MANAGERIAL UTILITY MAXIMISATION UNDER UNCERTAINTY[1]

by

GEORGE YARROW

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The recent economic literature contains several theories of the Firm in which the objectives of management and shareholders differ, perhaps the best known being those put forward by Baumol {2} Marris {6} {7} and O.E. Williamson {13} {14}(2). The fruitfulness of these new theories of the firm is still a matter of dispute, largely because many of their predictions are qualitatively similar to those derived from profit maximisation models, thus rendering the distinction between them unnecessary for many purposes {11} and making it difficult to devise convincing tests of the alternative hypotheses. In these circumstances there is a good case to be made for the profit maximising assumption on the grounds of its simplicity. It is argued below however, that many of the difficulties with managerial theories of the firm are due more to the unrealistic way in which the constraints on discretionary behaviour have been formulated than to the particular variables included in the objective functions. The predictions of the two models discussed are shown to be very sensitive to the form assumed by the constraint, indicating that if more attention is paid to this aspect of the problem it will be possible to derive results more sharply differentiated from those of profit maximising models(3).

The approach followed is to reformulate the managerial theories as problems of utility maximisation under uncertainty, the uncertainty arising from the assumption that the functional relationship between the security of managers and the policies they follow takes a probabilistic rather than a deterministic form (section 1). This makes possible a definition of managerial discretion and a discussion of the difficulties involved in trying to define what is meant by increases and decreases in discretion (section 2). Using the new approach the sales revenue maximisation model is examined and some of its comparative statics properties are shown to differ substantially from those of Baumol's original version. Further, the effect on the optimum output level of changes in discretion is split into 'security' and 'price' effects and is shown to be ambiguous in sign. i.e. an increase in discretion may lead to either an increase or a decrease in the firm's output (section 3). Finally predictions are derived from the Marris' growth maximisation model which again differ from those of the original formulation. The effect of changes in discretion on the growth rate is also ambiguous in sign (section 4).

(1) Alternative approaches to Managerial Utility Maximisation

In contrast to the amount written on the appropriate variables for inclusion in the managerial utility function, there have been relatively few suggestions about the functional form of the constraints on managerial behaviour[4]. Most models are based on lexicographic utility functions so that they can be expressed mathematically as:

$$\text{maximise} \quad f(\underset{\sim}{x}) \quad \text{such that} \quad g(\underset{\sim}{x}) \geq c$$

where $\underset{\sim}{x}$ is the vector of variables which contribute to utility and c is a constant. The 'constraint' $g(\underset{\sim}{x}) \geq c$ arises from the interests of shareholders. Typically $g(\underset{\sim}{x})$ is the level of profits or market value of the equity of the firm and it is assumed that there is a minimum safe value of this variable given by c. If $g(x) \geq c$

then managers expect with certainty to lose their positions because of either:

(a) a take over induced by the low price of the firm's shares, or

(b) a direct intervention to remove them by the shareholders or their representatives.

If $g(\underset{\sim}{x}) \geq c$ then managers feel that their jobs are perfectly secure. The appropriateness of lexicographic utility functions has been questioned by Borch {3} and Rosenberg {10} because they do not allow a smooth trade off between security and the values of the decision variables $\underset{\sim}{x}$. Intuitively the assumption of a smooth trade off would seem to be more realistic. One way round this problem suggested by Marris is to include $g(x)$ as a proxy variable for security in an objective function $f(\underset{\sim}{x}, g(\underset{\sim}{x}))$ which has the desired continuity properties. The managers would then simply maximise $f(\underset{\sim}{x}, g(\underset{\sim}{x}))$. Implicitly Marris, Borch and Rosenberg are assuming that the relationship between security and the decision variables is probabilistic, at least as perceived by management[5]. Otherwise, there would be some value of $g(\underset{\sim}{x})$, known to management, below which a take over or shareholder intervention would occur and a lexicographic utility function would be appropriate[6]. Assuming that such a probabilistic relationship exists [7], it follows that the problem should be set up directly as one of utility maximisation under uncertainty, rather than incorporating a security proxy in the utility function. It will also be shown below that this makes the derivation of predictions from the models much easier.

Consider therefore the following situation. Suppose managers are concerned only with the present period and assume that there is a probability p that they will be removed from office at the beginning of the period which depends on the level of profits they plan to make[8]. i.e. $p = p\,[g(\underset{\sim}{x})] = p(\underset{\sim}{x})$. $1 - p$ will be defined as the security of management and $S(\underset{\sim}{x}) = 1 - p(\underset{\sim}{x})$ will be called the managerial security function (MSF). If they are not sacked it is assumed that the utility of managers is a function of the variables $\underset{\sim}{x}$, say $U(\underset{\sim}{x})$ where $\frac{\partial U}{\partial x_i} > 0$, while if they are sacked their utility will be some constant $\bar{U}$. Without loss of generality $\bar{U}$ can be set equal to zero[9]. Under these conditions it can be assumed that managers maximise their expected utility:-

$$E\,[U(\underset{\sim}{x})] = U(\underset{\sim}{x})\,(1 - p(\underset{\sim}{x}) + \bar{U}.\,p(\underset{\sim}{x})$$

$$= U(\underset{\sim}{x})\,S(\underset{\sim}{x}) \qquad \ldots\ldots\ldots\ldots(1)$$

If the functions have the necessary continuity properties the first order conditions for a maximum are S when $\underset{\sim}{x}$ is a n-vector[10];

$$0 = \frac{\partial U}{\partial x_i}\,S(\underset{\sim}{x}) + \frac{\partial S}{\partial x_i}\,U(\underset{\sim}{x}) \qquad i = 1,2,\ldots.n$$

$$\ldots\ldots\ldots\ldots(2)$$

Generalising the problem to the many period case the elements of the choice set become stochastic processes. For the purposes of this paper the following simplifying assumptions will be made:

(a) the function $p(\underset{\sim}{x})$ does not shift from period to period[11]

(b) $\underset{\sim}{x}$ is chosen at the beginning of the first period and remains constant thereafter [12]

(c) management maximise the discounted present value of the expected utility in each period [13] $V(\underset{\sim}{x}, T, i)$ where:

$$V(\underset{\sim}{x}, T, i) = \sum_{t=0}^{T-1} U(\underset{\sim}{x})\, S(\underset{\sim}{x}) \left(\frac{S(\underset{\sim}{x})}{1+i}\right)^t \qquad \ldots\ldots.(3)$$

where T is the management time horizon

and i is the management discount rate.

More complex models can be produced by relaxing some or all of the above assumptions.

The first order conditions for a maximum are:

$$\frac{\partial V(x, T, i)}{\partial x_i} = \sum_{t=0}^{T-1} \left[\frac{\partial U}{\partial x_i} S + (t+1) \frac{\partial S}{\partial x_i} U \right] \frac{S}{1+i}^t$$

$$= 0 \qquad \text{for } i = 1, 2, \ldots.n. \qquad \ldots\ldots\ldots(4).$$

(2) <u>The meaning of Managerial Discretion</u>

Before going on to consider specific models of the firm it is necessary to discuss the shape of the security function and relate it to the concept of managerial discretion. If $g(\underset{\sim}{x})$ is the level of profits or market value of the firm's equity then $\underset{\sim}{x}^*$ will be the profit maximising vector of $g(\underset{\sim}{x}^*) \geq g(\underset{\sim}{x})$ for all $\underset{\sim}{x}$. The simplest assumptions to make about the security function is that it has a maximum at $\underset{\sim}{x}^*$ and that $S(\underset{\sim}{x}^*) > 0$ [14]. In other words, managers are most secure when they are following profit maximising politices. [15] Managerial discretion can then be defined in the following way:

Management is said to have discretion if there exists at least one policy $\underset{\sim}{x} \neq x^*$ such that $S(\underset{\sim}{x}) > 0$. i.e. discretion is the ability to pursue non profit maximising objectives without the consequence of certain and immediate dismissal.

Three security functions are shown below for the case in which only one variable x enters the utility function. They indicate respectively situations in whihh (i) there is no discretion (ii) discretion exists (iii) the MSF approach is equivalent to a lexicographic objective function.

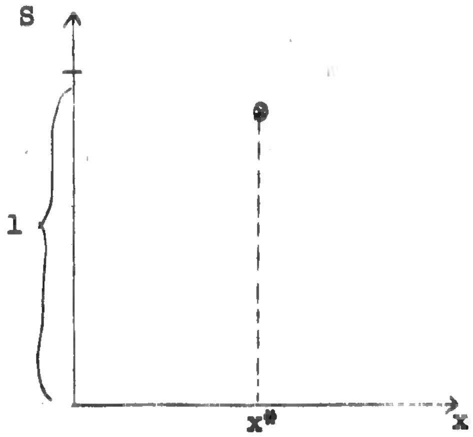


Fig. (i)

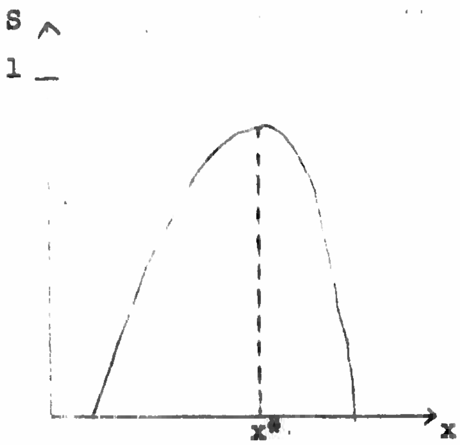
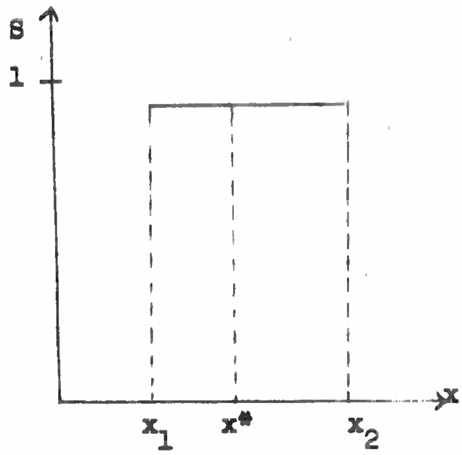


Fig (ii)



$$C = g(x_1) = g(x_2)$$

Fig. (iii)

In each case x^* is the profit maximising value of x.

While the definition of discretion is straightforward it is not so obvious how an increase in discretion should be defined. With lexicographic utility functions it is usually taken to mean a reduction in the minimum safe level of profits or share price, C. The nearest definition to this using the MSF would be to say that an increase in discretion is an enlargement of the set of $\tilde{x}$'s for which $S(\tilde{x}) > 0$. Comparing two situations A and B with MSF's $S^A(\tilde{x})$ and $S^B(\tilde{x})$ respectively, management would be said to have more discretion in situation A if $\{ \tilde{x} \mid S^B(\tilde{x}) > 0 \}$ were a proper subset of $\{ \tilde{x} \mid S^A(\tilde{x}) > 0 \}$.

However, this definition leads to immediate problems. Comparing $S^A(\tilde{x})$ and $S^B(\tilde{x})$ in figure (iv) it implies that the former allows more discretion than the latter for all $\varepsilon >$ no matter how small, a ranking which is at variance with the idea of increased discretion as an expansion of the choice set (which consists of all points on or below the MSF). To avoid this problem the following stronger definition of discretion will be adopted.

Comparing two situations A and B with MSF's $S^A(\tilde{x})$ and $S^B(\tilde{x})$ managers will be said to have more discretion in situation A if $S^A(\tilde{x}) \geq S^B(\tilde{x})$ with strong inequality at least one point.

Clearly the definition does not give an ordering over all security functions because if two cross for example, it will not be possible to say that one allows more discretion than the other. In [illegible] below the analysis will be restricted to shifts in the

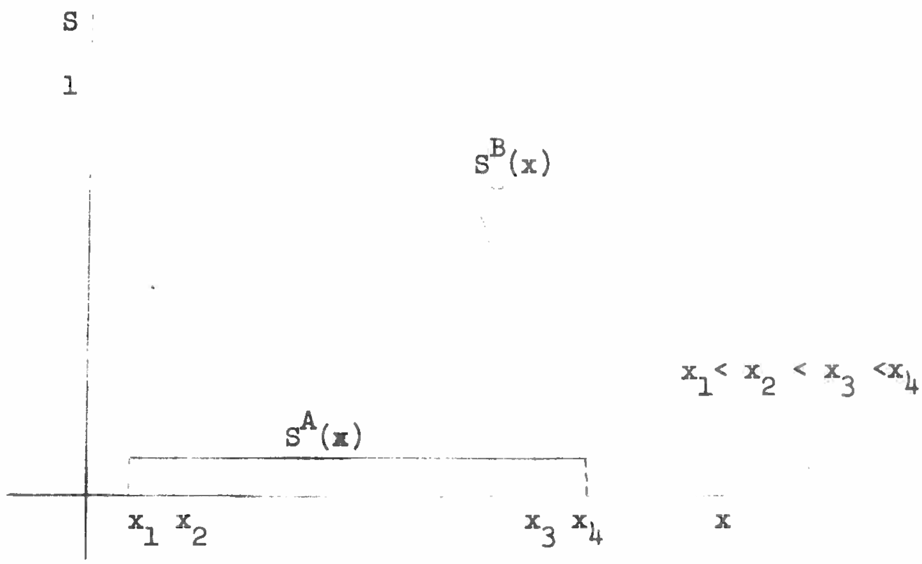


Fig. (iv)

security function which increase its value in the neighbourhood of the optimum value of $\underset{\sim}{x}$. Specifically S will be assumed to be a function of $\underset{\sim}{x}$ a shift parameter where $S(p, \bar{Z}) \in C[2]$,

$\frac{\partial S}{\partial \bar{Z}} > 0$ for values of $\underset{\sim}{x}$ in the neighbourhood of the optimum point. $\frac{\partial S}{\partial \bar{Z}} \geqslant 0$ everywhere. An increase in $\bar{Z}$ therefore implies an increase in discretion as defined above.

(3) <u>Sales Revenue Maximisation</u>

Baumol has put forward the hypothesis that managers maximise the sales revenue of their firm subject to a minimum profits constraint, from which it can be deduced that:

(i) an increase (decrease) in the rate of a proportional profits tax causes the firm to decrease (increase) output.

(ii) an increase (decrease) in overhead costs, or a lump sum profits tax, causes the firm to decrease (increase) output.

The results are crucially dependent on the assumption that the changes in tax and costs do not affect the minimum safe level of profits. Result (i) can be shown to be reversed using the lexicographic utility function when more realistic assumptions about the constraint are made {15}. Here we shall examine the effects of tax and overhead costs changes using the decision making under uncertainty approach with a security function of the type shown in figure (ii).

Thus suppose managers have a utility function $U(R)$, $U'(R) > 0$, $U''(R) < 0$[16] where R is sales revenue = $p(q)$, q. q being the output level of the firm and p the price per unit of that output. It will be assumed that the probability that managers lose their jobs is a function of the losses $L(q)$ that are attributable to the pursuit of non profit maximising policies[17]. Consider first a one period model Then:

$$L(q) = (1-t)\ (\Pi\ (q^*) - \Pi\ (q)) \ \ldots\ldots\ldots\ldots\ldots\ (5)$$

where t is the rate of proportional profits tax

and $\Pi(q)$ is the level of profits at output q.

$\Pi(q^*)$ is the maximum level of profits

$$p = p\ \left[\, L(q),\ \cancel{z}\, \right] \ , \qquad \frac{\partial p}{\partial L} > 0$$

It is assumed that Π and R are strictly concave functions of q and that $S = 1-p$ is a strictly concave function of L in the interval given by $S(L,\cancel{z}) > 0$[18]. It follows that S and therefore U.S are strictly concave functions of q in the interval given by $S(L, \cancel{z}) > 0$.

Managers maximise:

$$V(q,t,\bar{Z}) = U\left[R(q)\right] \; S\left[(1-t)(\Pi(q^*) - \Pi(q)), \bar{Z}\right] \quad .....(6)$$

The first order conditions for a maximum are:

$$\frac{\partial V}{\partial q} = \frac{dU}{dq} S + U \frac{\partial S}{\partial q}$$

$$= \frac{dU}{dR} \frac{dR}{dq} S + U \frac{\partial S}{\partial L} \frac{dL}{dq} \quad ....(7)$$

$$= 0$$

and the above assumptions guarantee that it is a maximum and not a minimum.

Now at the profit maximising output level:

$$\frac{dR}{dq} = \text{marginal cost} > 0$$

and

$$\frac{\partial L}{\partial q} = -(1-t)\frac{d\Pi}{dq} = 0$$

$$\therefore \quad \frac{\partial V}{\partial q} > 0.$$

But since V is a strictly concave function of q this implies that the sales revenue maximising output level $\bar{q}$ say, is greater than the profit maximising output level, as expected.

At $\bar{q}$:

$$\frac{\partial S}{\partial q} = -\frac{\partial S}{\partial L} \frac{d\Pi}{dq} (1-t)$$

$$> 0 \quad ....(8)$$

and therefore from (7)

$$\frac{dR}{dq} > 0 \qquad \qquad ....(9)$$

Marginal revenue is positive at the optimum level of output and therefore $\bar{q}$ is less than the output level which maximises R, say q_1.

The optimum output level can be determined geometrically from the point of tangency between the security function and an iso - V curve in the security/output plane, as shown below in fig. (V)[19].

Now consider the effects of small increases in (a) the rate of profits tax t, (b) overhead costs (c) the shift parameter Z indexing an increase in discretion,

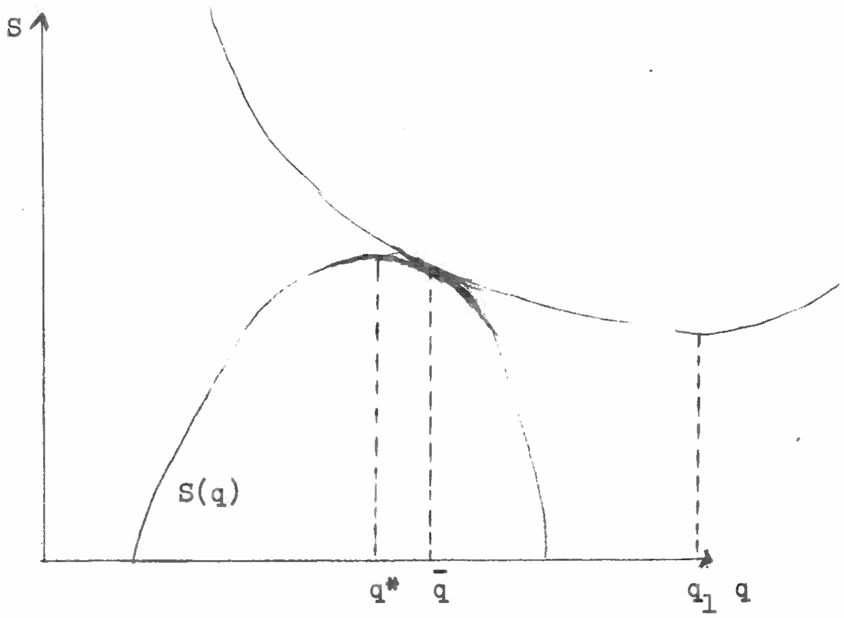


Fig. (v)

Differentiating (7) totally with respect to t gives:

$$\frac{\partial^2 V}{\partial q^2} \frac{d\bar{q}}{dt} + \frac{\partial^2 V}{\partial t \, \partial q} = 0 \quad \text{at } \bar{q} \quad ....(10)$$

and since

$$\frac{\partial^2 V}{\partial q^2} < 0$$

$$\text{sign} \left(\frac{d\bar{q}}{dt} \right) = \text{sign} \left(\frac{\partial^2 V}{\partial t \quad \partial q} \Big|_{\bar{q}} \right) \quad .. (11)$$

Differentiating $\frac{\partial V}{\partial q}$ partially with respect to t.

$$\frac{\partial^2 V}{\partial t \partial q} = \frac{dU}{dR} \frac{dR}{dq} \frac{\partial S}{\partial t} + U \frac{\partial^2 S}{\partial t \, \partial q} \quad .. (12)$$

Now

$$\frac{\partial S}{\partial t} = - \frac{\partial S}{\partial L} \left(\Pi \left(q^*\right) - \Pi \left(q\right)\right)$$

$$> 0 \quad \text{at } \bar{q} \text{ by the concavity of}$$

S with respect to L .... (13)

and

$$\frac{\partial^2 S}{\partial t \partial q} = \frac{\partial^2 S}{\partial q \partial t}$$

$$= + \frac{\partial^2 S}{\partial L^2} (1-t) \frac{d\Pi}{dq} \left(\Pi(q^*) - \Pi(q)\right)$$

$$+ \frac{\partial S}{\partial L} \frac{d\Pi}{dq} > 0 \quad \text{at} \quad \bar{q} \quad ... (14)$$

(9), (13), and (14) show that:

$$\frac{\partial^2 V}{\partial t \partial q} \Big|_{\bar{q}} > 0$$

and hence

$$\frac{d\bar{q}}{dt} > 0 \quad ..... (15)$$

i.e. increases in the profit tax rate lead to increases in output, the reverse of result (i) quoted above. The change in the tax rate would not of course affect the output level of a profit maximising firm.

What happens is that at a given level of output the losses L(q) caused by non profit maximising policies are lower the higher is the tax rate, thus shifting the MSF upwards ($\frac{\partial S}{\partial t} > 0$). (In the limit of a 100% profits tax there would be no reason for shareholders to intervene at any q such that $\Pi(q) \geq 0$). The increase in the tax rate also has the effect of increasing the slope of the MSF at each rate of output q ($\frac{\partial}{\partial t} \frac{\partial S}{\partial q} > 0$) thus reducing the cost (in terms of security) of extra output. The combination of both these effects leads to an increased output level.

(b) A change in overhead costs (or a lump sum profits tax) has no effect on L(q). None of the functions are shifted and there is therefore no change in the optimum output level. The result is the same as for a profit maximising firm but again differs from the prediction derived from the Baumol formulation of sales revenue maximisation (ii).

(c) Differentiating (7) totally with respect to Ξ and using $\frac{\partial^2 V}{\partial q^2} < 0$ we have that :

$$\text{sign}\left(\frac{d\bar{q}}{d\Xi}\right) = \text{sign}\left(\frac{\partial^2 V}{\partial \Xi \partial q} \Bigg|\, q = \bar{q}\right) \quad \ldots\ldots (16)$$

Differentiating $\frac{\partial V}{\partial q}$ partially with respect to Ξ:

$$\frac{\partial^2 V}{\partial Z \partial q} = \frac{dU}{dR} \frac{dR}{dq} \frac{S}{Z} + U \frac{\partial^2 S}{\partial Z \partial q} \qquad \text{......(17)}$$

By assumption $\frac{\partial S}{\partial Z} > 0$ and therefore from (9) the first term on the right hand side of (17) is positive. However, without knowledge of $\frac{\partial^2 S}{\partial Z \partial q}$ the sign of $\frac{\partial^2 V}{\partial Z \partial q}$ is intermediate.

Substituting for $\frac{dU}{dR} \frac{dR}{dq}$ from (7) into (17) gives at $\bar{q}$:

$$\frac{\partial^2 V}{\partial Z \partial q} = - \frac{U}{S} \frac{\partial S}{\partial q} \frac{\partial S}{\partial Z} + U \frac{\partial^2 S}{\partial Z \partial q}$$

$$= \frac{US}{q} \frac{\partial}{\partial Z} \left[\frac{q}{S} \cdot \frac{\partial S}{\partial q}\right] \qquad \text{.......(18)}$$

$$\therefore \text{sign} \left(\frac{d\bar{q}}{dZ}\right) = \text{sign} \left\{\frac{\partial}{\partial Z} \left[\frac{q}{S} \frac{\partial S}{\partial q}\right]\right\} \qquad \text{....... (19)}$$

An increase in discretion, as defined above and of the type represented by an increase in Z, therefore leads to an increase, no change or a decrease in the optimum output level according as to whether the elasticity of the security function is increased, unaffected or decreased by the change in discretion.

In equation (17) the first and second terms onthe right hand side, each divided by $-\frac{\partial^2 V}{\partial q^2}$, can be regarded respectively as 'security' and 'price' effects. The security effect is the change in output induced by a shift in the MSF (greater or lesser security) when there is no change in the slope of the MSF, so that there is no change in the security/output trade off. For the one period model we have shown that the security effect is always positive. Now the slope of the MSF at any point is minus the marginal cost of output in terms of security, or minus the price of marginal output in terms of security. $-\frac{\partial}{\partial Z}\frac{\partial S}{\partial q}$ is therefore simply the rate of change of the price of output with respect to discretion. The 'price' effect could be split further into the normal income and substitution effects of consumer theory, but we are not concerned with that problem here. The point to note is that, in the one period model, holding the level of security constant at $\bar{q}$, a reduction (increase) in the price of output always leads the firm to increase (decrease) its output level.

The multi-period version of this model, in which there is no growth and managers maximise:

$$\sum_{t=0}^{T-1} U(q) \quad S(q)\left(\frac{S}{1+i}\right)^{t}$$

can be shown to produce the following results:

(i) ceteris paribus an increase in the time horizon of managers (T) leads to a decrease in output,

(ii) ceteris paribus an increase in the managers discount rate (i) leads to a decrease in output,

(iii) when the time horizon is infinite the security effect is negative iff $1 + i - 2S < 0$,

(iv) ceteris paribus a reduction (increase) in the 'price' of output $-\frac{\partial S}{\partial q}$ always leads to an increase in output

(v) Increases in discretion may lead to increases or decreases in output,

(vi) When the time horizon is infinite an increase in the tax rate can lead to either increased or decreased output if $1 + i < 2S$. If

$1 + i \geq 2S \quad \frac{d\bar{q}}{dt} \geq 0.$

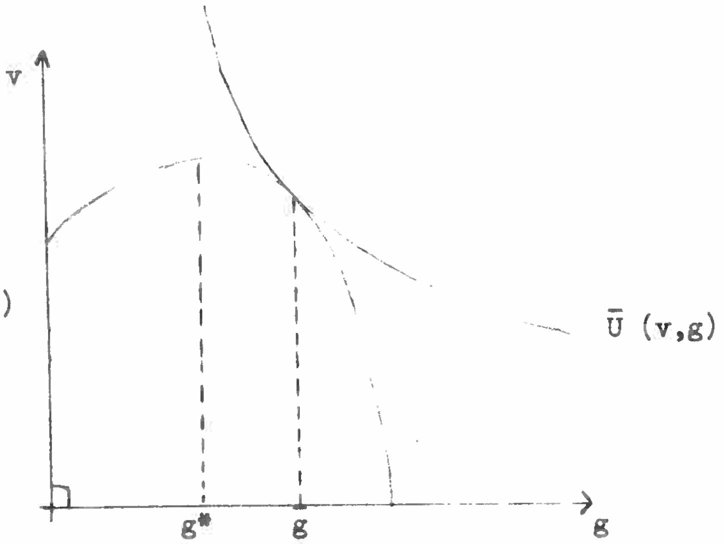
The proofs are not given here since, except for (vi) which follows readily from (iii) and (iv) (see note 32), they are exactly equivalent to those in section (4) on the growth maximisation model, where the results have more interesting consequences.

(4) Growth Maximisation [20]

It is now assumed that the initial output, capital stock, labour force, etc. of the firm are all fixed and the problem for management is to choose between alternative steady state growth rates of the firm. There is a relationship between the valuation ratio v, defined as the ratio of the market value of the firm's equity to the book value of its net assets, and the steady state rate of growth of the firm, which is of the form shown in figure (vi).

To avoid the problems associated with a lexicographic utility function, Marris assumes that managers maximise a 'smooth' function $U(g,v)$, where v is included as a proxy for security. The optimum growth rate is then $\bar{g}$ (see fig. (vi)) which is greater than g^* the profit maximising growth rate.

fig.(vi)



In addition to covering up the decision making under uncertainty aspect of the problem, the limitation of the approach is that it provides no answers to questions such as:

What is the effect on $\bar{g}$ if managers discount the future at a lower rate?(21)

What is the effect on $\bar{g}$ when discretion is increased?(22)

On the other hand answers to these questions are possible if the approach of section (2) is adopted. Thus assume that managers maximise:

$$V(g, T, i, Z) = \sum_{t=0}^{T=1} U(g).\ S(g, Z) \left(\frac{S(g, Z)}{1 + i}\right)^t \qquad .....(20)$$ (23)

where $U'(g) > 0$

T & i are defined as in (3)

$S(g, Z)$ has a maximum in g at $S(g^*, Z)$ for all Z(24)

It will be assumed that $V(g, T, i, Z)$ is strictly concave function of g for all values of the other variables (i.e. $\frac{\partial^2 V}{\partial g^2} < 0$) and that $S(g, Z)$ is a strictly concave function of g for all Z. We are then able to derive the following results:

(a) The longer is the time horizon of managers, ceteris paribus, the lower is the optimum growth rate.(25) (26)

The proof is by induction. Let the optimum growth rate when the time horizon is T be $\bar{g}$ (T) (when i & Z are fixed at some constant levels).

We first prove that if $\bar{g}\,(T-1) < \bar{g}\,(T-2)$ then $\bar{g}\,(T) < \bar{g}\,(T-1)$.

Assume therefore that:

$\bar{g}\,(T-1) < \bar{g}\,(T-2)$ ..... (21)

From (20):

$$V(g,\ T-1) = V(g,\ T-2) + US\left(\frac{S}{1+i}\right)^{T-2} \quad ..... \ (22)$$

for $T \geq 2$

Differentiating (22) with respect to g:-

$$\frac{\partial V(g,\ T-1)}{\partial g} = \frac{\partial V(g,\ T-2)}{\partial g} + \left(\frac{S}{1+i}\right)^{T-2} \left(\frac{dU}{dg} S + (T-1)U \frac{\partial S}{\partial g}\right)$$

Now $\left.\frac{\partial V(g,\ T-1)}{\partial g}\right|_{\bar{g}(T-1)} = 0$ ..... (24)

$\left.\frac{\partial V(g,\ T-2)}{\partial g}\right|_{\bar{g}(T-2)} = 0$ ..... (25)

& $\bar{g}(T-1) < \bar{g}(T-2)$ by assumption

Therefore since $\frac{\partial^2 V(g,\ T-2)}{\partial g^2} < 0$

$\left.\frac{\partial V(g,\ T-2)}{\partial g}\right|_{\bar{g}(T-1)} > 0$ ..... (26)

Substituting (24) & (26) in (23) shows that:

$\frac{dU}{dg} S + (T-1)U \frac{\partial S}{\partial g} < 0$ at $\bar{g}(T-1)$ ..... (27)

By assumption $\frac{dU}{dg} > 0$ & $\therefore \frac{dS}{dg} < 0$ at $\bar{g}(T-1)$ (27)..... (28)

$$\therefore \frac{dU}{dg} S + TU \frac{\partial S}{\partial g} < \frac{dU}{dg} S + (T-1)U \frac{\partial S}{\partial g} < 0 \text{ at } \bar{g} \, (T-1) \ldots\ldots (29)$$

From (23):

$$\frac{\partial V(T, g)}{\partial g} = \frac{\partial V(T-1, g)}{\partial g} + \left(\frac{S}{1+i}\right)^{T-1} \left(\frac{dU}{dg} S + TU \frac{\partial S}{\partial g}\right) < 0$$

at $\bar{g}$ (T-1) using (24) & (29) ..... (30)

Since $\frac{\partial^2 V(g, T)}{\partial g^2} < 0$ it follows that:

$\bar{g}$ (T) < $\bar{g}$ (T-1) ..... (31)

The next step in the proof is to show that $\bar{g}(2) < \bar{g}(1)$.

From (20):

$$V(g, 2) = V(g, 1) + US\left(\frac{S}{1+i}\right) \ldots\ldots (32)$$

$$\therefore \frac{\partial V(g, 2)}{\partial g} = \frac{\partial V(g, 1)}{\partial g} + \left(\frac{S}{1+i}\right) \left(\frac{dU}{dg} S + 2U \frac{\partial S}{\partial g}\right)$$

$$= \left(\frac{S}{1+i}\right) \left(\frac{dU}{dg} S + 2U \frac{\partial S}{\partial g}\right) \text{ at } \bar{g} \, (1) \text{ using } (25) \ldots\ldots (33)$$

Now $V(g, 1) = US$

$$\therefore \frac{dU}{dg} S + U \frac{\partial S}{\partial g} = 0 \text{ at } \bar{g} \, (1) \ldots\ldots (34)$$

Again by assumption:

$$\frac{dU}{dg} > 0 \;\&\; \therefore \frac{\partial S}{\partial g} < 0 \text{ at } \bar{g} \, (1) \ldots\ldots (35)$$

$$\therefore \frac{dU}{dg} S + 2U \frac{\partial S}{\partial g} < \frac{dU}{dg} S + U \frac{\partial S}{\partial g} = 0 \text{ at } \bar{g} \text{ (1) } \ldots.. (36)$$

$$\therefore \left. \frac{V(g, 2)}{g} \right|_{\bar{g}(1)} < 0$$

and by the strict concavity of V(g, 2) it follows that:

$$\bar{g}\ (2) < \bar{g}\ (1)$$

Hence $\bar{g}$ (T) < $\bar{g}$ (T-1) for all T

It can be shown on the other hand that if managers maximise profits then an increase on the shareholder's time horizon leads to an <u>increase</u> in the optimum growth rate. Since the proof is very similar to the above it has been relegated to the appendix.

(b) As the rate at which management discount the future rises the optimum growth rate increases (for all T $\geq$ 2)[28] i.e. $\frac{d\bar{g}(T, i)}{di} > 0$

<u>Proof</u> Differentiating (20) with respect to g:

$$\frac{\partial V(g, T, i)}{\partial g} = \sum_{t=0}^{T-1} \left[\frac{dU}{dg} S + (t+1)U \frac{\partial S}{\partial g} \right] \left(\frac{S}{1+i} \right)^t$$

$$= 0 \text{ at } \bar{g} \text{ (T, i) } \ldots.. (37)$$

Using (28):

$$\frac{dU}{dg} S + (t+1) \frac{\partial S}{\partial g} < \frac{dU}{dg} S + t \frac{\partial S}{\partial g} \text{ at } \bar{g} \text{ (T, i) } \ldots.. (38)$$

Hence:

$$0 > \sum_{t=j}^{T-1} \left[\frac{dU}{dg} S + (t+1) \frac{\partial S}{\partial g} U\right] \left(\frac{S}{1+i}\right)^t \text{ for all } j \geq 1 \; ..... \; (39)$$

Differentiating $\frac{\partial V}{\partial g}$ partially with respect to i:

$$\frac{\partial^2 V(g, T, i)}{\partial i \; \partial g} = \sum_{t=0}^{T-1} \left(\frac{-t}{1+i}\right) \left[\frac{dU}{dg} S + (t+1) \frac{\partial S}{\partial g} U\right] \left(\frac{S}{1+i}\right)^t =$$

$$\sum_{j=0}^{T-1} \sum_{t=j}^{T-1} \left(\frac{-1}{1+i}\right) \left[\frac{dU}{dg} S + (t+1) \frac{\partial S}{\partial g} U\right] \left(\frac{S}{1+i}\right)^t$$

$$> 0 \text{ from (37) \& (39) } ..... \; (40)$$

Differentiating (37) totally with respect to i we have as before:

$$\text{sign}\left(\frac{d\bar{g}(i)}{di}\right) = \text{sign}\left(\frac{\partial^2 V}{\partial i \; \partial g}\right) \text{ at } \bar{g}(i) ..... \; (41)$$

$$\therefore \quad \frac{d\bar{g}(i)}{di} > 0 \text{ for all } T \geq 2$$

Again it is proved in the appendix that in the profit maximising case the optimum rate of growth falls as the rate of discount of the shareholders rises, as is to be expected.

The above two theorems indicate that the growth maximising model has interesting implications quite different from any of those derived from the profit maximisation assumption. They demonstrate that, ceteris paribus, the largest deviations from the profit maximising growth rate g* will be associated with those managements who have short time horizons & discount the future most heavily.(30) When either the rates of discount or the time horizons of both management and shareholders change together in response to other factors, the final effect on the optimum growth rate $\bar{g}$ will depend on shifts in g* and the MSF as well as the effect discussed above. While an increase in interest rates for example would always reduce g*, it is quite possible that $\bar{g}$ would be increased. In any case it might be expected that the rate of growth maximising firms would be less responsive to changes in interest rates than that of profit maximisers.

(c) Ceteris paribus the effect on $\bar{g}$ of an increase in discretion (indexed by z) can be either positive or negative (or zero).

<u>Proof</u> Differentiating (37) totally with respect to z gives at $\bar{g}$.

$$\text{sign}\left(\frac{d\bar{g}(z)}{dz}\right) = \text{sign}\left(\frac{\partial^2 V}{\partial z \partial g}\right) \qquad \ldots\ldots (42)$$

Differentiating $\frac{\partial V}{\partial g}$ partially with respect to z at $\bar{g}(z)$:

$$\frac{\partial^2 V}{\partial z \partial g} = \left\{ \sum_{t=0}^{T-1} \left[\frac{dU}{dg} s + (t+1) \frac{\partial s}{\partial g} U \right] \frac{t \, s^{t-1}}{(1+i)^t} \right.$$

$$\left. + \sum_{t=0}^{T-1} \frac{dU}{dg} \left(\frac{s}{1+i}\right)^t \right\} \frac{\partial s}{\partial z} + \left\{ \sum_{t=0}^{T-1} (t+1)U \left(\frac{s}{1+i}\right)^t \right\} \frac{\partial^2 s}{\partial z \partial g}$$

$$\ldots\ldots\ldots (43)$$

The terms in $\frac{\partial s}{\partial z}$ and $\frac{\partial^2 s}{\partial z \partial g}$, each divided by $\left(- \frac{\partial^2 V}{\partial g^2}\right)$, may again be called respectively the security and price effects. By definition

$\frac{\partial s}{\partial z} > 0$ but the expression in brackets multiplying it may be positive or negative depending on the values of the other variables. For example for T = 1 it reduces to $\frac{dU}{dg} > 0$, whereas when T is infinite it is shown below to be negative if $1 + i - 2s < 0$.

The sign of the second term on the right hand side of (43) is the same as the sign of $\frac{\partial^2 s}{\partial z \partial g}$ which may be either positive or negative when discretion increases.

Hence sign ($\frac{\partial^2 V}{\partial z \partial g}$) and therefore sign $(\frac{d\bar{g}}{dz})$ may be positive or negative for increases in discretion.

(d) Ceteris Paribus a reduction (increase) in the 'price' of growth $(-\frac{\partial s}{\partial g})$ always leads to an increase (decrease) in the optimum growth rate.

Again using z as a shift parameter in the security function, we want to determine the effect of increase in z which reduces $(-\frac{\partial s}{\partial g})$, the 'price' of growth, but leaves the level of security at $\bar{g}$ constant. We therefore have:

$$\frac{\partial}{\partial z}\left(-\frac{\partial s}{\partial g}\right) < 0$$

$$\text{and} \therefore \quad \frac{\partial^2 s}{\partial z \partial g} > 0 \qquad \ldots\ldots\ldots (45)$$

$$\text{and} \quad \frac{\partial s}{\partial z} = 0 \qquad \ldots\ldots\ldots (46)$$

Substituting (45) and (46) in (43) gives:

$$\frac{\partial^2 V}{\partial z \partial g} > 0$$

$$\text{and from (42)} \therefore \quad \frac{d\bar{g}}{dz} > 0$$

(e) When the time horizon is infinite the security effect is negative iff $1 + i - 2s < 0$.

Proof When T is infinite:

$$V(g,i,Z) = \sum_{t=0}^{\infty} U(8)s(g,Z)\left(\frac{s}{1+i}\right)^t$$

$$= \frac{(1+i)Us}{1+i-s} \qquad \text{.........(47)}$$

$$\therefore \frac{\partial V}{\partial g} = \frac{(1+i-s)(1+i)\left(\frac{dU}{dg}s + U\frac{\partial s}{\partial g}\right) + (1+i)Us\frac{\partial s}{\partial g}}{(1+i-s)^2}$$

$$= 0 \quad \text{at } \bar{g}(\infty) \qquad \text{.........(48)}$$

Differentiating $\frac{\partial V}{\partial g}$ with respect to Z at $\bar{g}(\infty)$ and simplifying

$$\frac{\partial^2 V}{\partial Z\, \partial g} = \frac{(1+i)}{(1+i-S)^2}(1+i-2S)\frac{\partial S}{\partial Z} + \frac{(1+i)^2}{(1+i-S)^2}U\frac{\partial^2 S}{\partial Z\, \partial g} \qquad \text{..... (49)}$$

The security effect is the first term on the right hand side of (49) divided $\left(-\frac{\partial^2 V}{\partial g^2}\right)$. By definition $\frac{\partial S}{\partial Z} > 0$ and therefore the security effect is negative iff $1+i-2S < 0$[31] Assuming a managerial rate of discount of 10%, the probability of a take-over or shareholder intervention would have to be greater than .45 per period for this inequality not to be satisfied. Remembering result (a) that, ceteris paribus, managements with long term horizons will opt for slower growth rates, and therefore greater security, than those with shorter horizons, it seems reasonable to assume that $1+i-2S < 0$ at $\bar{g}$ (∞).[32] It can be inferred then that managers with long term horizons will tend to react to increased security by reducing the growth rate and increasing security still more. In this case growth can be regarded as an inferior good. Further, only if upward shifts of the security function are associated with a fall in the marginal price of growth, large enough for the price effect to dominate the security effect, will increased discretion be associated with faster growth.

Conclusions

Using the approach to managerial utility maximisation developed above it has been shown that the predictions of two of the new theories of the firm are very sensitive to the way in which the constraints are formulated and are quite different from those of profit maximising models. It will be necessary therefore to pay more attention to the constraints or the MSF if further testable predictions are to be derived from the theories.

The discussion of the concepts of discretion and increases in discretion in section (2) is not very encouraging for empirical work. Previous writers have assumed that increases in discretion would lead to increases in the values of those variables appearing in the utility function and have used this in attempts to test their theories (egs. {9} {13} {14}). While this is usually true for increases in discretion defined as the relaxing of the constraint in a lexicographic utility function, it does not hold in the more general approach. The straightforward result may be restored by adopting a different definition of increases in discretion, but this merely emphasises ambiguities in the concept itself.

However the analysis has shown that it is possible to derive quite interesting predictions using the new approach. For example a model using the MSF in which managers derive utility from the level of sales revenue of their firm relative to that of other firms, rather than the absolute level of sales itself, would lead to testable predictions concerning the relationships between the size, growth and profitability of firms, which differ in important respects from those of both profit maximising models and the two theories discussed above. Such a model is at present being developed.

In all models using the utility maximisation under uncertainty approach, one of the factors influencing the size of the deviations from profit maximising values of the variables is the attitude to risk of management. It is to be expected that the greater the risk aversion of managers the closer will be the policies they follow to profit maximisation. Again this has interesting implications in that although the size of firms may be positively correlated with discretion the more bureaucratic organisation of large firms could tend to produce managers with greater risk aversion {8} . It may well be, therefore, that the largest firms are more nearly profit maximisers than their smaller rivals.

Finally, the only uncertainty introduced into the models above has been that connected with job security. It would be desirable also to introduce uncertainty linked to variables such as the level of sales, profits, growth of demand, etc. and further developments of the new theories could incorporate such considerations into a more general model. Work on the effects of these other uncertainties has already been done by Lintner (see for eg {5}).

In the real world there is no doubt that managerial discretion, as defined above, exists. The simple facts of differing attitudes to risks among shareholders and differing marginal rates of taxation will ensure that the MSF cannot be of the shape allowing no discretion. It follows that the important questions are how large are the deviations from the maximum point of the MSF and what factors influence their size? The above analysis has outlined some answers and possible directions for further research.

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Appendix

We consider here the direction of change of the optimum growth rate with respect to changes in the time horizon and discount rate of shareholders, on the assumption that managers are profit maximisers. The initial size of the firm is given so let the opening book value of equity assets by K_o. The objective function is then:

$$V\ (g,\ T,\ i)\ =\ K_o \sum_{t=0}^{T-1} \left[\pi(g) - g\right] \left(\frac{1+g}{1+i}\right)^t \quad \ldots\ldots (50)$$

where $\pi(g)$ is the rate of return on equity assets, g is the steady state rate of growth, T is the time horizon of shareholders and i is the shareholders discount rate.

It is assumed that $\frac{\partial^2 V(g,\ T,\ i)}{\partial g^2} < 0$ and that $g^*(T,\ i)$ is the optimum growth rate. Two results will be proved:

(a) A lengthening of the time horizon leads to faster growth.

As in section (4) the proof is by induction.

Assume that $g^*(T-1) > g^*(T-2)$

Now:

$$V\ (g,\ T-1,\ i)\ =\ V\ (g,\ T-2,\ i) + K_o\ (\pi - g)\left(\frac{1+g}{1+i}\right)^{T-2} \quad \ldots\ldots (51)$$

Differentiating (51) with respect to g:

$$\frac{\partial V\ (g,\ T-1,\ i)}{\partial g}\ =\ \frac{\partial V\ (g,\ T-2,\ i)}{\partial g} + K_o \frac{(1+g)^{T-3}}{(1+i)^{T-2}}\left[\left(\frac{d\pi}{dg} - 1\right)(1+g) + (T-2)\ (\pi - g)\right] \quad \ldots\ldots (52)$$

At g*(T-1, i):

$$\frac{\partial V(g,\ T-1,\ i)}{\partial g} = 0 \quad \ldots\ldots (53)$$

& $$\frac{\partial V(g,\ T-2,\ i)}{\partial g} < 0 \quad \ldots\ldots (54) \quad \text{since} \quad \frac{\partial^2 V(g,\ T-2,\ i)}{\partial g^2} < 0 \ \&$$

g*(T-1) > g*(T-2) by assumption.

$$\therefore \left(\frac{d\pi}{dg} - 1\right)(1+g) + (T-2)(\pi-g) > 0 \quad \ldots\ldots (55)$$

Now $\pi - g > 0$ otherwise $V(g,\ T,\ i) \leqslant 0$

$$\therefore \left(\frac{d\pi}{dg} - 1\right)(1+g) + (T-1)(\pi-g) > \left(\frac{d\pi}{dg} - 1\right)(1+g) + (T-2)(\pi-g) > 0 \text{ at } g^*(T-1) \quad \ldots\ldots (56)$$

From (52):

$$\frac{\partial V(g,\ T,\ i)}{\partial g} = \frac{\partial V(g,\ T-1,\ i)}{\partial g} + K_o \frac{(1+g)^{T-2}}{(1+i)^{T-1}} \left[\left(\frac{d\pi}{dg} - 1\right)(1+g) + (T-1)(\pi-g)\right] > 0 \text{ at}$$

g*(T-1) using (53) & (56) ..... (57)

Since $\frac{\partial^2 V(g,\ T,\ i)}{\partial g^2} < 0$ this implies that:

$$g^*(T) > g^*(T-1)$$

To complete the proof it is necessary to show that g*(2) > g*(1)

$$V(g,\ 1,\ i) = K_o(\pi-g)$$

$$\therefore \quad \text{at } g^*(1) \quad \frac{d\pi}{dg} = 1 \quad \ldots\ldots (58)$$

From (52) $$\frac{\partial V(g,\ 2,\ i)}{\partial g} = \frac{\partial V(g,\ 1,\ i)}{\partial g} + K_o\left(\frac{d\pi}{dg} - 1\right)\left(\frac{1+g}{1+i}\right) + \frac{K_o(\pi - g)}{1+i} > 0 \text{ at } g^*(1) \text{ using (58)} \quad \ldots\ldots(59)$$

Since $\frac{\partial^2 V(g,\ 2,\ i)}{\partial g^2} < 0$ it follows that:

$g^*(2) > g^*(1)$

Hence $g^*(T) > g^*(T-1)$ for all T

(b) As the rate of discount falls, the optimum growth rate increases

i.e. $\frac{dg^*}{di} < 0$

As in section (4) it is easily shown that:

$$\text{sign}\left(\frac{dg^*}{di}\right) = \text{sign}\left(\frac{\partial^2 V}{\partial i\, \partial g}\bigg|_{g^*}\right) \quad \ldots\ldots (60)$$

Differentiating (50) with respect to g:

$$\frac{\partial V(g,\ T,\ i)}{\partial g} = K_o \sum_{t=0}^{T-1} \left\{\frac{(1+g)^{t-1}}{(1+i)^t}\left[\left(\frac{d\pi}{dg} - 1\right)(1+g) + t(\pi - g)\right]\right\}$$

$$= 0 \text{ at } g^*(T,\ i) \quad \ldots\ldots (61)$$

At $g^*(T,\ i)$ since $\pi - g > 0$ $\left[\left(\frac{d\pi}{dg} - 1\right)(1+g) + t(\pi - g)\right]$ is a strictly increasing function of t.

$$\therefore \quad 0 < \sum_{t=j}^{T-1} \frac{(1+g)^{t-1}}{(1+i)^t}\left[\left(\frac{d\pi}{dg} - 1\right)(1+g) + t(\pi - g)\right] \text{ for all}$$

$$j \geq 1 \text{ at } g^*(T,\ i) \quad \ldots\ldots (62)$$

Differentiating $\frac{\partial V}{\partial g}$ partially with respect to i:

$$\left.\frac{\partial^2 V(g,\ T,\ i)}{\partial i\ \partial g}\right|_{g^*(T,\ i)} = -\frac{K_o}{1+i}\sum_{t=0}^{T-1}\frac{t(1+g)^{t-1}}{(1+i)^t}\left[\left(\frac{d\pi}{dg}-1\right)(1+g)+t(\pi-g)\right]$$

$$= -\frac{Ko}{1+i}\sum_{j=0}^{T-1}\sum_{t=j}^{T-1}\frac{(1+g)^{t-1}}{(1+i)^t}\left[\left(\frac{d\pi}{dg}-1\right)(1+g)+t(\pi-g)\right]$$

< 0 using (61) and (62) ..... (63)

Hence from (60):

$$\frac{dg^*}{di} < 0$$

Notes

(1) This paper is based on work completed at the University of Warwick. I would like to thank Professor Keith Cowling for his constructive comments and encouragement during that period.

(2) For a comparison and a discussion of the profit, growth and revenue maximisation theories of the firm see {11}.

(3) In multi period models profit maximisation will be interpreted to mean maximisation of the present value to shareholders of the firm's dividend stream.

(4) That is to say that while there has been a good deal of discussion on the causes of limited discretion such as take overs, proxy fights, etc. little attention has been paid to the form of the function linking them to the decision variables.

(5) Although there might be a particular value of $g(\underset{\sim}{x})$ at and above which managers are perfectly safe and below which they are replaced, it would not necessarily be known to them. They would then have a subjective probability distribution for c upon which they based their decisions.

(6) There is an analogy here with the limit price for new entry into particular markets. Since this paper was written an article has appeared in which new entry is treated in a probabilistic rather than a deterministic way: M.I. Kamien & N.L. Schwarz - Limit Pricing and Uncertain Entry - Econometrica May 1971, 39, 441-454.

(7) {6} is an empirical study which tries to estimate a probabilistic function.

(8) It is assumed that all plans are realised if management survive and that shareholders have full knowledge of management intentions. Needless to say this is unrealistic and it would be better to introduce some sort of learning process by the shareholders in which they gradually gain knowledge of management decisions and their effects. Such an analysis is beyond the scope of the present paper.

(9) Assuming managerial preferences satisfy the von Neumann-Morgenstern axioms they can be represented by a utility function which is unique up to

a positive linear transformation.

(10) It will be assumed that the second order conditions for a maximum are satisfied.

(11) Thus excluding any learning process.

(12) A more realistic analysis would make use of the calculus of variations or dynamic programming. The assumption is made both for mathematical simplicity and because the iterature on growth maximisation has focused on the choice between alternative steady state growth paths for the firm.

(13) For a brief discussion of decision making when the choice set consists of stochastic processes {3} Chapter 12.

(14) It will in fact be assumed below that $\underset{\sim}{x}^*$ is unique and that $s(\underset{\sim}{x}^*) > s(\underset{\sim}{x})$ for all $\underset{\sim}{x}$.

(15) When the dividend stream is itself uncertain and shareholders have differing attitudes to risk, the determination of the policy which maximises $s(\underset{\sim}{x})$ becomes in itself an interesting economic problem. In general the security maximising policy will not be the same as that $\underset{\sim}{x}$ which maximises the market value of the firm's equity.

(16) Management is therefore assumed to be risk averse. This is not a necessary condition for the maximisation of expected utility and could easily be dropped.

(17) This seems to be the logical assumption to make if potential take over bidders as well as shareholders are profit maximisers and there are costs involved in take over bids, proxy fights, etc. However if potential bidders are revenue maximisers they will be interested in the opportunity cost of extra sales in terms of security, rather than the level of profits as such. The probability of take over would then not be a function of L(q) alone.

(18) These together with $U'' > 0$ ensure a unique interior optimum.

(19) q_1 is the output level which maximises total revenue. Therefore

$$\frac{dU}{dq} = U' \frac{dR}{dq} < 0. \text{ for } q > q_1.$$

(20) For the full development of Marris's original model see {6} and {7} .

(21) In fact the time dimension is suppressed altogether.

(22) It is not easy to see how an increase in discretion would be defined in this case.

(23) When the time horizon is not infinite, the restriction to choice between steady state growth rates is highly dubious since it implies that managers maintain the same growth rate in every period including the last even though they are not interested in what happens after that period. (See note (12)). When the time horizon is infinite however the dynamic programming approach will produce a constant growth rate as the optimal solution as can be seen by noting that the objective function of management at the end of the first period is the same as the objective function at the beginning. Using the principle of optimality this implies that the optimal growth rate is constant from period to period.

(24) The same assumptions are made about $s(g, \underline{z})$ as were made for $s(q, \underline{z})$ in section (3).

(25) The result will not necessarily be true in a continuous time analysis.

(26) Note that the time horizon and rate of discount of shareholders are being held constant so that the MSF does not shift in any way.

(27) Hence $\bar{g}(T-1) > g^*$

(28) Again note that the time horizon and discount rate of shareholders are held constant.

(29) Except of course when $T = 1$ since then $\frac{\partial v}{\partial i} = 0$.

(30) It is likely that these conclusions would be reinforced in the MSF shifted downwards through time at non profit maximising growth rates, due to a learning process on the part of shareholders. Long sighted managements would attach more weight to the higher probabilities of job loss in the

future than would short sighted managements, causing them to pursue more secure growth rates.

(31) Replacing g by q and z by t in equation (49) we can prove theorem (vi) of section (3). The losses due to the pursuit of non profit maximising policies will be the discounted present value to shareholders of $(1-t)(\pi(q^*) - \pi(q))$, say $k(1-t)(\ (q^*) - \ (q)) = L(q)$. $(\pi(q^*) - \pi(q))$

Then $$\frac{\partial s}{\partial t} = -\frac{\partial s}{\partial L} k(\pi(q^*) - \pi(q)) > 0$$

and $$\frac{\partial^2 s}{\partial t \partial q} = k^2 \frac{\partial^2 s}{\partial L^2} (1-t) \frac{d\pi}{dq} (\pi(q^*) - \pi(q)) + k \frac{\partial s}{\partial L} \frac{d\pi}{dq} > 0$$

Looking at equation (49) it can be seen that $\frac{\partial^2 v}{\partial t \partial q}$ (and hence $\frac{dq}{dt}$) will always be positive if $1 + i - 2s \geq 0$ but could possibly be negative otherwise.

(32) In a continuous time analysis it can be shown that the security effect is always negative.