rewritten as

$$\nabla t = \hat{t}^{-1} (I - A)^{-1} \hat{t} \nabla x \tag{36}$$

From equation (9)

$$\nabla t = \hat{t}^{-1} M \hat{t} \nabla x$$
$$= \hat{t}^{-1} M_3 M_2 M_1 \hat{t} \nabla x$$
$$= (\hat{t}^{-1} M_3 \hat{t}) (\hat{t}^{-1} M_2 \hat{t}) (\hat{t}^{-1} M_1 \hat{t}) \nabla x \tag{37}$$

Now letting

$$N = \hat{t}^{-1} M \hat{t} \tag{38}$$

be the normalized form for the aggregate multipliers, it then follows that N_1, N_2 and N_3 can be defined to be the normalized forms for the own effects and cross effects multipliers, in an analogous way. Furthermore, noting that

$$N = \hat{t}^{-1} (I - A)^{-1} \hat{t}$$
$$= (I - \hat{t}^{-1} A \hat{t})^{-1}$$
$$= (I - B)^{-1} \tag{39}$$

it is clear that N corresponds to an inverse of Leontief form, based on
a coefficient matrix B, which is derived from the social accounting flow
matrix by dividing row elements by row sums. This contrasts with matrix A
which is derived by dividing column elements by column sums. It is important
to note, however, that the systems defined by equation (36) (with (39)) and
equation (34) are equivalent; they are dependent on the same behavioural
assumptions.

The outcome of the normalizing procedure as applied to the Sri
Lanka multipliers is shown in Tables 6 and 7. The normalized multiplier
matrix N_1 is set out in Table 6 whilst that for N is shown in Table 7.
Clearly, the scaling of rows and columns of the multiplier matrices by
account totals, as in equation (38), leaves all diagonal elements unchanged.
The effect on the off-diagonal elements is obviously quite marked. Referring
to Table 6, for example, the own-direct effects of institution transfers,
$N_1(2,2)$, shows that a unit percentage change in income of both urban and
rural households leads to a percentage change in income of the public sector
(public companies plus government) which is larger than would be apparent
from the unscaled matrix $M_1(2,2)$. Not surprisingly, the estate households
give rise to virtually no multiplier effect, even on the normalized basis,
due to their low level of connectedness within the institutional transfer
matrix. The Leontief inverse for production activities in its normalized
form is shown as $N_1(3,3)$. The obvious change is, on the one hand, to lower

NORMALIZED MULTIPLIER MATRIX M_r

		RECEIPTS →																						
		FACTORS						**INSTITUTIONS**						**ACTIVITIES**										
		Labour			Capital			Households			Comp.		Govt	Tea	Rub.	Coco.	Rice	O.Agr	Food	Manuf	Min+C	Pwr	Serv	Dwel	Gov.S
	↓	Urb	Rur	Est	Pub	OPr	Hsg	Urb	Rur	Est	Prv	Pub	Gov	13	14	15	16	17	18	19	20	21	22	23	24
Urban (L)	1	1.00																							
Rural (L)	2		1.00																						
Estate (L)	3			1.00																					
Public (K)	4				1.00																				
O.Priv (K)	5					1.00																			
Housing (K)	6						1.00																		
Urban (HH)	7							1.04	*	*	.15	*	.04												
Rural (HH)	8							*	1.00	*	.03	*	.02												
Estate (HH)	9							*	*	1.00	-.01	*	*												
Private (Co)	10							.01	*	*	1.01	*	.01												
Public (Co)	11							.08	.04	*	.08	1.02	.59												
Government	12							.14	.06	*	.15	.03	1.03												
Tea	13													1.00	*	*	*	*	*	*	*	*	*	*	*
Rubber	14													*	1.00	*	*	*	*	*	*	*	*	*	*
Coconut	15													*	*	1.63	*	*	*	*	*	*	*	*	*
Rice	16													*	*	*	1.95	*	*	*	*	*	*	*	*
Other agric	17													.01	*	*	*	1.06	.01	*	*	*	*	*	*
Food, D.&T.	18													*	*	*	*	.04	1.03	*	*	*	*	*	*
Manufacturing	19													.05	*	.03	.04	.03	.03	1.37	.22	.10	.02	*	.02
Mining + Constr.	20													*	*	*	*	*	.01	.01	1.09	.01	*	*	*
Power, Trade, Tra.	21													.03	*	.04	.05	.03	.03	.01	.04	1.06	.10	*	.02
Services	22													.01	*	1.01	.04	.01	.01	.04	.01	.11	1.04	.01	.01
Dwellings	23													*	*	.02	.05	.02	.03	.03	.02	.10	.06	1.06	.02
Govt. Services	24													.01	*	*	.01	*	*	.02	*	.03	.03	*	1.00

TABLE 7: NORMALIZED MULTIPLIER MATRIX N

EXPENDITURES

RECEIPTS (MATRIX N)		FACTORS						INSTITUTIONS						ACTIVITIES											
		LABOUR			CAPITAL			HOUSEHOLDS			COMPANIES		GOVERNMENT	Tea	Rubber	Coconut	Rice	Other agric	Food,D&T	Manufac.	Mining,Const	Power,Trade,Tr	Services	Dwellings	Govt.services
		URBAN	RURAL	ESTATE	HOUSING	OTHER PRIVATE	PUBLIC	URBAN	RURAL	ESTATE	PRIVATE	PUBLIC													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	URBAN	1.20	.37	.08	.07	.56	.01	.79	.36	.09	.15	.02	.57	.10	.04	.14	.37	.23	.09	.36	.35	.66	.10	.03	.47
2	RURAL	.19	1.37	.09	.54	.01	.04	.34	.36	.15	.20	.47	.57	.11	.07	.24	.33	.29	.08	.31	.32	.72	.05	.08	.37
3	ESTATE	.43	.14	1.08	.03	.20	*	.12	.31	.04	.13	.10	.70	.07	.15	.10	.11	.03	.07	.07	.09	.14	1.17	.03	.0
4	HOUSING	.26	.14	.03	1.13	.10	*	.04	.09	.20	.13	.15	.33	.15	.09	.28	.55	.36	.13	.36	.36	.20	.11	.08	.21
5	OTHER PRIVATE	.35	.50	.10	.08	1.74	*	.65	.46	.11	.14	.01	.31	.05	.05	.15	.35	.55	.15	.15	.34	.55	.10	.21	.29
6	PUBLIC	.79	.44	.10	.14	.99	1.00	.01	.77	.16	.06	*	.13	.02	.10	.17	.41	.14	.04	.21	.32	.18	.15	.06	.10
7	URBAN	.23	.18	.94	.07	.30	*	1.42	.40	.12	.17	.01	.50	.64	.18	.25	.69	.41	.11	.33	.37	.62	.17	.07	.13
8	RURAL	.09	.18	.94	.14	1.12	.01	.99	1.99	.12	.42	.01	.42	.13	.14	.18	.69	.41	.04	.09	.11	.14	.01	*	.26
9	ESTATE	.55	.13	.19	1.63	.01	.48	1.19	1.15	.08	.03	.37	.15	.08	.27	.63	.51	.12	.35	.19	.69	.27	.20	.13	
10	PRIVATE	.27	.55	.05	.54	.43	.36	.70	.15	1.03	*	.79	.05	.04	.08	.19	.41	.15	.08	.15	.19	.49	.27	.07	.13
11	PUBLIC	.10	.32	.06	.04	.42	1.42	.07	.15	1.15	1.03	.04	.17	.02	.10	.13	.15	.08	.10	.42	.49	.60	.23	.06	.10
12	GOVERNMENT	.16	.10	.05	.02	.04	.43	.04	.07	.06	*	4.17	1.02	.17	.08	.17	.14	.04	.42	.23	.07	.13			
13	Tea	.03	.07	.02	.01	.09	*	.15	.02	.02	*	.04	.04	1.00	*	.01	.01	*	.03	.03	.05	.02	.01	.02	
14	Rubber	*	.01	*	*	*	*	.01	*	*	.01	*	1.00	*	.01	.01	.01	*	*	*	*				
15	Coconut	.21	.47	.12	.07	.63	*	.01	.01	.01	.26	.05	4.77	.28	.26	.22	.37	.18	.09	.18					
16	Rice	.27	.70	.18	.12	.9	.49	1.49	.19	.16	.16	2.33	.28	.26	.33	.16	.28	.52	.12	.35					
17	Other agric	.33	.68	.17	.12	.93	.58	1.44	.19	.17	.17	.41	1.39	.13	.31	.07	.13	.17	.13						
18	Food, D&T	.34	.69	.16	.13	.95	.60	1.50	.18	.16	.40	.22	.43	1.13	.36	.07	.07	.18	.13	.26					
19	Manufacturing	.32	.45	.11	.08	.63	.40	.99	.12	.16	.07	1.46	.33	.26	.02	.26									
20	Mining Const	.03	.04	.01	.01	.06	*	.04	.01	.02	.01	.34	.15	.20	.35	.02	.07	1.06	.01	.03	.01	.09	.18		
21	Power,Trade,Tr	.25	.52	.13	.09	.72	*	.46	1.13	.14	.14	.35	.06	.17	.34	.27	.26	.10	.33	.47	1.49	.10	.20		
22	Services	.43	.70	.14	.13	1.03	.01	.77	1.52	.16	.43	.03	.15	.23	.44	.17	.37	.26	.01	.12	.17	.23	.07	.96	1.10
23	Dwellings	.16	.20	.04	.05	.42	.29	.42	.04	.01	.01	.31	.10	.05	.08	.15	.17	.04	.10	.02	.12	.10	1.10		
24	Govt. services	.16	.20	.04	.05	.42	1.17	.04	.08	.17	.18	.34	.26	.14	.04	.12	.23	.07	.96	1.10					

the apparent multiplier effects accruing to large sectors such as manufacturing, shown in the row for account 19, whilst on the other hand a unit percentage exogenous change in output of the manufacturing sector will raise the output multiplier effects on all other activities, relative to those shown in $M_1(3,3)$.

The total normalized multiplier effects, N, are shown in Table 7. These correspond to the unscaled version previously shown in Table 4. Here the normalized multipliers are particularly interesting. For example, $N(2,3)$ shows the percentage income effects arising out of a unit percentage change in activity outputs. Unlike $M(2,3)$, the effect on urban and rural household incomes from an increase in output of the Tea sector is small relative to that of estate households. In fact, reading across the row for estate households, subject to the rigid behavioural assumptions we have adopted, there is no activity other than Tea or Rubber which would redistribute income in favor of estate households. As one might expect, it appears that other primary activities tend to redistribute in favor of rural households whilst the secondary and tertiary activities favor the urban households. A similar result holds in relation to factoral incomes. Estate households receive little transfer income, so the normalized multipliers for estate labor is not too different from those of estate households. However, urban and rural labor show lower multipliers, in general, than those of the corresponding household groups simply because the latter are also recipients of non-factor transfer payments from companies and government.

5. Redistribution and Sectoral Expansion

The multiplier matrices that we have derived from the structure of the Sri Lanka SAM permit some preliminary analysis of redistribution of income between institutions which might arise out of sectoral expansion. The distinction between the multiplier and normalized multiplier matrices allow for some alternative sectoral rankings in relation to their effects on the distribution of income and on the distribution of household income in particular.

We have previously noted that the component M(2,3) of matrix M,[1] showing non-standardized multipliers, depicts the incremental change in income shares of households and other institutions which would arise out of an extra rupee of output of the twelve production activities. Table 8 summarizes these effects for households. Two sets of effects are shown; one relates to the generated income that is received by all households, and the other shows only the generated income to estate households. In each case, the rank order for sectors is also shown. Not surprisingly, estate households benefit most in terms of income generation from an extra rupee of output of tea production. This sector is followed by the Rubber and Coconut sectors but these apart, most others generate a similar amount of income for estate households. If all households are considered, the rank order of production activities in terms of income generation is quite different. The government service sector which, it must be remembered refers only to the value added payments of government expenditure, generates the highest

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1/ See Table 4.

Table 8:   INCOME DISTRIBUTION AND UNIT SECTORAL EXPANSION

Activity	All Households (1)		Estate Households (2)	
	Income Change	Rank	Income Change	Rank
Tea	2.03	8	.58	1
Rubber	2.19	2	.31	2
Coconut	2.18	3	.12	3
Rice	2.10	6	.05	10
Other agriculture	2.12	5	.06	5
Food, Drink & Tobacco	1.06	12	.03	12
Manufacturing	1.45	11	.03	11
Mining and Construction	1.96	9	.05	8
Power, Trade, Transport	2.07	7	.05	9
Services	1.64	10	.05	7
Dwellings	2.16	4	.09	4
Government service	2.42	1	.05	6

aggregate household income. Its ranking for estate household income alone however is only six. Tea production ranks quite low in aggregate income terms, although the rubber and coconut sectors rank just as high as they do for estate income generation. The rank correlation between estate and aggregate income generation is very high for most other sectors; notably, Manufacturing ranks quite low in both instances, and the Food, Drink and Tobacco sector ranks last of all.

Sectoral rankings based on the non-standardized multipliers show the relative effect on household incomes of the marginal unit of output. However, a potentially more interesting question concerns the redistributive effects of growth in output of each production activity. This reflects a general concern not simply with income shares but rather with the level and growth of income in the lower income groups. [1] In the Sri Lanka context, where households have been aggregated into urban, rural and estate sectoral groups, there is clearly much intrasectoral inequality of incomes, [2] but mean incomes also vary considerably between the three sectors. Per capita incomes of the estate households are less than half those of urban households. [3]

The analysis can most usefully proceed within the framework recently proposed by Ahluwalia and Chenery. [4] They work in the context of an overall

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[1] See Chenery, et al. (1974).

[2] In Jain (1974), urban households have a Gini coefficient of 0.41 and rural households have a lower one of 0.35.

[3] The per capita income of urban households in 1970 was 1385 rupees; that of rural households was 781 rupees, whilst that of estate households was 615 rupees.

[4] See Ahluwalia and Chenery (1974) pp.50-51.

performance measure, or weighted index of welfare, the weights being set to reflect the degree of distributional emphasis desired. They use several weighting schemes, all of which are appropriately viewed within the context of an index of welfare improvement from a social utility function of the semilogarithmic type:

$$U \;=\; \sum_i w_i \log y_i \tag{40}$$

where U is the level of social welfare, y_i is the level of the ith household group, [1] and w_i is a weight for the ith group. [2] For our analysis, therefore, we assume that the level of social welfare is related to the level of income or urban, rural and estate households, according to equation (40). Expressing

$$\frac{\Delta y_i}{y_i} \;=\; \nabla y_i \tag{41}$$

as before, then equation (40) becomes

$$\Delta U \;=\; \sum_i w_i \, \nabla y_i \tag{42}$$

so that with this utility function an increment in social welfare is a linear function of the growth in income of each household group. Various measures of the household income weights, w_i, can be used. We shall use a weighting system where the w_i's all sum to one, so that the utility increment is

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[1] The y_i's are the first three elements of the six element vector t_2.

[2] This is not strictly as in Ahluwalia and Chenery, since their household groups are income groups.

measured up to a scalar multiple. If the social objective is to maximize

GNP, then the appropriate weights would be the income shares of the three

household groups. For an egalitarian utility index the weights would be

proportional to population size.

The normalized multiplier matrix N, and in particular the matrix

N(2,3), which shows the institutions' income growth arising out of a unit

growth in output of each of the production activities, has some interesting

implications in terms of the changes in social welfare. More specifically,

a unit growth in output of any particular production activity leads, via

the multiplier effects, to an implied growth in income for each of the urban,

rural and estate household groups. These are captured by the appropriate column

of the submatrix N(2,3). Applying the weights of the social welfare function,

the social utility increment arising out of a unit growth in output of that

sector can easily be calculated.

Table 9 shows four sets of calculations for all twelve production activities

identified in our SAM. The sets differ according to the social welfare weights

that were used. For the first set the weights were simply the income shares

of the urban, rural and estate households. The utility index increments [1]

and sectoral rankings show quite interesting results. The Trade, Transport

and Power sector made the largest contribution to the social welfare function,

so defined, from a unit growth in sectoral output. Its rank is quite different

from that indicated in Table 8, where the sector had a consistently low ranking.

Table 9 also shows the Tea, Rubber and Coconut sectors to have a low ranking

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[1] The calculations omit any scaling adjustments and hence simply weight
growth rates of household income by standardized weights.

Table 9: INCOME DISTRIBUTION AND SECTORAL GROWTH IN SRI LANKA

Activity	Income Weights (1)		Equal Weights (2)		'Poverty' weights (3)		'Estate only' weights (4)	
	Utility index	Rank	Utility index	Rank	Utility index	Rank	Utility index	Rank
Tea	.165	8	.182	8	.230	8	.637	1
Rubber	.077	12	.081	12	.088	12	.145	4
Coconut	.232	7	.236	7	.232	7	.176	3
Rice	.445	2	.442	2	.391	2	.136	6
Other agriculture	.371	3	.369	3	.345	3	.140	5
Food, Drink & Tobacco	.100	11	.099	11	.092	11	.038	12
Manufacturing	.312	5	.303	5	.279	5	.093	8
Mining & Construction	.339	4	.334	4	.312	4	.113	7
Power, Trade & Transport	.595	1	.577	1	.532	1	.178	2
Services	.162	9	.158	9	.147	9	.066	11
Dwellings	.139	10	.136	10	.129	10	.075	10
Government services	.289	6	.275	6	.250	6	.080	9

with the income shares weighted utility index, whilst Rice and Other Agriculture have high rankings. Bearing in mind that the first eight sectors are those which have high policy content (the last four are tertiary, or related-tertiary, sectors) it is perhaps quite significant that the Rice and Other Agriculture sectors yield the highest rankings under this weighting system.

The second set of columns show similar calculations except that population weights were chosen for the w_i's. Under this weighting system, corresponding to equal social welfare weights for individuals regardless of the household grouping, the numerical results for the utility increments were little different to those for the first set of calculations. The sectoral rankings were invariant. This slightly surprising result is probably due to the dominance of the rural household sector in both weighting schemes, where it represents 71.9 percent of the total population receiving 64.5 percent of total household income.

Moving away from 'GNP maximization' or 'egalitarian' weights, the third set of calculations, the 'poverty' weights, were designed so as to impose the proposition (admittedly arbitrary) that a rupee of income to an individual in an estate household is worth four times a rupee to an individual in an urban household. Combining this arbitrary social valuation with the population size of the two groups leads to weights for the urban, rural and estate households of 1 : 7 : 2. These we refer to as 'poverty' weightings, in the spirit of the Ahluwalia and Chenery analysis referred to earlier. In spite of reweighting utility increments in this way, the rank order of sectors is again invariant. This is perhaps not surprising because the rural household group still dominates the other two by about the same amount.

The final columns show a weighting system where the social welfare function is assumed to depend only on the growth of estate households. It is interesting to observe that the Trade and Transport sector remains second highest rank, but that only in this quite extreme weighting situation does the Rubber sector rise significantly in rank order. The most interesting comparison to make, however, is the rankings that emerge between the weightings appropriate to GNP maximization and those from 'Estates only' weightings.

6.   Some Conclusions

The empirical results of the exercises in this paper depend crucially on assuming a constant A matrix. The multiplier analysis based on a fixed A is probably a good guide to a number of issues in the context of Sri Lanka in 1970 because production was not limited by capacity in most sectors. However, in other country situations, this would not be the case, and one or both of two modifications would then be needed. These would be to allow the A matrix to change, say, as a result of price changes or a change in import dependence. [1/] Alternatively, some allowance would need to be made for increasing capacity in response to increased demand on a sector. Following this approach, investment becomes endogenous within the structure, thus increasing its interdependencies and the multipliers attached to the remaining exogenous forces. [2/] How best to pursue these alternative lines of development is a question we leave open. For now the point is simply that one can go only so far with simple models as a guide to either policy or prognosis.

Just as we recognize the limitations of a fixed coefficient approach to sectoral and institutional linkage, we also realize that there are deficiencies with the analysis underlying sectoral growth and its distributional impact. The social utility function we have used is of a particular form and is convenient for the use to which it is put. Clearly, it would be an interesting question to pose the robustness of our results to alternative

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[1/]   This is the development explored tentatively in Pyatt et al. (1972)

[2/]   See Karunaratne (1973).

functions as well as to non-linear behaviour within the model itself. This, too, is a question which, for the present, we leave open. The analysis does, however, serve to highlight the interdependencies within the economy and the extent to which poverty groups cannot be treated in isolation from the rest of the economy.

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