LIQUIDITY, SPECULATION, AND THE DEMAND FOR MONEY : A TEMPORARY GENERAL EQUILIBRIUM MODEL

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1. Introduction and summary of results

This paper studies some general equilibrium aspects of the demand for money. A model of the demand for money is developed in the context of an intertemporal economy in which money is the institutional medium of exchange and the asset whose nominal yield is certain. The distinction of money as the medium of exchange is formally represented by an expenditure constraint which reflects the relative illiquidity physical commodities, and is responsible for the transactions demand for money. The presence of an alternative financial asset (a long-term bond) whose future market value, and hence its resultant yield, is uncertain gives rise to a speculative demand for money. The decision on how much to consume, how much to save, and the form in which savings should be held, is a joint decision, and the choice is based on intertemporal preferences for consumption streams.

It is found that, as long as there is a positive subjective probability of a positive return (nominal interest payment plus change in capital value) on holding bonds, a temporary equilibrium will exist and, in equilibrium, money will have a positive exchange value provided there is an incentive for intertemporal modification of the initial endowment allocation. The positive value of money is due in an essential way to its role as the means of payment. The requirement of a positive return on bonds is simply to make the individual's decision problem well-defined (avoiding unbounded short-sales for purposes of speculation) and could be removed by restricting the search for an equilibrium to the region of prices in which the condition is satisfied (cf. Green (1973)).

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The rather natural requirement that there is also a positive subjective probability of a net loss on bond-holding, so that money is not dominated absolutely by bonds as a store of value, does not have to be introduced to ensure existence of a temporary monetary equilibrium (again because of money's role in exchange), but it could be argued that, unless this were the case, the social acceptance of money as the means of payment would be eroded over time.

It is found also that the interest rate (the inverse of the price of long-term bonds) will be positive if the subjective probability of a capital loss goes to one as the interest rate goes to zero, in which situation the demand for money becomes arbitrarily large. This suggests that positivity of the interest rate is due to the existence of a speculative demand for money rather than to a "liquidity trap" per se, or to inelastic expectations, the latter being sufficient but not necessary for the result. If the subjective probability of a capital loss goes to one at a positive rate of interest, there will be a proper liquidity trap at that rate and the above result will hold a fortiori.

The framework used here should be amenable to further elaboration and, likewise, the technical approach of more general applicability. There are several features in common with the model of Green (1973) and it is therefore not surprising to find that several of the conditions required to prove the existence of a temporary equilibrium correspond very closely. A major difference, however, is that, in Green's model, future trading is in contracts for goods which are desirable in themselves. The assumptions that future prices

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are expected to be positive, important for the proof of existence of a temporary equilibrium with positive prices, is justified by monotonicity of preferences. In contrast, the only assets here are financial assets, which are of value only through their purchasing power for real goods. The corresponding assumption in this context, that money is expected to have a positive value in the future, is not particularly satisfactory. This is avoided by the approach taken here, and the relevant assumptions concern only the real commodities and their prices.

2. Liquidity, speculation, and the demand for money

The environment for this analysis is a pure exchange economy operating over a sequence of time periods (t=1,2...). In each period there are N perishable consumption goods, traded in Walrasian markets, i.e. the commodity prices adjust to equate supply and demand. There is also fiat money, which is the unit of account (the price of money is fixed at unity in each period), the medium of exchange, and a store of value. The total quantity of money is constant over time. Finally, there is an alternative financial asset, a long-term bond, which is a note promising to pay the bearer one unit of money in each succeeding period, in perpetuity. 1/ The bond market clears by price adjustment ; the inverse of the bond price in any given period is "the" rate of interest in that period, the economy's measure of willingness to lend. In the present context, bonds are perhaps best thought of as claims on (or debts to) a central clearing-house.

The two essential aspects of the demand for money with which the analysis will be concerned are (i) the transactions demand, arising from money's role as the means of payment, and (ii) the speculative demand, arising from the uncertainty about future bond prices (equivalently, future interest rates) and thus the resultant yield of a bond. In order to distinguish money as the medium of exchange 2/ and provide a transactions motive for holding money, it will be assumed that real commodities (in this model, the perishable goods) are relatively illiquid, in the sense that money must be on one side of any transaction (goods must be bought with money ; goods are sold in return for money) and that there

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is a delay between the supplying of a good and the opportunity to spend the sales proceeds. More specifically, it is assumed that money received from sales conducted in one trading period is not available for expenditure until the next trading period. (This corresponds formally to an exchange process in which there is one round of actual trading per period and, in each round, offers to buy and sell are expressed simultaneously on each market, an offer to buy being valid only if backed by cash. In practice, it corresponds most nearly to the situation of an individual who sells his labor in order to buy food, but receives his wages in arrears at periodic intervals). In this framework, the decision with respect to purchases in the current period determines the <u>transactions demand</u> for cash. The value of goods (in particular, labor) supplied during the period constitutes an <u>income demand</u> for cash (cf. Clower {1967}), which will be allocated, at the start of the next period, between expenditure and savings.

Bonds, like consumption goods, must be purchased with cash. But, in order to isolate the speculative motive for holding money, it will be assumed that bonds, unlike consumption goods, are instantly convertible into cash. In this sense, bonds are as liquid as money [3/].

It will be assumed that adjustments to portfolios (comprising the amount of money in excess of transaction needs, and the (positive or negative) level of bond-holdings) are made, if at all, only at the beginning of each period. So, at this time, the individual must decide on his new portfolio, his purchases in the current period, and the income to be generated for the next period.

Let $x_t \in R^N_+$, $b_t \in R$, and $m_t \in R_+$ denote respectively the individual's final demands for consumption goods, bonds and money balances in period t, with $p_t \in R^N_+$, $q_t \in R_+$ and 1 the corresponding (absolute) prices. In each period t, the individual receives an initial endowment of goods, $\omega_t \in R^N_+$, assumed to be known with certainty for all relevant periods*. Then the <u>expenditure constraint</u> on the individual's action $a_t \equiv (x_t, b_t, m_t)$ in period t, reflecting money's role as the means of payment, is

$$(2.1) \qquad p_t \cdot (x_t - \omega_t)^+ + q_t b_t \leq (q_t + 1) b_{t-1} + m_{t-1} ,$$

where $(x_t - \omega_t)^+$ is the vector of purchases. By definition,

$$m_t = m_{t-1} + b_{t-1} - q_t(b_t - b_{t-1}) - p_t \cdot (x_t - \omega_t)$$

which is usually seen as the budget equation :

$$(2.2) \qquad p_t \cdot x_t + q_t b_t + m_t = p_t \cdot \omega_t + (q_t + 1) b_{t-1} + m_{t-1}$$

As a store of value, money's distinguishing feature is that, due to its being the unit of account, its <u>nominal</u> value is constant

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* and has a portfolio $(b_{t-1}, m_{t-1}) \in R \times R_+$ as a result of his action in the previous period.

over time. (Its real value, like the real value of any financial asset, will, of course, depend on the level of commodity prices). A bond, even though its nominal yield is fixed in money terms, will be subject to capital gain or loss in terms of money, and hence of relative purchasing power - its ultimate use - regardless of the commodity price level. The resultant yield of a bond, the sum of the fixed nominal yield and the change in capital value, depends on its market price (equivalently, the interest rate) in the future and is therefore uncertain. Without this uncertainty, and the possibility of a capital loss greater than the interest payment, there could be no speculative demand for money.

It will be assumed here that the individual chooses jointly his current consumption and his portfolio, and that the utility function used for evaluating the alternative actions is one which is based on his intertemporal preferences for consumption. An action taken by the individual in period t will have consequences for his choice alternatives in future periods. Of particular interest here is the possibility of a capital gain or loss which would permit greater future consumption or less. It will be assumed that, in order to take account of the effects of an action in the current period, the individual considers only the consequent position in the succeeding period and evaluates it as if any credit (or debt) were actually to be realized. In this way, his consequent financial position is evaluated in real (consumption) terms; the appropriate intertemporal utility function is therefore one which represents his preferences for combinations of current consumption and quasi future consumption.

(2a) Let these intertemporal preferences be represented by a von Neumann-Morgenstern utility function, $u : R_*^{2N} \to R$, which is bounded, concave, continuously differentiable, and strictly monotonic.

The space of admissible prices (the money prices of consumption goods and bonds) in each period will be denoted by Π, defined by

$$\Pi = \left\{ \pi = (p,q) \in R_*^{N+1} \;\middle|\; p >> 0 \text{ and } q > 0 \right\}$$

In period t, the individual's expectations about the prices which will prevail in period $t+1$ will depend on the prices observed in the current period (and also on the fixed sequence of past prices). Denote by $\mathcal{B}(\Pi)$ the Borel σ-algebra of Π, and by $\mathcal{M}(\Pi)$ the set of all probability measures on $(\Pi, \mathcal{B}(\Pi))$.

(2b) Let the individual's price expectations be represented by a mapping $\psi : \Pi \to \mathcal{M}(\Pi)$, which is continuous in the topology of weak convergence of probability measures.

For each B in $\mathcal{B}(\Pi)$, $\psi(B;\pi_t)$ denotes the probability assigned to B when π_t is the price system observed in period t.

When considering an action $a_t = (x_t, b_t, m_t)$ in period t, the individual will therefore anticipate the following constraints on his choice in period $t+1$, 4/ for each

$\pi_{t+1} = (p_{t+1}, q_{t+1})$ which he considers possible (i.e. for each π_{t+1} in supp $\psi(\pi_t)$:

(2.3) $$p_{t+1} \cdot (x_{t+1} - \omega_{t+1})^{+} \leqq (q_{t+1} + 1)\ b_t + m_t$$

and

(2.4) $$p_{t+1} \cdot x_{t+1} = p_{t+1} \cdot \omega_{t+1} + (q_{t+1} + 1)\ b_t + m_t$$

Finally, the individual's action in period t must be such that

(2.5) $$(q_{t+1} + 1)\ b_t + m_t \geqq 0, \ \forall \ \pi_{t+1} \in \text{supp } \psi(\pi_t) \ ;$$

i.e., an individual who borrows in period t must expect to have enough money in period $t+1$ to cover his debt.

The following procedure for choosing an optimal action in period t is well-known (see, e.g., Grandmont {1974}). Corresponding to an action $a_t = (x_t, b_t, m_t)$ in period t and a price system $\pi_{t+1} = (p_{t+1}, q_{t+1}) \in \Pi$ in period $t+1$, the optimal choice in period $t+1$ would be an $x_{t+1} \in R^N_+$ which maximizes $u(x_t, x_{t+1})$, subject to (2.3) and (2.4). Let $x_{t+1}(a_t, \pi_{t+1})$ denote such an optimal choice. Then in period t, if $\pi_t \in \Pi$ is the price system, the optimal a_t is that which maximizes

$$v(a_t, \pi_t) = \int_{\Pi} u\left[x_t\, ,\ x_{t+1}(a_t, \pi_{t+1})\right] d\ \psi(\pi_{t+1}\ ; \pi_t),$$

subject to (2.1), (2.2) and (2.5). (The function V is continuous in both arguments, concave and strictly monotone in the first. See Grandmont {1974}).

The analysis (in section 4) will be concerned with the interaction of all the individuals in the economy, when each determines an optimal action in this way. It will be helpful to derive first the necessary conditions for an optimal consumption-portfolio choice.

3. Optimality conditions for the consumption-portfolio choice

Consider the decision problem of an individual choosing simultaneously his current consumption and a portfolio of money and bonds. The division of his current wealth between current consumption and savings, and the form in which savings are held, will depend jointly on his intertemporal preference pattern for consumption and on the yield of the portfolio. The individual does not derive utility from asset-holding per se, but only from the consumption stream to which the assets will ultimately give rise. As the future real value of any portfolio is uncertain (because of the uncertainty of future prices of both bonds and consumer goods), so is the eventual consumption stream. The current alternatives will be evaluated according to their expected utility. The individual desires to achieve the best possible time-pattern of consumption, having obtained any possible capital gains from speculating on the bond market. To simplify notation in what follows, let the current period be period 1.

In period 1, the individual has open to him the following basic alternatives :

(i) transfer wealth from period 1 to period 2 by holding money;

(ii) transfer wealth between periods 1 and 2 by buying or selling bonds;

(iii) speculate on a capital gain, by selling bonds in period 1 and holding the proceeds in the form of cash.

Let $\pi_1 = (p_1,q_1)$ be the price system in period 1, and $a_1 = (x_1,b_1,m_1)$ an arbitrary position. Given a_1, to each price system $\pi_2 = (p_2, q_2)$ in period 2 there corresponds an optimal consumption, $x_2 = x_2(a_1, \pi_2)$. (The actual choices of a_1 and x_2 are constrained, but the constraints need not be considered explicitly at this point).

In case (i) above, if final wealth in period 1 is carried over to period 2 in the form of a cash balance, with no change in nominal value, λx_{1k_1} units of any commodity k_1 in period 1 can be given up in return for an additional $\frac{\lambda x_{1k_1} p_{1k_1}}{p_{2k_2}}$ units of some commodity k_2 in period 2. At the margin, the change in expected utility would be

$$\Delta v_1(k_1, k_2) = \int_{\Pi} \left[-\lambda x_{1k_1} u'_{k_1} + \frac{\lambda x_{1k_1} p_{1k_1}}{p_{2k_2}} u'_{N+k_2} \right] d\psi(\pi_2; \pi_1)$$

$$= \lambda x_{1k_1} p_{1k_1} \int_{\Pi} \left[\frac{u'_{N+k_2}}{p_{2k_2}} - \frac{u'_{k_1}}{p_{1k_1}} \right] d\psi(\pi_2; \pi_1)$$

where u' is evaluated at $(x_1, x_2(a_1, \pi_2))$. The position (x_1, b_1, m_1) cannot be optimal if $\Delta v_1 > 0$ for any k_1. (Note that at $(x_1, x_2(a_1, \pi_2))$, the first-order conditions for the choice of x_2 imply that $\frac{u'_{N+k_2}}{p_{2k_2}}$ is equalized for all k_2). So a necessary condition for intertemporal optimality is that $\Delta v_1 \leq 0$ for all (k_1,k_2); i.e. for all (k_1,k_2),

$$(3.1) \qquad \int_{\Pi} \frac{u'_{k_1}}{p_{1k_1}} \, d\psi(\pi_2;\pi_1) \geq \int_{\Pi} \frac{u'_{N+k_2}}{p_{2k_2}} \, d\psi(\pi_2;\pi_1) :$$

the expected ratio of marginal utility to own price of any commodity in period 1 must be at least as great as the expected ratio of marginal utility to own price of any commodity in period 2.

In case (ii), the purchase or sale of a bond in period 1 enables the exchange of $\frac{q_1}{p_{1k_1}}$ units of commodity k_1 in period 1 for $\frac{q_2+1}{p_{2k_2}}$ units of commodity k_2 in period 2. Equivalently, λx_{1k_1} units of k_1 can be exchanged for a quantity $\frac{\lambda x_{1k_1} p_{1k_1}}{q_1}$ of bonds, which is in turn exchangeable for $\frac{\lambda x_{1k_1} p_{1k_1}}{q_1} \cdot \frac{q_2+1}{p_{2k_2}}$ units of k_2. If the flow is from period 2 to period 1 (respectively, from period 1 to period 2) the change in expected utility at the margin is $\Delta v_2(k_1,k_2)$ (respectively, $-\Delta v_2(k_1,k_2)$), where

$$\Delta v_2(k_1,k_2) = \frac{\lambda x_{1k_1}}{q_1} \int_{\Pi} \left[\frac{q_1}{p_{1k_1}} u'_{k_1} - \frac{q_2+1}{p_{2k_2}} u'_{N+k_2} \right] d\psi(\pi_2;\pi_1)$$

The position (x_1,b_1,m_1) cannot be optimal if $\Delta v_2(k_1,k_2) \neq 0$ for any pair (k_1, k_2). A necessary condition for intertemporal optimality is therefore

$$(3.2) \qquad \int_{\Pi} \frac{u'_{k_1}}{p_{1k_1}/q_1} \, d\psi(\pi_2;\pi_1) = \int_{\Pi} \frac{u'_{N+k_2}}{p_{2k_2}/(q_2+1)} \, d\psi(\pi_2;\pi_1),$$

for all (k_1, k_2), where u' is evaluated at $(x_1, x_2(a_1, \pi_2))$. This condition requires the equalization of the expected ratios of marginal utility to corresponding price; the "price" is that of the consumption good relative to the value of a bond in the same period.

In case (iii), the change in expected utility as a consequence of a speculative sale of a quantity λ of bonds (implying a change in the level of consumption of k_2 in period 2 equal to $\frac{\lambda}{p_{2k_2}}$ times the difference of q_1 and (q_2+1), is

$$\Delta v_3 \; (k_2) = \int_{\Pi} \lambda \, \frac{q_1-(q_2+1)}{p_{2k_2}} \, u'_{N+k_2} \, d\psi \; (\pi_2;\pi_1) \quad ,$$

where u' is evaluated at $(x_1, x_2(a_1, \pi_2))$. (Note again that $\frac{u'_{N+k_2}}{p_{2k_2}}$ is equal for all k_2). So a necessary condition for optimality is that $\Delta v_3 \leq 0$; i.e. for any k_2,

$$\int \frac{q_1 - (q_2 + 1)}{p_2 \, k_2} \; u'_{N+k_2} \; d\psi \; (\pi_2; \pi_1) \leq 0 \tag{3.3}$$

In summary, conditions (3.1) - (3.3) must be satisfied by a consumption-portfolio choice (x_1, b_1, m_1) if it is to be optimal, given the individual's expectations about the future.

4. Temporary Monetary Equilibrium and its Implications

In this section the main concern will be the implications of the additional restrictions which have to be imposed on the behaviour of the individual agents in the economy in order to ensure the mutual consistency of their actions in the current period (t=1), i.e. to ensure the existence of a temporary equilibrium. In the present context, a temporary equilibrium is a system of absolute (money) prices of consumer goods and bonds, and corresponding actions for each individual, such that these actions are individually optimal and all markets are cleared. It will be required further that the equilibrium prices of consumer goods be finite, so that money has positive exchange value, i.e. that it be a temporary monetary equilibrium. The requirement that the equilibrium price of bonds be finite, i.e. that the equilibrium interest rate be positive, will also be examined.

Given the assumptions made in section 2, which continue to hold throughout, and the additional assumptions specified in each case, the following preliminary results can be straightforwardly established. (These, and other results below of a more standard nature, are formally stated and proved in the Appendix, as indicated by the parenthetical references.)[5/] (Lemma A.1). For any current price system π_1 in Π, the set $A(\pi_1)$ of feasible choices in period 1 is convex; it is compact if and only if

(4a) for every $\pi_1 = (p_1, q_1) \in \Pi$, there exists $\bar{\pi}_2 = (\bar{p}_2, \bar{q}_2) \in \text{supp}\, \psi(\pi_1)$ such that $\bar{q}_2 + 1 > q_1$;

i.e., if and only if $\Psi(B,\pi_1) > 0$, where $B = \{\pi_2 \in \Pi | q_2 + 1 > q_1\}$. Condition (4a), requiring a positive probability of a positive net yield on bonds, will be used to rule out unbounded borrowing (speculative selling of bonds). It corresponds in part to condition 2.6(ii) in Green (1973) which (if interpreted in the present context) would also require a positive probability of a negative return on bonds, a requirement which is not needed here because of money's other role as the medium of exchange.

The correspondence A(.) is continuous on Π (Lemma A.2) if the correspondence supp $\Psi(.)$ is continuous on Π. Since the weak continuity of Ψ, already assumed, implies the upper hemicontinuity of supp $\Psi(.)$ (see Remark 3.1 in Green (1973)) it is necessary to assume only that

(4b) the correspondence supp $\psi(.)$ is lower hemicontinuous on Π.

Given the assumption (2a) on preferences, and the above results concerning A(.), the individual's demand correspondence, $\xi(.)$, is nonempty-, convex-, compact- valued, and upper hemicontinuous on Π, and $\tilde{\pi}_1 \cdot \xi(\pi_1) = \tilde{\pi}_1 \cdot w_1$, for all π_1 in Π, where $\tilde{\pi}_1 = (\pi_1, 1)$ and $w_1 = (\omega_1, b_0, m_0)$ is the initial endowment (Lemma A.3).

The proof of existence of a temporary monetary equilibrium uses a fixed-point argument and the main requirement is that excess demand vectors "point inward" at the boundary of the set of admissible prices. (See Grandmont (1975)). In the framework here, the "boundary" corresponds to price vectors which either havec one or more components zero or have one or more components arbitrarily large. So it must be shown that (i)

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the excess demands for consumer goods and bonds become arbitrarily large as their respective prices approach zero; and (ii), the (excess) demand for money becomes arbitrarily large when its "price" approaches zero, i.e. when the prices of goods (or, possibly, bonds) become arbitrarily high. In other words, the system must be such as to generate counter-forces when prices approach the boundary : upward pressure (resulting, directly or indirectly, from excess demand) on prices approaching zero; downward pressure (resulting from excess supply) on prices approaching infinity.

It will be assumed that price expectations are bounded uniformly away from zero, i.e.,

(4c) there exists $\sigma >> 0$ such that, for all π_1 in Π,

$$\text{supp}\,\psi(\pi_1) \geqq \sigma$$

With assumption (4c), the unboundedness of excess demands when prices go to zero (Lemma A.4) is straightforward to establish, given monotonicity of preferences and, in the case of bonds, the certainty of a positive return. Attention will be focussed now on the situations in which prices become arbitrarily high.

If the prices of goods or bonds become arbitrarily high then, in the limit, if the appropriately specified optimality conditions of section 3 are not satisfied, the individual will want to take an action such as would generate the necessary counter-pressure on prices.

This is the essence of the argument applied to obtain the next results; the sufficiency conditions are derived from the optimality conditions, (3.1) to (3.3). Also, the constraint on money expenditure has two important implications. First, money expenditure on bonds is bounded above and so, if the bond price is arbitrarily high, nobody can buy a positive quantity of bonds. Consequently, there will be an aggregate excess supply of bonds if anyone desires to sell bonds in this situation. Second, the proceeds of any sales of consumer goods will constitute part of the individual's demand for money. Consequently, if some individual desires to sell a positive quantity of a good whose price is arbitrarily high, the demand for money will be arbitrarily large.

(4d) for any sequence $< \pi_1^n >$ in Π with $p_{1k}^n \to +\infty$ for all k, there exists a commodity k such that

either (i) $$\frac{u'_k(\omega)}{u'_{N+k}(\omega)} < \lim_{n \to \infty} \int_\Pi \frac{p_{1k}^n}{p_{2k}} \, d\psi(\pi_2; \pi_1^n),$$

or (ii) $$\frac{u'_k(\omega)}{u'_{N+k}(\omega)} > \lim_{n \to \infty} \int_\Pi \frac{p_{1k}^n}{p_{2k}} \frac{q_2+1}{q_1^n} \, d\psi(\pi_2; \pi_1^n)$$

Theorem 4.1 :

Let $\langle \pi_1^n \rangle$ be any sequence in Π with $|| p_1^n || \to + \infty$,

and $\langle a_1^n \rangle$ any sequence with $a_1^n \in \xi (\pi_1^n)$ for all n.

If (4d) is satisfied, then either $m_1^n \to + \infty$ or

$| q_1^n b_1^n | \to \infty$. 6/

Proof :

[It is necessary to consider only the "worst" possible situation, where $p_{1k}^n \to + \infty$ for all k and $p_{2k}^n \to \infty$ for all k, for all $\pi_2^n \in \text{supp}\ \psi(\pi_1^n)$].

Suppose that the proposition is false. Then $\exists$ sub-sequences $\langle x_1^n \rangle$, $\langle b_1^n \rangle$, $\langle m_1^n \rangle$ such that $x_1^n \to x_1^o$, $|| x_1^o || < + \infty$; $b_1^n \to 0$, if $q_1^n \to + \infty$, or $b_1^n \to b_1^o < + \infty$, if $q_1^n \to q_1^o < + \infty$; and $m_1^n \to m_1^o < + \infty$. Thus, for large n, $m_1^n < m_1^o + 1$. Since $p_{1k}^n \to + \infty$ for all k, $x_1^o \leqq \omega_1$; this, together with $m_1^o < + \infty$, implies $x_1^o = \omega_1$. Also then, $\lim_{n\to\infty} x_2^n = \omega_2$, for any $x_2^n = x_2(a_1^n, \pi_2)$ with $\pi_2 \in \text{supp}\ \psi(\pi_1^n)$ for all n.

Suppose (i) of (4d) holds. Define $\bar{x}_1^n = x_1^n - \frac{1}{\bar{p}_{1k}^n} e_k$ 7/

$\bar{b}_1^n = b_1^n$, $\bar{m}_1^n = m_1^n + 1$, and $\bar{x}_2^n = x_2^n + \frac{1}{p_{2k}} e_k$

for all (x_2^n, π_2) such that $\pi_2 \in \text{supp } \psi(\bar{\pi}_1^n)$ and

$x_2^n = x_2(a_1^n, \pi_2)$. For large n, $(\bar{x}_1^n, \bar{x}_2^n)$ is feasible, and

$$v(\bar{a}_1^n, \pi_1^n) - v(a_1^n, \pi_1^n) = \int_\Pi \left[u(x_1^n - \frac{1}{p_{1k}^n} e_k, \; x_2^n + \frac{1}{p_{2k}} e_k) - u(x_1^n, x_2^n) \right] d\psi$$

$$\approx \int_\Pi \left[-\frac{1}{\bar{p}_{1k}^n} u'_k(\omega) + \frac{1}{p_{2k}} u'_{N+k}(\omega) \right] d\psi$$

> 0, by (i), contradicting $a_1^n \in \xi(\pi_1^n)$.

Similarly, suppose (ii) holds and define $\bar{x}_1^n = x_1^n + \frac{1}{p_{1k}^n} e_k$,

$\bar{m}_1^n = m_1^n$, $\bar{b}_1^n = b_1^n - \frac{1}{q_1^n}$, $\bar{x}_2^n = x_2^n - \frac{1}{q_1^n} \frac{q_2+1}{p_{2k}} e_k$.

For large n,

$$v(\bar{a}_1^n, \pi_1^n) - v(a_1^n, \pi_1^n) \approx \int_\Pi \left[\frac{1}{p_{1k}^n} u'_k(\omega) - \frac{q_2+1}{q_1^n p_{2k}} u'_{N+k}(\omega) \right] d\psi$$

> 0, by (ii), contradicting $a_1^n \in \xi(\pi_1^n)$.

q.e.d.

Theorem 4.1 shows that, under the alternative assumptions, $m_1 \to +\infty$ when $\| p_1 \| \to +\infty$ and, a fortiori, when $q_1 \to +\infty$ along with p_1. But the result is not sufficient to establish that $|q_1 b_1| \to +\infty$ (and hence $m_1 \to +\infty$) when $q_1 \to +\infty$ but p_1 converges. When p_1 converges it is no longer the case that ω is the reference point (in the limit) for the individual's evaluation of an adjustment in his intertemporal position. With finite commodity prices, the consumption stream can be adjusted without using an arbitrarily large amount of money. In order to guarantee that $|q_1 b_1| \to +\infty$ when $q_1 \to +\infty$, a further assumption is required.

(4e) Let $\langle \pi_1^n \rangle$ be any sequence in Π with $q_1^n \to +\infty$; then $\lim_{n \to +\infty} \psi(B^n; q_1^n) = 1$, where $B^n = \{\pi_2 \in \Pi \mid q_2 + 1 < q_1^n\}$;

i.e., as the current bond price gets arbitrarily high, high, the individual becomes subjectively certain that there will be a fall in the capital value of a bond in excess of the nominal interest payment. 8/ This is a fairly weak assumption. It should be noted that it does not require that bond-price (or, equivalently, interest-rate) espectations be inelastic.

Theorem 4.2 :

Let $\langle \pi_1^n \rangle$ be any sequence in Π with $q_1^n \to +\infty$, and $\langle a_1^n \rangle$ any sequence with $a_1^n \in \xi(\pi_1^n)$ for all n. If (4e) is satisfied, then $| q_1^n b_1^n | \to +\infty$.

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Proof :

Assume the proposition false. Then there exist subsequences $\langle x_1^n \rangle$, $\langle b_1^n \rangle$ and $\langle m_1^n \rangle$ such that $x_1^n \to x_1^o$, $b_1^n \to 0$, $m_1^n \to m_1^o$.

Define $\bar{x}_1^n = x_1^n$, $\bar{b}_1^n = b_1^n - \dfrac{1}{q_1^n}$ and $\bar{m}_1^n = m_1^n + 1$.

Then the effective initial cash balance in period 2, corresponding to $\bar{m}_1^n$, if π_2 occurs, is $\bar{m}_1^n - \dfrac{q_2+1}{q_1^n} = m_1^n + \dfrac{1}{q_1^n}\left[q_1^n - (q_2+1)\right]$.

For (x_2^n, π_2) such that $\pi_2 \in \operatorname{supp} \psi(\pi_1^n)$ and $x_2^n = x_2(a_1^n, \pi_2)$, define $\bar{x}_2^n + \dfrac{q_1^n - (q_2+1)}{q_1^n\, p_{2k}}\, e_k$, for some k.

For large n, $(\bar{x}_1^n, \bar{x}_2^n)$ is feasible, and

$$v(\bar{a}_1^n, \pi_1^n) - v(a_1^n, \pi_1^n) \cong \int_{\Pi} \frac{q_1^n - (q_2+1)}{q_1^n\, p_{2k}}\, u'_{N+k}\, d\psi$$

where u' is evaluated at $(x_1^o, x_2(a_1^o, \pi_2))$. So for n sufficiently large, $v(\bar{a}_1^n, \pi_1^n) > v(a_1^n, \pi_1^n)$ in view of (4e), contradicting $a_1^n \in \xi(\Pi_1^n)$.

q.e.d.

When all traders in the economy behave as the representative individual of the preceding analysis, the respective results on individual choice carry over to the aggregate situation (Lemmas A.5 and A.6). A standard procedure, adapted to the present context (Lemma A.7 and Theorem A.8), then establishes the existence of a temporary monetary equilibrium, i.e. a temporary equilibrium in which the consumption goods prices are finite and thus the exchange value of money positive. As in Green (1973), even though the method here is somewhat different, it is found necessary to assume the existence of a set of future prices which all agents always consider possible : 9/

(4f) there exists a $C \subset \Pi$ such that

(i) int C is nonempty ;

(ii) $\forall\ i \in I$ and $\forall\ \pi_1 \in \Pi$, $C \subset \text{supp}\ \psi_i(\pi_1)$;

The set C might be regarded as a region of prices which are considered "normal", based upon past observations.

If (4e) is satisfied, the equilibrium price of bonds also is finite (Theorem A.9), i.e. the equilibrium interest rate is positive. Unless (4e) is satisfied, it is possible that $q_1 \to +\infty$ without $|q_1 b_1| \to +\infty$; the temporary equilibrium in this case is one in which there is no trading on the bond market. It can be seen from Theorem 4.2 that (4e), together with (2.5) and (4a), implies a "liquidity trap" at a zero rate of interest, i.e. the demand for money becomes arbitrarily large as the interest rate goes to zero. Assumption (4e) does not require that interest-rate expectations be inelastic - it can be satisfied with unit-elastic

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expectations. [10/] It would have been possible to strengthen (4e) to require that

$$pr\ (q_2 + 1 < q_1) \to 1 \quad \text{as } q_1 \to \bar{q}_1 < +\infty \ ;$$

i.e. subjective certainty of a speculative gain by selling bonds when the current price is at least $\bar{q}_1$. Then, a fortiori, a temporary monetary equilibrium exists with a positive interest rate ; and a liquidity trap exists at a positive rate of interest : the demand for money becomes arbitrarily large as $q_1 \to \bar{q}_1$ (as the interest rate $\to \frac{1}{\bar{q}} > 0$). But Theorem A.9 suggests that it is the existence of a speculative demand for money, rather than either inelastic expectations or a proper liquidity trap per se, which is fundamentally responsible for a positive (equilibrium) rate of interest.

In order to guarantee a positive exchange value for money in equilibrium, it was sufficient to assume (4d). Condition (i) of (4d) is a sufficient condition for the same result when no borrowing or lending is permitted (see Hool (1975)) ; it requires only that price expectations be no more than unit-elastic. The addition of alternative (ii) of (4d) indicates that introduction of the possibility of borrowing does not diminish the likelihood that money will have a positive exchange value. In either situation, the positive value of money is due in an essential way to its function as the medium of exchange.

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APPENDIX

It is assumed throughout that $b_0 = 0$ (see footnote 5)
Recall the constraints on the choice of an action $a_1 = (x_1, b_1, m_1)$ in period 1, when the price system is $\pi_1 = (p_1, q_1)$:

(2.1) $\quad p_1 \cdot (x_1 - \omega_1)^+ + q_1 b_1 \leqq m_0$

(2.2) $\quad p_1 \cdot x_1 + q_1 b_1 + m_1 = p_1 \cdot \omega_1 + m_0$

(2.5) $\quad (q_2 + 1)\, b_1 + m_1 \geqq 0 \,, \ \forall\, \pi_2 \varepsilon \text{ supp } \psi(\pi_1)$

The feasible choice set in period 1, corresponding to π_1, is

$$A(\pi_1) = \{a_1 = (x_1, m_1, b_1) \mid (2.1)\,,\ (2.2) \text{ and } (2.5)\}$$

(4a) $\forall\, \pi_1 \ \varepsilon\ \Pi,\ \exists\, \bar{\pi}_2 \ \varepsilon \text{ supp } \psi(\pi_1)$ such that $\bar{q}_2 + 1 > q_1$.

Lemma A.1: $\quad \forall\, \pi_1 \varepsilon\, \Pi$, $A(\pi_1)$ is non-empty and convex ;

$A(\pi_1)$ is compact if and only if (4a).

Proof: $\quad \forall\, \pi_1 \ \varepsilon\ \Pi,\ (\omega_1, 0, m_0)\ \varepsilon\ A(\pi_1)$, so $A(\pi_1)$ is non-empty.
It is clear from the definition of $A(\pi_1)$ that it is convex and closed. $A(\pi_1)$ is bounded if and only if (4a): $(b_1)^+$ is bounded above by $\dfrac{m_0}{q_1}$;
$m_1 \leqq p_1 \cdot \omega_1 + m_0 + q_1 (b_1)^-$ and $\therefore$, $\forall\, \pi_2\ \varepsilon$ supp $\psi(\pi_1)$,
$(q_2 + 1)\,(b_1)^- \leqq p_1 \cdot \omega_1 + m_0 + q_1 (b_1)^-$, i.e. , $(q_2 + 1 - q_1)\,(b_1)^- \leqq p_1 \cdot \omega_1 + m_0$. If (4a) holds, $\exists\, \bar{q}_2$ such that $\bar{q}_2 + 1 > q_1$ and

$$\therefore \quad (b_1)^- \leqq \frac{1}{\bar{q}_2 + 1 - q_1}\,(p_1 \cdot \omega_1 + m_0),$$

Thus $\frac{-1}{\bar{q}_2 + 1 - q_1} (p_1 \cdot \omega_1 + m_0) \leqq b_1 \leqq \frac{1}{\bar{q}_1} m_0 \cdot x_1$ is bounded below by 0 and, since b_1 is bounded, x_1 is bounded above because of (2.1). m_1 is bounded below by 0 and, since x_1 and b_1 are bounded, m_1 is bounded above because of (2.2). q.e.d.

<u>Lemma A.2:</u> If supp ψ (.) is continuous on Π, then A(.) is continuous on Π.

<u>Proof:</u> (i) A(.) is upper hemicontinuous on Π: let $\langle\pi_1^n\rangle \to \pi_1^o$, and $\langle a_1^n\rangle \to a_1^o$ with $a_1^n \in A(\pi_1^n)$ for all n. Then a_1^o clearly satisfies (2.1) and (2.2). Since (b_1^n, m_1^n) satisfies (2.5) for all n, $(q_2 + 1)b_1^o + m_1^o \geqq 0$, $\forall\ \pi_2 \in$ supp ψ (π_1^o), follows from the continuity of supp $\psi(.)$. So A(.) has a closed graph. Any such sequence $\langle a_1^n\rangle$ is bounded, therefore A(.) is u.h.c.

(ii) A(.) is lower hemicontinuous on Π: take any sequence $\langle\pi_1^n\rangle \to \pi_1^o$ and $a_1^o \in A(\pi_1^o)$; it is necessary to find a sequence $\langle a_1^n\rangle$ such that $a_1^n \to a_1^o$ and $a_1^n \in A(\pi_1^n)$ for all n sufficiently large. (For earlier n, it is possible to take $a_1^n = (\omega_1, 0, m_0)$.). Define $a_1^n = (x_1^n, b_1^n, m_1^n)$ by $x_1^n = (1 - \frac{1}{n})\ x_1^o$;

$b_1^n = (1 - \frac{1}{n})\ b_1^o$, if $b_1^o \geqq 0$, and $= b_1^o$, if $b_1^o < 0$;

$m_1^n = m_o + p_1^n \cdot (\omega_1 - x_1^n) - q_1^n b_1^n$. Then $a_1^n \to a_1^o = (x_1^o, b_1^o, m_1^o)$

as $n \to \infty$ $p_1^o \cdot (x_1^o - \omega_1)^+ + q_1^o b_1^o \leqq m_0$, so

$p_1^o \cdot (x_1^n - \omega_1)^+ + q_1^o b_1^n < m_0$, and $\therefore$ $p_1^n \cdot (x_1^n - \omega_1)^+ + q_1^n b_1^n \leqq m_0$

for n sufficiently large. $p_1^n \cdot x_1^n + q_1^n b_1^n + m_1^n = p_1^n \cdot \omega_1 + m_0$,

from the definition of m_1^n. Clearly, $(q_2 + 1)\ b_1^n + m_1^n \geqq 0$,

$v(a_1^o \; \pi_1^o) \geq v(a_1, \pi_1^o)$; ie, $a_1^o \; \varepsilon \xi(\pi_1^o)$. That ξ (.) is u.h.c. follows from the fact that $<\tilde{a}_1^n>$ is bounded, since A(.) is compact-valued and uhc on Π.

(ii) follows directly from the budget equation, (2.2) q.e.d.

(4c) $\exists \; \sigma >> 0$ such that, for all $\pi_1 \; \varepsilon \; \Pi$, supp $\psi \; (\pi_1) \geq \sigma$

<u>Lemma A.4:</u> Assume (4c). Let $<\pi_1^n>$ be any sequence in Π, and $<a_1^n>$ any sequence with $a_1^n \; \varepsilon \; \xi \; (\pi_1^n)$ for all n. If $\pi_1^n \rightarrow \pi_1^o \; \varepsilon \; \bar{\Pi} \setminus \Pi$ then $||a_1^n|| \rightarrow + \infty$.

<u>Proof:</u> (cf. Grandmont [1974].) We can assume without loss of generality that there exists some compact subset of Π which contains supp $\psi \; (\pi_1^n)$ for all n, and therefore that $\psi \; (\pi_1^n)$ converges weakly to some ψ^o in $\mathcal{M}(\Pi)$. Then we can define $v^o(a_1) = \int_\Pi u[x_1, x_2(a_1, \pi_2)] \; d\psi^o(\pi_2)$; lim $v(a_1, \pi_1^n) = v^o(a_1)$, and $v^o(.)$ is continuous and strictly increasing. We consider two cases : (i) $p_{k1}^o = 0$ for some k ; in this case we can apply the reasoning of Grandmont [1974], Proposition 2(i) of Section 3.8; (ii) $q_1^o = 0$; assume that the sequence $<a_1^n>$ converges to $a_1^o = (x_1^o, b_1^o, m_1^o)$. with $b_1^o < + \infty$. a_1^o is feasible for π_1^o , and $v^o(a_1^o) \geq v^o(a_1)$ for all a_1 feasible for π_1^o . But $\bar{a}_1 = (x_1^o, \bar{b}_1, m_1^o)$, with $\bar{b}_1$ arbitrarily large, is also feasible for π_1^o . Since $v^o(.)$ is strictly increasing, $v^o(\bar{a}_1) > v^o(a_1^o)$. a contradiction.

q.e.d.

$\forall\ \pi_2 \in \text{supp}\ \psi\ (\pi_1^n)$, if $b_1^o \geq 0$. Suppose $b_1^o < 0$; then

$m_1^o = m_0 + p_1^o \cdot (\omega_1 - x_1^o) + q_1^o\ (b_1^o)^- < m_o + p_1^o \cdot (\omega_1 - x_1^n) + q_1^o (b_1^n)^-$,

since $x_1^n << x_1^o$ and $b_1^n = b_1^o$, for all n. $\therefore\ m_1^o < m_1^n =$

$m_0 + p_1^n \cdot (\omega_1 - x_1^n) + q_1^n\ (b_1^n)^-$, for n sufficiently large.

Since $a_1^o \in A(\pi_1^o)$, $m_1^o + (q_2 + 1)b_1^o \geq 0, \forall\ \pi_2 \in$ supp $\psi(\pi_1^o)$,

$\therefore\ m_1^n - (q_2 + 1)(b_1^n)^- > 0, \forall\ \pi_2 \in$ supp $\psi(\pi_1^o)$. For n sufficiently large, $m_1^n - (q_2 + 1)(b_1^n)^- \geq 0, \forall\ \pi_2 \in$ supp $\psi(\pi_1^n)$, follows from the continuity of supp ψ (.). Thus $a_1^n \in A(\pi_1^n)$ for n sufficiently large. q.e.d

The individual's demand correspondence is defined by

$$\xi(\pi_1) = \{a_1 \in A(\pi_1) \mid v(a_1, \pi_1) \geq v(a_1', \pi_1)\ \forall\ a_1' \in A(\pi_1)\}.$$

<u>Lemma A.3</u>: (i) ξ (.) is nonempty - , convex - , compact-valued and upper hemicontinuous on Π :

(ii) $\tilde{\pi}_1 \cdot \xi\ (\pi_1) = \tilde{\pi}_1 \cdot w_1$, $\forall\ \pi_1 \in \Pi$.

<u>Proof:</u> (i) That ξ (.) is nonempty - , convex - , compact-valued follows from the fact that A (.) has these properties and v is continuous in both arguments, and concave and monotone in the first. ξ (.) has a closed graph: take any sequence $<\pi_1^n>$ in Π, with $\pi_1^n \to \pi_1^o$, and any sequence $<\tilde{a}_1^n>$ such that $\tilde{a}_1^n \in \xi(\pi_1^n)$, for all n, and $\tilde{a}_1^n \to a_1^o$. Since the graph of A(.) is closed, $a_1^o \in A(\pi_1^o)$. Take any $a_1 \in A(\pi_1^o)$; there exists a sequence $<a_1^n>$ such that $a_1^n \in A(\pi_1^n)$ for all n, and $a_1^n \to a_1^o$. $v\ (\tilde{a}_1^n, \pi_1^n) \geq v(a_1^n, \pi_1^n)$, for all n.
So, from the continuity of v in both arguments,

Let $\mathcal{I} = \{1, \ldots, i, \ldots, I\}$ be the index set of the individuals in the economy. Characteristics of the ith individual will be distinguished by the subscript i, and aggregations will be over the index set $\mathcal{I}$. The aggregate excess demand correspondence, $\zeta : \Pi \to R^{N+2}$ is defined by $\zeta(\pi_1) = \sum \xi_i(\pi_1) - \sum w_{i1} = \sum \xi_i(\pi_1) - (\sum w_{i1}, 0, M)$, where M is the money supply (constant over time). An element of $\zeta(\pi_1)$ is $z_1 = (x_1 - \omega_1, b_1, m_1 - M)$, where, $x_1 = \sum x_{i1}$, $\omega_1 = \sum \omega_{i1}$, $b_1 = \sum b_{i1}$, , and $m_1 = \sum m_{i1}$. A temporary monetary equilibrium in period 1 is an (I + 1) - tuple $(\tilde{\pi}_1^*, a_{11}^*, \ldots, a_{I1}^*)$ of points in R^{N+2} such that $\tilde{\pi}_1^* = (\pi_1^*, 1)$ with $\pi_1^* \in \Pi$, $a_{i1}^* \in \xi_i(\pi_1^*)$ for all $i \in \mathcal{I}$, and $\sum a_{i1}^* = \sum w_{i1}$.

<u>Lemma A.5:</u> (i) ζ is nonempty -, convex -, compact-valued and upper hemicontinuous on Π ;

(ii) $\tilde{\pi}_1 \cdot \zeta(\pi_1) = 0$.

<u>Proof:</u> This follows directly from Lemma A.3.

<u>Lemma A.6:</u> Let $\langle\pi_1^n\rangle$ be any sequence in Π;

(i) if $\pi_1^n \to \pi_1^o \in \bar{\Pi} \setminus \Pi$, then $|| \zeta(\pi_1^n) || \to +\infty$;

(ii) if $||p_1^n|| \to +\infty$, then either $|| \zeta(\pi_1^n) || \to +\infty$ or $|q_1^n b_1^n| \to +\infty$.

<u>Proof</u> : This follows from Lemma A.4 and Theorem 4.1.

(4e) $\exists$ $C \subset \Pi$ such that (i) int C nonempty, and (ii) $\forall i \in \mathcal{I}$ and $\forall \pi_1 \in \Pi$, $C \subset \text{supp } \psi_i(\pi_1)$.

Lemma A.7: Let $\langle \Pi^n \rangle$ be a non-decreasing sequence of compact, convex subsets of Π such that $\Pi \subset \bigcup_{n=1}^{\infty} \Pi^n$ and each Π^n has a nonempty interior. Let $\langle \pi_1^n \rangle$ and $\langle z_1^n \rangle$ be sequences such that, for each n, $\pi_1^n \in \Pi^n$, $z_1^n \in \zeta(\pi_1^n)$, $\pi_1^n \cdot z_1^n = 0$, and $\tilde{\pi}_1 \cdot z_1^n \leqq 0$ for all $\pi_1 \in \Pi^n$.

Then (i) $\langle z_1^n \rangle$ is bounded; (ii) if $q_1^n \to +\infty$, $|q_1^n \ b_1^n|$ is bounded.

Proof: (i) (a) . $\langle b_1^n \rangle$ is bounded. Choose $\bar{\pi}_2 = (\bar{p}_2, \bar{q}_2) \in \text{int } C$; then

(1) $(\bar{q}_2 + 1) b_1^n + m_1^n \geqq 0, \forall n$. Choose $\bar{\pi}_1 = (\bar{p}_1, \bar{q}_1)$ such that $\bar{\pi}_1 \in \Pi^n$, for n large, and $\bar{q}_1 > \bar{q}_2 + 1$; then, since $\tilde{\pi}_1 \cdot z_1^n \leqq 0$, for all $\pi_1 \in \Pi^n$, $\bar{p}_1 \cdot x_1^n + \bar{q}_1 \ b_1^n + m_1^n \leqq \bar{p}_1 \cdot \omega_1 + M$, and therefore

(2) $\bar{q}_1 \ b_1^n + m_1^n \leqq \bar{p}_1 \cdot \omega_1 + M$, for n large. Similarly, choose $\bar{\bar{\pi}}_1$ such that $\bar{\bar{\pi}}_1 \in \Pi^n$ for large n, and $\bar{\bar{q}}_1 < \bar{q}_2 + 1$; then

(3) $\bar{\bar{q}}_1 \ b_1^n + m_1^n \leqq \bar{\bar{p}}_1 \cdot \omega_1 + M$, for large n. (2) - (1) implies $[\bar{q}_1 - (\bar{q}_2 + 1)] \ b_1^n \leqq \bar{p}_1 \cdot \omega_1 + M$; (1) - (3) implies $[(\bar{q}_2 + 1) - \bar{\bar{q}}_1] \ b_1^n \geqq -(\bar{\bar{p}}_1 \cdot \omega_1 + M)$. Thus $-\underline{b}_1 \leqq b_1^n \leqq \bar{b}_1$, where

$$\underline{b}_1 = \frac{1}{\bar{q}_2 + 1 - \bar{\bar{q}}_1} (\bar{\bar{p}}_1 \cdot \omega_1 + M), \text{ and}$$

$$\bar{b}_1 = \frac{1}{\bar{q}_1 - (\bar{q}_2 + 1)} (\bar{p}_1 \cdot \omega_1 + M);$$

i.e., b_1^n is bounded.

(b) $\langle m_1^n \rangle$ is bounded. Clearly, $m_1^n \geqq 0$. Take $\pi_1 \in \Pi^1$; then $m_1^n \leqq p_1 \cdot \omega_1 + M - q_1 \ b_1^n \leqq p_1 \cdot \omega_1 + M + q_1 \ \underline{b}_1 = \bar{m}_1$, say . So m_1^n is bounded.

(c) $\langle x_1^n \rangle$ is bounded. Clearly $x_1^n \geqq 0$. Again, take $\pi_1 \in \Pi^1$; then $p_1 \cdot x_1^n \leqq p_1 \cdot \omega_1 + M + q_1 \ \underline{b}_1$. Since $p_1 >> 0$, x_1^n is bounded.

(ii) If $q_1^n \to +\infty$, then $|q_1^n \; b_1^n|$ is bounded; i.e. $b_1^n \to 0$. Constraint (2.5) and assumption (4a) together imply that, for all i and all n, $q_1^n \, b_{i1}^n + m_{i1}^n \geqq 0$. Therefore $-q_1^n \, b_1^n \leqq m_1^n$, for all n. Since m_1^n is bounded above, $q_1^n \, b_1^n$ must be bounded below. From aggregation of constraints (2.1), $q_1^n \, b_1^n \leqq M$. So it must be the case that $b_1^n \to 0$ if $q_1^n \to +\infty$.

q.e.d.

The economy $\mathcal{E}$ consists of the set $\mathcal{I}$ of traders satisfying all the assumptions (2a) through (4f), with the exception of (4e). The economy $\mathcal{E}'$ is the same as $\mathcal{E}$ except that the traders satisfy (4e) as well.

Theorem A.8: $\mathcal{E}$ has a temporary monetary equilibrium in period 1.

Proof: The proof is the same as that given below for Theorem A.9, with the following modification: by Lemma A.6(ii), the sequence $\langle p_1^n \rangle$ (but not necessarily the sequence $\langle \pi_1^n \rangle = \langle (p_1^n, q_1^n) \rangle$) must be bounded, and therefore $\exists$ a subsequence converging to $p_1^* \gg 0$. There is a corresponding subsequence of $\langle \pi_1^n \rangle$ converging to $\pi_1^* = (p_1^*, q_1^*)$ (with q_1^* possibly infinite) and a subsequence of $\langle z_1^n \rangle$ converging to π_1^* (with $b_1^* = 0$ if q_1^* is infinite).

Theorem A.9: $\mathcal{E}'$ has a temporary monetary equilibrium in period 1, with a positive interest rate $(r_1^* = \frac{1}{q_1^*} > 0)$.

Proof: Let $\langle \Pi^n \rangle$ be a sequence of subsets of Π as in the statement of Lemma A.7. For each n, take Z^n to be a compact, convex set containing the range of ζ^n. Then, using the result of Debreu [1956], $\exists$, $\forall n$, a pair (π_1^n, z_1^n) such that $z_1^n \in \zeta^n(\pi_1^n)$, $\tilde{\pi}_1^n \cdot z_1^n = 0$, and $\tilde{\pi}_1 \cdot z_1^n \leq 0 \; \forall \pi_1 \in \Pi^n$. By Lemma A.7, the sequence $\langle z_1^n \rangle$ is bounded and $b_1^n \to 0$ if $q_1^n \to +\infty$. The sequence $\langle \pi_1^n \rangle$ must therefore be bounded, otherwise Lemma A.6(ii) would be contradicted. So $\exists$ a convergent subsequence of $\langle \pi_1^n \rangle$ converging to π_1^*. Since the corresponding subsequence of $\langle z_1^n \rangle$ is bounded, Lemma A.6(i) implies that $\pi_1^* \notin \bar{\Pi} \setminus \Pi$. Therefore $\pi_1^* \in \Pi$. Since ζ has closed graph and $\langle z_1^n \rangle$ is bounded, $\exists$ a subsequence of $\langle z_1^n \rangle$ converging to $z_1^* \in \zeta(\pi_1^*)$. $\tilde{\pi}_1^* \cdot z_1^* = 0$ since $\tilde{\pi}_1^n \cdot z_1^n = 0 \; \forall n$; $\tilde{\pi}_1^* \cdot z_1^* = 0$ prohibits both $z_1^* > 0$ and $z_1^* < 0$. So if $z_1^* \neq 0$ and $\tilde{\pi}_1^* \cdot z_1^* = 0$, $\exists \, \pi_1 \in \Pi$ such that $\tilde{\pi}_1 \cdot z_1^* > 0$ and hence, for n sufficiently large, $\tilde{\pi}_1 \cdot z_1^n > 0$. But this contradicts $\tilde{\pi}_1 \cdot z_1^n \leq 0 \; \forall \pi_1 \in \Pi^n$. Therefore $z_1^* = 0$.

q.e.d.

Notes

1. Bonds are issued by individuals, but are considered to be homogeneous, no account being taken of differences in default risk, etc.

2. That money is the medium of exchange is taken as an institutional fact.

3. But, in another sense (cf. Hicks [1974] , for example), they have imperfect liquidity because of the uncertainty of their future market value.

4. These are the constraints used by the individual for <u>planning</u> purposes. It is not implied that the economy ends, or that the individual expires, at the end of period t+1.

5. If an individual is in debt at the start of the current period, there is the possibility that he will be bankrupt, in the sense that there is no action which he could take in the current period that would enable him to pay off his (expected) debt within his planning horizon. In order to avoid the technical complications of the equilibrium existence proof in this situation, it is assumed that all individuals start out with zero bond-holdings. This simplification affects only the bankruptcy possibility; in all other respects, the results would be unaffected by the existence of initial credit and debt.

6. Given the positive probability of a positive net return on bonds (assumption (4a)) and the solvency requirement ((2.5)), it follows that $m_1^n \to +\infty$ whenever $|q_1^n b_1^n| \to +\infty$ (i.e., when bonds are sold at an arbitrarily high price). So the phrase "or $|q_1^n \; b_1^n| \to +\infty$" in the statement of Theorem 4.1 is superfluous in the present context. However, the result as stated is more basic (cf. also Lemmas A.6 and A.7).

7. e_k denotes the unit vector with 1 in the k-th position and 0 elsewhere.

8. A violation of the optimality condition (3.3) would lead to a condition of the form

$$\exists k \text{ such that } \lim_{||\pi_1^n|| \to \infty} \int_{\Pi} \frac{q_1^n - (q_2 + 1)}{p_{2k}} u'_{N+k} \, d\psi \, (\pi_2; \pi_1^n) > 0.$$

Such a condition could have been used as a further alternative to (i) and (ii) of (4d) for the proof of Theorem 4.1. However it is not a meaningful assumption when p_1 and p_2 are bounded. Condition (4e) is actually a strengthening of this condition which removes the dependence on p_2.

9. Cf. Green (1973), Assumption (4.2). The assumption of "common expectations" used here is somewhat stronger than Green's since it refers to absolute, rather than relative, prices.

10. The term "elasticity", being rather loosely applied here to probabilistic expectations, refers to the average.

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