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UNBALANCED GROWTH IN THE WELFARE STATE

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.

## I. Introduction

Public Policy Economics has benefited recently from a revival of interest in the problem of Optimal Taxation. A feature of much of this work is its concentration on the use of the taxation powers of government as an instrument for improving social welfare. In the models of Phelps (1973a), the use to which the government puts its net tax revenue does not affect anyone's utility, and in other models, for example those of Atkinson (1973a), net revenue (taxes minus handouts) is actually constrained to be zero. In the simple model of a 'Welfare State' proposed in the present paper attention is focussed on the expenditure of taxation revenues as a means of achieving the ends of social policy. Although the results of the optimal taxation theorists may yet yield important implications for future policy, it is arguable that in attempting to understand the recent history of an economy such as the United Kingdom one does not violate reality too drastically by assigning to the tax system a merely revenue-raising role. Despite our income tax schedules that are, on paper, progressive, it is probably fair to infer from the results of Nicholson (1967) that if the government of the United Kingdom has been trying to engineer a fundamental re-distribution of income by means of taxation policies, then it has failed.

Diamond and Mirrlees (1971) have included in their paper a discussion of the optimal production of public or collective goods, such as Defence, and the Police Force. Of a similar order of magnitude in a Welfare State is government expenditure on the provision, at zero or subsidised prices, of 'welfare' goods and services, including such things as hospital services, education, and housing, which are typically consumed privately. These activities are of considerable importance. In the U.K., in 1961, net public expenditure on housing and social services was a fraction 0.165 of GNP, and public expenditure in other areas, including the production of collectively consumed

goods, was a fraction 0.195. By 1971, the figure for housing and social services had risen to 0.221, and that for other public expenditure had also grown, though less rapidly, to 0.221 of GNP. <sup>2</sup>

The purpose of this paper is to discuss the workings of the welfare system which disposes of such a large and increasing fraction of national output. In Section II a model, which draws heavily on the methods of the Optimal Taxation theorists, of a Welfare State is proposed; in Section III the dynamics of the system under 'balanced' growth are revealed, but the realism of the implications of balanced growth are challenged with illustrative data for the U.K., and in Section IV a resolution of the problem in terms of 'unbalanced' growth is suggested. In V the worthwhileness, in terms of justice and efficiency, of the Welfare State system is examined, and in the following section an alternative system is briefly discussed. Section VII summarises and concludes the paper.

## II. A Simple Model of a Welfare State

We assume that the economy has two goods, 1 and 2, which are produced entirely by labour. Workers differ in their productive capacities. The least productive worker receives a money income  $\bar{y}$ . The prices of goods 1 and 2 are set at  $p_1 = \bar{y}/\bar{x}_1$ ,  $p_2 = \bar{y}/\bar{x}_2$ , where  $\bar{x}_i$  is the amount of good  $i$  that the worker can produce when producing none of the other. The worker can produce a combination of 1 and 2, moving, say, from production of good 1 to production of good 2 at a constant marginal rate of transformation equal to  $-p_1/p_2$ . The other workers also receive incomes,  $y$ , equal to the value of their output and may also change from 1 to 2 at the same rate,  $-p_1/p_2$ . That is, the production-possibility frontiers of the workers, though at differing distances from the origin, are all linear with the same slope. Following Atkinson (1973a) we assume that the distribution of incomes is Pareto in form : <sup>3</sup>

$$f(y) = \mu \bar{y}^{-\mu} y^{-\mu-1} \quad (1)$$

The aim of the Welfare State is to attack the 'giants' of 'Want, Disease, Ignorance, Squalor and Idleness', (Beveridge, 1942) that result from  $\bar{y}$  being too low, and it does this in a paternalistic way by ensuring that everyone consumes a certain amount of the appropriate goods and services - food, health services, schooling, housing, which we summarize in the model as good 1 - rather than by redistributing incomes. There is no Social Welfare Function to be optimized in the model - we assume that the ends of the Welfare State are given and fixed; specifically, that the State will give away one unit of good 1 to anyone who wants it, and requires (by means of legislation such as the school leaving age) that a worker who chooses to buy good 1 from the private sector also consumes a minimum of one unit. For

simplicity, we will assume that one unit is also a maximum. So, the per capita consumption of good 1, whether obtained free from the state, or privately, is fixed at one unit. <sup>4</sup>

Although their production costs are the same,  $p_1$ , it is assumed that the utility,  $\theta_s$ , gained from consuming the State's version of good 1 is less than the utility,  $\theta_p$ , derived from the privately produced article. This is because, for example, people can indulge their sectarian preferences in a private school, but not in a secular state school, or can choose their neighbours when buying a house privately, but not when they are assigned a place in a local authority scheme. Everyone has the same utility function,  $U$ , assumed to be additively separable, and written

$$U = \begin{cases} U_s & = \theta_s + x_2^\delta \\ U_p & = \theta_p + x_2^\delta \end{cases}, \quad \theta_p > \theta_s, \quad 1 > \delta > 0 \quad (2),$$

which assumes independence between consumption of goods 1 and 2, positive but diminishing marginal utility from consumption,  $x_2$ , of 2, and has as the unit of measurement the utility derived from the consumption of one unit of 2. Good 2 summarizes all the goods in the economy that the state considers to be non-essential.

The curves of the utility function are drawn on the upper part of Figure 1. Consider an individual with disposable income  $y_d'$ . By accepting good 1 from the state the individual can buy  $x_2' = y_d'/p_2$  of good 2, and achieve a utility of  $\theta_s + x_2'^\delta$ . Alternatively, he/she could buy good 1 privately, for  $p_1$ , consume only  $x_2' - p_1/p_2$  of 2, and receive utility  $\theta_p + (x_2' - p_1/p_2)^\delta$ ; but, as drawn, this is less than  $\theta_s + x_2'^\delta$ , so he/she wouldn't. However, a rich enough individual, say one with  $y_d''$ , would

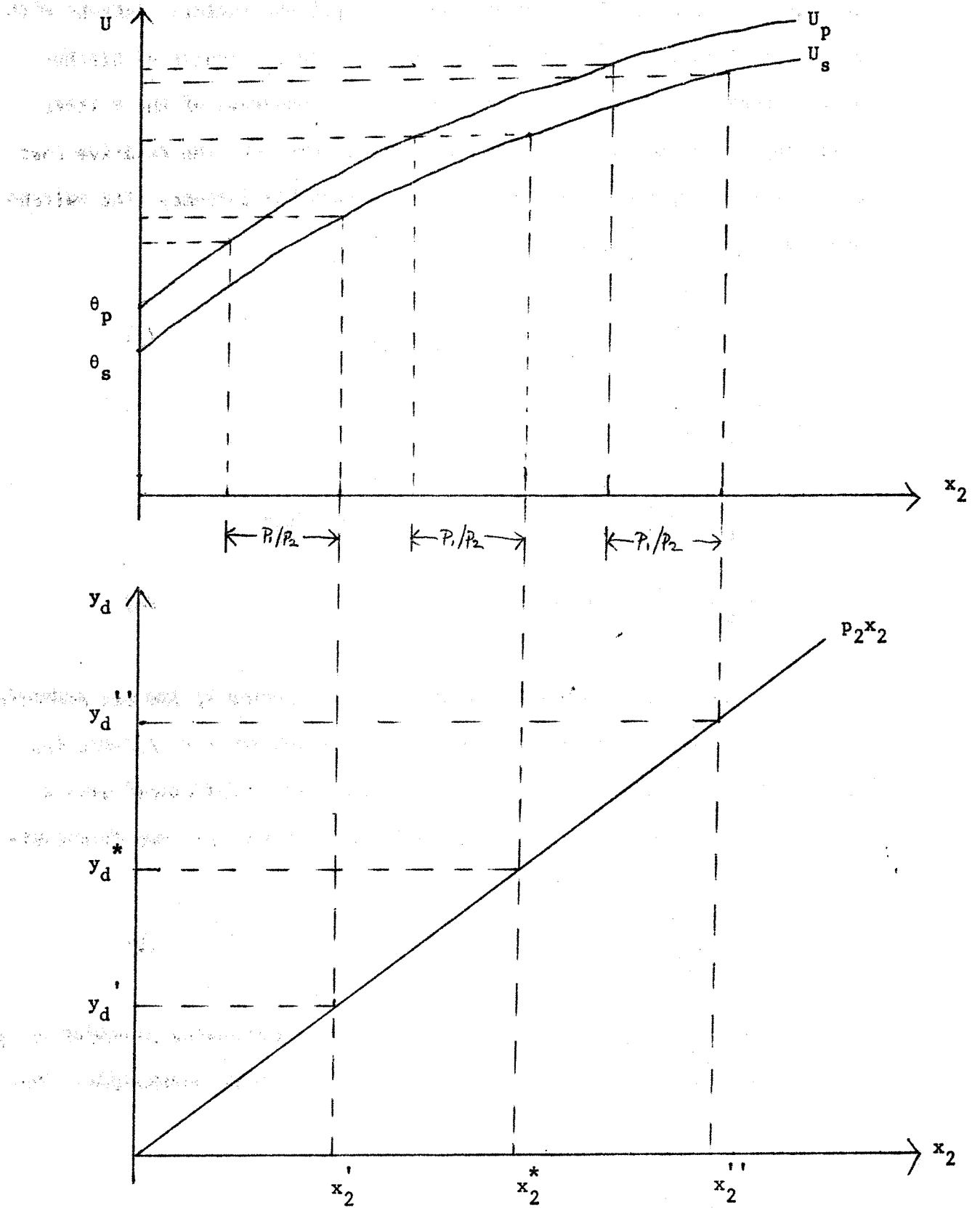


Figure 1 ; The choice between private and public sectors



get more satisfaction from buying 1 from the private sector. Someone with disposable income  $y_d^*$  would be indifferent as to the source of his/her unit of good 1. Thus, the demands made on the resources of the Welfare State depend on the parameters of the utility function, the relative cost of goods 1 and 2, and the distribution of disposable incomes. The switch-over value of  $x_2$ ,  $x_2^*$ , satisfies

$$\theta + x_2^{*\delta} = \theta_p + (x_2^* - p_1/p_2)^\delta \quad (3)$$

so that  $y_d^*$  satisfies

$$\theta_s + (y_d^*/p_2)^\delta = \theta_p + (y_d^*/p_2 - p_1/p_2)^\delta$$

or 
$$y_d^{*\delta} - (y_d^* - p_1)^\delta = (\theta_p - \theta_s) p_2^\delta \quad (4)$$

Of course, disposable income will be affected by the tax schedule, which in turn will be determined by the need of the state to finance its expenditure on giving away good 1. We assume that an individual pays a constant proportion  $(1 - \beta)$  of his/her income in tax, so that disposable income

$$y_d = \beta y \quad (5)$$

It would probably be more realistic to have  $\beta$  a decreasing function of  $y$ , but I do not expect that the added complication would be worthwhile. The switch-over level of income,  $y^*$ , is then

$$y^* = y_d^*/\beta = p_2 x_2^*/\beta \quad (6)$$

The amount of tax revenue required,  $T_r$ , is therefore,

$$T_r = N p_1 \int_{\bar{y}}^{p_2 x_2^* / \beta} f(y) dy \quad (7)$$

where  $N$  is the total number of workers in the economy,<sup>5</sup> and the integral term is the proportion of  $N$  receiving good 1 from the State. Equation (7) solves to

$$T_r = N p_1 (1 - (\beta \bar{y} / p_2 x_2^*)^\mu) \quad (8)$$

The amount of tax revenue forthcoming,  $T_s$ , obeys

$$T_s = N \int_{\bar{y}}^{\infty} (1 - \beta) y f(y) dy \quad (9)$$

which reduces to

$$T_s = \frac{N \mu}{\mu - 1} (1 - \beta) \bar{y} \quad (10)$$

The tax rate,  $(1 - \beta)$ , must be chosen so that  $T_r = T_s$ . On Figure 2,  $T_r$  and  $T_s$  are plotted as functions of  $\beta$ . The  $T_s$  schedule is simply the straight line through  $N \frac{\mu}{\mu - 1} \bar{y}$  on the  $T$  axis, and 1 on the  $\beta$  axis. I have not tried to plot the  $T_r$  curve precisely, but we can deduce its main properties. If  $T_r = 0$ , then

$$\beta = p_2 x_2^* / \bar{y} = y_d^* / \bar{y} \quad (11)$$

which will be greater than one if the minimum income is such that there is some demand for the services of the welfare state. The  $T_r$  curve has its

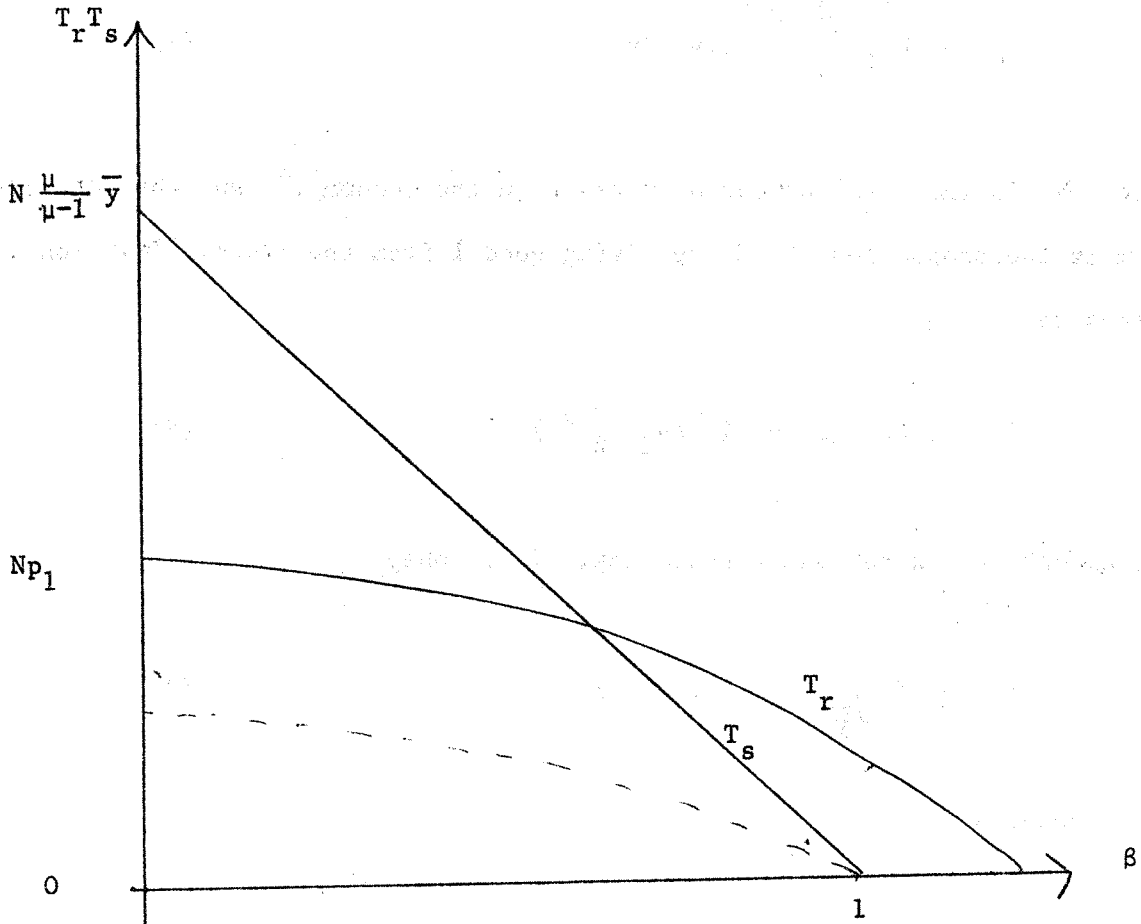


Figure 2 : Tax supply and demand schedules

intercept at  $T_r = Np_1$ , which is below the intercept of the  $T_s$  schedule if

$$\frac{\mu}{\mu - 1} \bar{y} > p_1 \quad (12) ;$$

that is, if the Welfare State is not infeasible (since  $\frac{\mu}{\mu - 1} \bar{y}$  is the average income in the economy).

So, if the economy is rich enough to be able to afford a Welfare State, and not so rich that it doesn't want one, there will be a tax rate,  $1 - \beta$ , between 0 and 1, such that the demand for the services of the state is equal to the supply of tax revenue.

### III. The Dynamics of the Welfare State

Suppose that there is technological advance in the economy, which with money wages held constant, reveals itself in a steady reduction in the unit costs,  $p_1$  and  $p_2$ , of production. Consider the situation of 'balanced' productivity growth; that is, of  $p_1$  and  $p_2$  declining at the same percentage rate. Then the marginal rate of transformation,  $p_1/p_2$ , will remain constant, and so too will the value of  $x_2^*$  in Figure 1. However, the slope,  $p_2$ , of the line  $p_2x_2$  will be decreasing, so that the value of  $y_d^*$  corresponding to the constant  $x_2^*$  will be falling. As growth goes on, fewer and fewer people will choose to take advantage of the services offered by the Welfare State. This trend is reinforced by the effect of a reduced demand for welfare services on the amount of tax needed. As demand falls, so too will  $1 - \beta$ , which will give more people, after tax, incomes greater than  $y_d^*$ . The  $T_r$  curve of Figure 2 shifts downwards, eventually reaching the position of the dotted line, when taxation ceases and the Welfare State is dissolved. <sup>6</sup>

However, looking at a particular example of a Welfare State, the U.K., in which there has been some growth in GNP per head, it is clear that, far from withering away, the relative economic importance of the welfare sector has recently been increasing. We noted in Section I that between 1961 and 1971 the share of public expenditure on welfare services took of national product had risen, and this trend is confirmed in Table 1, which shows the output of the three main welfare services - housing, health and education - at the start and the end of this ten year period.

It does not seem adequate to explain the trend simply in terms of income elasticities of demand - of people becoming more appreciative of good education, health care, and housing as their incomes rise - since the table reveals that there has been, overall, a marked substitution of publicly supplied for privately supplied welfare goods. Between 1961 and 1971 the ratio of private to public activity in education and housing declined markedly (the latter ratio despite a sudden, and probably non-permanent, swing towards private sector building initiated by the Conservative government in 1970) and in hospital services stayed about constant, contrary to the prediction of the balanced growth model of this section.

If the state has been increasing the minimum quantity of good 1 to be consumed, by, for example, raising the school leaving age and tightening building standards, so that  $p_1$  has risen relative to  $p_2$ , without much appreciation on the part of the populace, so that  $\theta_p$  and  $\theta_s$ , or, at least, their difference, have not altered much, then, from Figure 1, it is clear that the effect will have been to raise  $y_d^*$ , and so increase the demands made on the state. As well, as the Welfare State has become established and perhaps, more efficient, some of the stigma attached by the middle classes to the use of its services may have gone, reducing the difference between  $\theta_p$  and  $\theta_s$ .

Table 1 ; U.K. Private and public sector activity; 1961-71

	Private		Public		Ratio Private Public	
	1961	1971	1961	1971	1961	1971
Total number of pupils in nursery, primary and secondary schools; assisted and independent against public sector, thousands :	674	614	8096	9475	0.083	0.065
Permanent dwellings completed; private sector against public sector, thousands :	181	196	122	168	1.476	1.168
Discharges and deaths from NHS hospitals; private against non-private patients, thousands :	84	115	4768	6103	0.018	0.019

Sources : various issues of Social Trends and Annual Abstract of Statistics (HMSO)

These suggestions may have some validity, but they are not pursued further in this paper. In the next section we propose an explanation in terms of 'unbalanced growth', which appears to have an empirical plausibility, and yields some interesting insights into the workings of welfare state systems.

#### IV. Unbalanced Growth

Baumol (1967) pointed out that productivity growth does not proceed at the same rate in all sectors of the economy; in particular, that it tends to be slowest in service and other labour-intensive activities. Since these sectors generate the bulk of publicly provided output - defence, police, sanitation, administration, housing, health and education - there is a tendency for the opportunity cost of providing a given level of public services to increase remorselessly, which Baumol blames for a lot of the worsening financial situation of American public authorities, especially of city governments, which are susceptible to 'vicious circle' declines in their revenue-raising powers. Table 2 gives the percentage increase in some cost and income variables in the U.K., over the decade 1961-1971. If there had been zero productivity growth in a labour-intensive sector we would expect costs in this sector to have risen by about the same amount as the incomes of its labour force. In the U.K. economy as a whole there does appear to have been growth in productivity, since the All Items Index of Retail Prices has gone up by substantially less than either the index of wage rates or of average salaries. However, the welfare goods sector has not been doing so well. The building cost per square foot figure has actually grown faster than the wage rate, as has education expenditure per full-time teacher equivalent compared with the growth in salaries, and in hospitals, the cost per in-patient case seems to have just about matched the index of salaries, on average. Of course, these figures give a rather cursory

Table 2 ; Changes in costs

	Percentage change 1961-71
All items index of retail prices	60
Index of basic weekly wage rates, manual workers.	87.2
Index of average salaries; non-manual employees.	102.7
Cost per square foot (five bedspace house; tenders approved for local authorities).	102.2
Expenditure per full-time teacher equivalent.	118.8
Cost per in-patient case :	
Outside London teaching hospitals	108.1
Non-teaching hospitals	92.8

Sources: various issues of ; Annual Abstract of Statistics, Housing Statistics, Statistics of Education, Health and Personal Social Service Statistics for England and Wales (all HMSO).



view of what has been going on, and factors other than changes in technical productivity, such as material prices in house building, and changes in relativities in the wages of public and private sector employees have probably also been important.

Whatever the reasons, there certainly seems to have been 'unbalanced growth' in costs in the United Kingdom economy over the last ten years or so. What effect should this have had on the Welfare State? Returning to our model, let it now be assumed that  $p_1$  and  $p_2$  fall at different exponential rates  $\rho_1$  and  $\rho_2$  from bases  $p_{10}$  and  $p_{20}$ . The switch-over disposable income,  $y_d^*$  satisfies

$$y_d^{*\delta} - (y_d^* - p_{10} e^{-\rho_1 t})^\delta = \Delta\theta (p_{20} e^{-\rho_2 t})^\delta \quad (13)$$

from (4), with  $\Delta\theta \equiv \theta_p - \theta_s$ . Rearranging (13), and defining the implicit function

$$f(y_d^*, t) \equiv y_d^{*\delta} - (y_d^* - p_{10} e^{-\rho_1 t})^\delta - \Delta\theta p_{20}^\delta e^{-\delta\rho_2 t} = 0 \quad (14),$$

In (14)

$$dy_d^*/dt = -f_t/f_{y_d^*} \quad (15)$$

Now,

$$f_{y_d^*} = \delta (y_d^{*\delta-1} - (y_d^* - p_{10} e^{-\rho_1 t})^{\delta-1}) < 0 \quad (16)$$

since  $0 < \delta < 1$ , and  $y_d^* > p_1$ ;

$$f_t = -\delta\rho_1 (y_d^* - p_{10} e^{-\rho_1 t})^{\delta-1} + \delta\rho_2 \Delta\theta p_{20}^\delta e^{-\delta\rho_2 t} \quad (17)$$

(17) may be positive or negative. The smaller is  $\rho_1$  relative to  $\rho_2$  the more likely it is that (17) is positive. Suppose that we make the assumption, which, in view of Table 2, does not seem unreasonable, that  $\rho_1 = 0$ ; that is, that there is no technological progress in the production of the welfare good. Then  $f_t$  becomes

$$f_t = \delta p_2 \Delta \theta p_{20}^\delta e^{-\delta \rho_2 t} \quad (17)'$$

which is positive. Given (15) and (16), if  $f_t$  is positive so too is  $dy_d^*/dt$ . As unbalanced growth proceeds, more and more people will choose to accept the welfare goods provided by the State. The income effect from a cheaper good 2 as  $\rho_2$  declines is outweighed by the substitution effect of the increasing opportunity cost of units of good 2 sacrificed in order to consume good 1 privately.  $y_d^*$  rises, and fewer people can 'afford' to refuse the services of the Welfare State, this trend being reinforced by the higher tax rates needed to finance more pervasive welfare services. The Welfare State will expand, not wither away.<sup>7</sup>

#### V. Is the Welfare State Worthwhile ?

The set of people whose utilities are higher because of the Welfare State will be a subset of the set of people who use the Welfare State. Might this subset be empty ? Let us suppose that society is restricted to a choice between the sort of Welfare State assumed in this paper and a system with no public expenditure on welfare services (and no taxes), but which retains the laws making consumption of one unit of good 1 compulsory. For this choice to be possible it must be that minimum income  $\bar{y}$ , is greater than  $p_1$ .<sup>8</sup> The possible utility functions are, from (2)

$$U_s = \theta_s + (\beta y / p_2)^\delta \quad (18)$$

$$U_p = \theta_p + ((\beta y - p_1) / p_2)^\delta \quad (19)$$

if there is a Welfare State, and

$$U_p' = \theta_p + ((y - p_1) / p_2)^\delta \quad (20)$$

if there is not. For given  $\beta$ ,  $p_1$  and  $p_2$ , these utility functions are drawn as in Figure 3. The curves are only drawn for non-negative consumption of good 2.

If the minimum income,  $\bar{y}$ , is greater than  $y^*$  we have the situation mentioned in Section II, in which the economy is so rich (and/or income so equally distributed) that there is no demand for the services of the Welfare State. In this case, even if the State should, mistakenly, set up an apparatus for providing free welfare services, it will not be utilised.<sup>9</sup>

If  $\bar{y}$  is less than  $\hat{y}$ , the intersection of  $U_p'$  and  $U_s$ , then the lowest income people are better off with a Welfare State than without it. In this case a Welfare State is 'just' in Rawls' sense - it serves better the interests of the least well-off group.<sup>10</sup>

In the third possible case, when  $\bar{y}$  is between  $\hat{y}$  and  $y^*$ , the Welfare State is neither just nor Pareto efficient. Not only would those with incomes above  $y^*$  who do not use the services of the State in any case be better off without it (and pay no taxes), but so too would the poorest people.

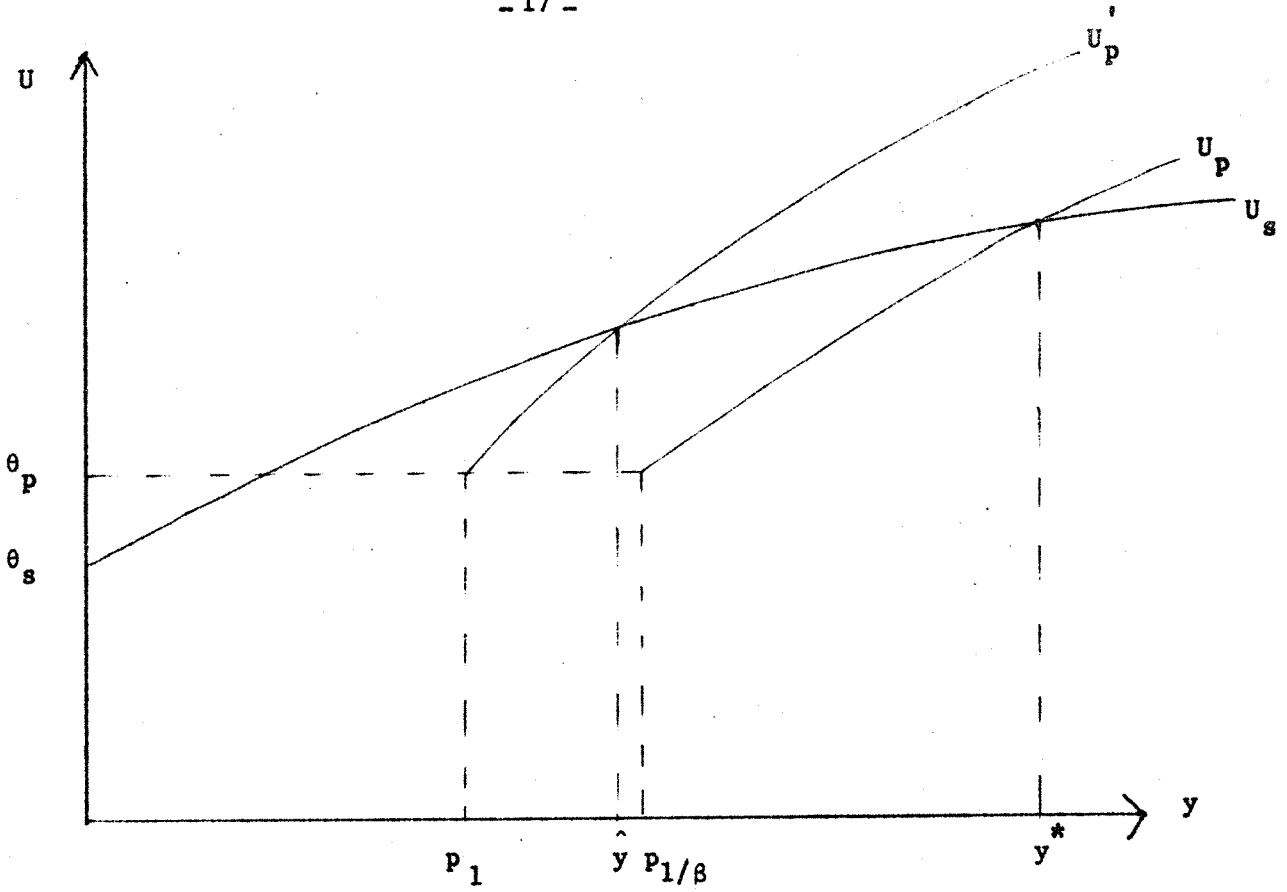


Figure 3 ; The decision to have a welfare state

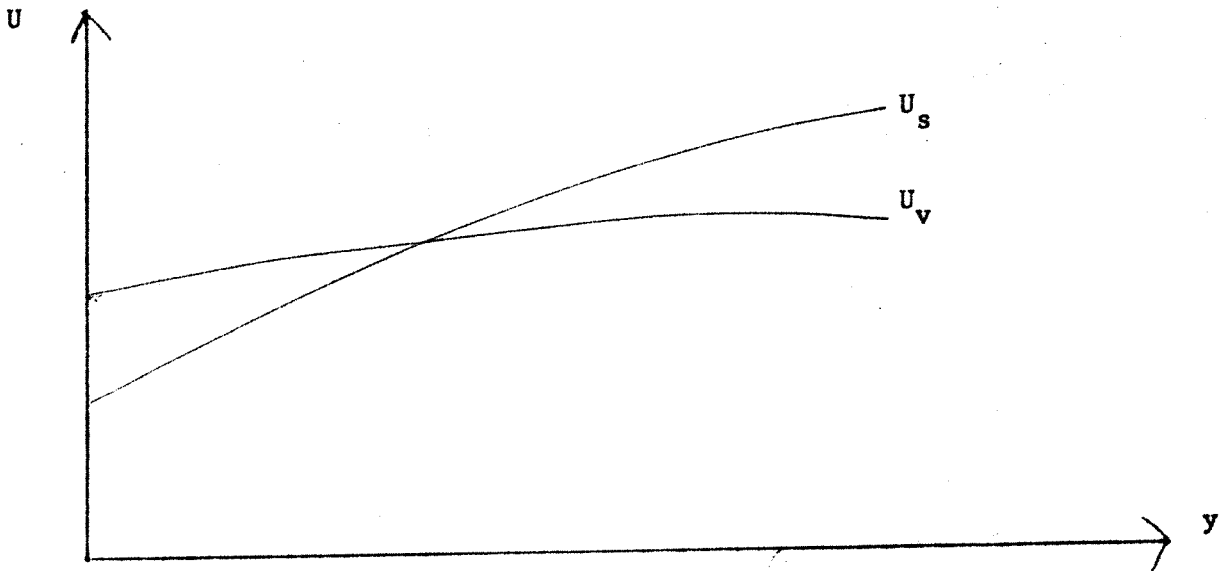


Figure 4 ; Public production and the voucher system

Given the existence of a Welfare State, each person with an income of less than  $y^*$  would use it, since taxes have to be paid whether the service is used or not, but everyone would be happier in a system in which no welfare services were provided free by the State, and no taxes were levied.

Without numerical values of the parameters of the utility functions, we cannot work out precisely where  $\hat{y}$  and  $y^*$  are in relation to  $\bar{y}$ . However, if we made our functions more realistic by assuming that lower income groups are typically unafflicted by middle-class gentility, so that for them  $\theta_s$  is about equal to  $\theta_p$ , and that they pay little or no taxes, so that  $\beta$  is nearly 1 at low  $y$ 's then  $\hat{y}$  would undoubtedly be greater than  $\bar{y}$ , and the Welfare State would not be unjust and inefficient. In any case, it can be shown that as  $p_2/p_1$  falls with unbalanced growth, the intersection between  $U'_p$  and  $U_s$  moves to the right, so that the number of people made better off by the Welfare State increases.

Nevertheless, there is nothing 'progressive' about the Welfare State modelled in this paper. Setting up the welfare system may involve a once-for-all redistribution when taxes are raised to finance it, though Beveridge at least intended that the system be a self-financing insurance scheme -

"The Plan for Social Security .. is... a plan of insurance - of giving in return for contributions, benefits up to subsistence level " 11

- but the effect of unbalanced growth is to continually erode this redistribution as increasingly better off people begin to take advantage of free Welfare State services, increasing the amount of taxes paid by all, including lower income groups, who do not, however, receive an increased quantity of the welfare good in return.

## VI. A Voucher System

An alternative to State production and distribution of welfare goods, at least for housing and education, is for the State to issue vouchers which could be exchanged for certain quantities of specified welfare goods. These could then be produced privately, so generating higher utility to the extent that  $\theta_p$  is greater than  $\theta_s$ . Would this system be 'just' in Rawls' sense, compared with the public production Welfare State? If there is no means test everyone will take up their (free)voucher, so that unless  $p_2$  has become low enough for everyone to use publicly produced welfare goods in the Welfare State system, the tax rate in the latter system,  $(1-\beta_s)$  will be less than the tax rate  $(1-\beta_v)$  in the voucher system.

The utilities  $U_s$  and  $U_v$  of a person whose income,  $y$ , is less than  $y^*$  in the two systems are therefore, as drawn in Figure 4,

$$U_s = \theta_s + (\beta_s y / p_2)^\delta \quad (18)'$$

$$U_v = \theta_p + (\beta_v y / p_2)^\delta \quad (21)$$

The voucher system is more 'just' than the public production system if  $\bar{y}$  is to the left of the intersection of the  $U_s$  and  $U_v$  curves. This is less likely to be so the larger the difference between  $\beta_s$  and  $\beta_v$  - that is, the smaller the proportion of the population who would make use of the services of a public production Welfare State - and the closer  $\theta_p$  and  $\theta_s$  are for low income people. With unbalanced growth, however, the difference between  $\beta_s$  and  $\beta_v$  will be decreasing so that the intersection point will eventually move to the right of  $\bar{y}$ , by which time the voucher system will generate higher utilities for the least well-off members of society.

When  $p_2$  becomes so low that everyone in the society would choose to make use of the Welfare State, were its services available, then  $\beta_s = \beta_v$ , and there is no intersection between  $U_s$  and  $U_v$ . The voucher system would be Pareto superior - it would generate higher utility for all.

Against this conclusion it can be argued that vouchers for education would not be in the interests of lower income groups, since they would probably extend the present system, whereby the middle and upper classes perpetuate inequality through their domination of the grammar and 'public' school systems, and hence the universities and other points of access to well paid and agreeable work. The strength of this objection must be admitted, although it can be pointed out that in a more fundamentally egalitarian system than, say the U.K. at present, income differentials would be narrower, and determined so that the private returns from investing in a 'good' or lengthy education would be much lower than they are now.

VII. Summary and Conclusions

In this paper a simple model of a Welfare State was used to explain the choices made by individuals that determine the relative importance of the public and private sector in the provision of welfare goods. Figures were shown which suggested that the proportion of people in the economy making use of the services of the Welfare State has been increasing recently in the United Kingdom, and an explanation of this in terms of 'unbalanced growth' - below average productivity growth in the production of welfare goods and services - was put forward.

The analysis suggests that, although the Welfare State may be of some net benefit to recipients of the lowest incomes, its redistributory effects are weakened as unbalanced growth proceeds. It is possible that an alternative 'voucher' system would make everyone happier.



## Footnotes

1. The author would like to thank participants in the Warwick Economics Staff Seminar for their comments and suggestions.
2. The numbers are calculated from information in the 1971 Annual Abstract of Statistics (HMSO). Expenditure on housing and social services is net of local authorities' rental income from dwellings.
3. This distribution function appears, at best, to be accurate only for the upper tail of the income distribution. For a critique of its use, and a counter-proposal, cf. Harrison (1974).
4. Recent work by Green and Sheshinski (1972) and Arrow (1971) has made the supply to individuals of educational services a variable to be set optimally. These interesting analyses seem most relevant to the provision of tertiary education. For other welfare services, our assumption that supply is set with reference to social and political 'norms' is probably quite realistic.
5. Having a finite number of workers is not strictly consistent with the continuous distribution function  $f(y)$ .
6. It can be shown that drawing the  $T_r$  curve which takes value 0 at  $\beta = 1$  with a slope less steep than that of the  $T_s$  curve at this point implies that  $\mu p_1 N$  is less than national income. Since the value of  $\mu$  seems to be, at most, no more than about 2, and  $p_1 N$  - total expenditure on welfare goods - is well below  $\frac{1}{2}$  of GNP, the assumption seems reasonable.
7. This prediction is sensitive to the choice of utility function. If  $U$  were the multiplicative
$$U_s = \theta_s x_1^\beta x_2^\delta, \quad U_p = \theta_p x_1^\beta x_2^\delta,$$
we can show that the switchover value of  $y_d^*$ ,  $y_d^*$ , is not a function of  $p_2$ , and so is not affected by unbalanced growth. Our defense for using (2) can only be the inductive one that it does generate empirically valid predictions.
8. A useful definition of  $\bar{y}$  must be something like 'the full-time earnings of an unskilled manual worker', rather than the zero or negative figures associated with a few permanently unemployed or bankrupt unfortunates.
9. So that  $\beta = 1$ , and  $U_p$  is the same as  $U'_p$ .
10. cf. Rawls (1967) Phelps (1973a).
11. Beveridge (1942), page 2. Peacock and Browning (1954) discuss the proposition of Weaver (1950) that the poor pay for their welfare services.

## Bibliography

- Arrow, K.J., 1971, 'The Utilitarian approach to the concept of equality in public expenditure', Quarterly Journal of Economics, 1971, pp. 409-415, reprinted in Phelps, 1973b.
- Atkinson, A.B., 1973a, 'How Progressive should income-tax be?', in M. Parkin (ed.), Essays on Modern Economics, Longman, reprinted in Phelps, 1973b.
- , 1973b, (ed.), Wealth, Income and Inequality, Penguin.
- Baumol, W.J., 1967, 'Macroeconomics of Unbalanced Growth : The Anatomy of Urban Crises', American Economic Review, June, pp. 415-26.
- Beveridge, W., 1942, Social Insurance and Allied Services, HMSO.
- Diamond, P.A. and J. A. Mirrlees, 1971, 'Optimal taxation and public production II', American Economic Review, June, pp. 261-278.
- Green, Jerry R. and Eytan Sheshinski, 1972, 'A Note on the Progressivity of Optimal Public Expenditures', Stanford Institute for Mathematical Studies in the Social Sciences Technical Report No. 65.
- Harrison, A.J., 1974, 'Inequality of Income and the Champernowne Distribution', paper presented to the Association of University Teachers of Economics Annual Conference, Manchester, April 1974.
- Lydall, H.F., 1968, The Structure of Earnings, Oxford University Press.
- Mirrlees, J.A., 1971, 'An exploration in the theory of optimum income taxation', Review of Economic Studies, April, pp. 175-208.
- Nicholson, R.J., 1967, 'The distribution of personal income', Lloyds Bank Review, No. 83, pp. 11-21; reprinted in Atkinson, 1973b.
- Peacock, Alan T. and P. R. Browning, 1954, 'The Social Services in Great Britain and the Redistribution of Income', in Alan T. Peacock (ed.), Income Redistribution and Social Policy, Jonathan Cape.
- Phelps, E.S., 1973a, 'Taxation of wage income for economic justice', Quarterly Journal of Economics, August, pp. 331-354.
- , 1973b, (ed.), Economic Justice, Penguin.

Rawls, J., 1967, 'Distributive justice', in Phelps, 1973b.

Weaver, Findlay, 1950, 'Taxation and redistribution in the United Kingdom',  
Review of Economics and Statistics, pp. 201-13.

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