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CONTINGENT PRICES FOR THE PROPERTY RIGHTS
OF NATURAL RESOURCES

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.

I.

There has recently been increased interest in the economics of the development of mineral and other natural resources. This has been stimulated partly by large changes in the traded prices of minerals, other raw materials and land, and partly by the wish to find a 'correct' inter-temporal pricing policy for those commodities which are exhaustible.⁽¹⁾ As well as the obvious international redistribution of income which has arisen from the formation of cartels to promote higher prices, the profit potential from both developed and undeveloped natural resources has increased implying different incomes, actual or potential, to either or both of the original owner of the property rights to the resource and the resource developer who has or will acquire these rights. The original owner and the resource developer will often be different economic agents. To develop natural resources, whether they are known resources or simply a possibility awaiting exploration, generally will require the services of a specialist developer: rarely can it be achieved by the original owner of the rights to the resource acting alone.

The distribution of any profit between the two agents may be contingent upon the outcome of the development according to a contract. Thus the contract would distribute risk in addition to expectations of incomes. It is useful to distinguish between pure ex ante contracts and contracts which involve ex post adjustments of terms. The former can be defined as one where the terms specify the distribution of profit under all contingencies and are binding on both parties. This can be contrasted with a contract where the terms can be adjusted as a result of the outcome of the development and thus ex post adjustments are involved. A contract consisting only of a profit sharing agreement with share α is a pure ex ante contract if α is binding on both parties to the contract. If one party, however, can and does force an ex post adjustment by changing

α in response to profitability and thus controls the other's profit ex post, then the contract is not purely ex ante. Examples of ex post adjustment include nationalisation and the 'windfall tax' currently being proposed in the U.S.A. Garnaut and Clunies Ross (p.274) state one of the difficulties of using ex post adjustments, even if one of the parties to the contract has the legal power to do so, as:

"... investors assess political risk and the stability of negotiated tax systems, by reference to ex post treatment of established investments ... (this) causes ex post adjustment in one project to affect ex ante expectations of the after-tax profitability of future projects. Ex post adjustments that are based on clearly articulated principles might, in principle, cause new investors to expect consistent treatment, but in reality the fact of ex post adjustments would probably contribute to some uncertainty about the stability of all systems ... and, given investors' aversion to risk, would probably raise the supply price of investment ..."

Garnaut and Clunies Ross go on to propose a Resource Rent Tax (RRT) as an appropriate contingent price in the context of a pure ex ante contract. This paper is also confined to considering contingent prices compatible with pure ex ante contracts. A 'price' will be considered as an amalgam of an immediate transfer of a given sum of money from one party to the other followed by a continuous stream of payments from the developer to the owner until time L when the property rights of the resource that remains revert to the owner. The two extreme cases are first when the developer just pays a lump sum to the owner, and second when the developer is paid a fixed fee and simply acts as the owner's agent, transferring all profits (positive or negative) to the owner. In the first case the owner's income is non-stochastic and in the second the developer's income is non-stochastic. A problem with the second case is that the developer has no incentive to adopt a policy which is Pareto

efficient in respect of the owner's and developer's utilities. A price that does provide this incentive when the uncertainties have been resolved is an ideal price and can be shown to have welfare advantages over other policies.⁽²⁾ Also the price should be such as to produce policy decisions taken in anticipation of stochastic events which are Pareto efficient in terms of expected utilities.

We will see in the next section that an 'optimal' price should have four properties:

- P1 The price should be ideal.
- P2 The price should produce decisions, which
 have to be taken by the developer in anticipation
 of stochastic events, that are Pareto efficient
 in terms of expected utilities.
- P3 The price should approximately optimally
 distribute risk.
- P4 A price satisfying P1 \rightarrow P3 can be constructed
 for any particular distribution (i.e. ratio)
 of expected utilities.

These properties also relate to such diverse economic phenomena as sharecropping (see Reid {7} and Stiglitz {9}), agent-principal relationships (Ross {8}), penalty systems (Ireland {4}) and incentives (Mirrlees {6}). An added dimension in the present case is that the contingent element of the price is a stream of contingent payments rather than a single payment. It is the time profile of such payments that is of particular interest. Should payments be made only after a reasonable return on capital has been earned by the developer as embodied in the RRT? Or should they be a

constant or varying proportion of (positive or negative) profit flow?

We are not concerned primarily with the distribution of expected income between the owner and the developer although the nature of the contingent prices available may effect the supply and demand for developers' services. P4 above, however, states that the price should be capable of satisfying any feasible distribution of expected utilities.

In Section II a model is constructed on the basis of a risk premium approach and Proposition I states a price satisfying P1 → P4 on the assumption that both the owner and the developer have the same discount rate. Section III extends the analysis to differentiated discount rates. The final section discusses the RRT in the light of the analysis and draws conclusions concerning the relative merits in a normative context of different policy prescriptions.

II

The model is constructed upon the following assumptions.

A1 The owner wishes to maximise his expected utility where his utility is $U_G(Y_G)$ and the developer wishes to maximise his expected utility where his utility is $U_D(Y_D)$
 U_G, U_D are concave monotonic increasing functions and Y_G, Y_D are the present values of the net receipts, discounted at rate ρ , of the owner and developer respectively from the transactions in the

contract and the development of the resource.

A2 Y_G, Y_D can be written:

$$Y_G = X_G + P \quad (1)$$

$$Y_D = \Pi - P \quad (2)$$

where
$$P = \int_0^L p(t) e^{-\rho t} dt + h_0 \quad (3)$$

$$\Pi = \int_0^L \pi(t) e^{-\rho t} dt \quad (4)$$

$$X_G = \int_0^L x(t) e^{-\rho t} dt \quad (5)$$

P is the discounted stream of payments to the owner where $p(t) = p(t, u, \theta)$ plus an initial transfer h_0 . Π is the discounted stream of net receipts from the project where $\pi(t) = \pi(t, u, \theta)$. X_G is the present value of money-equivalent receipts other than P (positive or negative) of the owner from the development. Examples may be tax receipts if the owner is the government and changes in the value of neighbouring land under the same ownership in the case of land development. Again $x(t) = x(t, u, \theta)$. u is a vector of decision variables under the control of the developer, and is partitioned into $u = [u_1; u_2]$. θ is a vector of random variables with $E(\theta) = 0$ and

$E(\theta \theta')$ = $[c_{ij}]$. The decisions involving choice of u_1 are taken to maximise $E[U_D]$ in anticipation of the random effects θ and with knowledge of $E(\theta)$ and $E(\theta \theta')$. The decisions involving choice of u_2 are taken to maximise U_D given θ .

A3 Risk premiums R_G, R_D can be found such that

$$E[U_G(Y_G)] \approx U_G(Y_G^0 - R_G) \quad (6)$$

$$E[U_D(Y_D)] \approx U_D(Y_D^0 - R_D) \quad (7)$$

where superscript 0 denotes evaluation at $\theta = \bar{\theta}$, where $\bar{\theta}$ is a vector of fixed values of θ .

R_G, R_D are found by expanding $U_G(Y_G)$ and $U_D(Y_D)$ around $\theta = \bar{\theta}$ and taking expectations.

The expansion of $U_G(Y_G)$ yields:

$$\begin{aligned} U_G \approx & U_G^0 + \left(\frac{dU_G}{dY_G} \right)^0 \sum_i \left\{ \left(\frac{\partial Y_G}{\partial \theta_i} \right)^0 (\theta_i - \bar{\theta}_i) \right\} \\ & + \frac{1}{2} \left(\frac{d^2 U_G}{dY_G^2} \right)^0 \sum_i \sum_j \left\{ \left(\frac{\partial Y_G}{\partial \theta_i} \right)^0 (\theta_i - \bar{\theta}_i) (\theta_j - \bar{\theta}_j) \left(\frac{\partial Y_G}{\partial \theta_j} \right)^0 \right\} \\ & + \frac{1}{2} \left(\frac{dU_G}{dY_G} \right)^0 \sum_i \sum_j \left\{ \left(\frac{\partial^2 Y_G}{\partial \theta_i \partial \theta_j} \right)^0 (\theta_i - \bar{\theta}_i) (\theta_j - \bar{\theta}_j) \right\} \end{aligned}$$

(8)

Taking expectations:

$$\begin{aligned}
 E[U_G] &\approx U_G^o - \left(\frac{dU_G}{dY_G} \right)^o \sum_i \left(\frac{\partial Y_G}{\partial \theta_i} \right)^o \bar{\theta}_i \\
 &+ \frac{1}{2} \left(\frac{d^2 U_G}{dY_G^2} \right)^o \sum_i \sum_j \left\{ \left(\frac{\partial Y_G}{\partial \theta_i} \right)^o (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \left(\frac{\partial Y_G}{\partial \theta_j} \right)^o \right\} \\
 &+ \frac{1}{2} \left(\frac{dU_G}{dY_G} \right)^o \sum_i \sum_j \left\{ \left(\frac{\partial^2 Y_G}{\partial \theta_i \partial \theta_j} \right)^o (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \right\}
 \end{aligned} \tag{9}$$

Also by expanding $U_G(Y_G^o - R_G)$ around $R_G = 0$ we have

$$U_G(Y_G^o - R_G) \approx U_G(Y_G^o) - \left(\frac{dU_G}{dY_G} \right)^o R_G \tag{10}$$

Substituting (9) and (10) into (6) we have:

$$\begin{aligned}
 R_G &\approx \sum_i \left(\frac{\partial Y_G}{\partial \theta_i} \right)^o \bar{\theta}_i + \frac{1}{2} A_G \sum_i \sum_j \\
 &\quad \left\{ \left(\frac{\partial Y_G}{\partial \theta_i} \right)^o (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \left(\frac{\partial Y_G}{\partial \theta_j} \right)^o \right\} \\
 &- \frac{1}{2} \sum_i \sum_j \left\{ \left(\frac{\partial^2 Y_G}{\partial \theta_i \partial \theta_j} \right)^o (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \right\}
 \end{aligned} \tag{11}$$

where $A_G = - \left(\frac{d^2 U_G}{dY_G^2} \right)^o / \left(\frac{dU_G}{dY_G} \right)^o$ is the (assumed constant) coefficient

of absolute risk aversion of the owner evaluated at $\theta = \bar{\theta}$. Thus

$$Y_G^o - R_G \approx E[Y_G] - \frac{1}{2} A_G \sum_i \sum_j \left\{ \left(\frac{\partial Y_G}{\partial \theta_i} \right)^o (c_{ij} + \bar{\theta}_i - \bar{\theta}_j) \left(\frac{\partial Y_G}{\partial \theta_j} \right)^o \right\} \quad (12)$$

Similarly

$$Y_D^o - R_D \approx E[Y_D] - \frac{1}{2} A_D \sum_i \sum_j \left\{ \left(\frac{\partial Y_D}{\partial \theta_i} \right)^o (c_{ij} + \bar{\theta}_i - \bar{\theta}_j) \left(\frac{\partial Y_D}{\partial \theta_j} \right)^o \right\} \quad (13)$$

where $A_D = - \left(\frac{d^2 U_D}{dY_D^2} \right)^o / \left(\frac{dU_D}{dY_D} \right)^o$ and again is assumed constant.

Thus maximising (12) is (nearly) equivalent to maximising $E[U_G]$ and maximising (13) is (nearly) equivalent to maximising $E[U_D]$. The substance of A3 is that (12) and (13) are sufficiently near equivalents for the following analysis to hold. We proceed by proving Proposition 1 and then by considering the policy implications of various forms of X_G .

Proposition 1

Provided the owner is risk averse ($A_G > 0$), a price consisting of an initial payment of h_0 and a subsequent stream of payments of the form $p^*(t) = \lambda \pi(t) - (1 - \lambda) x(t)$ where λ is constant and equal to $A_D / (A_D + A_G)$ has properties P1 \rightarrow P3 and a value of h_0 can be found to satisfy P4.

Proof of Proposition 1

First note that if changing $p(t)$ increases $Y_G^0 - R_G$ by less than it decreases $Y_D^0 - R_D$ then such a change is (Pareto) inferior to changing h_0 . Thus an optimal payments stream maximises Z where Z is (12) plus (13).

$$Z \approx E[\Pi + X_G] - \frac{1}{2} \sum_i \sum_j \left\{ \left\{ A_G \begin{pmatrix} \frac{\partial Y_G}{\partial \theta_i} & \frac{\partial Y_G}{\partial \theta_j} \end{pmatrix}^0 + A_D \begin{pmatrix} \frac{\partial Y_D}{\partial \theta_i} & \frac{\partial Y_D}{\partial \theta_j} \end{pmatrix}^0 \right\} (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \right\} \quad (14)$$

$$\text{Write } p(t) = p^*(t) + \epsilon h(\theta, t) \quad \text{all } t, \theta \quad (15)$$

where $h(\theta, t)$ is any arbitrary variation. If (14) is maximised by $p^*(t)$ then the derivative of (14) with respect to ϵ should be zero for any $h(\theta, t)$ at $\epsilon = 0$. Let $H = \int_0^L h(\theta, t) e^{-\rho t} dt$.

Now u_1^* , u_2^* the developer's optimal decisions are functions of ϵ for any given arbitrary $h(\theta, t)$: $u_1^* = u_1^*(\epsilon)$ and $u_2^* = u_2^*(\epsilon|\theta)$.

Thus

$$\frac{dZ}{d\epsilon} = \frac{\partial Z}{\partial u_1} \frac{du_1^*}{d\epsilon} + \frac{\partial Z}{\partial u_2} \frac{du_2^*}{d\epsilon} + \frac{\partial Z}{\partial \epsilon} \quad (16)$$

Consider $\frac{\partial Z}{\partial \epsilon}$:

$$\begin{aligned} \frac{\partial Z}{\partial \epsilon} &\approx \sum_i \left\{ \frac{\partial Z}{\partial \left(\frac{\partial Y_G}{\partial \theta_i} \right)^o} \left(\frac{\partial H}{\partial \theta_i} \right)^o - \frac{\partial Z}{\partial \left(\frac{\partial Y_D}{\partial \theta_i} \right)^o} \left(\frac{\partial H}{\partial \theta_i} \right)^o \right\} \\ &\approx - \sum_i \sum_j \left\{ \left(A_G \left(\frac{\partial Y_G}{\partial \theta_j} \right)^o - A_D \left(\frac{\partial Y_D}{\partial \theta_j} \right)^o \right) \left(\frac{\partial H}{\partial \theta_i} \right)^o \right. \\ &\quad \left. (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \right\} \quad (17) \end{aligned}$$

(17) is zero for any $h(\theta, t)$ if

$$A_G \left(\frac{\partial Y_G}{\partial \theta_j} \right)^o - A_D \left(\frac{\partial Y_D}{\partial \theta_j} \right)^o = 0 \quad \text{all } j \quad (18)$$

i.e.

$$A_G \left(\frac{\partial X_G}{\partial \theta_j} \right)^o + A_G \left(\frac{\partial P^*}{\partial \theta_j} \right)^o - A_D \left(\frac{\partial \Pi}{\partial \theta_j} \right)^o + A_D \left(\frac{\partial P^*}{\partial \theta_j} \right)^o = 0 \quad \text{all } j \quad (19)$$

integrating with respect to θ_j :

$$\begin{aligned} P^{*o} &= (A_D \Pi^o - A_G X_G^o) / (A_D + A_G) + h_o \\ &= \lambda \Pi^o - (1 - \lambda) X_G^o + h_o \quad (20) \end{aligned}$$

(20) is necessary only at the locality of $\theta = \bar{\theta}$. However if it holds for any θ , then $\frac{\partial Z}{\partial u_1} = \frac{\partial Z}{\partial u_2} = 0$ providing $A_G > 0$, by the following argument.

$$\text{If } P = P^* \text{ where } P^* = \lambda \Pi - (1 - \lambda) X_G + h_0 \quad (21)$$

then u_2^* maximises $\Pi - P^* = (1 - \lambda) (\Pi + X_G) - h_0$. i.e. u_2^* coincidentally maximises $\Pi + X_G$. But Z is only a function of u_2^* via $\Pi + X_G$, as, given $P = P^*$, we can write:

$$Z \approx E[\Pi + X_G] - \frac{1}{2} (A_G \lambda^2 + A_D (1 - \lambda)^2) S \quad (22)$$

where
$$S = \sum_i \sum_j \left\{ \left(\frac{\partial (\Pi + X_G)}{\partial \theta_i} \right)^0 \left(\frac{\partial (\Pi + X_G)}{\partial \theta_j} \right)^0 (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \right\}$$

Thus
$$\frac{\partial \Pi - P^*}{\partial u_2} = 0 \quad \text{implies} \quad \frac{\partial Z}{\partial u_2} = 0$$

Now u_1^* is chosen such that
$$\frac{\partial (Y_D^0 - R_D)}{\partial u_1} = 0$$

but

$$Y_D^0 - R_D \approx (1 - \lambda) E(\Pi + X_G) - \frac{1}{2} A_D (1 - \lambda)^2 S - h_0 \quad (23)$$

$$\approx (1 - \lambda) Z - h_0 \quad (24)$$

Thus maximising $Y_D^0 - R_D$ with respect to u_1 coincidentally maximises Z and so $\frac{\partial Z}{\partial u_1} = 0$.

Thus if $P = P^*$, $\frac{\partial Z}{\partial u_1} = \frac{\partial Z}{\partial u_2} = \frac{\partial Z}{\partial e} = 0$ and $\frac{dZ}{de} = 0$.

Second order conditions follow from maximising behaviour by the developer and by:

$$\frac{\partial^2 Z}{\partial e^2} = - (A_G + A_D) \sum_i \sum_j \left\{ \left(\frac{\partial H}{\partial \theta_i} \right)^0 (c_{ij} + \bar{\theta}_i \bar{\theta}_j) \left(\frac{\partial H}{\partial \theta_j} \right)^0 \right\} \quad (25)$$

(25) is negative as $[c_{ij} + \bar{\theta}_i \bar{\theta}_j]$ is a positive definite matrix. (3)

Finally note that h_0 is an arbitrary constant of integration and can thus be used to satisfy P4 and also that a special form of (21) is an initial payment h_0 followed by a sequence of payments $p(t)$ where

$$p(t) = \lambda \pi(t) - (1 - \lambda) x(t) \quad (26)$$

Q.E.D.

Equation (26) is the special case where the payments are made according to current flows and no 'indebtedness' exists. This formulation is preferred as such indebtedness may change the distribution of risk in a way not included in the model.

The assumption of the constancy of A_D and A_G simplified the proof of Proposition 1, but is not the most general assumption that can be made. For instance if $A_D = R/(Y_D + W_D)$ and $A_G = R/(Y_G + W_G)$ Proposition 1 still holds and $\lambda = \frac{W_G}{W_D + W_G}$; W_G, W_D could be interpreted in 'other wealth' or 'other income' terms. In general, however

non-constancy of A_D and A_G will imply that (26) is a less close approximation to an optimal payments sequence.

Despite this the simple policy implications of Proposition 1 are appealing, partly because the probability distributions of the random variables do not need to be known for the construction of P^* , although they are required for the developers control decisions u_1^* , and also for choosing amongst various prices offered by a number of potential owners/developers.

P^* is composed of an initial payment h_0 (positive or negative) followed by a share in the aggregate net revenues $\pi(t) + x(t)$. If, for instance $X_G \equiv 0$, then apart from h_0 which is the subject of bargaining, this is exactly the case of the owner giving up his property rights for a λ share of the profit (or loss). If $A_D \approx A_G$, then an optimal policy for the owner might be to ask all potential developers if they would be interested in forming a joint company with half the stock held by the owner and half by the developer, but under the direction of the developer. Thus the developer would contribute entrepreneurship and expertise and the owner the property rights. In a competitive context, the owner would form the joint company with that developer who was willing to make the highest (positive or negative) side payment (h_0) to the owner. If owners and developers have different coefficients of absolute risk aversion but are equally efficient in providing development services, then the opportunity set from which the owner can choose the pair (λ, h_0) can be found by asking each developer to bid a value of h_0 for each of a number of values of λ . The expected utility from each (λ, h_0) can be calculated and the pair which yields the highest expected utility is chosen. The owner will then choose (λ^G, h_0^G) to maximise $h_0 + \lambda Z$,

while the developer will choose (λ^D, h_o^D) to maximise $-h_o + (1 - \lambda) Z$ from the pairs (λ, h_o) associated with owners with identical resource prospects. Adding additional elements M relating to the quality of entrepreneurial abilities and N relating to the quality of the resource implies that the owner will accept that contract defined by (λ^G, h_o^G, M^G) and the developer that contract defined by (λ^D, h_o^D, N^D) . Thus an equilibrium schedule of contracts can be found as the set $\{\lambda, h_o, M, N\}$ such that for each element in the set there is some developer who weakly prefers that to any other and also some owner who weakly prefers that element to any other. It is obvious that for given M, N , a relatively risk averse developer will contract with a relatively non risk-averse owner etc.

Once the contract has been settled, both the owner and developer wish to maximise a monotonic increasing function of Z . This commonality of objective function is related to the Principle of Similarity (see Ross {8}). It implies that policy decisions would be agreed between the two parties to the contract.

X_G can in principle be any function. It can relate to externalities of the project suffered or gained by the owner: appreciation or depreciation of neighbouring sites in land development or the present value of capital stock at time L are examples. If the owner is the government representing community welfare then employment created (in present value money equivalent terms), taxes raised or pollution caused may be incorporated as X_G . Also X_G may relate to the amount of an exhaustible resource used up or extracted and sold during the lifetime of the project. The cases we will consider in detail here are chosen partly because they exemplify the role of the X_G function and partly because they are of

particular relevance to the development of exhaustible mineral resources where the property rights are owned by the government. The first case is that of an owner-collected profits tax/loss subsidy.⁽⁴⁾ Suppose this is of rate τ of before tax profits Π^b . Thus from (26)

$$\begin{aligned} p^*(t) &= \lambda(1 - \tau) \pi^b(t) - (1 - \lambda) \tau \pi^b(t) \\ &= (\lambda - \tau) \pi^b(t) \end{aligned} \quad (27)$$

$$\text{i.e. } P^* = (\lambda - \tau) \Pi^b + h_0 \quad (28)$$

(27) and (28) imply that the share of the enterprise taken by the owner should be the share that would be taken in the absence of taxation minus the tax rate.

The second case upon which we will focus is when the owner benefits according to the size of the stock of the resource remaining at L . Let the initial stock be K and total use be $Q = \int_0^L q(t) dt$. Then the stock at L is $K - Q$. Let the owner value this stock at present value price ϕ . Then $X_G = -Q\phi$ and is the loss in the value of terminal stocks due to the development operation and $\phi = \phi(\theta)$. Thus from (26):

$$p^*(t) = \lambda \pi(t) + (1 - \lambda) q(t) \phi e^{\rho t} \quad (29)$$

$$\text{and } P^* = \lambda \Pi + (1 - \lambda) Q \phi + h_0 \quad (30)$$

(29) and (30) imply that an extraction tax should be levied by the owner and that the (specific) rate should increase by the rate of discount. The

initial rate should be $(1 - \lambda)$ times the present (time L) value ϕ , which might be related to the world price of the resource at time L and the costs specific to the extension of the project. The incorporation of such a tax may be necessary to prevent too fast exploitation due to the finite horizon of the property rights lease. Of course, the nature of the ϕ function will be known when determining h_0 , and so the distribution of expected utility is unaffected by it. ϕ should in fact be set equal to the uncertain present value shadow price of having more resource at time L if Z were being maximised over an infinite horizon.

III

The application of Proposition 1, although general, is limited to situations where both parties have the same rate of discount. A contrary situation can occur if the two parties are involved in separate capital markets or involve different risks to creditors. In particular, differential discount rates are not compatible with capital transactions between the two parties; otherwise the one with the higher discount rate would wish to borrow an unbounded amount from the other. To abstract out of this difficulty constraints have to be placed upon the class of price profiles permissible in order to prove Proposition 2. An alternative and preferable approach would be to extend the model so that the discount rate is endogenous or to change to a discounted-stream-of-utility approach implying limits, on borrowing or lending, which are themselves partly endogenous. It is hoped that such an approach will be the subject of a later paper.

Proposition 2

A contingent payments stream $p^*(t) = \lambda(t) \pi(t) - (1 - \lambda(t)) x(t)$

where $\lambda(t) = A_D e^{-\rho t} / (A_D e^{-\rho t} + A_G e^{-rt})$ and where ρ, r are the discount rates of the developer and the owner respectively is Pareto superior in the set of all payments streams

$$p(t) = p^*(t) + \epsilon h(\theta, t) \text{ for any } \epsilon \text{ and arbitrary } h(\theta, t)$$

$$\text{where } \int_0^L \left(\frac{\partial h(\theta, t)}{\partial \theta_i} \right) (e^{-\rho t} - e^{-rt}) dt = 0 \quad \text{all } i \quad (30)$$

$$\text{and } E \left[\int_0^L h(\theta, t) (e^{-\rho t} - e^{-rt}) dt \right] = 0 \quad (31)$$

Note that the constraints eliminate the possibility of arbitrage in risks and expected incomes respectively.

Proof of Proposition 2

$$\text{Let } P^D = \int_0^L p(t) e^{-\rho t} dt + h_0$$

$$P^r = \int_0^L p(t) e^{-rt} dt + h_0$$

$$H^D = \int_0^L h(\theta, t) e^{-\rho t} dt$$

$$H^r = \int_0^L h(\theta, t) e^{-rt} dt$$

$$\Pi = \int_0^L \pi(t) e^{-\rho t} dt$$

$$X_G = \int_0^L x(t) e^{-rt} dt$$

Then (12) + (13) becomes Z' where

$$Z' = E [P^r - P^p + \Pi + X_G] - \frac{1}{2} \sum_i \sum_j \left\{ \left\{ A_G \left(\frac{\partial Y_G}{\partial \theta_i} \right)^0 \left(\frac{\partial Y_G}{\partial \theta_j} \right)^0 \right. \right. \\ \left. \left. + A_D \left(\frac{\partial Y_D}{\partial \theta_i} \right)^0 \left(\frac{\partial Y_D}{\partial \theta_j} \right)^0 \right\} \right. \\ \left. \{ c_{ij} + \bar{\theta}_i \bar{\theta}_j \} \right\} \quad (32)$$

Again construct $p(t) = p^*(t) + \epsilon h(\theta, t)$ all θ, t where $h(\theta, t)$ is any arbitrary variation satisfying the constraints (30) and (31).

Now

$$\frac{dZ'}{d\epsilon} = \frac{\partial Z'}{\partial u_1} \frac{du_1^*}{d\epsilon} + \frac{\partial Z'}{\partial u_2} \frac{du_2^*}{d\epsilon} + \frac{\partial Z'}{\partial \epsilon}$$

Again we show that $\frac{\partial Z'}{\partial u_1} = \frac{\partial Z'}{\partial u_2} = \frac{\partial Z'}{\partial \epsilon} = 0$ at $\epsilon = 0$

Consider $\frac{\partial Z'}{\partial \epsilon} : \frac{\partial E [P^r - P^p + \Pi + X_G]}{\partial \epsilon} = 0$ due to the constraints

on $h(\theta, t)$. Thus:

$$\frac{\partial Z'}{\partial \epsilon} = -\frac{1}{2} \sum_i \sum_j \left\{ \left\{ A_G \left(\frac{\partial Y_G}{\partial \theta_i} \right)^0 \left(\frac{\partial H^r}{\partial \theta_j} \right)^0 - A_D \left(\frac{\partial Y_D}{\partial \theta_i} \right)^0 \left(\frac{\partial H^p}{\partial \theta_j} \right)^0 \right. \right. \\ \left. \left. + A_G \left(\frac{\partial Y_G}{\partial \theta_j} \right)^0 \left(\frac{\partial H^r}{\partial \theta_i} \right)^0 - A_D \left(\frac{\partial Y_D}{\partial \theta_j} \right)^0 \left(\frac{\partial H^p}{\partial \theta_i} \right)^0 \right\} \right. \\ \left. \{ c_{ij} + \bar{\theta}_i \bar{\theta}_j \} \right\} \quad (33)$$

For $\frac{\partial Z'}{\partial \epsilon} = 0$ for any permissible $h(\theta, t)$ we require

$$A_G \left(\frac{\partial Y_G}{\partial \theta_i} \right)^0 - A_D \left(\frac{\partial Y_D}{\partial \theta_i} \right)^0 = 0 \quad \text{all } i \quad (34)$$

integrating with respect to θ_i :

$$p^*(t)^0 = (A_D e^{-\rho t} \pi(t)^0 - A_G e^{-rt} x(t)) / (A_D e^{-\rho t} + A_G e^{-rt}) \quad (35)$$

$$\frac{\partial Z}{\partial u_1} = \frac{\partial Z}{\partial u_2} = 0 \quad \text{if } p^*(t)^0 \text{ holds over all } \theta \text{ can be shown}$$

in the same way as in the proof of Proposition 1. The second-order conditions also follow similarly.

Thus
$$p^*(t) = \lambda(t) \pi(t) - (1 - \lambda(t)) x(t)$$

Q.E.D.

Proposition 2 states that if $\rho \neq r$, λ will change with t as the party with the higher rate of discount will discount risk faster and it will be optimal to allocate him an increasing proportion of the risk.

Note that

$$\frac{d\lambda(t)}{dt} \begin{matrix} > \\ < \end{matrix} 0 \quad \text{as } r \begin{matrix} > \\ < \end{matrix} \rho.$$

Any difference in expected utility distribution due to the time profile of $\lambda(t)$ would be countered by a changed initial payment.

IV

It is of interest to consider the RRT in the light of the preceding analysis, although it should be stressed that the analysis is normative and a different approach or formulation may alter its implication. In essence the RRT consists of a tax on the return earned on capital above a given reasonable rate. As the tax is not symmetric with regard to all policies that could be pursued by the developer, the higher the tax the less relatively appealing will be policies the outcome of which is very uncertain, as very high profits would be taxed away but very high losses would not be subsidised. If the owner is less risk averse than the developer, this implies that too cautious a policy will be followed: for instance investment would take place in established coal seams rather than a search being undertaken for new seams.

Arguments have been advanced elsewhere (see Heal {2}, p.216) concerning why the owner is likely to be risk averse even if the owner is the government. However, if the owner is much less risk averse than the developer, then the optimal contract (by Proposition 1) is for the developer to act almost as the owner's agent with a small shareholding to act as an incentive,⁽⁵⁾ thus most of the risk is borne by the owner. The RRT cannot allocate risk in this way; it simply reduces the probability of the developer making very high profits. It does not reduce the probability of the developer making very low or negative profits.

The RRT concentrates payments to the owner (government) in the later years of the project. This is stated to be advantageous if "the supply price of loan capital to the government is below the supply price of investment" ({1}, p.276), i.e. $r < \rho$. But Proposition 2 states

that $\frac{d\lambda(t)}{dt} < 0$ if $r < \rho$. This is partly because Proposition 2 disallows the use of the contract to transfer finance - surely there is a case for treating this separately - and partly because variations in profit in later years are relatively unimportant to the developer if he has the higher rate of discount.

The above points are made in a purely normative spirit as is the analysis in this paper. Implementation costs may prohibit the adoption of the price given in (26), and in particular it may not be possible for the government to find the finance to provide its share of initial costs from either taxation or capital markets if a large number of development projects are to be undertaken. However the analysis in Sections II and III does incorporate the rate of discount and the degree of risk aversion of the government explicitly and it does solve for all the parameters in the system with the exception of the distribution of expected utility (i.e. h_0). The choice of the values and number of parameters in the RRT is left open due to the lack of a normative framework. Also it is stated ([1], p.284) that "the advantage for the government is being able to get the benefits of high returns without actually purchasing equity in that it removes the risk of absolute loss that equity investment entails." This statement ignores the premium that the developer is willing to pay for a reduction in his risk due to equity investment by the owner specific to the one project. If this premium is greater than the government's premium for accepting this risk, the joint expected utility is increased. The advocacy of a consortium of developers ([1], p.275 fn2) is in a similar risk-sharing spirit, but the promotion and organisation of such a consortium plus the need to arrive at optimal development policy decisions may prove problematic. Also competition for the development contract may be reduced and this may produce a reduction in the expected utility of the government.

Finally any pricing or taxation policy must be able to take account of outcomes of the project other than simply its profitability. Revenue paid directly to the owner is rarely the sum total of his concern.

In this paper, it has been argued that owner or government participation in the development of natural resources as proposed in Propositions 1 and 2 is optimal in three ways. First, it produces Pareto efficient decisions in terms of expected utility by the developer in anticipation of stochastic events. Second, it produces Pareto efficient decisions in terms of utility given the stochastic events. Third, it produces an (approximately) optimal distribution of risk. It is also argued that there is a contract of this nature for any feasible distribution of expected utility (choice of h_0). The argument is based on an assumption of the existence of approximate risk premiums and implementation costs are ignored. However, the conclusion remains that there exists considerable theoretical foundation for a policy of joint owner-developer participation in projects with uncertain outcomes. Such a policy can be seen to exist or be emerging in several cases of present resource development, not least the case of North Sea Oil.

Footnotes

- (1) See for instance Heal {3}.
- (2) For a discussion of ideal prices and an analysis of their welfare advantages see Ireland {5}.

(3) The term inside the summation in (25) can be written

$$E \left[\left(\sum_i \theta_i \left(\frac{\partial H}{\partial \theta_i} \right)^0 \right)^2 \right] + \left(\sum_i \bar{\theta}_i \left(\frac{\partial H}{\partial \theta_i} \right)^0 \right)^2 \text{ which is}$$

positive for non-trivial $\left[\frac{\partial H}{\partial \theta_i} \right]$.

- (4) It is assumed that the developer can use any loss incurred in this development to offset tax liabilities in others.
- (5) Note that there does not exist an 'optimal' contract in the sense of Proposition 1 if $A_G = 0$. Z increases as λ approaches unity, but at unity the lack of incentive for (jointly) optimal policy prevents this being optimal.

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