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LIBERALISM, PARETO PRINCIPLE AND THE CORE
OF A SOCIETY

by

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Abstract

The paper argues that where individual choice in personal activities is involved, the liberal principle of freedom of choice must include the right for the individual to enter voluntary trade with others regarding the personal activities. Once the problem is so conceived, it can be shown to belong to the general class of n -person games for which Scarf (Journal of Economic Theory, 1971) has proved that there is always a non-empty α -core, on certain convexity assumptions.

Liberalism, Pareto Principle and the Core of a Society

By S.K. Nath

1. Introduction

It has often been argued recently (see [16], [17] or [18]) that the Pareto principle and the liberal principle of individual freedom of choice in what might be described as personal matters, necessarily conflict with each other in the sense that it is not possible by any rule which obeys both these principles to select a non-empty social choice set from every possible subset of feasible social states - for all possible profiles of individual preferences over the set of all feasible social states. We shall argue (in section 2) that the liberal principle of individual freedom of choice in personal matters must include the right of voluntary exchange regarding these matters; and that, then, the liberal principle necessarily implies the use of the Pareto principle. Once the problem is so conceived, it invites a game theoretic approach. Such an approach is developed in section 4. We point out there that the society we describe meets all the requirements of Scarf's [14] general class of n -person games, so that his theorem about the existence of a non-empty α -core for such games, is immediately applicable. Section 3 briefly reviews Sen's formulation of the Paretian-liberal paradox. In the concluding section, we argue that some liberals' dislike of the Pareto principle may be unjustified.

2. Freedom of Choice and the Pareto Principle

Though it is too much to hope for exact agreement, even among persons of liberal outlook, on a detailed list of actions that the liberal principle of individual freedom of choice should encompass, it is hoped that all such persons would agree that the principle lays down the following general requirement: society should permit each individual an area of autonomy in which he can dispose of the relevant matters - by himself alone, or in voluntary trade with others - according to his own wishes, so long as his exercise of this right is consistent with the others' exercise of their similar rights. (Cf. [12], p.60), Gibbard [7] has also emphasized the importance of the freedom for an individual to engage in voluntary exchange regarding some matter which belongs to his personal sphere. As he says, "There is a strong libertarian tradition of free contract, and on that tradition a person's rights are his to use or bargain away as he sees fit." [7], p.397. We take the view that individual's right to the freedom of choice over some activity is in the nature of a property right - which can be enjoyed on its own, or used as the wherewithal in voluntary exchange to acquire other property rights. (Cf. [5].)

Consider an example. Ralph and Alan live opposite each other across a road, the banks of which rise steeply on both sides. Thus, sitting down in a front room of either house, each can see the front garden of the neighbour opposite but not his own. Let us assume that what each person grows in his front garden is judged to be his personal matter. Ralph loves roses and Alan loves alpines. However, Ralph prefers a state in which Alan's garden has some roses and his own some alpines to one in which his own garden has only roses and Alan's none. Similarly, Alan

prefers a state in which Ralph's garden has some alpine and his own some roses to one in which his own garden has only alpine and Ralph's none. There can be no doubt that the liberal principle of freedom of individual choice in personal matters, must include for each of the two individuals the right to enter trade with each other, and voluntarily agree, if it suits each of them, to grow in his garden some or all of what the neighbour opposite would like, provided that the neighbour does the same in return, or provides some other quid pro quo.

An individual's actions in what should clearly be regarded as an area of his personal autonomy are likely often to affect the utility levels of others, favourably or unfavourably - through external effects. Compare Robbins' remarks: "There is scarcely anything I do outside the privacy of my home which has not some overtone of indiscriminate benefit or detriment. The clothes I wear, the shows I frequent, the flowers I plant in my garden, all directly, or through the mysterious influence of fashion, influence the enjoyments and satisfactions of others." [13], p.20. Whenever an economic agent's action gives rise to an external diseconomy, the question arises: do those who get the external diseconomy have a right of redress? If they do have such a right, we might say that what Starret [20] calls the common rule prevails: but if they do not have such a right, then what he calls the possession rule prevails. Under the latter rule though those who get the external diseconomy do not have a right of redress, they may still negotiate with the agent whose actions produce the external diseconomy, in order to make it worth his while for him to reduce the level of the activity which gives rise to the external diseconomy (see [8]); or they may undertake or threaten to undertake such action in their personal spheres as they hope would induce

the first individual to reduce the level of his personal activity which gives rise to the external diseconomy.

If an action is deemed to be in an individual's sphere of personal autonomy, then it follows that the society in question takes the view that in the event of any external diseconomies arising from such an action, the possession rule should prevail. We shall describe the external diseconomies which arise from any individual's personal activities, and about which the possession rule must necessarily prevail, as non-essential. For example, though others have preferences about the length of John's hair, yet - at least if John is an adult - the length of his hair might be deemed to be John's personal matter in most societies, because any external diseconomies are considered non-essential. This suggests that what are declared to be the personal issues of any individual cannot be chosen arbitrarily. The first requirement that they must satisfy is that if there are any external diseconomies, then they are all judged to be of the non-essential type, so that the possession rule can prevail regarding them; if the possession rule is not considered appropriate about the external diseconomies arising from some action, then such an action cannot be deemed to be personal in any meaningful sense. Hence what is taken to be the personal issue of an individual does have to be, according to some acceptable criteria, personal to him. This also suggests another fundamental requirement: one individual's exercise of his rights in his personal sphere must be, as Nozick [9] says, "co-possible" with any other individual's exercise of his rights in his personal sphere. Or in Seidl's [15] terminology, what are declared to be the personal activities of a group of individuals must be technologically (one may also say logically) compatible.

When any two persons i and j trade together and agree to take some joint actions in their personal spheres, some other persons may not like the actions they are contemplating. Though these other individuals do not have a right to directly prevent i and j from their contemplated joint action, they have as much right to form a voluntary trading group, i.e., the coalition complementary to the coalition of i and j , and take such actions in their personal spheres as suit them and which they think might deter i and j from taking their contemplated action. Thus the situation invites a game-theoretic approach.

Turning now to the Pareto principle, we note that it has a strong and a weak form. Its strong form is that if between any two alternatives x and y , all individuals consider x at least as good as y , and some prefer x to y , then x should be socially accepted as better than y . Its weak form is that if everybody prefers x to y , then x should be socially accepted as better than y . Now, if the social state vectors under consideration are identical in all respects except the particular values of the personal activities of the individuals comprising the society, and if one of these social states is not Pareto optimal, then it can never be a stable social outcome because by definition either all individuals can move to higher levels of utility, or at least some can while nobody's utility level is lowered - by mutual voluntary trade; and the liberal individual right about personal activities, as we have seen, must include the right to mutual voluntary exchange, in order to be a meaningful right. Hence, at least with regard to a set of feasible social states which vary only regarding the particular values of individuals' personal activities, if any social state is to be a stable outcome in any sense, it must be Pareto optimal. The outcomes in the

core of the game model of a society are always Pareto optimal. Our task in section 4 will be to construct a model of a society where the core is non-empty, even though it obeys both the strong Pareto principle and a restriction formulated to protect the individual freedom of choice in personal activities.

3. Sen's Paradox

Sen proved (see [16], [17] or [18]) that there is no social decision function which satisfies the weak Pareto condition (defined in the previous section), his condition of liberalism (see below), and the condition of unrestricted domain (that all possible profiles of individual preference orderings over the set of feasible social states are permitted): a social decision function being a method of choosing for each possible profile of individual orderings, an acyclic social preference relation over the set of feasible social states. Sen's condition of liberalism is: For every individual, there is at least one pair of distinct alternative social states, say (x, y) , such that if he prefers x to y , then society should prefer x to y , and if he prefers y to x , then society should prefer y to x .

For a number of reasons, we consider Sen's condition of liberalism unsatisfactory as a representation of the liberal principle of individual freedom of choice in personal matters. First, as Farrell points out, Sen "makes no distinction between a man deciding whether to sleep in a prone or a supine position, and a religious leader dictating whether he

does so" [6], p.5. Secondly as Seidl [15] notes, Sen does not require that what are regarded as personal activities of different individuals are at least technologically (or logically) compatible. (In Sen's by now well-known example (see [16]) of two individuals and one copy of the book, Lady Chatterly's Lover, which he assumes only one person can read, if reading (all, any part or none of) the book is regarded as a personal activity of each of the two persons, then these two personal activities of the individuals are not always technologically compatible, so that there is no need to investigate any further the possibility of the individual freedom of choice in personal matters in that example.) Thirdly, Sen does not include the right to voluntary exchange regarding personal matters as part of the liberal individual right. (One consequence of this is that though the condition of the independence of irrelevant alternatives is not explicitly imposed by Sen on his social decision function, it is nevertheless implicitly used in every step of the proof of Sen's proposition - as Blau [3] points out.)

Fourthly, and this is the most important criticism of Sen's condition of liberalism, though Sen quite rightly takes individual preferences to be defined over social states, he grants each individual too much authority when he makes him decisive in the social choice over a pair of social states: though individual preferences must be taken to be defined over social states (especially when externalities are admitted), all that the liberal principle of individual freedom of choice in personal matters requires is that no components of a social state vector which concern the personal activities of an individual should ever be changed without his voluntary consent. (Cf [3] and [4]). We shall not discuss this criticism further here, but its full import will become obvious in

in the next section, in connection with our Definitions 1,2 and 3.

4. The α -Core of a Liberal Society.

A social state is a vector of decisions or activities, some of which are recognized to be personal activities of the individuals - in the sense explained in section 2. All other activities are to be described as non-personal. Our analysis starts at the stage where particular values for all non-personal activities have been selected some how or other. After the manner suggested by Farrell [6], we may envisage that the set of all feasible social states has been partitioned into non-personal-activities equivalence classes, and that a social consensus has already emerged on one of these equivalence classes. We are then left with the further social choice over the social states in the one non-personal-issues equivalence class. Alternatively, we may assume that the model to be developed here applies to each one of such possible equivalence classes, one at a time. Either way, in our model all non-personal activities are assumed to remain constant; the social states under discussion vary only in the values of personal activity vectors.

Let us then consider a set of social states in which they all have the same values for the non-personal activities; hence we can ignore those components of each of its social state vectors which specify the values of such activities. We shall denote such a set of social states by ξ . Let the society have a set I of n individuals. For us then a social state vector, $\underline{x} = (x^1, \dots, x^n)$ is an n -tuple of

particular values of vectors x^i of each individual $i \in I$. $X^i \ni x^i$ represents a non-empty activity set for person i , where each X^i is a subset of a finite-dimensional Euclidian space. Our typical non-personal-activities equivalence class is then $\xi = X^1 \times \dots \times X^n$; and a typical element of ξ is a social state vector \underline{x} .

An activity which is deemed to be personal to an individual, may be a production, consumption or exchange activity. Of course only such personal activities are feasible for an individual as are consistent with his initial endowment of goods and rights. An individual's property right to his initial endowment is assumed to be exclusive. Any personal activity of one individual is assumed to be logically compatible with any personal activity of another person. However, a personal activity of one person may affect the utility of another person through non-essential externalities of the kind discussed in section 2. What was described there as the possession rule prevails regarding these externalities. Each individual has the liberty to do what he likes regarding production, consumption or voluntary exchange with respect to his personal activities.

Apart from the usual reasons for voluntary exchange, in this model there would be the additional reason for individuals to enter into trade with one another, in response to one another's externalities. If the number of individuals is greater than two, the possibility arises for coalitions to form. We shall make the usual (implicit) assumptions of the theory of co-operative games, namely that there is costless communication among individuals about possibilities of trade, so that any mutually gainful coalitions will be formed; and that each individual

assumes the others to be truthful and trustworthy, so that all agreements are expected to be binding. (Notice that these assumptions imply that a prisoner's dilemma like problem cannot arise in our model.) In addition, we shall require the following two assumptions.

Assumption 1: Each X^i is a closed bounded convex set in a finite-dimensional Euclidian space.

Assumption 2: Each individual has a continuous quasi-concave utility function $U^i(\underline{x})$, defined on the product space ξ ; or equivalently, each individual has a continuous, convex weak preference pre-ordering over the set ξ .

The essential part of the two assumptions is that about the convexity of the respective sets, Starret [19] has argued that external diseconomies tend to introduce non-convexities in the production and preference sets. We may, however, justify our assumptions by pointing out that the only external diseconomies admitted in our model are of what we have described as the non-essential type, for which what is called the possession rule is assumed to apply. Therefore, external diseconomies admitted in the model are likely to be mild enough not to intrude any non-convexities.

An activity vector of an individual is a plan of action which can be regarded as a strategy. Recall that any personal activity of an individual is assumed to be compatible with any personal activity of any other individual; hence the same is true of individual strategies. Joint strategies of the members of any coalition cannot be arbitrary; they are constrained by the initial endowments of the members.

A social state is in the α -core if, first, its associated vector

of the utility levels of all individuals is achievable by the members of the grand coalition, $K = I$, and, secondly, no coalition, K , can achieve a higher level of utility for each of its members independently of the actions of the members of the complementary coalition, \bar{K} . (For the distinction between α -core and β -core see [14]. For a game model which admits interdependence among the individual utility functions, the concept of the α -core seems to be more relevant than that of the β -core.)

Since the second condition applies to all possible coalitions - i.e., all possible subsets of I - a social state in the α -core is obviously Pareto optimal according to the strong Pareto condition. Thus the α -core is a subset of the Pareto optimal social states, which consists of such social outcomes as are stable in the sense defined by the two foregoing conditions, and as also satisfy the constraint of the initial endowments that is placed on the joint strategies of the members of any coalition. In order to see the connection between this usual restriction on joint strategies, and the restriction required by the individual freedom of choice in personal activities, we need to formulate the following definitions.

Definition 1: For any person i , T_i , is a binary relation in ξ , such that $\underline{x} T_i \underline{x}' \leftrightarrow x^i = x'^i$, where vector x^i is a component of \underline{x} , and vector x'^i is a component of \underline{x}' .

Clearly T_i is reflexive, transitive and symmetric. Therefore T_i is an equivalence relation, which induces a partition of ξ into non-empty disjoint and exhaustive equivalence classes of ξ , such that every social state belongs to some equivalence class. We shall call such a class a personal-activities-equivalence class of person i , and represent it by $[\underline{x}]^i$.

Definition 2: A personal-activities-equivalence class of a person i , $[\underline{x}]^i$, is that subset of ξ , which consists of those social states in which the values of i 's personal activity vectors are the same.

Remark 1: Apart from the activity vectors of person i , there is no restriction on the other components of social states in $[\underline{x}]^i$; they can vary.

Remark 2: The difference between any two social states in $[\underline{x}]^i$ from the standpoint of the individual i can only be with respect to the activity vectors of other persons.

Remark 3: There is a partition of the set ξ into personal-activity-equivalence classes, for each of the n individuals.

Remark 4: Given any arbitrary initial social state \underline{x} , a change in the joint-strategy choice of the members of any coalition K , will necessarily shift the society into another social state, say \underline{x}' . So long as the new social state, \underline{x}' , is in the same personal-activities-equivalence class of every person i who is not a member of K , as the initial social state \underline{x} , no individual's liberal right of freedom of choice in his personal activities would have been affected. Of course the utility levels of persons who are not members of K may well be affected, because of externalities, and they may want to make a response to the changes (or the threat of changes) in the personal activities of the members of the coalition K . But that is a different matter.

The last remark suggests the restriction to impose on the joint strategies of any coalition for the individual liberal right to personal activities to be honoured. Clearly, we should require that for the members of any coalition, the joint strategies should be restricted to those which cannot shift the society into a social state that is not in the same personal-activities-equivalence class of every individual who is not a member of K , as is the initial social state.

Definition 3: The liberal principle of individual freedom of choice requires that the set of joint strategies of any coalition $K \subset I$ be restricted to the following.

$$\xi^K = \{ \underline{x} \in \xi : \underline{x} \in \bigcap_{i \in K} [\underline{x}']^i \text{ for all } i \in K \}$$

where \underline{x}' is the initial social state

It now emerges that this restriction which the liberal principle requires is essentially the same restriction as that which is required in any theory of co-operative games - namely that members of any coalition can form their joint strategies only within the constraint of their own initial endowments; in other words, that members of a coalition are not to take any of the initial endowments of the complementary coalition (with whom, by definition, they are not trading). (see Bergstrom [1]). Our restriction in Definition 3 requires essentially the same thing: namely that no joint strategy of the members of any coalition is to change the endowment of personal activities in the initial social state of the members of the complementary coalition (with whom, by definition, they are not trading).

A society which satisfies assumptions 1 and 2, and the restriction of Definition 3, meets all the requirements of Scarf's [14] general class of n-person games. Therefore, we can apply his theorem directly to our society.

Theorem (Scarf): For a society such as that described above, which satisfies assumptions 1 and 2, there exists a social state which

belongs to the α -core.

Any social state in the α -core is clearly Pareto optimal; its selection also obeys the condition regarding the liberal rights of individuals; and the existence of a non-empty core is proved without imposing any restriction on the possible configurations of individual preferences over the set of all feasible social states. However, since voluntary exchange is permitted in the model, the condition of the independence of irrelevant alternatives is violated, because voluntary exchange among individuals is influenced by the interpersonal differences in the intensity of preferences (without actually providing a means for measuring these differences). However, nobody has ever argued, nor do we think that a remotely convincing argument can be made, to the effect that voluntary exchange among individuals regarding activities which are judged to be their own concern, should be disallowed because such exchange is influenced by the interpersonal differences in the intensity of preferences.

5. Conclusions

We have argued that the liberal principle of individual freedom of choice over personal issues must include the right for the individual to enter voluntary trade with others regarding them, and that, therefore, we cannot jettison the Pareto principle even in its strong form; because whenever a social state is not Pareto optimal, some individuals can, by definition, through voluntary agreement take certain joint actions which

raise their levels of utility (while others' levels of utility are not lowered) - thus moving the society to a Pareto optimal social state.

We also argued that what are to be regarded as personal activities cannot be chosen arbitrarily. They need to be such that any external diseconomies arising from them can be judged to be what we described as non-essential. Moreover, any personal activity of any individual needs to be logically compatible with any personal activity of any other individual.

Viewed thus, the problem of social choice from among a set of social states which differ from one another only regarding the values of personal activities of individuals, can be treated as that of determining the solution for an n -person co-operative game. We showed that the α -core of such a game is non-empty on certain convexity assumptions.

The Pareto principle needs to be exonerated on two other counts. As Osborne [10] judiciously points out, the Pareto principle - in its weak or strong form - is often misunderstood to be a biconditional normative proposition incorporating "if and only if"; whereas it is only a conditional normative proposition incorporating "if". This means that adherence to the Pareto principle does not preclude ranking a social state \underline{x} over a social state \underline{x}' , even though \underline{x} is not superior to \underline{x}' according to either the weak or the strong version of the Pareto principle. It is true that the principle is sometimes used as a biconditional normative proposition. But one can quite consistently be against its use as a biconditional, and for its use as a conditional normative proposition.

Where alternative distributions of initial endowments are to be compared, one needs to know how the individual preference relations are to be interpreted before the Pareto principle can be used. Say between any two alternative distributions of initial endowments x and y , the only difference is that in x the rich are richer than in y , whereas the poor have the same absolute share of initial endowments in both x and y . Now it is possible to so (perhaps implicitly) define the individual binary preference relation that one claims that the rich "prefer" x to y while the poor are "indifferent" between x and y , so that x is socially superior to y according to the Pareto principle in its strong form. Though the Pareto principle has been used here only as a conditional normative proposition, yet such a use of it may be obnoxious to a liberal, for - as Peacock and Rowley [11] point out - liberal values and a concern for equity are quite compatible. However, if the individual preference relation is, in this context, more broadly interpreted to reflect all individual values (including those of distributive justice), then one cannot be sure that the rich would necessarily "prefer" x to y , or that the poor would necessarily be "indifferent" between x and y ; therefore, one cannot be sure that x would necessarily be superior to y according to the Pareto principle.

Hence if the Pareto principle is understood to be a normative conditional proposition, and if individual preference relation is appropriately interpreted according to the problem being analyzed, a liberal need have no qualms about assenting to it.

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