PUBLIC FINANCE IN A KEYNESIAN TEMPORARY EQUILIBRIUM*

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1. Introduction

The theory of public finance has always been separated into two compartments. In the first of these we find general equilibrium models of allocative and distributive effects of government activity, and in the second we have income-expenditure models for analysing the effects of such activity on employment and output. While each approach contributes important insights, each in isolation has serious limitations. Differential incidence models, to be useful, must rely on rapid adjustment to equilibrium, and there are well known difficulties concerning that. Income-expenditure models are notorious for their neglect of relative prices. In examining the effects of various taxes and government expenditures on the economy, it has therefore been necessary to use these two models separately, and supplement them by ad hoc judgements.

Policy discussions in practice usually rely on crude income-expenditure models alone. The key considerations are the size and the balance of the budget. Except for some attention paid to differences in saving propensities, important distinctions among different fiscal measures are thus blurred. For example, it is recognised that indirect taxes can have price-inflationary effects in addition to demand-deflationary ones, but we have no systematic framework for analysing the two together.

The means for providing a unified approach are now available. Following the work of Clower {4}, there has been a great deal of activity in building models of short-run equilibrium with inter-related markets, where some of the markets clear by quantity rationing and others by price adjustment. The dual decision framework of demand in these models, where the realised transactions in rationed markets can influence effective demand in other markets, provides a way to combine a Keynesian consumption function concept with the usual price-theoretic analysis of consumer behaviour. Such models therefore seem well suited for joint analyses of price and employment effects of government activity. Thus far, these models have been studied at an abstract level, with emphasis on testing their overall consistency and therefore leading to results consisting largely of existence theorems.[(1)] In this paper I shall put one such model to use in studying the effects of government policy. I hope this will prove to be a useful first step in integrating the two approaches to public finance, and also in providing economists the same kind of intuition about the neo-Keynesian models as they now have for the general equilibrium and income-expenditure models.

I should emphasise one point at the outset. While I shall obtain some new and interesting results using this approach, the model itself is too simplified to be directly applicable. This paper should therefore be seen as an indication of the potentialities of the approach, and should not be contrasted unfavourably with the sophisticated level that the general equilibrium and income-expenditure models have reached after years of research. In the concluding section, I shall comment on the shortcomings of the model used here, and also suggest lines for future development.

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(1) See Benassy {2}, Grandmont and Laroque {6} and Barro and Grossman {1}.

2. Equilibrium with Rationing

The model I shall use is that of Grandmont and Laroque {6}, with minor modifications and different notation. The transactors in this model are concerned with two periods, but prevailing markets only provide a Hicksian temporary equilibrium for goods and services for the first period. Agents must therefore form expectations concerning the state of markets that will prevail when the second period arrives. For simplicity, it is assumed that such expectations are held with subjective certainty, but of course there is no presumption that they will turn out to be correct. The expectations are functions of the current market experience.

Current markets may clear either by quantity rationing or by price adjustment, according to specified rules. In particular, it is interesting to consider the case where the labour market clears by rationing and all other markets by price adjustment.[2] Much of the difficulty of proving existence arises from the problems of proper piecing together of possible regimes, but I shall circumvent the difficulty here by assuming that the rationing constraint in the labour market is binding in the first period and expected to remain binding in the second. The quantity of labour services traded is then set by the short side of the market, i.e. by the firms' demand. This is rationed among the sellers by some rule that need not be specified. Commodity markets clear by price adjustment; in fact I shall make matters simple by assuming that at each date there is only one commodity.

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(2) See Bliss {3} for a defence of this interpretation of Keynesian unemployment.

This suffices to illustrate some of the important new features of the model, and complications arising from cross-substitution among several commodities are best left for future elaboration. There is no possibility of borrowing or lending, but each agent enters the economy with a fixed endowment of money which he can use in the first period or carry over to the second, and I shall assume that the endowments of money are sufficient to ensure that no one finds the constraint of carrying over a non-negative amount binding, i.e. there is no desire to borrow at a zero rate of interest. I shall also assume that all consumers and all firms are identical within their respective groups, and thus in effect that there is only one of each.

Consider the consumer first. He comes to the first period markets with an endowment M_1 of money. This includes any amount received from the government, and his expectations of distribution of profit by the firm, but it is assumed that the current actions of firms do not affect such expectations. This seems reasonable in a short run model. The consumer faces prices q_1 for commodities and v_1 for labour. His employment is constrained at e_1, and he demands x_1 of commodities and plans to carry m_2 of money to the second period. He then expects to face prices q_2 and v_2, to be constrained to sell e_2 of labour and purchase x_2 of commodities. Thus his budget constraints are

$$q_1 x_1 - v_1 e_1 + m_2 = M_1$$

$$q_2 x_2 - v_2 e_2 = m_2$$

and he wishes to maximise utility

$$u(x_1, x_2, e_1, e_2).$$

Since utility does not depend explicitly on m_2, and since I am assuming that the constraint $m_2 \geq 0$ is not binding, the constraints can be collapsed into one:

$$q_1 x_1 + q_2 x_2 = M_1 + v_1 e_1 + v_2 e_2 \tag{1}$$

With e_1 and e_2 fixed, the choice variables are x_1 and x_2. It is then convenient to describe the consumer's choice by means of the 'partial' expenditure function

$$E(q_1, q_2, e_1, e_2 u) = \min \{ q_1 x_1 + q_2 x_2 \mid u(x_1, x_2, e_1, e_2) \geq u \} \tag{2}$$

This will be increasing in all arguments, concave and homogeneous of degree one in q_1 and q_2, and convex in e_1 and e_2. Then, given the quantities which are taken as parameters by the consumer, his utility level and his first period demand for commodities will be defined by

$$E(q_1, q_2, e_1, e_2, u) = M_1 + v_1 e_1 + v_2 e_2 \tag{3}$$

$$x_1 = E_1(q_1, q_2, e_1, e_2, u) \tag{4}$$

Subscripts on functions, as usual, indicate partial derivatives with respect to the appropriate argument.

As mentioned before, q_2, v_2 and e_2 will be functions of the current market observations. I shall assume for simplicity that each is a function only of the corresponding variable for the first period. Grandmont and Laroque assume that e_2 is constant. This has to do with ensuring continuity for existence proofs where it is not specified in advance whether rationing will occur, and it is therefore important to have continuity across possible regimes. I shall neglect this problem.

As in the first model of Grandmont and Laroque, I shall assume that the producer is not rationed on any market, and that he wishes to maximise profit given his price expectations. He enters period 1 with a given amount Y_1 of commodities and enough money to finance his activities. He can combine commodities y_1 and labour ℓ_1 to produce commodities y_2 available in period 2. As I shall consider the economy with taxation, the consumer and the producer can face different prices. Suppose the producer faces a price p_1 for commodities in period 1 and a wage rate w_1 for labour in period 1, and expects the price of commodities in period 2 to be p_2, where p_2 is a function of p_1. He will determine his own demand for commodities y_1 (and hence his supply $Y_1 - y_1$ to the rest of the economy) and his demand for labour ℓ_1 to maximise

$$p_1 (Y_1 - y_1) + p_2 y_2 - w_1 \ell_1 = p_1 Y_1 + (p_2 y_2 - p_1 y_1 - w_1 \ell_1)$$

This choice is best described by means of a cost function. While a more general treatment is no more difficult, I shall be able to interpret the results somewhat more easily by using a cost function of the form

$$y_2^n \;\; C(p_1, w_1)$$

where C is increasing, concave and homogenous of degree one and $n > 1$. Profit-maximisation will then require

$$p_2 = n \, y_2^{(n-1)} \;\; C(p_1, w_1) \tag{5}$$

and the input demands will be given by

$$y_1 = y_2^n \;\; C_1(p_1, w_1) \tag{6}$$

$$\ell_1 = y_2^n \;\; C_2(p_1, w_1) \tag{7}$$

Finally, consider the government. I shall allow five types of policies. The government can levy taxes in period 1 markets, at specific rates t_1 on commodities and r_1 on labour. It can purchase amounts x_g of commodities and ℓ_g of labour in the first period markets. Finally, it can hand out m_1 units of money to consumers. Any budget deficit is financed by creation of money, and any surplus is destroyed. Thus the deficit, defined as the rate of net money creation, is given by

$$D = p_1 \, x_g + v_1 \;\; \ell_g - t_1 \, x_1 - r_1 \, \ell_1 + m_1 \tag{8}$$

Note that the expression excludes the taxes on the government's purchases from both its expenditures and its revenues; alternatively they could have been included in both.

Suppose that the market wage w_1 cannot change within the period. We can then write down the conditions for an equilibrium with rationing:

$$p_1 = q_1 - t_1 \tag{9}$$

$$v_1 = w_1 - r_1 \tag{10}$$

$$x_1 + x_g + y_1 = Y_1 \tag{11}$$

$$e_1 = \ell_1 + \ell_g \tag{12}$$

As a check, note that in (3)-(7) and (9)-(12) we have nine equations, and nine unknowns: u, x_1, y_1, y_2, ℓ_1, e_1, q_1, p_1 and v_1. The second period (expected) values of the last four are uniquely related to the first period values, and therefore need not be considered as separate variables.

Note the assumption that it is the market wage before taxation that is 'sticky', producing an equilibrium in which the sellers of labour are rationed. This need not always be the case, and stickiness may appear in v_1, the wage net of tax. This is a possibility worth separate treatment.

3. Comparative Statics

To study the effects of government policy, it will be necessary to find out how the equilibrium shifts in response to various policy changes. The calculations become quite complex, and it is convenient to begin with the comparative statics of each agent considered in isolation.

First the consumer. For him the endogenous variables are x_1 and u, and the exogenous ones are q_1, v_1, e_1 and M_1. Total differentiation of (3) and (4) yields

$$E_1\, dq_1 + E_2\, q_2'(q_1)\, dq_1 + E_3\, de_1 + E_4\, e_2'(e_1)\, de_1 + E_5\, du$$

$$= dM_1 + v_1\, de_1 + v_2\, e_2'(e_1)\, de_1 + e_1\, dv_1 + e_2\, v_2'(v_1)\, dv_1$$

$$dx_1 = E_{11}\, dq_1 + E_{12}\, q_2'(q_1)\, dq_1 + E_{13}\, de_1 + E_{14}\, e_2'(e_1)\, de_1 + E_{15}\, du$$

Substituting from the first of these into the second, we can write

$$dx_1 = -A\, dq_1 + B\, de_1 + \Gamma\, dv_1 + \Delta\, dM_1 \tag{13}$$

where

$$-A = (E_{11} - E_1 E_{15}/E_5) + (E_{12} - E_2 E_{15}/E_5)\, q_2'(q_1) \tag{14}$$

$$B = \left[E_{13} + (v_1 - E_3)\, E_{15}/E_5 \right] + \left[E_{14} + (v_2 - E_4) \right.$$

$$\left. E_{15}/E_5 \right] e_2'(e_1) \tag{15}$$

$$\Gamma = (E_{15}/E_5) \left[e_1 + e_2\, v_2'(v_1) \right] \tag{16}$$

$$\Delta = E_{15}/E_5 = \partial x_1 / \partial M_1 \tag{17}$$

We see at once that provided x_1 is a normal good, Δ will be positive. Then, provided $v_2'(v_1)$ is non-negative, i.e. provided an increase in wage in period 1 does not generate expectations of a decrease in wage in period 2, Γ will be positive. Both these assumptions seem sensible. I shall similarly assume $q_2'(q_1)$ and $e_2'(e_1)$ (and later $p_2'(p_1)$) to be non-negative, but the signs of A and B are still not so clear.

By the Slutsky-Hicks equations, we recognise the derivatives in (14) as the (uncompensated) demand derivatives $\partial x_1/\partial q_1$ and $\partial x_1/\partial q_2$ respectively. The first will be negative if x_1 is a normal good, and the second will be negative if x_1 and x_2 are not too strong substitutes, in fact if they are gross complements. In view of this, it seems reasonable to assume that A will be positive.

Finally, consider B. Observe that E_3 is the marginal rate of substitution between e_1 and M_1, i.e. the supply price of labour in period 1. Under conditions of excess supply and rationing, i.e. involuntary unemployment, in the labour market, we must have $(v_1 - E_3)$

positive. As the same state is expected to prevail in period 2, $(v_2 - E_4)$ will similarly be positive. Now E_{13} will be positive if an increase in e_1, i.e. a decrease in period 1 leisure, increases the compensated demand for x_1, i.e. if x_1 is a substitute for period 1 leisure. Similarly, E_{14} will be positive if it is a substitute for period 2 leisure. Thus, if we rule out strong complementarity between x_1 and leisure in either period, we can assume B to be positive. This seems a reasonable assumption at this level of aggregation; in a disaggregated model, we would wish to allow some particular commodities to be complementary to leisure.

The assumption of a positive B may be interpreted as a Keynesian consumption function in Clower's dual decision framework. In fact the terms $(v_1 \, de_1 + v_2 \, de_2)$ in the expression for dx_1 are really the ones that are peculiar to the dual decision framework. The other terms would appear even in a general equilibrium framework, but of course the actual values of de_1 and de_2 would be different in the two cases.

The comparative statics of the producer's decisions is easier except that the formation of expectations introduces a link between p_1 and p_2. Define $\varepsilon = d \log p_2 / d \log p_1$, the elasticity of expectations. Then, taking small proportional changes in (5)-(7), we find

$$\varepsilon \; dp_1/p_1 = dp_2/p_2 = (n - 1) \; dy_2/y_2 + \theta \; dp_1/p_1$$

$$dy_1/y_1 = n \; dy_2/y_2 - (1 - \theta) \; \sigma \; dp_1/p_1$$

$$d\ell_1/\ell_1 \quad = \quad n \; dy_2/y_2 \; + \; \Theta\sigma \; dp_1/p_1$$

where Θ is the imputed share of period 1 commodity inputs in factor cost, and σ is the elasticity of substitution between commodities and labour in production. Thus the responses of derived demands to parameter changes are given by

$$dy_1/y_1 \quad = \quad \left[\; (\varepsilon - \Theta) \; n/(n - 1) \; - \; (1 - \Theta)\sigma \; \right] \; dp_1/p_1 \qquad (18')$$

$$d\ell_1/\ell_1 \quad = \quad \left[\; (\varepsilon - \Theta) \; n/(n - 1) \; + \; \Theta \, \sigma \; \right] \; dp_1/p_1 \qquad (19')$$

In the 'normal' case, we would like the coefficient in (18') to be negative and that in (19') to be positive. The first would give a downward sloping demand curve for commodities as inputs to production, i.e. an upward sloping supply curve of period 1 commodities to the rest of the economy, while the second would mean that a fall in the price of period 1 commodities received by producers would cause them to reduce labour demand. However, it is quite easy for one of these to be false. The first can fail since sufficiently elastic expectations can lead producers to expand production as p_1 increases, while the second can fail since the fall in the marginal cost curve as p_1 falls can cause increased production. The two require opposing circumstances, and it is easy to see that one of the signs must be normal. However, if n is close to one, the first term in each coefficient is large in absolute value, and this increases the possibility of one of the two ceasing to be normal. This is because returns to scale are then nearly constant, the marginal cost curve is very flat, and any vertical shifts in it or in the expected price produce large output effects.

In this paper I shall assume that both effects are 'normal', as I would like to emphasise some other anomalies that remain as an essential feature of an equilibrium with rationing. However, the other cases also deserve attention.

Introducing some more abbreviations, I shall accordingly write

$$dy_1 = -\Phi \ dp_1 \tag{18}$$

$$d\ell_1 = \Psi \ dp_1 \tag{19}$$

where Φ and Ψ are both positive.

One more condition can be imposed on these expressions, by considering the total demand for period 1 commodities as a function of q_1, assuming the appropriate quantity adjustment in the labour market and holding all other variables fixed. Now, corresponding to a change dq_1, we have the same change in p_1, and a change of $\Psi \ dq_1$ in e_1. Using (13) and (18), we see that the change in the total demand for period 1 commodities is

$$dx_1 + dy_1 = -A \ dq_1 + B \ \Psi \ dq_1 - \Phi \ dq_1$$

If we assume a downward sloping demand curve for stability in this market, we have

$$A + \Phi - B \ \Psi > 0 \tag{20}$$

I shall write Σ for the expression on the left hand side.

We can now put the pieces together to study the comparative statics of the whole equilibrium. Suppose the tax rates change by dt_1 and dr_1, the government purchases by dx_g and $d\ell_g$, and the money transfers to consumers by dm_1 (and of course $dM_1 = dm_1$). From (9) - (12) we have

$$dp_1 = dq_1 - dt_1, \qquad dv_1 = - dr_1,$$

$$de_1 = d\ell_1 + d\ell_g$$

and then

$$\begin{aligned} 0 &= dx_1 + dx_g + dy_1 \\ &= \left[- A\,(dp_1 + dt_1) + B\,(d\ell_1 + d\ell_g) - \Gamma\, dr_1 + \Delta\, dm_1 \right] \\ &\quad - \Phi\, dp_1 + dx_g \end{aligned}$$

and therefore

$$dp_1 = \Sigma^{-1} \left[- A\, dt_1 - \Gamma\, dr_1 + dx_g + B\, d\ell_g + \Delta\, dm_1 \right] \quad (21)$$

$$dq_1 = \Sigma^{-1} \left[(\Phi - B\,\Psi)\, dt_1 - \Gamma\, dr_1 + dx_g + B\, d\ell_g + \Delta\, dm_1 \right] \quad (22)$$

$$de_1 = \Sigma^{-1} \left[- A\,\Psi\, dt_1 - \Gamma\,\Psi\, dr_1 + \Psi\, dx_g + (A + \Phi)\, d\ell_g \right.$$

$$\left. + \Delta\,\Psi\, dm_1 \right] \qquad (23)$$

$$dx_1 = \Sigma^{-1} \left[- A\,\Phi\, dt_1 - \Gamma\Phi\, dr_1 - (A - B\Psi)\, dx_g \right.$$

$$\left. + B\,\Phi\, d\ell_g + \Delta\,\Phi\, dm_1 \right] \qquad (24)$$

Finally, we can calculate the effect of various policies on the budget deficit. After some substitution and simplification, we find

$$dD = (q_1 + \Omega)\, dx_g + (v_1 + B\,\Omega)\, d\ell_g + (1 + \Delta\,\Omega)\, dm_1$$

$$- (x_1 + A\,\Omega)\, dt_1 - (e_1 + \Gamma\,\Omega)\, dr_1 \qquad (25)$$

where

$$\Omega = \Sigma^{-1} (x_g - t_1\,\Phi - r_1\,\Psi) \qquad (26)$$

The sign of Ω is ambiguous.

In the two sections that follow, these formulae will be put to use in examining and comparing various policy measures.

4. Prices and Quantities

Consider first the simplest (and the most reassuring) result. If the government increases its purchases of labour slightly, we have

$$de_1 = \Sigma^{-1} (A + \Phi) \; d\ell_g > d\ell_g \tag{27}$$

This is the conventional employment multiplier. However, the conventional output multiplier does not arise quite as clearly. If the government increases its purchases of commodities slightly, we have

$$d(x_1 + x_g) = \left[1 - \Sigma^{-1} (A - B \, \Psi) \right] dx_g$$

$$= \Sigma^{-1} \Phi \; dx_g > 0 \tag{28}$$

but we cannot say in general that the output multiplier will exceed one. The problem is that the increased government demand raises the price of commodities, and this decreases the quantity demanded by the consumers. If this demand is price-elastic, i.e. if A is large, we can have

$$dx_1 = - \Sigma^{-1} (A - B \, \Psi) \; dx_g < 0$$

This is the first example of a phenomenon that the conventional fixed-price income-expenditure models miss.

The output multiplier would be stronger if the supply of commodities could be increased within the first period very elastically in response to the price increase. This would be the case in a model which allowed unused capacity and assumed a zero production lag.

Having mentioned price changes, let us examine them in greater detail. Begin with the question of the incidence of the commodity tax. We have

$$dp_1 = -A \ \Sigma^{-1} \ dt_1 \ , \quad dq_1 = \Sigma^{-1} \ (\Phi - B \ \Psi) \ dt_1 \qquad (29)$$

It is now possible that an increase in the tax rate will lower the price to consumers. The price to producers will then fall by more than the amount of the tax, i.e. the incidence of the tax can be more than 100% on the producer. In a conventional partial equilibrium model, this cannot happen with a downward sloping demand curve and an upward sloping supply curve. Even in a general equilibrium model, such a result is somewhat paradoxical, and can in fact be ruled out using natural sufficient conditions for the uniqueness of equilibrium (see Dixit {5}). In a temporary equilibrium with rationing, however, such a result can arise quite naturally. The reason is that the fall in the price received by the producer causes a fall in the amount of labour demanded, and this shifts the demand curve for commodities further to the left, thus causing a further fall in the price to the producer. The cumulative outcome of this process can easily lower the price to the consumer.

Conversely, it is possible that a reduction in indirect taxes, by stimulating demand, will actually increase prices 'at a stroke'. This is another example of a possibility that the unified approach to public finance brings to our attention.

This leads us naturally to a comparison of the price and employment effects of various policies. Three of the policies - the wage tax, commodity purchases and money supply changes - are identical in this regard. For each of them, we have

$$dq_1/de_1 = 1/\Psi \tag{30}$$

Labour purchases cause a smaller change in commodity prices per unit of employment change; for this policy we have

$$dq_1/de_1 = B/(A + \Phi) \tag{31}$$

and it is easy to verify that the right hand side of (31) is less than $1/\Psi$. For the commodity tax we have

$$dq_1/de_1 = (B\Psi - \Phi) / (A\Psi) \tag{32}$$

This may be of either sign, but will always be less than $1/\Psi$.

These comparisons are important not only for their policy implications, but also because they have an indirect bearing on the possibility of finding stable relations of the Phillips curve type in an economy in which discretionary macro-policy is pursued. Of course a model which will allow us to investigate such a curve must be much more sophisticated - in particular we must allow a flexible market wage and compare temporary equilibria in successive periods, taking into account changes in expectations - but it is unlikely that the necessary complications will eliminate the differences among policies.

5. Budget Deficits and Employment

The standard question in general equilibrium models of public finance is the policy choice which, subject to the appropriate constraints, maximises utility. Given a revenue constraint, for example, we look for a tax structure which in an interior optimum equates the marginal dead-weight loss per unit of revenue. In a temporary equilibrium framework, however, utility maximisation loses much of its welfare significance. This is the case since consumers maximise utility given their expectations, and these expectations are in general inconsistent and incorrect. In fact the welfare economics of temporary equilibria has hardly been studied, and therefore it seems more appropriate to catalogue various comparisons of policies rather than to try to reduce them all to a scalar measure of welfare. One comparison of particular interest is the expansionary effect of policies in relation to the effect on the budget deficit. Since the constraint on employment is the main new feature of this model, it seems natural to consider expansionary effects on employment. In considering the budget deficit, we must of course include both direct and indirect effects of each policy, and the expression (25) does this.

If we could be assured of the existence of in interior solution to the problem of finding an optimal policy mix for employment, we would write down first order conditions equating their effects on employment per unit of added budget deficit at the margin. However, interior solutions do not always exist, and I shall pose the problem as one of the various binary comparisons of employment expansions per unit of marginal budget deficit.

The first such comparison will be between the government's commodity purchases and transfers of money to consumers. For the former, we see from (23) and (25) that

$$de_1/dD \quad = \quad \Sigma^{-1}\, \Psi \,/\, (q_1 + \Omega)$$

For the latter, the corresponding expression is

$$de_1/dD \quad = \quad \Sigma^{-1}\; \Delta\, \Psi \,/\, (1 + \Delta\, \Omega)$$

Therefore commodity purchases are more expansionary per unit of deficit at the margin if

$$\Sigma^{-1}\, \Psi \,/\, (q_1 + \Omega\,) \;>\; \Sigma^{-1}\, \Delta\, \Psi \,/\, (1 + \Delta\, \Omega\,)$$

This reduces to

$$q_1\, \Delta \;<\; 1 \tag{33}$$

Now $q_1 \;\; \partial x_1/\partial M_1$ is the marginal propensity to consume first period commodities, and therefore (33) is true under the standard assumption.

This is the familiar result about the balanced budget multiplier. It should not be surprising that the appropriate comparison is with transfers of money, since the simple income-expenditure analysis treats taxes as lump sum transfers. Comparisons of commodity purchases with other tax reductions yield related results, but in the new framework the price effects of these taxes must be considered.

Proceeding in a similar way, we find that commodity purchases are more expansionary per unit deficit at the margin than a reduction in the wage tax if

$$(q_1 \; \partial x_1/\partial M_1) \left[1 + (e_2/e_1) \; v_2'(v_1) \right] < 1 \tag{34}$$

This is equivalent to (33), and therefore true, if wage expectations are totally inelastic. Otherwise it is less likely to be true.

Comparing commodity purchases with a commodity tax reduction, we find the former more expansionary if

$$- \left(\frac{q_1}{x_1} \; \frac{\partial x_1}{\partial q_1}\right) - \left(\frac{q_2}{x_1} \; \frac{\partial x_1}{\partial q_2}\right) \cdot \left(\frac{q_1}{q_2} \quad q_2'(q_1)\right) < 1 \tag{35}$$

This amounts to requiring a price-inelastic demand, in an overall sense including the effect on expectations.

Next compare the government's labour purchases with money transfers. The condition for the former to be more expansionary per unit deficit at the margin is

$$\Sigma^{-1} \; (A + \Phi) \; / \; (v_1 + B \; \Psi) > \Sigma^{-1} \; \Psi \; \Delta \; / \; (1 + \Delta \; \Omega)$$

For this, it is sufficient to have

$$\Sigma^{-1} \; B \; \Psi \; / \; (v_1 + B \; \Omega) \; \geq \; \Sigma^{-1} \; \Psi \; \Delta \; / \; (1 + \Delta \; \Omega)$$

which reduces to

$$B \geq v_1 \Delta \tag{36}$$

This, too, can be interpreted as a balanced budget multiplier if we think of $v_1 de_1$ as the equivalent money income increase in response to an increase of de_1 in employment. For the marginal propensity to consume is then $q_1 B/v_1$. If we identify this with $q_1 \Delta$, then (36) holds with equality.

However, this interpretation is not very satisfactory, and there is in fact some difference between commodity purchases and labour purchases. In comparisons involving the former, we consider the expansionary effects of two ways of expanding commodity demand, the first being direct purchases and the second involving various inducements to consumers. Conditions involving the marginal propensity to consume and price-elasticities thus arise naturally. The same cannot be said of labour purchases.

We can proceed differently by examining (15). If commodities at date 1 are gross substitutes for leisure at either date in the sense that an increase in the quantity of e_1 or e_2 will increase the demand for x_1, then $(E_{13} - E_3E_{15}/E_5)$ and $(E_{14} - E_3E_{15}/E_5)$ will be positive. Then $B > v_1 + v_2\, e_2'(e_1)$, and (36) is true. Thus we have a sufficient condition with a ready interpretation, but it is very restrictive.

Using the same method and assuming gross substitutes, we find that labour purchases are more expansionary than a wage tax reduction if

$$e_1 \, e_2'(e_1) \;/\; e_2 \;>\; v_1 \, v_2'(v_1) \;/\; v_2 \tag{38}$$

i.e. if employment expectations are more elastic than wage expectations. This seems sensible, and the only surprise is that the condition is only sufficient.

The condition for a wage tax cut to win in the comparison with money transfers is

$$\Gamma \;>\; e_1 \, \Delta \tag{39}$$

which is always true. If the market wage showed some flexibility, this effect would be further strengthened through substitution in production.

Finally, a wage tax cut wins against a commodity tax cut if

$$-\left(\frac{q_1}{x_1}\frac{\partial x_1}{\partial q_1}\right) - \left(\frac{q_2}{x_1}\frac{\partial x_1}{\partial q_2}\right)\left(\frac{q_1}{q_2} \quad q_2'(q_1)\right) < q_1 \frac{\partial x_1}{\partial M_1}\left(1 + \frac{e_2}{e_1} \quad v_2'(v_1)\right) \tag{40}$$

Thus a high marginal propensity to consume and elastic wage expectations favour the wage tax cut; price-elastic demand favours a commodity tax cut.

6. Modifications and Extensions

The familiar and new results of the preceding sections are, I hope, sufficient to persuade readers of the fruitfulness of this approach. However, the model has serious limitations which must be removed in order to make it more realistic. The most important modification is the introduction of borrowing and lending, and it will affect the model in several ways.

First, it will enable us to give a better treatment of investment. Commodities as inputs to production are a form of investment in the present model, but the theory of this investment is a very crude one, with only internal finance and only the opportunity cost of sales to consumers. With financial capital markets we can give a much richer theory of this important component of aggregate demand. Also, the theory of producer behaviour itself is weak in absence of such markets. I have assumed profit-maximisation, but the basis on which the producer compares p_2 and p_1 is at best unclear. In fact any behaviour that leads to (18) and (19) will serve for the purpose of the model, but borrowing and lending possibilities will be important if we are to make sense of most types of behaviour.

The second point is that financial capital markets will remove the need to assume that the constraint $m_2 \geq 0$ was never binding on the consumers. This will be particularly important if we are to introduce heterogeneity among consumers, as must be done if we are to study the structure and not merely the magnitude of involuntary unemployment.

Finally in this connection we have the issue of finance of the government's budget deficit. In the present context, there is little reason for having any concern about the magnitude of the deficit per se. However, if the deficit is financed by borrowing; it affects interest rates and thus has other repercussions on the economy. The exercises of finding fiscal measures which yield most expansion per unit of deficit acquire more meaning in such a setting.

On a related point, there is equally little reason in the present model why expansion should not be carried to a point where the rationing constraint on labour ceases to be binding. In reality, second or third-best policies with regard to the balance of payments provide the reason for maintaining some involuntary unemployment. It would be desirable to include these considerations directly in the model.

Another modification that may be thought desirable is to allow quantity adjustment in the commodity market as well. This can happen, for example, if firms are rationed as to the quantity of commodities they can sell. This dual rationing of firms and consumers is an attractive way to think about processes of cumulative deflation like the multiplier. This approach is taken, for example, by Barro and Grossman {1} and by Grandmont and Laroque {6} in a second, and in their opinion more properly Keynesian, model. However, we have seen that the approach adopted here, with workers rationed but firms able to act as price-takers, also leads to a multiplier. Thus the modification is not essential. Also, for what it may be worth, it is clear that the present approach accords with Keynes' intensions quite explicitly stated in Chapter 2 of the General Theory.

Finally, of course, it is important to extend the horizon and consider a succession of temporary equilibria, allowing for changes in expectations and speculative action in anticipation of such changes. That, however, seems a rather ambitious task at this stage.

I would like to defend the model against one possible charge. It can be said that the model is too specific, assuming as it does a particular situation in which some constraints are binding and others are not. The model in conditions of excess demand, for example, would have to be recast and would not be symmetric. However, I believe that such specificity is desirable, and that more progress will result from building different models applicable to different situations than can be expected from a very general model that tries to encompass the whole range of the underlying theory.

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