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PRICE, QUALITY AND MARKET SHARE

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There is a very extensive literature covering the general area of demand analysis but most of this deals with commodities or commodity groups and implicitly treats them as homogeneous products when in reality a commodity may define a wide spectrum of varieties, models or brands. To investigate consumer behaviour at the sub-commodity level is to investigate the nature of the demand function faced by the firm or the division of a firm whereas traditionally empirical market demand analysis has been concerned with the characteristics of the industry demand function. The results of commodity level demand analysis have got obvious relevance to government tax and trade policy and to indicative national planning but only limited relevance to the firm. To understand the behaviour of firms and to assess the degree of competition among firms, we need information on the demand function faced by that firm - that is the demand function for that firm's brand of the commodity. Despite this, little published work has been forthcoming on the demand for branded goods and two obvious explanations come to mind - first the problem of quality variation among brands of the same commodity and second the lack of data on market share behaviour. This contribution is largely concerned with developing an operational version of a demand for brand model in which quality variation is explicitly recognized. This is accomplished by following up a suggestion made by Griliches [1] and generating implicit prices for the range of qualitative attributes a commodity possesses. This allows us to construct quality adjusted prices for each brand which are the correctly specified prices for the demand equation. The model is tested out in the U.K. market for farm tractors and found to give satisfactory results.

The Theoretical Structure

Demand analysis, both theoretical and applied, has traditionally been concerned with commodities which are assumed to be homogeneous. There has been no recognition of quality variations within a commodity or of the existence of brands, varieties or models of a particular commodity, or of the entry of new products or the disappearance of old ones. The only hard prediction from the orthodox theory of consumer behaviour has been the sign of the substitution effect and even this cannot be applied with new products since the theory assumes that consumers have always had access to all commodities. Further, traditional theory allows for no predictions about complementarity or substitutability among commodities - a very real indictment of the theory when we remember

how these relationships among commodities are often ~~so~~ obvious to us as consumers. The story on the production side is symmetric with that on the demand side with the theory of the firm saying nothing about quality of inputs, about the choice between brands of machine or types of labour or of the basic relationships between these various inputs.

Recently there have been some theoretical developments mainly concerned with recognising commodities as bundles of characteristics (Lancaster [6], Ironmonger [3]) and a new theoretical structure has evolved with a linear programming framework - an approach used previously almost exclusively for production problems*.

- * The early use of linear programming in diet problems did in fact recognise food commodities as bundles of characteristics, i.e. various nutrients - calories, proteins, fats, etc. In this case the constraints that had to be met were relatively simply defined on physiological grounds.

Actually a theory of consumer behaviour where varieties of a commodity are available was set out by Houthakker [2] as long ago as 1951/2. His consumer utility function included quality levels as well as commodity levels and he assumed price differences among varieties are exactly determined by quality differences. The same assumption was made in some empirical work by Theil [11] done at about the same time where he took price as an indicator of quality and then estimated a demand function for quality using budget data.

These theoretical developments can indicate long-run equilibrium situations and the variables which will cause movements in the equilibrium but in examining the demand for brands or varieties of a commodity we do not expect instantaneous adjustment by consumers or by firms. Thus a programming model without further constraints would for example immediately replace a brand appearing in the solution by a new product if that new product had the same quality mix and a lower price. Because of brand loyalty and imperfect information we will not expect an immediate adjustment to the new equilibrium but instead a gradual adjustment over time with the rate of adjustment probably being determined by the size of the price and quality differences between existing and new products. In fact it would be possible to put 'adjustment' or 'flexibility' constraints into the model which would limit the change in patterns of expenditure to what had been observed historically. This would be the same idea as is embodied in the recursive programming approach to production response. However to be

operational in the field of demand for brands, the programming approach requires us to be completely specific about the consumers objective and about his array of constraints something which does not appear as a feasible proposition at the moment.

Similarly the work of Houthakker [2] has very real operational limitations as it stands. The present paper tries to release the assumption made by Houthakker of exact concordance between price variation and quality variation, while recognising that an important, but stochastic relationship exists. Thus, while it is reasonable to suggest that interaction between consumer and firm behaviour will result in adjustments in price and quality mix that will lead to a long-run equilibrium where all qualities have the same implicit prices in all brands, in the short-run it is likely that we will get substantial departures from this situation so that some brands appear as relatively cheap or relatively expensive sources of quality characteristics as compared with the existing array of brands.

The only empirical study of demand for branded goods in the economics literature is the one by Telser [10]. In trying to explain short-run behaviour in the demand for brands Telser sets up a model which explicitly examines brand loyalty and switches in brand loyalty. He sets out a matrix of transition probabilities relating consumers demand for particular brands in two time periods where the element in i^{th} row and the j^{th} column is the probability of buying the j^{th} brand in a specific time period having previously bought the i^{th} brand in the previous time period; but since it is frequently impossible to observe these transition probabilities directly Telser proposes a procedure for using least squares regression to get estimates of the probabilities knowing the market shares for different brands over time. His theory of demand for brands then explains these probabilities in terms of the variation in relative prices for the different brands. His model boils down to estimating the relation between the current market share, and (i) price, (ii) lagged market share, and (iii) the product of price and lagged market share. The practical difficulty in estimation results from the collinearity existing among the explanatory variables but more basically the classical regression model does not force the coefficients to sum to one across brands and thus a Bayesian or programming approach is called for.

Summarising our review of the theoretical and empirical work we conclude that the theoretical literature has limited relevance to

the examination of short-run demand behaviour for brands and therefore of market shares, whereas the limited empirical work in the economics literature* does not take up the problem of quality variation among

* Obviously an extensive literature on demand for brands exists in the market research field but this is largely concerned with estimating the impact of different promotional activities. This will comprise only a facet of the present contribution.

brands which is developed in the theoretical contributions.**

** In the case of Telser's study [10] the problem of quality variation among brands in the markets studied - coffee, frozen orange juice and margarine may have been relatively unimportant. In the case of consumer and producer durables the problem looms very large; there is a considerable array of specifications in the car, truck and tractor markets, and among refrigerators, houses and computers. The case for an explicit recognition of 'quality' variation (in its widest sense) is obvious.

As we have argued elsewhere (Rayner and Cowling [7]) a neglected area in empirical demand analysis at the aggregate level has been the adjustment of price changes over time for quality changes over time. Equally important, in demand analysis at the disaggregate level, is the adjustment of price differences across brands of a commodity for quality differences, both at a specific point in time and at different points in time. The problem remains of getting a weighting system for the different qualitative attributes of a commodity which reflects the utility to the average consumer of that particular characteristic. One way of doing this is to estimate implicit prices for the different characteristics by relating the variation in prices among brands of a particular commodity to variations in the bundles of characteristics associated with the different brands. Such price-quality relations can be interpreted as cost functions from the supply side, but cost functions derived from a range of brands with particular characteristics and particular prices which have survived the competitive environment at that point in time. Thus we can argue that the individual characteristics, and the bundle of characteristics being offered by firms and the prices they are being offered at are derived from supply and demand factors. The price-quality relationship would become more

obviously demand oriented if each brand observation were weighted by the market share captured by that brand. However, it is very unlikely that the price of a particular brand is exactly determined by its quality mix with each quality being weighted by its estimated implicit price. It is more likely that the relationship between price and quality will leave a significant amount of residual variation. This is to be expected in any market where there is not instantaneous adjustment by producers and consumers. Lags in adjustment can be explained by : (1) the cost of information and the resultant limited search by consumers, (2) brand loyalty and brand ties through distribution networks, (3) rigid administered prices, and (4) the technological lag in response to the quality innovations of rivals.

As regards the cost of information it can be seen that through informational advertising of price and quality the direct cost of information to the consumer is reduced but advertising expenditure in turn may be considered as one of the characteristics explaining the price of the product. An interesting prediction can be derived from a consideration of information costs [Stigler (9)]. If these costs of search are independent of the price of the commodity we are seeking information about, then we would expect an inverse relationship to exist between price discrepancies among brands, after correcting for quality differences, and the average price of the commodity. This prediction comes quite simply from the assumption of a maximising consumer and a version of this prediction has been found to hold in the Chicago retail market for cars and refrigerators where the coefficient of price variation for cars exceeded that for refrigerators [Jung (4)(5)]. We are interested in brand prices of a particular commodity and manufacturers price and quality policies. We are postulating that widespread search activity by consumers, of both price and quality, will lead to rapid sales adjustments to any discrepancies in manufacturers price or quality policies and therefore rapid adjustment by manufacturers in response. Usually we would expect the adjustment to come from those manufacturers whose brands have a positive price discrepancy and whose sales are falling. The prediction does not require adjustment by those firms whose sales are rising but if excess demand for their products exists then they may raise prices and eliminate their negative price discrepancies. The response by consumers, and hence market share adjustments, will be more rapid for large items of expenditure and therefore we would expect that price-quality relationships estimated across brands will have a bigger explanatory power (lower residual variance) for commodities forming a

major part of the consumers' budget than for those commodities which are less important. In explaining market share behaviour we expect a bigger response to price discrepancies for these same important commodities than for other commodities of lesser importance.

On the second item, brand-loyalty, we expect a high degree of autocorrelation over time in demand for a specific brand by a particular consumer. Temporary, positive price discrepancies will not throw the consumers habitual behaviour off course. Similarly brand ties may exist for a particular consumer because of a local distribution outlet and local service facilities. The third and fourth items reflect rigidities from the firms' point of view. List prices generally show more limited variation than actual retail prices although they can be varied more easily than quality-mix which may involve wholesale changes in production technology. A change in advertising policy may come more slowly than a change in price policy but faster than a change in quality. Thus an innovation by one firm can result in disequilibrium in the market for a considerable period. Some of the discrepancies created may be reduced by price and advertising adjustments but equilibrium will not generally be reached until a similar product is offered by the other firms.

If therefore, for the above reasons, we accept that a commodity market can realistically be characterised as one where differences in price between brands cannot be fully explained by quality differences then we can begin to see how the price or quality-mix of a particular brand can be 'out of line' compared with the other brands available.

The elements of our model are : (i) an equation expressing brand price as a function of the brands' qualitative characteristics, and (ii) an equation expressing quantity demanded of a particular brand as a function of the quality adjusted price of that brand relative to the average quality adjusted price for the commodity in that time period and other variables. The quantity demanded of all brands (i.e. commodity demand) is obtained by summing over brands. The model is set out in symbolic form below :

$$(1) \quad P_{it} = f(V_{it} ; U_{it})$$

$$(2) \quad q_{it} = g(U_{it} ; X_t ; Z_{it} ; e_{it})$$

Equation 1 expresses the price of the i^{th} brand in time period t as a function of a vector of characteristics associated with that brand in that time period (V_{it}) and a disturbance term (U_{it}). In equation 2 the quantity

demanded of the i^{th} Brand (q_{it}) is related to the disturbance term from the first equation (U_{it}), representing that part of brand price not explained by its qualitative characteristics, a vector of variables (X_t) explaining the total market for all brands of the commodity, a vector of non-price variables specific to the sales of the i^{th} brand (Z_{it}) and a disturbance term (e_{it}). Under certain assumptions we will be able to convert equation 2 into a market share equation and drop the X_t vector.*

* This would obviously be appealing from a statistical viewpoint as we would be able to economize on degrees of freedom and perhaps avoid multicollinear situations. However it does pose other statistical problems created by bounded dependent variables (see later).

If we take as an example a semi-logarithmic version of the brand demand equation :

$$q_{it} = e^{\beta_0 + \beta_1 U_{it} + \beta_2 X_t + \beta_3 Z_{it} + e_{it}}$$

and assume a single, non-stochastic relationship for commodity demand :

$$q_t = e^{\alpha + \beta_2 X_t}$$

then we get out a market share equation with a reduced set of variables on the right-hand side which are all specific to the i^{th} brand :

$$\frac{q_{it}}{q_t} = \frac{-\alpha + \beta_0 + \beta_1 U_{it} + \beta_3 Z_{it} + e_{it}}{e}$$

Let us now look rather more closely at the price variable which appears in the demand or market share equation. What we want to measure is the ratio of the i^{th} brand price to the commodity price in a particular period, where the commodity price is the weighted mean of the prices of the different brands offered. But we are also concerned in making these prices comparable by adjusting for quality differences that may exist between a specific brand and the average. The quality adjusted price may be defined as the difference between the actual price of the brand and the expected price given its specific quality characteristics. For a simple, linear price-quality relationship $P_{it} = c_{0t} + c_{1t}' V_{it} + U_{it}$ the quality adjusted price will be U_{it} and in the case of the average brand this will be zero if we minimise the sum of squared deviations. To be able to

define a price ratio variable we could add a constant to each U_{it} . But this will change the relationship. If the price difference is relevant then we simply include U_{it} as the relevant price variable. We may choose to make the average price comparable with the price of a particular brand by setting the quality characteristics at the average level in which case the price ratio variable would be

$(1 + \frac{U_{it}}{\bar{p}_t})^*$ for the price quality relationship defined above. Where

$$\frac{P_{it} - \alpha_{lt}'(V_{it} - \bar{V}_t)}{\bar{p}_t} = \frac{\alpha_{0t} + \alpha_{lt}' \bar{V}_t + U_{it}}{\alpha_{0t} + \alpha_{lt}' \bar{V}_t} = \frac{\bar{p}_t + U_{it}}{\bar{p}_t}$$

\bar{p} is the average brand price. Alternative deflators to \bar{p}_t could be defined by holding quality constant at alternative levels. The choice of deflator is empirical but its presence is theoretically required where a price ratio is considered relevant. The need for a deflator could be explained by the observation that our estimate of the disturbance term (U_{it}) is in terms of current market prices and these may rise through time due to quality improvements not caught in the specified price-quality relationship.

The discussion up to this point has centred on the problem of explaining consumer behaviour in the selection of brands. Equally well we can turn the discussion towards the firm's demand for input and the problems posed when we have several varieties of a specific input each with a specific set of qualities. In programming terminology each variety would form a specific activity and the problem would be to choose the best set of activities. This may involve producing a planned output at minimum cost or output may be a variable. This approach requires us to know the input-output coefficients at a quality level rather than at a commodity level. Equally well Houthakker's theoretical structure could be converted to a theory of the firm by specifying a production function where output is determined by the level of commodity inputs and quality inputs. The solution would give optimal commodity and quality levels which would define the optimal variety of each commodity. In factor markets, as distinct from product markets, monopsonistic and oligopsonistic situations are quite feasible in which case the quality prices, as well as commodity prices, will depend on the quantity demanded.

Other Variables influencing Demand for Specific Brands

(1) Time Since the Product was Introduced

We would expect the market share of a specific brand to decay over time, with its relative price-quality level held constant, because the new products entering the market will have a more "relevant" quality mix. More generally we would expect a gradual adjustment by consumers to a new product. Thus we would expect an initial growth in market share rising to a maximum and then decaying. The relevance of this type of adjustment would depend on the product and in empirical work would depend on the length of the period covered by each observation - annual observations may cover the whole period of positive growth so that the next observation will be of the first phase of decline of the product. An important aspect of this for consumer behaviour would be evolving fashions and styles - reflecting automoncus or induced changes in tastes which are exemplified by bandwagon effects. From the firms' viewpoint rising wages could lead to larger machines becoming optimal. The decay could of course be countered by allowing brand price to decay over time relative to other newer products.

(2) Advertising

There is a substantial literature which attempts to measure the impact of advertising on brand sales. From the standpoint of our model advertising associated with a particular brand might be viewed as one of the characteristics (v_i 's) in the price/quality relation but this has little relevance from the consumers' point of view. Even informational advertising only helps in the decision about which brand to buy - the information offered about one brand may turn people towards other brands. It seems more appropriate to include advertising as a shifter in the market share equation. There then exists a possible identification problem when the time comes to estimate the relationship. We are interpreting the relationship as reflecting consumer behaviour with advertising expenditure associated with a particular product being exogenously determined. However it may be quite reasonable to suggest that the relationship may partly reflect the behaviour of firms in setting their advertising budget. Thus large product sales may result in a substantial flow of resources into advertising whereas low sales firms will not "be able" to spend large sums on advertising. This pattern would certainly emerge from a typical industry size

distribution where there are a few big firms and a long tail of small firms. In the short-run however one may observe firms reacting to a fall in market share by increasing their advertising budget, and also by price and quality adjustments, in some cases through more rapid product innovations, which leads to further problems of simultaneity since we can realistically regard sales, advertising, price and quality as all being endogenous variables. At the very least these points would lead us to treat any estimate we get rather carefully - more generally they would require us to formulate and estimate a more complete model.

(3) Lagged Variables

As argued previously we would not in general expect instantaneous adjustment to price and quality levels which were out of line - more likely we would expect a gradual adjustment to the desired or equilibrium level of consumption. A convenient way of handling this phenomenon is to treat equation 2 as a desired level of brand consumption equation. If we call this unobserved quantity q_{it}^* we might then say that the adjustment function will look something like :

$$(4) \quad q_{it} - q_{it-1} = \lambda (q_{it}^* - q_{it-1})$$

where $0 < \lambda < 1$

which says that the actual change in consumption of a particular brand is some fraction of the difference between the actual consumption in the previous period and the current desired level of consumption. The working equation is then obtained by substituting for q_{it}^* in equation 4

$$(5) \quad q_{it} = \lambda g (U_{it}; X_t; Z_{it}; e_{it}) + (1-\lambda) q_{it-1}$$

Allowing for such an adjustment function as this is another reason why including aggregate sales (q_t) as an explanatory variable would give operational problems since in many markets there is likely to be a high sample correlation between q_t and q_{it-1} , lagged brand sales. This is likely to be most acute where brand sales means total sales by a firm in a particular market. It may be less troublesome where firms are multi brand sellers, as is often true.

The variables previously discussed would be candidates for consideration in any study of brand demand. In any particular commodity market there may be additional variables to be considered. For example, in the tractor market study to be reported, a trend term is introduced

which allows a different parameter to be estimated depending on a large - small classification for tractor horsepower. Thus we might expect the market share for large horsepower tractors to trend up over time as wage costs rise. In consumer demand studies we might expect the income elasticity of demand for different brands to be different so that income increases would not only have an impact on aggregate demand but also on the brand pattern of that demand, i.e. on market shares.

Estimation Problems

1. Price-Quality Relationships

We might estimate price-quality relations among brands using ordinary least squares regression procedures. These could be interpreted as basically cost functions. The demand side would be reflected in the brands on offer and the prices the firms remaining in the market felt were appropriate to meet a target rate of sales. However to obtain a more accurate indication of the value put on particular characteristics by consumers it would be more realistic to weight each observation by the importance of the particular brand in the market. Using weighted regression procedures we would then get relatively smaller residuals (absolute values) for big sellers and relatively bigger residuals (absolute value) for small sellers.

Therefore the size of the coefficients in the market share equation will be different according to whether a weighted or unweighted price-quality relation was estimated.

A problem remains in making predictions where sales of a new product has to be known before estimating the price-quality relation from which we get a variable to be used in the market share equation. The only way out of this would be to iterate to some solution by feeding back the first sales prediction into the price-quality relation, re-estimating until, hopefully one converged to a solution. With unweighted regression prediction would still require re-estimation of the price quality relationship with the new product's price and set of characteristics making a new observation otherwise we may predict a market share for the new product which would be inconsistent with the continuance of the existing market shares resulting from the same residuals being associated with each of the old brands.

2. Market share Equation

The problem raised by a regression equation where the dependent variable is inherently bounded is one of biased parameter estimates. This will arise when there is a concentration of observations of the dependent variable at the bound, i.e. when the market share for a series of brands is at zero.* In such cases we can no longer make the assumption that $E_{ui} = 0$ since negative market shares are infeasible.

* The upper bound is not effective since only one brand can be there and this is a trivial problem.

We are unlikely to encounter such an extreme situation in the real world but we are likely to encounter many situations where brand market shares are very small. This is of course less likely when we define brand on a broad basis, as by manufacturer. Therefore this is an extra source of possible error and one which must be recognised. Alternative estimation procedures have been developed but are difficult to handle.

The problem can be circumvented by replacing the market share equation by a demand for brand equation which is not subject to such rigid bounds. However to preserve the same theoretical model we will now have an aggregate demand variable on the right-hand side with all its associated problems.

There is also a potential simultaneity problem associated with the market share equation. The equation itself is intended to represent consumer demand behaviour in a particular commodity market, but it may also reflect decisions made by manufacturers in response to movements in the market shares of their products. Thus a falling market share may generate action by the manufacturer to reduce price, increase quality or put up advertising expenditure. Such a response will usually take time and the assumption of one-way causation may be realistic if the time period covered by each observation is reasonably short.

An Empirical Case Study :

The Tractor Market in the U.K.

One of the reasons for the rarity of brand demand studies compared with studies of the demand for commodities or commodity groups has been the scarcity of data on sales by brand. We were very fortunate in getting access to the U.K. sales by all the major U.K. tractor manufacturers (David Brown, Ford, International Harvester,

Massey-Ferguson and Nuffield (British Motor Corporation)). The actual behaviour of market shares must remain confidential but we are able to report the regression results.

Price-Quality Relationships for Tractors

Relationships between the price of tractor models and their quality characteristics were estimated for each year from 1948 through 1965. The number of models available varied over the period but was typically between fifteen and twenty. To avoid problems resulting from large numbers of explanatory variables the prices to be explained were the prices of a basic model defined as one with self starter, simple hydraulics and power take-off. It was possible to do this by using the available prices of optional extras. These adjusted prices were related to the belt horsepower ratings of the different models and to whether they had a diesel engine or not. Other attributes which might be considered relevant such as drawbar pull and fuel economy were in fact highly correlated with the included variables. A detailed account of the methods and results is available in Rayner [8]. A typical result is quoted below which was obtained from data for the 1953 model range using weighted regression :

$$p_i = 223.5 + 8.120 \text{ h.p.}_i + 85.5 D_i + \hat{U}_i \quad R^2 = 0.83$$

where p_i is the price of a basic model i (£), h.p._i is the belt-horsepower rating for that model and D_i is a dummy variable taking the value one if the i th model has a diesel engine and zero if it has not. The residual \hat{U}_i is the difference between the actual price of the basic model, and the estimated price based on the models' quality mix. Thus a positive value of \hat{U}_i indicates a relatively "pricey" model and a negative value indicates a relatively cheap model. The cross-section regression of price on quality gives residuals for each model in each year for the period 1948-65. It is interesting to note the high correlation between price and the quality attributes which could easily give rise to problems if they were both to be used as explanatory variables in the market share equations.

Aggregate Demand for Tractors

This paper is not directly concerned with aggregate demand problems but it will be generally true that movements in aggregate demand will result in movements in brand sales and can indeed be

decomposed into them. We are concerned to identify those variables influencing aggregate demand which do not cause any change in the composition of that demand. An investigation of aggregate demand indicated the ratio of tractor price to agricultural wages to be a key variable, with lagged stock of tractors and investment allowances to be also significant :-

$$q_t = 344.5 - 55.23 \log_e \left[\frac{P_T(t)}{P_L(t-1)} \right] + 1.163 \log_e I_t$$

$$- 27.50 \log_e S_{t-1}$$

$$R^2 = 0.94$$

$$\text{Von Neumann Ratio} = 1.84$$

where q_t is aggregate sales of tractors (deflated value in millions of pounds), $\frac{P_T(t)}{P_L(t-1)}$ is the ratio of the index of current constant quality price of

tractors to an index of labour earnings in the previous year, I_t is the percentage investment allowance, and S_{t-1} is the stock of tractors in ten millions of pounds (deflated). Tractor price relative to crop price may also be important but is closely correlated with the tractor/labour price ratio variable in the sample period. [see Rayner and Cowling (7)]

Demand for Tractor Models

The theoretical discussion earlier in the paper has justified the use of the residual from the price-quality relation, in some form, as a quality adjusted price ratio of a specific tractor model to the average model available at that time. Thus the optimum model depends on the prices of different models and their relative productivities which are reflected in their qualitative characteristics. To explain demand for a brand of tractor over the post-war period we must also allow the tractor-labour ratio to change in response to relative price changes and also take the planned output of agriculture as a variable. If we assume that these variables, which determine aggregate demand for tractors, do not affect the brand composition of that aggregate then we can replace them by q_t , aggregate sales of tractors and then divide through the brand equation by aggregate sales since a one per-cent change in aggregate sales will give a one per-cent change in brand sales. If we do this then we have a market share equation in which the variables which determine aggregate sales can be ignored. This is a very useful trick since

the bane of short time-series analysts is a multitude of highly inter-correlated explanatory variables. If we form a market share equation in this way we are then only concerned with variables which cause people to switch between brands. Unfortunately this may mean dragging out of retirement some of the very same variables we have just eliminated. Thus the increasing cost of labour relative to tractors will result in a switch to bigger tractors since this is one way in which we can substitute tractor power for labour power. The other obvious variables likely to influence market share behaviour are advertising and years since model introduction. The second variable is intended to represent the decay of the market share of a tractor model as time passes and it becomes less relevant to the current needs of the agricultural sector. Advertising was not measured as a quantitative variable in all the equations because (a) it is difficult to get information on promotion for each tractor model offered and (b) there remains the problem of identification since it is likely that large scale manufacturers like Ford and Massey Ferguson with large sales will have large advertising expenditures simply due to the practices followed by firms in determining advertising expenditure. Advertising expenditure was included in the manufacturers share equations but the above reservations apply. As a proxy for the advertising splurge accompanying a new model launch it was decided to include the new model year as a dummy variable. The analysis of market share proceeded at two levels :- (1) by manufacturer, and (2) by model (all manufacturers).

The demand curve estimated in (1) is that facing the firm (or the tractor division of the firm). The demand curves estimated in (2) and (3) refer to the several interdependent demand curves facing the multi-brand firm.

The general model of brand demand (in linear form) looks like

$$\frac{q_{it}}{q_t} = s_{it} = \beta_0 + \beta_1 \hat{U}_{it} + \beta_2 N_{it} + \beta_3 t_i + \beta_4 TL + \beta_5 s_{it-1} + e_{it}$$

where s_{it} is the market share of the i th model in t , \hat{U}_{it} (deflated or undeflated) represents the quality adjusted price ratio (or difference) of the i th model to all models available at that time, s_{it} has a value of 1 in the year of introduction of the i th model and a zero in other years, t_i is the number of years since the i th model was introduced, and TL is an interaction variable between time and large

wages increase relative to tractor prices.

Manufacturers Market Share

We are here concerned in explaining the market share performance of five manufacturers over a period of seventeen years - we are in effect pooling seventeen cross-sections of manufacturers. Since the price-quality regressions used models offered by the five manufacturers as observations we have to average the residuals obtained for each manufacturer to get the appropriate quality adjusted price. In this analysis of the demand function facing the firm the variables concerned with demand for specific tractor models will be ignored so that we are left with the price-quality residuals and lagged sales as explanatory variables. We will also experiment with a variable for total advertising expenditure on tractor promotion by these manufacturers.

Preliminary work involved experimentation with two functional forms of the market share equation, linear, and semi-log, with the semi-log formulation giving the best fit. Also two alternative forms of the residual were tried, (1) deflated by the current average price of tractors (\bar{P}_t) to represent a quality adjusted price ratio and (2) undeflated to represent a quality adjusted price difference. The undeflated formulation gave a consistently better fit and also a bigger t value for the coefficient associated with U_{it} . Market share was alternately defined on the basis of number and value but the results were not significantly different. Table 1 summarizes the more interesting results.

In Table 1 the first equation indicates that the quality adjusted price difference explains a third of the variation in market shares (measured as numbers) among the five manufacturers over the post-war period. The coefficient is more than six times as big as its standard $\%/\sqrt{n}$. Equation 2 has as dependent variable share defined in value terms but the results are very similar - further results refer to numbers only, perhaps a variable about which management is generally more sensitive. Compared with the linear form of the first two equations the third is non-linear with the log value of share related to the original value of the price variable. This formulation gives a better fit to the data than the linear form with forty-five per cent of the variance in market share being explained.

TABLE 1 MANUFACTURERS SHARE RESULTS

Equation	Dependent Variable	Constant	\hat{U}_{it}	$\frac{\hat{U}_{it}}{D_t}$	$\Delta \hat{U}_{it}$	A_{it}	$\log S_{it-1}$	R^2	V.N. Ratio
1	S_{it}	25.0202	-0.1920 (-6.41)					0.334	1.98
2	S_{it}	25.1106	-0.1830 (-6.08)					0.311	1.96
3	$\log S_{it}$	1.2338	-0.0060 (-8.20)					0.450	2.02
4	$\log S_{it}$	0.4019	-0.0019 (-4.17)					0.6944 (15.64)	1.77
5	$\log S_{it}$	1.1783		-1.7927 (-3.27)				0.5975 (13.81)	1.39
6	$\log S_{it}$	0.3132	-0.0013 (-2.43)		-0.0015 (-2.37)			0.7566 (14.98)	1.72
7	$\log S_{it}$	0.9406	-0.0050 (-4.75)			0.0256 (4.00)	0.4873 (11.29)	0.882	1.71

N.B. Figures in parentheses are the t-values of the associated parameter estimates.

The Von Neumann ratios give no evidence of the presence of auto-correlated disturbances and we might conclude that this very simple model of market share adjustment does a pretty good job in explaining a significant part of the variation among eight-four observations of market share. If we now discard the notion of instantaneous adjustment to price and quality discrepancies and substitute the idea of a gradual adjustment to a desired or equilibrium level of demand for a particular brand then we find that we can explain 86% of the variation in market shares as in equation 4. The obvious explanation for this is that the series on market shares for a particular manufacturer is highly autocorrelated and including lagged share as an explanatory variable will obviously help a lot. However the price variable remains highly significant and the coefficients on lagged share indicates a rate of adjustment to equilibrium of about 30% per annum. Replacing the price difference variable (\hat{U}_{it}) by the ratio version (\hat{U}_{it}/\hat{P}_t) gives a somewhat worse result as shown in equation 5. The coefficient is of course bigger (market share elasticities will be presented towards the end of the paper), but is only three times its standard error. Equation 6 in Table 1 includes a variable for change in price ($\Delta \hat{U}_{it}$) and although significant there is little contribution to the explanatory power of the equation. In Equation 7 we include a variable for advertising expenditure by manufacturer and the parameter estimate appears positive and significant but the explanatory power of the equation is only marginally increased. Its inclusion has boosted the impact of quality adjusted price on market share and has reduced the effect of last year's market share. This former effect may be explained by the fact that advertising provides information about the price and quality attributes of the tractors available. At the same time advertising expenditure is related to sales through the budgeting behaviour of firms and thus lagged share is picking up part of the effect of differentials in advertising expenditure if an advertising variable is omitted. Thus by including an advertising variable we are going some way toward solving the problem of specification bias in the adjustment coefficient but we are left with the problem of identifying the advertising parameter. If current advertising is partly determined by current sales then we have a two equation model and ordinary least squares estimators will be biased and inconsistent and the parameter cannot be identified as a demand parameter. However it seems reasonable to assume that advertising expenditure is determined by previous sales in which case we may view advertising as a predetermined variable. in which case ordinary least squares possesses

the usual optimal properties, and the parameter can be identified as a demand parameter and not a mixture of consumer and producer behaviour. In the expectation of biased estimates of the market share parameters because of observations fairly close to the zero bound on market share it was thought useful to run a demand for brand equation as a check. Unfortunately we cannot generally expect the market share parameter on price to equal the sales parameter on price; the sales elasticity w.r.t. price will in general exceed (in absolute value) the market share elasticity.* The result reported below confirms the

Since $q_i = m_i q$, $\frac{\partial q_i}{\partial p_i} = \frac{\partial m_i}{\partial p_i} q + \frac{\partial q}{\partial p_i} \cdot m_i$

and therefore the sales elasticity $\frac{\partial q_i}{\partial p_i} \cdot \frac{p_i}{q_i}$ is equal to the

market share elasticity $\frac{\partial m_i}{\partial p_i} \cdot \frac{p_i}{m_i}$ plus the elasticity of

aggregate sales w.r.t. the i^{th} price, $\frac{\partial q}{\partial p_i} \cdot \frac{p_i}{q}$.

significance of quality adjusted price and lagged sales but aggregate sales are shown to be unimportant, and this cannot be explained by its collinearity with lagged sales. Also the structure of the model is not satisfactory since variables determining brand sales will automatically determine aggregate sales and yet this is treated as a predetermined variable.

$$\log \left(\frac{q_{it}}{10} \right) = 2.6775 - 0.0053 \hat{U}_t + 0.0225 \log \left(\frac{q_t}{10^3} \right) + 0.5868 \log \left(\frac{q_{it-1}}{10} \right)$$

$$R^2 = 0.871 \quad V.N. = 1.53.$$

Model Market Shares (All Manufacturers)

We now try to explain market share movements for all the models offered by all the five manufacturers over the period 1948 to 1965. The number of observations is thus increased to 227 and three further variables are added to the ones considered in the manufacturers' share equations : (1) the model decay variable (t_i) (2) new model year, N_i

(year of introduction of the i th model), which is a dummy variable taking a value of 1 in the year in which the model was introduced and zero otherwise, and (3) a trend term for larger h.p. models (TL). The new model variable was intended to catch the impact of the promotional activity surrounding the launch of a new model and therefore the expected sign of the coefficient is positive. The price variables in these equations are taken directly from the price-quality regression since we have a residual associated with every model in every year. In addition to the variables already mentioned we are including a conventional trend variable for a perhaps unconventional reason. In this analysis we are pooling a whole series of cross-sections relation demand to price among a range of models at a point in time and if we make no further adjustment we are forcing the same demand parameters on all years. To be completely flexible about this we would add a dummy variable for each year to allow the relation to shift up or down and also interaction terms with, for example, price to allow the slope of the relation to change. Without going to such empirical extremes we have decided that there is good reason to suspect that the demand curve facing particular manufacturers for their different models will become more elastic over time since there had been a continuous tendency for the equipment range offered by the different manufacturers to become more homogeneous and interchangeable over time. We therefore include a trend variable and expect a negative sign since for a constant slope ($\frac{\partial S_{it}}{\partial U_{it}}$) an

increasing ratio of price to market share ($\frac{U_{it}}{S_{it}}$) implies an increasing elasticity.

The results arrayed in Table 2 indicate that we can explain a large proportion of the variation in model market share with the set of variables at hand. The parameter estimates associated with the price and lagged share variables are consistently significant and

TABLE 2 MODEL MARKET SHARE RESULTS (ALL MANUFACTURERS)

Iteration	Dependent Variable	Constant	\hat{U}_{it}	t_i	$\frac{\hat{U}_{it}}{p_t}$	T	T.L.	S_{it-1}	$\log S_{it-1}$	R^2	V.N. Ratio
1	S_{it}	4.794	-0.0168 (-2.7996)	-0.9905 (-4.9748)	-2.2660 (-2.2064)	0.1262 (2.0998)	0.8314 (22.1706)	0.735	2.35		
2	S_{it+1}	4.087	-0.1603 (-2.7590)	-0.9821 (-4.4040)	0.0266 (0.0220)	0.1478 (2.5537)	0.8491 (22.9870)	0.751	2.19		
3	S_{it}	2.2878	-0.0079 (-2.17)	-0.4668 (-4.01)	-1.1833 (-1.92)	0.0425 (1.18)	0.9087* (41.75)	0.904	2.70		
4	S_{it}	2.2490	-0.4641 (-3.9785)	-1.2192 (-1.97)	-3.6273 (-1.91)	0.0371 (1.03)	0.9108* (41.79)	0.903	2.70		
5	$\log S_{it}$	1.5670	-0.0084 (-7.40)	-0.2001 (5.31)	-0.2012 (1.02)	0.0121 (1.07)	0.3962* (11.91)	0.566	1.88		
6	S_{it}	3.1442	-0.0098 (-2.54)	-0.4294 (-3.62)	-1.1251 (-1.82)	-0.0927 (-1.45)	0.0781 (1.80)	0.8990* (39.60)	0.904	2.71	
7	$\log S_{it}$	2.2914	-0.0099 (-8.69)	-0.1675 (-4.56)	-0.1667 (-0.89)	-0.0860 (-4.64)	0.0455 (3.49)	0.3789* (11.79)	0.603	2.08	

* Adjusted where appropriate for replaced model.

N.B. Figures in parentheses are the t-values of the associated parameter estimates.

of the expected sign, although it is interesting to note that the impact of the price variable is better determined in the semi-log regressions than in the linear. The model decay variable (t_i) is also consistently significant and negative as expected, reflecting the declining market share of a model over time as newer models, more relevant to the demands of the consumer, are introduced. Similarly there appears to be a trend towards bigger horsepower tractors as revealed by the positive and significant trend coefficient associated with large tractors. The negative coefficient on N_i appeared at first sight to have the wrong sign but the explanation is perhaps obvious. Although we might expect a bigger rate of sales in a new model year the annual sales figures will be pulled down by the time period in which there were zero sales - that is the time up to the date of actual introduction, or more precisely, actual distribution. New models are usually announced at either the Royal Agricultural Show in July or the Smithfield Show in November. The coefficient is picking up the net effect on market share of these various features associated with the year of introduction, and this is evidently negative as far as numbers sold is concerned.

The first equation in Table 2 refers to market share by number whereas equation 2 refers to share by value. The results are essentially similar except for the new model year variable which is non-significant in the value equation which perhaps reflects the fact that in any particular year the new models are the most expensive and what you lose on the numbers you gain on the price! Examination of the residuals indicated that we were making very poor predictions where a new model was an obvious and close replacement for an older model offered by the same manufacturer. A particularly clear example of this was when a particular manufacturer decided to replace a popular model and the old model dropped in one year from a market share of 35.8% to 0.8% and its replacement jumped to a 43% share. It was therefore decided that where new models were obviously replacing older ones then the lagged share variable would be the share of the older model in the previous year. The effects of this adjustment are seen in equation 3 - the explanatory power of the equation is much improved, the lagged share variable becomes even more significant and the price variable is reduced in size, but is still clearly significant. The other variables all have a reduced role. In equation 4 the price difference variable (\hat{U}_{it}) is replaced by the price ratio variable (\hat{U}_{it/\bar{P}_t}). The price coefficient

is now much bigger but the rest of the equation is unaffected and the explanatory power is unchanged. The fifth and last equation in Table 2 gives a result for a semi-log equation. Although the explanatory power is considerably reduced compared with the linear form the coefficients on the other variables are much better determined. The coefficient on price is now almost ten times its standard error. In a later section we will compare the elasticities generated by these different functional forms. Equations 6 and 7 report results where we are allowing the market share elasticity with respect to model price to vary over time. The negative coefficient on the trend term is as expected but is only clearly significant in the semi-logarithmic formulation (Equation 7). Especially in this case, the presence of the trend term does appear to improve the specification of the relationship. We can see in Equation 7 that all the variables, except for the new model variable, are clearly significant and the explanatory power is significantly improved. Despite its lower explanatory power the semi-log relation seems preferable : because it is doing a better job in explaining deviations from previous market share performance. We also ran a demand for model or model sales equation which is reported below. Qualitatively the results are very similar to the market share formulation and as with sales by manufacturer we find that total sales aggregated over all models, is not important in explaining sales of the i th model.

$$\log q_{it} = 10.476 - 0.0098 \hat{U}_{it} - 0.1725 t_i - 0.1936 N_i - 0.0677 T$$

(-8.61) (-4.67) (-1.03) (-3.12)

$$+ 0.0435 TL + 0.3726 \log q_{it-1} - 0.1614 \log q_t$$

(3.33) (11.85) (-0.26)

$$\tilde{R}^2 = 0.60$$

$$\begin{matrix} V.N. \\ \text{Ratio} \end{matrix} = 2.08$$

In defining the quality-adjusted price variable \hat{U}_{it} we have throughout used the residuals from weighted regressions of price on quality characteristics. We could instead have used residuals from unweighted regressions and our qualitative inferences would stand although we would expect to get a smaller intercept term. These predictions proved correct in some experiments we made with unweighted residuals.

We have analysed market share behaviour with little reference to the supply side when in fact there may be important identification problems. First of all market share movements take place within the constraints of productive capacity at manufacturer and model level and at various times excess demand may exist for a particular model or make of tractor. This may have been particularly true in the early post-war period. Two manufacturers mentioned shortages which existed at various times but on inspection we did not find large negative residuals in the market share equations, as would have been expected. Also since almost 80% of tractor output is exported it is comparatively easy for diversions to take place to fill temporary shortages in the domestic market. Another aspect of the supply side of the problem is highlighted by the improvement in prediction resulting from using the lagged share of a "replaced" model in explaining the share of a new model. It is quite likely that firms introducing a new product try to run down the older replaced product as rapidly as possible so as to maximize any "scale" economies that might exist. There are obvious exceptions to this sort of behaviour, but in the case of a durable input like the tractor it is not likely that a significant proportion of buyers would be closely attached to an old model as might be the case with a durable consumer good like the car. Therefore the firm might feel that it could run down an old line pretty fast without affecting the probability of repeat purchasing.

The other supply problem considered earlier in the theoretical discussion was the simultaneous determination of sales, price, quality and advertising. Of these the real problem lies with advertising; in the long run the firm is likely to set advertising expenditure in some relation to sales which raises identification problems in cross-sectional analysis and in the short-run the firm can adjust advertising rapidly according to sales experience which gives rise to identification problems for time series analysis. Price and quality adjustments (including new product innovations) cannot be made so rapidly by the firm but it is undoubtedly true that in cross-section sales determines price - large firms operating at a lower point on the long-run cost curve.

Market Share Price Elasticities

We are now concerned with taking various results for the relationships between market share and price and from these generating estimates of the elasticity of demand facing the firm. We can then

compare brand elasticities with those obtained in a previous study for aggregate tractor demand and we can also check on one of the predictions of monopoly theory that the firm will always produce within the elastic arc of the demand curve. In fact we are dealing with oligopolists who are obviously concerned about the reaction of their rivals to any decision they take. We will in fact generate market share elasticities but these have a clear relationship with demand elasticities as shown earlier. We can write down the average own-price elasticity of demand as $\frac{\partial q_{it}}{\partial p_{it}} \cdot \frac{p_{it}}{q_{it}}$. In our

case we substitute s_{it} for q_{it} to get the market share elasticity. In our market share results we have used a price variable adjusted for the difference in quality between the particular model and the average model. Where the price difference variable (\hat{U}_{it}) is used its coefficient is an estimate of $\frac{\partial s_{it}}{\partial p_{it}}$ in the linear equations and the

coefficient times s_{it} is the estimate in semi-log equations. Where \hat{U}_{it} / \bar{p}_t is used then our estimates of $\frac{\partial s_{it}}{\partial p_{it}}$ are the previous estimates

divided by \bar{p}_t . We can compute these partial derivatives and then multiply by the price-market share ratio for the manufacturer or model in question to get the short-run elasticities - the long-run elasticities being obtained by dividing the short-run elasticities by $1 - \alpha_6$ (the coefficient on lagged share) which is our estimate of the adjustment coefficient.

Our estimates of the market share elasticities for manufacturers which are reported in Table 3 have been derived from the regression results presented in Table 1. Equations 4 and 5 differ only in the form of the price variable, equation 4 being a price difference and equation 5 being a price ratio variable. Equation 7 includes advertising expenditure as well as a price difference variable together with lagged market share, in common with the other equations. All the estimates indicate an elastic demand for particular brands of tractor with short-run elasticities varying between -1.03 and -2.99 and long-run elasticities being considerably bigger and varying between -3.35 and -5.43 on average over the period 1947 to 1965. The situation in the latest year (1965) suggests demand is becoming more elastic with short-run elasticities going as high as -3.80 and long-run going up to -6.91. The price ratio variable suggests a higher elasticity than the price difference variable but the latter appears

TABLE 3: MARKET SHARE ELASTICITIES FOR MANUFACTURERS

(derived from equations 4, 5, and 7 in Table 1)

Firm (average over the period 1957-1965)	Equation 4			Equation 5			Equation 7		
	Short run	Long run	Short run	Long run	Short run	Long run	Short run	Long run	Long run
Massey-Ferguson	-1.03	-3.35	-1.65	-4.14	-2.48	-4.52			
Ford	-1.08	-3.51	-1.75	-4.88	-2.60	-4.73			
David Brown	-1.15	-3.76	-1.88	-4.69	-2.79	-5.07			
International Harvester	-1.20	-3.91	-1.95	-4.88	-2.90	-5.27			
Nuffield	-1.23	-4.03	-2.01	-5.03	-2.99	-5.43			
Average (All Manufacturers)	-1.10	-3.59	-1.79	-4.49	-2.66	-4.84			
<hr/>									
1955									
Massey-Ferguson	-1.62	-5.29	-2.00	-5.01	-3.92	-7.12			
Ford	-1.93	-6.30	-2.39	-5.97	-4.67	-8.48			
David Brown	-1.48	-4.82	-1.83	-4.57	-3.57	-6.49			
International Harvester	-1.41	-4.60	-1.74	-4.36	-3.41	-6.19			
Nuffield	-1.57	-5.13	-1.95	-4.48	-3.80	-6.91			
Average (All Manufacturers)	-1.45	-4.73	-2.36	-5.90	-3.50	-6.37			

preferable on statistical grounds. Adding in advertising expenditure more than doubles the short-run elasticities but only increases the long-run elasticities by a third. To some extent we could say the specification of the market share equation had been improved by adding in advertising but its effect may be accentuated by the coexistence of a positive relationship between advertising and market share reflecting firm behaviour. We might then argue that price elasticities derived from an equation including advertising may be overstated and would represent an upper bound with the estimates from the equations omitting advertising being a lower bound. The elasticities derived from the manufacturer's demand (sales) equation are bigger than those from the market share equation including advertising. The average elasticities over the whole period were -3.07 in the short-run and -7.42 in the long-run and in 1965 they were -4.04 and -9.78 respectively. These elasticities are the most closely comparable with the aggregate elasticity for gross-investment in tractors which was found in the earlier study [Rayner and Cowling (7)] to be -2.52 at the mean of gross investment. As expected the demand for specific brands of tractors is more elastic than for tractors in aggregate. The observed elasticities are obviously consistent with the behaviour of profit maximising firms.

Conclusions

We have attempted an integration of the notion of price-quality relationships with a model of demand for branded goods. Recalling earlier contributions which attempt to introduce the idea of quality into theories of consumer behaviour, we have developed an operational version of a model in which price and quality are jointly allowed to determine market share behaviour. The model is tested out in the market for a durable input, farm tractors, and quality adjusted price is found to be an important variable. Elasticities of demand facing specific manufacturers were calculated and found to be quite high - well within the elastic range in the long-run.

A similar two-stage model (the first stage requiring an estimate of the implicit prices on the qualitative brand characteristics) would seem appropriate to many markets for branded goods where quality variation among manufacturers and products is important. We are currently examining the car market but one might examine qualitative characteristics of a less conventional sort - for example finding the trade-off between

price, delivery delays and servicing in international markets. In such ways we could begin to examine Britains' international competitive position to see how far devaluation may be able to counter the qualitative objections foreigners have to British goods.

As well as moving into such less conventional fields we must explore the simultaneity of the model treating price, advertising and possibly quality as endogenous variables in addition to brand sales. Such developments may lead to an operational version of a more complete theory of oligopolistic competition.

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