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POWER RELATIONS IN THE INTERNATIONAL MONETARY
FUND: A STUDY OF THE POLITICAL ECONOMY OF A
PRIORI VOTING POWER USING THE THEORY OF SIMPLE
GAMES

D. Leech

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USING THE THEORY OF SIMPLE GAMES

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**POWER RELATIONS IN THE INTERNATIONAL MONETARY FUND: A
STUDY OF THE POLITICAL ECONOMY OF A PRIORI VOTING POWER
USING THE THEORY OF SIMPLE GAMES**

by

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October 1997

This paper is preliminary. Please do not quote it without permission.

It was presented to the conference “Non-State Actors and Authority in the Global System” at the University of Warwick in November 1997. I would like to thank participants in the discussion who made valuable comments: these will contribute to the next draft; I would also like to thank Miquel Manjon and participants in a Warwick Economics department staff seminar for stimulating comments and encouragement in the development of this work. All errors in data or analysis are my own.

Comments welcome.

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Abstract: In general in organisations whose system of governance involves weighted majority voting, power and voting weight differ. Power indices are a value concept for majority voting games which provide a means of analysing this difference. This paper provides new algorithms for computing the two classical power indices (the Banzhaf index and the Shapley-Shubik index) and applies them to the voting distribution in the two governing bodies of the IMF in each year since its foundation. The focus is both substantive, being an analysis of the political economy of the IMF, and methodological, as a study of the use of the power indices. Power relations are studied with respect to two types of decisions: ordinary decisions requiring a simple majority and decisions requiring a special majority of 80% or 85%. Clear cut results are obtained for the former: among the G5 countries discrepancies between power and voting weight have declined over time with the exception of the United States which continues to have much more power than its weight even though that weight has declined. In the nineteen forties the United Kingdom's power was considerably below its relatively large nominal voting power, similarly to some extent for France. Both power indices give results which are qualitatively comparable. For decisions requiring special majorities, however, few general results emerge because of conflict between the indices.

I. Introduction

The system of governance of the IMF is of continuing interest not only because of the obvious importance of the organisation to the management of the world economy in an era of increasing economic globalisation. It is also a worthwhile subject of research because the IMF is one of a small number of international organisations (the World Bank, the European Union Council of Ministers and regional development banks are others) that operate on the basis of the weighted voting power of their members (in contrast for example to the United Nations).

The inequality of voting power among members of the IMF reflects the inequality of their quotas and therefore their respective financial contributions. Inequality of power among members is therefore a fundamental feature of the design of the system of governance. However, this raises the question of precisely what is meant by unequal power. If power is defined as the ability to influence voting outcomes, to make a difference to decisions taken in this way, then it is not always true that there is an exact correspondence between power and nominal voting strength. Indeed it has been known for a long time (e.g. Banzhaf (1965), Coleman(1971), Lucas (1983)) that it is generally the case that in a body which uses weighted voting, there is no simple relationship between power and voting weight¹.

In many weighted voting bodies power is much more unequally distributed than voting weight so that the nominal data can give a misleading picture. Members with very large voting weight can possess a disproportionately greater voting power - there is an extra “invisible weighting” - and other members proportionately

¹ In the discussion about the IMF voting system the term voting power is used to denote the number of votes (or fraction of the total) commanded by a member country. Since in this paper we are making a fundamental distinction between this and actual power as defined, we will use the term voting weight instead from now on.

less. Moreover, voting paradoxes can and do arise, particularly when there are changes to the membership, the distribution of votes or the decision rule (e.g. Brams (1975), Brams and Affuso (1976), also Felsenthal and Machover (1995)). Cases have been shown to occur where a change in voting weight has led to a change in power in the opposite direction. Similarly it is possible for individual voters to have no power at all despite possessing an apparently significant number of votes (for example Luxembourg in the EC Council of Ministers before 1973).

It is therefore of intrinsic interest to consider the voting system in the IMF from this point of view, by analysing the distribution of *a priori* voting power². Despite its importance and interest relatively little research has been done on this aspect, in contrast to other voting bodies. We are aware of only two studies which have adopted a similar approach to the one employed here: those by Dreyer and Schotter (1980) and recently Strand et.al. (1997).

A secondary purpose of the present study is a methodological one of examining the application of two different indices used as measures of voting power, neither of which has yet been shown to be superior although they both possess inherently plausible theoretical foundations. In applications they often give results which are in agreement to a good degree although in some cases they conflict considerably. One issue which is evident from the literature on power indices has been the difficulty of calculating them in empirical applications with real data rather than simple examples. In this study we employ new algorithms for computing the indices.

Questions we investigate in this paper are:

² It is important to stress that this is fundamentally different from attempting to draw conclusions from analyses of observed voting behaviour.

(1) How does the voting power of individual countries compare with their nominal votes and how have these relations evolved over time?

(2) In particular, to what extent is the degree of inequality in the distribution of votes reflected in the distribution of voting power. Is voting power more equally or more unequally distributed than voting weight, and how has the inequality changed over time as new members have joined and quotas increased?

(3) Have voting paradoxes arisen?

(4) Are there important differences in the distribution of voting power between the two main IMF decision-making bodies, the Board of Governors and the Executive Directors?

(5) Different types of decisions use different decision rules, some requiring a special majority larger than a simple majority. We investigate the effect of different decision rules on the distribution of power.

(6) How do the results given by the two measures of power compare?

The analysis in this paper is entirely in terms of formal voting power and the formal constitution as laid down in the Articles of Agreement and its amendments; we take the allocation of voting weight among the members at face value. It is commonplace however to note that the organisation is in practice controlled by the United States and the advanced industrial nations since their combined voting weight gives them a majority over the developing countries. The analysis presented here is not concerned with such questions about the power of informal groupings of countries, although the methodology can obviously and usefully be employed to

evaluate and compare the voting power of groupings which did not have an actual majority.

For example it would be possible to use this approach to comment on the criticisms made by developing countries that the distribution of voting power has been too heavily weighted towards developed countries. This is a consequence of a fundamental aspect of the design of the IMF that dominant voting power should be in the hands of creditor nations who provide the resources. Our analysis should be able to illuminate the extent to which this aim is fulfilled in practice or whether the “invisible weighting” of the United States and other large creditors skews the power distribution even further away from the debtor countries. Another obvious application could be the power of the European Union countries as a group.³

We also note at this stage that in practice actual votes are rarely taken in meetings of the IMF⁴. Indeed they are deliberately avoided, especially by the Executive Directors, in order to avoid the element of confrontation which attends a contested vote, with decisions being taken after arriving at a consensus. It might therefore be suggested that an analysis of voting power is beside the point if all decisions are taken by consensus. However, formal voting procedures may fundamentally influence the de facto decision making process; power relationships are fundamentally determined by relative voting strength and the fact that votes are not taken in meetings is a reflection that these are well understood and determine the basis on which a consensus is reached. It is also true that the members of the

³ This kind of analysis could be enriched by considering the power of individual countries which are members of such groupings in terms of a two-stage process (a composed game) whereby voting is considered first within the grouping then the grouping votes as a block within the organisation. This would be feasible as an extension of the methodology used here.

⁴ There is a parallel between voting and power distributions among the countries which are members of IMF and those among shareholders of a public company (the Board of Governors corresponding to the shareholders' meeting) where analyses of power and control are conducted without regard for actual voting patterns, but are in terms of potential votes which could theoretically be taken. There is an important difference however in that weighted voting is a central feature of the day-to-day operations of the IMF, in the Executive Directors as well as the Board of Governors

IMF attach considerable importance to the question of voting power.(Zamora (1980)).

II. The Institutional Framework

There are two decision-making bodies: the Board of Governors and the Executive Directors⁵. All powers of the IMF are vested in the Board of Governors, which may delegate to the Executive Directors authority to exercise all except certain specified reserved powers. The Executive Directors are responsible for the general operations of the IMF. The Board of Governors has delegated the maximum powers that the Articles permit to the Executive Directors.

The powers exercised by the Board of Governors and expressly reserved to it by the Articles refer to matters of a fundamental or political nature or which may have a profound economic impact. The reserved powers of the Board of Governors include the power to: admit new members; require a member to withdraw; approve a revision of quotas; determine the extent of delegation of powers to the Executive Directors; determine the distribution of the net income of the Fund; determine the remuneration of Executive Directors; determine the number of Executive Directors to be elected; decide appeals against decisions of the Executive Directors; make arrangements to co-operate with other international organisations; decide to liquidate the Fund. Some powers are vested in the Executive Directors and the exercise of them does not depend on delegation from the Board of Governors, such as the election of the Managing Director and the power to suspend or terminate suspension of certain provisions. As well as exercising the powers which are either vested in them or delegated to them, an important role of the Executive Directors is

⁵ See Gold (1972).

to make recommendations to the Board of Governors about decisions which the latter is to take.

The Board of Governors comprises one governor (or one alternate governor in the absence of the governor) appointed by each member country, usually the minister of finance or the governor of the central bank. It meets once per year; the Articles provide for the possibility of additional meetings and for a vote to be taken without a meeting if the Executive Directors judge it necessary. Governors vote in person and there is no proxy voting. Each governor is entitled to cast the number of votes of his country and may not cast fewer; his choice is to cast the total votes or to abstain. Decisions, which take the form of resolutions, are taken by a majority of the votes cast except where the Articles require a special majority for a particular decision. In all cases a quorum for a meeting is a majority of the governors exercising not less than two thirds of the total voting weight of the members.

The Executive Directors, who are generally officials or diplomats rather than politicians, are appointed or elected by members of the Board of Governors. Like the Board of Governors, the Executive Directors, who function in continuous session, employ a system of weighted voting and therefore it is necessary to consider their composition and analyse the distribution of voting power among them as well as among the Board of Governors. The Articles provide for not fewer than twelve Executive Directors, five of whom are appointed by the five members having the largest quotas, two elected by the American republics not entitled to make an appointment and five elected by the other members. (Two additional directors are to be appointed by the two members whose currencies have been sold by the Fund in the largest amounts in the previous two years if they are not already entitled to appoint.) As the number of members has increased the Articles have been amended

to increase the number of elected directors and in the 1994 election there were 19 elected directors.

Elections of Executive Directors are held every two years. There is a minimum and a maximum percentage of the eligible votes that a nominee must receive in order to be elected. In practice the minimum percentage results in most cases in directors requiring the votes of more than one member in order to be elected. The principle behind the requirement of a minimum percentage vote is to encourage the formation of coalitions of members with common interests who elect directors to represent them while the requirement of a maximum percentage prevents great disparities in the voting strength of individual elected directors.

An Executive Director casts the votes of those members who voted for him as a unit and cannot split the vote. With the exception of the American republics, the Articles do not associate Executive Directors with predetermined regional or other groupings. However the operation of the system has led to the formation of many such relatively stable groupings which have each elected an Executive Director over several elections. Members which combine in this way engage in negotiations among themselves through channels which are outside the IMF. It might therefore be appropriate to study the power relationships in the Executive Directors in terms of a two-stage voting process: first members vote by weighted voting within their group, second their director votes their combined weight as a unit in the Executive Directors. This two-stage approach could have the effect of giving a greatly increased voting power to certain individual members in the Executive Directors whose voting weight in the Board of Governors was small. For example in 1994 Belgium had 2.1% of the votes in the Board of Governors, but was in a relatively dominant position, with over 40% of the votes, in a grouping whose combined weight in the Executive Directors was 5.1%. That approach would require the

groups to be well determined, which probably cannot be assumed in all cases, and an assumption that there is a similar process within each grouping, which cannot be assumed either. However it is an interesting question which is not considered here and remains for future work⁶.

The basic decision rule is that, except as specifically provided, all decisions are made by a simple majority of the votes cast. This applies to both the Board of Governors and the Executive Directors and most of the decisions of the Fund. The rule is in terms to votes cast rather than total votes but this distinction is ignored in this study. For certain categories of decision, however, special majorities, defined in terms of proportions of the total voting weight, whether the votes are cast or not, are required. These tend to be the most important types of decision where a degree of consensus is required to make them effective.

The original Articles provided for a range of different special majority requirements, as fractions of the total voting weight, from a simple majority to four fifths, for different categories of decision but this was amended (after the Second Amendment) effectively to two: 70 percent and 85 percent. Since 1969 a large number of types of decisions now require a special majority of 85 percent. The introduction of this higher figure effectively allowed the United States to keep its veto while reducing its contribution, but at the same time it meant that groups of other countries, as a bloc, such as the EU or the developing countries could also have a veto. The adoption of a higher special majority requirement is a move in the direction of unanimity and in terms of voting power will have resulted in greater equality.

⁶ Some member countries have provided Executive Directors virtually continuously: Australia, Brazil, Belgium, Canada, China, India, Italy, Netherlands.

III. Weighted Voting

Every member of the IMF has a quota expressed in United States dollars which is its subscription to the resources of the organisation and also determines its voting weight. The votes allotted to a member are equal to two hundred and fifty plus one vote for each part of its quota equivalent to one hundred thousand dollars. Thus voting weight varies linearly according to the size of the quota rather than proportionately. This is one important difference with a business corporation where votes are strictly proportional to contributions to equity capital. The existence of the 250 “basic” votes which every member has independently of its quota reflect concerns expressed at the Bretton Woods conference. It was felt that the radical move (in an international organisation) of adopting a system of weighted voting for the IMF, where the weights reflected economic and financial factors, should be tempered by the political consideration of the traditional equality of states in international law. To have allocated votes in proportion to quotas would have meant too close a similarity with a business corporation and might have given too high a degree of control to a small group of member countries.

Table 1. "Basic" Votes as a Percentage of Total Votes

	Number of members	Total basic votes	Total votes	Percentage basic votes
1946	39	9750	84475	11.54
1956	58	14500	101280	14.32
1966	103	25750	217657	11.83
1976	128	32000	324114	9.87
1986	149	37250	930300	4.00
1996	179	44750	1493331	3.00

The Articles do not provide for an increase in basic votes when quotas increase and for most countries their voting weight has become more closely proportional to their quota as the latter has increased over the years. The proportion of total votes represented by the combined basic votes has accordingly fallen substantially over the years (see Table 1) from a high of over 14% in 1956 to 3% in 1996. This decline has been in spite of the increase in the number of members, many of which are developing countries with very small quotas. However for the great majority of members their basic votes have become insignificant. For example for Belgium the share of its total voting weight represented by basic votes has fallen from 10% in 1946 to 0.7% in 1996, for Mexico it has fallen from 21.7% to 1.4% over the same period.

IV. The Measurement of Voting Power

In a weighted voting body the power a member exercises is not, in general, the same as its voting strength. This is a general feature of weighted voting and does not depend on the nature of relations among the members, such as groupings or

parties. In the context of the present analysis of the IMF we are not concerned, for example, with the power of the G7 countries acting in concert, but with the formal properties of the particular distribution of the voting weight between the member countries. We employ the method of power indices for Simple Games based on a simple abstract definition of power. The measure of a member's power does not depend on its preferences but only on its voting weight and the distribution of voting weight among the other members.

The approach is in principle very simple. It is assumed that power can be defined in such a way that it can be thought of as being shared among the members. Given a voting rule in terms of a winning quota, such as a simple majority or a larger special majority, if a member's votes exceed the quota, it obviously has all the power; otherwise power is divided among the members on the basis of their ability to influence decisions. This requires a precise definition in rigorous mathematical terms to lead to a quantitative measure.

The two classical power indices⁷ which are used here (Owen (1995), Straffin (1994)) are both based on the idea that a member's power is its capacity to change a vote from losing to winning by using its votes. In general its share of the power is defined as the number of times it can do this expressed as a proportion of the total number of times that any member can do it. Thus the approach does not take any account of the extent to which the voting body itself is able to take decisions; members of a voting body may be powerless because the body itself is incapable of making decisions. The method is confined to the analysis of power relationships among the members relative to those decisions which are made at least in principle.

⁷ Another index which has been employed in some studies is the Johnston index. This has been used in a study of the political economy of the IMF by Strand et.al.(1997), but is not used here.

Although the two classical power indices employ the same basic concept of a swing whereby a member, by joining a voting coalition changes it from losing to winning, they are mathematically quite distinct since they employ different coalition models. The Banzhaf index (Banzhaf (1965)) is based on considering coalitions as combinations of members in the sense of a list of countries arranged in no particular order; they might be arranged alphabetically, or in any other arbitrary order. A member's power index is then the number of such coalitions it can swing from losing to winning by joining them, expressed as a proportion of all such swings when all members are considered.

The Shapley-Shubik index (Shapley and Shubik (1954)) on the other hand, counts coalitions on the basis not only of whether it is a swing, but also the order in which members are listed. Thus, given that a member is able to swing a vote, the index takes into account both the number of orderings of the members of the winning coalition and the members not in the coalition. The index is defined by expressing this number as a proportion of the number of orderings of all members.

For example, consider a voting body with 10 members. Suppose we wish to measure the power of member i . Let us consider the effect of the size of the coalition on the measurement of power according to the two indices. Consider two swings, coalitions which are losing until i joins, one of 4 members and one of 7 members. The Banzhaf index treats these two swings equally: each counts as one swing. The Shapley-Shubik index, however, attaches very different importance to them. For the coalition of size 4, the number of orderings of its members and the remaining 5 members is $4!5! = 2,880$. The contribution of this swing to the index is then $2880/10! = 2880/3628800 = 0.000793$. For the other coalition, however, its contribution to the power index is $7!2!/10! = 10,080/3628800 = 0.002778$. Thus the number of members in a swing coalition has a considerable importance to the

computation of the Shapley-Shubik index, those where the winning coalition and its complement are relatively equal being given much less weight.

Despite being so different in the way they count swings, the two indices have given results which have not been very different in some applications. However they have given results which have differed considerably in others. There is no clear guidance from the literature on the relative merits of the two indices. While political scientists and lawyers have tended to prefer the Banzhaf index and criticised the coalition model underlying the Shapley-Shubik index, mathematicians and game theorists have tended to the reverse preference ordering since the Shapley-Shubik index has been shown to satisfy certain axioms. Both indices give rise to a power distribution among the members of the legislature. In addition, however, the Shapley-Shubik index can be interpreted directly as the probability of swing given its coalition model, so that power can be thought of as the probability that each member swings the vote, whereas the Banzhaf index cannot. Evaluating the probability of a swing for each player within the Banzhaf coalition model gives indices which do not sum to unity and therefore must be normalised to give a power distribution⁸.

One factor which has to some extent limited the study of power indices in empirical applications has been the difficulty of computing them when the number of members of the voting body is moderate or large. In this study we employ algorithms which enable the approach to be used to study the two bodies of the IMF. Indices are computed for the Board of Governors, which has a large number of members, between 39 in 1946 and 179 in 1996, using a modified multilinear

⁸ Authors who have addressed the question of the respective merits of the indices from various angles include Brams (1975), Leech (1990), Roth (1977), Straffin (1977), Felsenthal and Machover (1995).

extensions algorithm. Power relations within the Executive Directors are studied using a new direct enumeration algorithm. These are described below.

V. Computation of the Voting Power Indices

We characterise a voting body by the number of its members, n , their voting weights, $w_1, w_2, \dots, w_i, \dots, w_n$ and a decision rule in terms of a quota⁹, q . In the present case the weights are proportions so that $\sum w_i = 1$. The quota is the size of majority required for a decision; since any voting body usually has a decision rule which leads to a unique decision, as here, we require $q \geq 1/2$.

The power index of member i is the value for player i of the corresponding simple game $\{q; w_1, \dots, w_n\}$. The set of all players is denoted N . For member i , a swing is a coalition represented by a set, S , $S \subseteq N$, such that $i \notin S$, where

$$\sum_{j \in S} w_j < q \quad \text{and} \quad \sum_{j \in S} w_j + w_i \geq q. \quad (1)$$

The Banzhaf power index, denoted β_i , is the ratio of the number of swings for member i , $\eta_i = \sum_s 1$, to the total number of swings for all members,

$$\beta_i = \eta_i / \sum \eta_i. \quad (2)$$

The Shapley-Shubik index is defined in terms of the number of orderings of members for each swing. Letting $\theta_i = \sum_s s!(n-s-1)!$, where s is the number of members of the set S , and the summation is taken over swings, the index is defined as $\gamma_i = \theta_i/n!$. More usually the expression is written:

⁹ This is here used as a technical term and should not be confused with IMF quotas.

$$\gamma_i = \sum_s \frac{s!(n-s-1)!}{n!}. \quad (3)$$

Two algorithms are used to calculate the β_i 's and the γ_i 's in this study: (1) Direct Enumeration, and (2) a Modified MLE Approximation.

(1) The Direct Enumeration Algorithm

The algorithm finds every subset $M \subseteq N$ and tests it for a swing for each member. The subroutine that is the basis of this algorithm which guarantees that every subset of N is found exactly once is described in Nijenhuis and Wilf (1995).

Thus if $\sum_{j \in M} w_j \geq q$ and $\sum_{j \in M, j \neq i} w_j < q$ then there is a swing for member i and η_i is increased by one and γ_i increased by $\frac{(m-1)!(n-m)!}{n!}$ (where obviously m is the number of members of the set M). The Shapley-Shubik index is obtained directly when all coalitions M have been tested for swings. The Banzhaf index is found by (2).

This algorithm evaluates the power index directly but has the disadvantage that it is compute-intensive. The time taken to test all subsets of N increases exponentially with n , since the number of subsets is 2^n , computation time approximately doubling every time n increases by 1. However the algorithm has been found to work well for moderate-sized games such as the IMF Executive Directors, for which $n \leq 24$. For games much larger than this the computing time required would be prohibitive. An advantage of this algorithm is that it can be generalised to compute values of general n -person games, such as Shapley values, since it does not rely on exploiting the special features of Simple Games.

(2) A Modified MLE Approximation Method

The question of the difficulty of computing the values of large games was addressed by Owen (1972) who proposed a method of approximation based on the multilinear extension (MLE). Algorithms based on this approach have been used with a good degree of success in a number of applications (e.g. Owen (1975), Leech (1988, 1992), Widgren (1994)).

Consider first the Shapley-Shubik index, defined by (3). The term inside the summation is a Beta integral and hence we can write:

$$B(s, n-s-1) = \frac{s!(n-s-1)!}{n!} \quad (4)$$

$$= \int_0^1 t^s(1-t)^{n-s-1} dt \quad (5)$$

Both (4) and (5) can be interpreted probabilistically. (4) is the probability that the swing S occurs in the sense that the members of S and of $N-S-\{i\}$ can arrange themselves in any order, where all orderings are equally probable and independent.

Similarly the term inside the integral in (5) can also be thought of as a probability. Indeed that is the basis of the algorithm. If we let t be the probability of any member belonging to S , its complement is $(1-t)$, the probability that it will belong to $N-S-\{i\}$. Then $t^s(1-t)^{n-s-1}$ is the probability that the coalition S will occur assuming independence.

Taking the expression inside the integral and summing over all swings gives the probability of a swing for member i as a function of t . Denote this by $f_i(t)$.

$$f_i(t) = \sum_s t^s (1-t)^{n-s-1} dt \quad (6)$$

Substituting (6) into (3) using (5) gives $\gamma_i = \int_0^1 \sum_s t^s (1-t)^{n-s-1} dt$. Hence,

$$\gamma_i = \int_0^1 f_i(t) dt \quad (7)$$

The Banzhaf index can be obtained using (6) evaluated at $t=1/2$. $f_i(1/2)$ is the probability of a swing assuming $t=1/2$. Then, we obtain the index by normalising,

$$\beta_i = \frac{f_i(1/2)}{\sum f_i(1/2)}. \quad (8)$$

In large games we can evaluate $f_i(t)$ approximately. Letting x_j be a random variable which equals w_j with probability t and equals 0 with probability $1-t$, then $E(x_j) = tw_j$ and $\text{Var}(x_j) = t(1-t)w_j^2$.

Define the random variable $X_i = \sum_{j \neq i} x_j$; X_i is the total number of votes of the members other than i in the same sense as that of i and so its distribution can be used to find the probability of a swing.

The distribution of X_j will be approximately normal for large n provided the conditions for the central limit theorem hold, notably that none of the w_j 's is too large in relation to the others. The moments of the distribution are,

$$\mu_i(t) = E(X_i) = t \sum_{j \neq i} w_j,$$

$$\sigma_i^2(t) = \text{Var}(X_i) = t(1-t) \sum_{j \neq i} w_j^2.$$

Then, the required probability is

$$f_i(t) = \Pr[q - w_i < X_i < q]. \quad (9)$$

which, from the normal distribution function $\Phi(\cdot)$, is

$$f_i(t) = \Phi\left(\frac{q - \mu_i(t)}{\sigma_i(t)}\right) - \Phi\left(\frac{q - w_i - \mu_i(t)}{\sigma_i(t)}\right). \quad (10)$$

The probability (10) is a function of t and this parameter is integrated out using numerical quadrature on (7) after substitution, to give the Shapley-Shubik index, or set equal to 1/2 for the Banzhaf index in (8).

This describes the MLE approximation method which is quite accurate enough provided the votes are relatively dispersed so that there are no large voting weights. Where one member has a large voting weight, however, while the approximation works well for that member's Shapley-Shubik index, it is inaccurate for those of the other members; it is also inaccurate for all the Banzhaf indices.

Applying the method to the data for the IMF gave results which were unsatisfactory because of the very large voting weight of the United States. We have therefore used a modified MLE approximation method which treats the votes of the largest member as a discrete random variable and applies the normal approximation to the rest.

The Modified MLE Approximation Method used in this study is as follows. Consider the derivation of the indices for $i \neq 1$. (That for $i = 1$ is the same as before.) This is based on the distributions of $X_i = \sum_{j \neq i} x_j$ conditional on x_1 . Each of these conditional distributions is normal with the same variance and different expectations:

$$\mu_{i1}(t) = E(X_i | x_1 = w_1) = w_1 + t \sum_{j \neq 1, j \neq i} w_j$$

$$\mu_{i2}(t) = E(X_i|x_1=0) = t \sum_{j \neq i, j \neq 1} w_j$$

$$\omega_i^2(t) = \text{Var}(X_i|x_1) = t(1-t) \sum_{j \neq i, j \neq 1} w_j^2$$

The probability in (9) is then

$$\begin{aligned} f_i(t) &= \Pr[q - w_i < X_i < q|x_1=w_i]t + \Pr[q - w_i < X_i < q|x_1=0](1-t) \\ &= [\Phi(\frac{q - \mu_{i1}(t)}{\omega_i(t)}) - \Phi(\frac{q - w_i - \mu_{i1}(t)}{\omega_i(t)})]t \\ &\quad + [\Phi(\frac{q - \mu_{i2}(t)}{\omega_i(t)}) - \Phi(\frac{q - w_i - \mu_{i2}(t)}{\omega_i(t)})](1-t) \end{aligned} \quad (11)$$

Expression (11) is used in the computation of the Banzhaf index by setting $t=1/2$ and in the computation of the Shapley-Shubik index by integrating out t .

VI. The Data

Separate data sets have been used to analyse power relations in the Board of Governors and the Executive Directors. The former consists of percentage voting weights for each member and every year; the latter consists of the voting weights held by each executive director.

The voting weights for the Board of Governors for the first twenty six years were taken from Gold (1972) which lists all the voting weights for every member country and every year. The weights for later years (and for the Executive Directors in all years) have been calculated using figures taken from various issues of the IMF Annual Report. The figures presented in the Annual Reports describe the way in

which the voting weights of the individual Executive Directors have been determined through the voting system and therefore which member countries are represented by each director. In almost every year certain countries have been excluded because they did not participate in elections of Executive Directors; China and South Africa have not infrequently been in this group. Also certain new members have often not participated in the elections or been unable to do so because they joined the organisation too late. Such countries have been members and therefore with a vote in the Board of Governors but their voting weight has not been represented by an Executive Director. An example of the data source, appendices of the Annual Report, is in the Appendix in a slightly abridged form.

In order to make the figures complete it has therefore been necessary to adjust the reported figures by including these countries. There is therefore a difference between the data for the two bodies in that the percentage voting weights for the Board of Governors sum to 100 while the total for the Executive Directors is often less than that because of abstentions. We have used these figures to make a separate analysis of voting power in the Board of Governors for each year by the Modified MLE Approximation method. This method was employed because of the large number of members which made the exact Direct Enumeration Algorithm infeasible.

The Direct Enumeration Algorithm was used to analyse the Executive Directors however because the number of members of that body has always been small enough for it to be computationally feasible.

VII. Results

A separate analysis has been carried out for each voting body for each year over the whole history of the IMF. This approach gives a power distribution for any year which is of interest as well as enabling us to trace the evolution of voting power over time. Separate analyses are given for the two levels of the majority quota required for ordinary decisions (where a 50% majority is required) and special majorities (where the required quota was 80% until 1969 and 85% in later years). We present results for both 1946 and 1996 in numerical form and then the evolution of voting power over time graphically.

Table 2 presents the results for 1946 for ordinary decisions for both the Executive Directors and the Board of Governors. For each body the table lists the voting weight and both power indices for each member. The figures for the five largest members which appointed executive directors are presented side-by-side for comparison but the other Executive Directors were elected by groups of countries. Where a group had an apparently dominant member (of which the elected director was a national) we have identified that country in parentheses, although we cannot strictly assume that the figures refer to the voting weight or power of the country.

In both bodies the two indices are in broad agreement about the qualitative nature of the power distribution but differ in the quantitative values of the indices. Both indices give the USA a lot more power than its weight and all the other members less power than weight. The only slight exception to this is the executive director with the least weight, which we have labelled D12¹⁰, has more power than weight, and the same power as a group with a larger weight led by Belgium. The USA had about 33 percent of the voting weight and either 49 or 43 percent of the

¹⁰ In fact representing a group consisting of Egypt, Ethiopia, Greece, Iran, Iraq, Philippines.

voting power (depending on the index) in the Executive Directors. In the Board of Governors the power discrepancy is slightly larger, the USA's power index being either 59 or 46 percent.

Table 2 Power Indices for 1946 (Ordinary Decisions, q=50%)

Executive Directors				Board of Governors			
	Votes	Bz Index	SS-Index		Votes	Bz Index	SS-Index
USA	33.22	48.77	43.18	USA	33.14	59.35	45.84
UK	15.86	8.59	12.51	UK	15.83	5.96	12.18
France	6.58	5.11	5.54	France	5.67	3.76	4.56
India	5.09	4.21	4.33	India	5.08	3.40	4.08
China	6.88	5.44	6.00	China	6.87	4.46	5.54
(Canada)	4.79	3.87	3.94	Canada	3.88	2.64	3.11
(Netherland)	5.09	4.21	4.33	Netherlands	3.58	2.44	2.86
(Belgium)	3.72	3.54	3.48	Belgium	2.99	2.05	2.39
D9	5.48	4.66	4.94	Brazil	2.09	1.44	1.66
D10	5.23	4.43	4.63	Czechoslov.	1.79	1.24	1.42
D11	4.61	3.65	3.64	Poland	1.79	1.24	1.42
D12	3.46	3.54	3.48	South Africa	1.49	1.03	1.18
Sum	100	100	100	Mexico	1.37	0.95	1.09
Exec.Dirs	12			Denmark	1.11	0.77	0.88
				Yugoslavia	1.02	0.71	0.81
				Chile	0.9	0.62	0.71
				Colombia	0.9	0.62	0.71
				Cuba	0.9	0.62	0.71
				Norway	0.9	0.62	0.71
				Egypt	0.84	0.58	0.67
				Greece	0.78	0.54	0.62
				Iran	0.6	0.42	0.47
				Peru	0.6	0.42	0.47
				Philippines	0.48	0.33	0.38
				Uruguay	0.48	0.33	0.38
				Bolivia	0.42	0.29	0.33
				Luxembourg	0.42	0.29	0.33
				Iraq	0.39	0.27	0.31
				Ethiopia	0.37	0.26	0.29
				Costa Rica	0.36	0.25	0.28
				DominicanR	0.36	0.25	0.28
				Ecuador	0.36	0.25	0.28
				Guatemala	0.36	0.25	0.28
				El Salvador	0.33	0.23	0.26
				Honduras	0.33	0.23	0.26
				Nicaragua	0.32	0.22	0.25
				Paraguay	0.32	0.22	0.25
				Iceland	0.31	0.22	0.24
				Panama	0.3	0.21	0.24
				Sum	100.03	100	98.77
				Members	39		

Note: Votes do not agree in the two bodies because of the omission of the votes of Denmark from the Executive Directors.

By contrast the picture for the United Kingdom is quite different despite the country's large voting weight in 1946, almost 16%, half the votes of the United States and much more than any other member. The indices both give the UK a substantial power discrepancy: in the Executive Directors, it only had either 8.59% or 12.51% of the power; it had even less power in the Board of Governors, the Banzhaf index giving it less than 6%. The figures for France are less extreme although it still had less power than voting weight in both bodies¹¹. Comparing the indices for the countries which dominated their groupings in the Executive Directors (Canada, Netherlands and Belgium) in both bodies suggests that their executive directors had considerably more power than their governors in the respective bodies. The effect comes largely through the concentration of voting weight, however, rather than being especially a property of the power indices.

Table 3 is the equivalent analysis for 1996¹². The general qualitative picture is the same as it was in 1946, with the United States having an excess of power over weight in both bodies and every other member country having a power discrepancy. Apart from that for the USA which remains substantial, the difference between power and weight is generally fairly small. The USA had almost 18% of the votes and 21% or 23% of the power in the Executive Directors and almost 28% or 21% in the Board of Governors. While the total power is a lot less than in the past, the power discrepancy is still substantial in relation to the voting weight. The effect of the voting system in the Executive Directors appears to have had a substantial effect in enhancing the power of some countries, in particular Australia, Belgium and Netherlands.

¹¹ There is a large discrepancy between the voting weight of France in the two bodies, but these figures are correct according to Gold (1972) and the IMF Annual Report for 1946.

¹² Using the data in the Appendix. Because of the large numbers not all the countries have been listed.

Table 3. Power Indices for 1996 (Ordinary Decisions, q=50%)

	Executive Directors			Board of Governors			
	Votes	Bz Index	SS-Index	Votes	Bz Index	SS-Index	
USA	17.78	23.28	21.33	USA	17.78	27.81	21.08
UK	4.98	4.71	4.91	UK	4.98	4.30	4.91
France	4.98	4.71	4.91	France	4.98	4.30	4.91
Germany	5.54	5.24	5.51	Germany	5.54	4.74	5.49
Japan	5.54	5.24	5.51	Japan	5.54	4.74	5.49
China	2.28	2.16	2.20	China	2.28	2.02	2.19
Saudi Arabia	3.45	3.27	3.34	SaudiArabi	3.45	3.03	3.35
(India)	2.58	2.44	2.49	India	2.06	1.82	1.98
(Canada)	3.72	3.52	3.61	Canada	2.91	2.56	2.81
(Netherland)	4.88	4.62	4.81	Netherland	2.32	2.05	2.23
(Belgium)	5.09	4.82	5.03	Belgium	2.09	1.85	2.01
LatinAm(Brazil)	2.63	2.49	2.54	Brazil	1.47	1.30	1.40
LatinAm(Mexico)	4.31	4.08	4.21				
(Italy)	4.02	3.81	3.91	Italy	3.09	2.72	2.99
(Australia)	3.29	3.12	3.19	Australia	1.58	1.40	1.51
Scandinavia	3.47	3.29	3.36
LatinAm(Arg.)	2.14	2.02	2.06
Africa(exBritish)	2.5	2.37	2.41
Africa(exFrench)	1.34	1.27	1.28
D20	3.19	3.02	3.09
D21	2.9	2.75	2.80
D22	2.8	2.65	2.70
D23	2.76	2.61	2.66
D24	2.65	2.51	2.56
Sum	98.82	100	100.426	Sum	100	100	100.1047
Exec Dirs	24			Members	179		

Note: Votes do not sum to 100 in the Executive Directors, because members who did not cast their votes were not represented.

Tables 4 and 5 show the analyses for decisions requiring special majorities in the same two years. These decisions concern matters of a more fundamental nature and therefore these indices might be considered as containing more information about the true distribution of power. These results are much more difficult to interpret because the two power indices used disagree in many cases and also there are differences in the pattern in the two years. In general we would expect the effect

of a higher quota to be to make the distribution of power more equal¹³. That has undoubtedly been the effect on the 1946 data for the Executive Directors as measured by the Banzhaf index which indicates considerably less power for the United States and more power for all the other directors. The power of the USA drops to 16.9% (compared with 48.8% with respect to ordinary decisions) and that of the UK is almost the same¹⁴. The power distribution given by the Shapley-Shubik index, by contrast, is somewhat different, giving less power than weight to every director except the UK; these effects however are moderate. The results for the Board of Governors are in italics because of slight doubts about the accuracy of the MLE approximation method with a high quota¹⁵ The pattern is similar to that for the Executive Directors for both indices. No figures have been presented in Table 5 for the Board of Governors because of fears¹⁶ about the accuracy of the approximation method when $q=85\%$. For the Executive Directors the power distributions given by the two indices are conflicting in qualitative terms: that for the Banzhaf index being a lot more equal than the distribution of voting weight, while that for the Shapley-Shubik index appears to be more unequal. The main overall conclusion from this analysis is therefore that it matters greatly which index is used. It is necessary therefore to try to form a view on the relative merits of the indices.

¹³ Theoretically, increasing the quota - ultimately to unity - should make all the indices converge on the same value $1/39= 2.56\%$ in the 1946 Board of Governors, 0.56% in the 1996 Board of Governors, etc.

¹⁴ It is interesting that the power index for the UK is now almost equal to that of the USA. This is not surprising since if the quota had been 85% then the two countries would have had the same power, swings for a weight of 33.22% being the same as swings for 15.86%.

¹⁵ Since writing this I have formed the view that I have been unduly pessimistic on the question of accuracy of these indices. Further work will clear the matter up.

¹⁶ I now think these fears to be groundless. See footnote 14.

Table 4. Power Indices for 1946 (Special Majorities, q=80%)

Executive Directors				Board of Governors			
	Votes	Bz Index	SS-Index		Votes	Bz Index	SS-Index
USA	33.22	16.90	31.94	USA	33.14	<i>15.50</i>	<i>31.43</i>
UK	15.86	16.52	21.04	UK	15.83	<i>16.12</i>	<i>21.59</i>
France	6.58	9.03	6.26	France	5.67	<i>7.51</i>	<i>5.86</i>
India	5.09	6.71	4.90	India	5.08	<i>6.76</i>	<i>5.19</i>
China	6.88	9.94	6.79	China	6.87	<i>9.01</i>	<i>7.28</i>
(Canada)	4.79	6.06	4.52	Canada	3.88	<i>5.20</i>	<i>3.87</i>
(Netherland)	5.09	6.71	4.90	Netherlands	3.58	<i>4.80</i>	<i>3.55</i>
(Belgium)	3.72	4.26	2.63	Belgium	2.99	<i>4.02</i>	<i>2.93</i>
D9	5.48	7.23	5.20	Brazil	2.09	<i>2.82</i>	<i>2.02</i>
D10	5.23	7.10	5.13	Czechoslov.	1.79	<i>2.42</i>	<i>1.72</i>
D11	4.61	5.68	4.29	Poland	1.79	<i>2.42</i>	<i>1.72</i>
D12	3.46	3.87	2.40	South Africa	1.49	<i>2.01</i>	<i>1.42</i>
Sum	100	100	100.00	Mexico	1.37	<i>1.85</i>	<i>1.31</i>
Exec. Dirs.	12			Denmark	1.11	<i>1.50</i>	<i>1.05</i>
				Yugoslavia	1.02	<i>1.38</i>	<i>0.97</i>
				Chile	0.9	<i>1.22</i>	<i>0.85</i>
				Colombia	0.9	<i>1.22</i>	<i>0.85</i>
				Cuba	0.9	<i>1.22</i>	<i>0.85</i>
				Norway	0.9	<i>1.22</i>	<i>0.85</i>
				Egypt	0.84	<i>1.14</i>	<i>0.79</i>
				Greece	0.78	<i>1.05</i>	<i>0.74</i>
				Iran	0.6	<i>0.81</i>	<i>0.57</i>
				Peru	0.6	<i>0.81</i>	<i>0.57</i>
				Philippines	0.48	<i>0.65</i>	<i>0.45</i>
				Uruguay	0.48	<i>0.65</i>	<i>0.45</i>
				Bolivia	0.42	<i>0.57</i>	<i>0.39</i>
				Luxembourg	0.42	<i>0.57</i>	<i>0.39</i>
				Iraq	0.39	<i>0.53</i>	<i>0.37</i>
				Ethiopia	0.37	<i>0.50</i>	<i>0.35</i>
				Costa Rica	0.36	<i>0.49</i>	<i>0.34</i>
				Dominican R.	0.36	<i>0.49</i>	<i>0.34</i>
				Ecuador	0.36	<i>0.49</i>	<i>0.34</i>
				Guatemala	0.36	<i>0.49</i>	<i>0.34</i>
				El Salvador	0.33	<i>0.45</i>	<i>0.31</i>
				Honduras	0.33	<i>0.45</i>	<i>0.31</i>
				Nicaragua	0.32	<i>0.43</i>	<i>0.30</i>
				Paraguay	0.32	<i>0.43</i>	<i>0.30</i>
				Iceland	0.31	<i>0.42</i>	<i>0.29</i>
				Panama	0.3	<i>0.41</i>	<i>0.28</i>
				Sum	100.03	100	<i>103.54</i>
				Members	39		

Note: Votes do not precisely agree in the two bodies because of the omission of the votes of Denmark from the Executive Directors. Figures in italics should be treated as approximate.

Table 5. Power Indices for 1996 (Special Majorities, q=85%)

Executive Directors			
	Votes	Bz Index	SS-Index
USA	17.78	6.20	18.60
UK	4.98	5.25	5.04
France	4.98	5.25	5.04
Germany	5.54	5.51	5.72
Japan	5.54	5.51	5.72
China	2.28	2.96	2.21
Saudi Arabia	3.45	4.18	3.52
(India)	2.58	3.29	2.49
(Canada)	3.72	4.44	3.81
(Netherlands)	4.88	5.19	4.96
(Belgium)	5.09	5.31	5.17
LatinAmer(Brazil)	2.63	3.35	2.56
LatinAmer(Mexico)	4.31	4.85	4.37
(Italy)	4.02	4.66	4.17
(Australia)	3.29	4.01	3.24
Scandinavia	3.47	4.20	3.55
LatinAmer(Arg.)	2.14	2.79	2.06
Africa(exBritish)	2.50	3.20	2.42
Africa(exFrench)	1.34	1.87	1.34
D20	3.19	3.92	3.10
D21	2.90	3.66	2.87
D22	2.80	3.53	2.75
D23	2.76	3.50	2.72
D24	2.65	3.37	2.59
Sum	98.82	100	100.00
Members	24		

Note: Votes do not sum to 100 because

members who did not cast their votes were excluded.

No indices are given for the Board of Governors because the errors of

approximation were too large¹⁷.

Figures 1 to 12 show historical trends in power as measured by the indices for the G5 countries. Similar graphs could be drawn for any member country. Besides the values of the two power indices we also show the corresponding power ratios; for each index this is the ratio of the index to the voting weight. A power ratio provide a simple indication of the extent to which power differs from weight, the power discrepancy. It can be used to indicate transparency of the voting system: where power and weight are equal the power ratio is equal to unity. (In our case the power ratios have been normalised to 100 so that the power discrepancy is expressed as a percentage). Figures 1 to 6 show the results for ordinary decisions and Figures 7 to 12 those requiring special majorities.

Figure 1 shows the results for the United States: 1(a) shows the voting weight and two power ratios for every year in the history of the organisation for the Executive Directors, 1(b) the corresponding power ratios, 1(c) the weights and indices for the Board of Governors, and 1(d) the corresponding power ratios. We see that power has been consistently above weight in every year. As the weight has declined there has been a corresponding decline in power. The power ratios have fallen slowly but they are still both considerably above 100. The Shapley-Shubik index gives a positive power discrepancy of at least 20% (which is down from over 30% in the 1940s), and the effect according to the Banzhaf index is even greater. The results are broadly similar for both bodies although there is generally a greater power discrepancy in the Board of Governors than in the Executive Directors. There is no evidence of any voting paradoxes.

Figure 2 shows the results for the UK. There was initially in the 1940s a great excess of weight over power in both bodies: there was a power discrepancy of either almost 20% or 40% in the Executive Directors and up to almost 60% in the Board of Governors. This is perhaps somewhat surprising in view of the very large voting weight of the UK at that time. Over time the power discrepancy has declined: the Shapley-Shubik index has been virtually equal to weight since the mid 1960s, although the Banzhaf index has continued to indicate a power deficit (substantial in

¹⁷ Since drawing up this table I have formed a different view.

the Board of Governors). There have been some voting paradoxes, although small in magnitude. For example in 1947 voting weight rose but power according to both indices fell in both bodies. In 1966 weight increased and the two indices went in opposite directions in the Board of Governors, the Banzhaf index falling and the Shapley-Shubik index rising. A number of other similar changes have occurred but they are all small in magnitude.

Figure 3 shows the trends for France. The power indices track the voting weight, at first falling until the mid 1960s then rising slightly. There has always been a small negative power discrepancy in both bodies. Small voting paradoxes occurred at several points according to one or the other index: 1947, 1957, 1976, 1981 (in the Executive Directors). Figure 4 shows the results for Germany which joined the IMF in 1953 and had its own executive director from 1955. The general pattern is one of fluctuating voting weight with the power indices following fairly closely, with a relatively small power discrepancy tending to get smaller over time. Some small voting paradoxes occurred in the Executive Directors in 1959, 1970 and 1976. Figure 5 shows the results for Japan, which joined in 1953 and has had its own director since 1971. The results show power to have been only slightly less than weight in every year. Figure 6 shows the power ratios for each index and each body. The diagram brings out the contrast between the power of the United States and the other G5 countries. Although the power ratios for the latter have converged on something only slightly below 100, that of the former has remained well above. This effect is produced by both the indices; only its size is always greater according to the Banzhaf index than the Shapley-Shubik index.

The results shown in Figures 1 to 6 are clear. However they are all for ordinary decisions requiring a majority of 50% of the votes. The results are much less clear when the analysis is repeated for decisions requiring special majorities. Figures 7 to 12 show historical trends for the two power indices for these cases. The majorities assumed are of 80% up to 1969 and 85% thereafter¹⁸. Figure 7 shows the US results. The Shapley-Shubik power ratio fluctuates around 100 in a range of $\pm 20\%$ while the Banzhaf index give a large power discrepancy. Both power ratios fell in 1970 after the special majority was increased to 85%.

¹⁸ No results are given for the Board of Governors because of initial doubts about the accuracy of the computed indices.

Figure 8 shows the picture for the UK with conflicting results according to the two indices. Generally the Shapley-Shubik index gives a positive power discrepancy and the Banzhaf index a negative one but these are quantitatively small effects. Figure 9 shows France where again there is a tendency for the indices to conflict. The Banzhaf index gives a positive power discrepancy. Similar results are found for Germany and Japan in the next two figures. All power ratios, however with the exception of that for the Banzhaf index for the United States have converged to close to 100 (Figure 12).

VIII. Conclusions

In Section I we set out the questions we sought to answer in this study: substantive ones about the distribution of voting power and methodological ones about the empirical measurement of power. We claim to have removed some of the obstacles in applying the power indices approach due to difficulties of computation (or the lack of available and appropriate software). Whatever other conclusions we reach we have developed new algorithms which appear to be very effective.

We proposed to answer six questions. The first and second were to compare the voting power of individual countries with their nominal votes and to study the relative inequality of power and votes over time. We have been able to give a partial answer in that our results are qualitatively unambiguous for all the countries we have looked at when we considered ordinary decisions requiring a 50% majority: the power of the USA has been much greater than its voting weight in every year and every other country has had correspondingly less power than its voting weight. Over time this discrepancy has fallen for the other G5 countries while remaining

large for the USA. Interestingly there was a large power deficit for the United Kingdom in the 1940s, its big block of votes not entitling it to as much power¹⁹.

While these results are interesting, it is really of more importance to look at voting power with respect to the more fundamental decisions requiring a special majority, such as adjustments to members' quotas (hence voting weights), the admission of new members and constitutional revisions. Unfortunately the results obtained for this case are difficult to interpret, the two power indices used often disagreeing. There is a need to resolve the ambiguity between the different power indices. We would suggest that by applying them to a wider range of voting bodies and historical periods it may be possible to gain insight into their respective performance.²⁰

We were also interested in whether voting paradoxes have occurred. At the most general level we sought to find cases where a redistribution or the accession of new members led a country's votes to change while its voting power changed in the opposite direction. A number of such cases were found, although the quantitative effects were often quite small.

We made separate analyses of the two decision-making bodies, the Executive Directors and the Board of Governors. Generally we found power to have been more unequal in the latter than in the former among the countries which appointed their own executive directors. Among the other countries which were not able to appoint their own directors and took part in the election process, a number have in fact provided executive directors continuously. These directors all cast much larger block votes in the Executive Directors than their own country does in the Board of Governors because they were elected by a group of countries and cast the votes of all those who voted for them. How this process can be analysed and in particular whether it can be regarded as a composed game is an interesting topic for further research.

¹⁹ Although it remained of course the second most powerful member because power indices are monotonic functions of weight.

²⁰ The effect of different decision rules on the distribution of power is profound. In general a larger special majority requirement leads to a more equal power distribution. In the limit a decision rule of 100% - unanimity - gives equal power to all members. The relationship between the decision rule and the power distributions in the IMF voting bodies is an interesting topic for future investigation.

The main question of methodological interest (apart from the feasibility of calculating them in practice) is to compare the performance of the two classical power indices, the Banzhaf index and the Shapley-Shubik index. The Banzhaf index has been used in more applications largely due to its comparative ease of computation. In this study this has not been an issue. The results indicate that the Banzhaf index tends to be more sensitive to the data than the Shapley-Shubik index and often gives more extreme results. How to relate this to the concept of power is not obvious however. It is possible that by such studies as this, of real institutions for which there exists independent knowledge based on experience, light will be cast on the respective ability of the indices to reflect power relations.

Figure 1: Historical Trends (Ordinary Decisions) USA

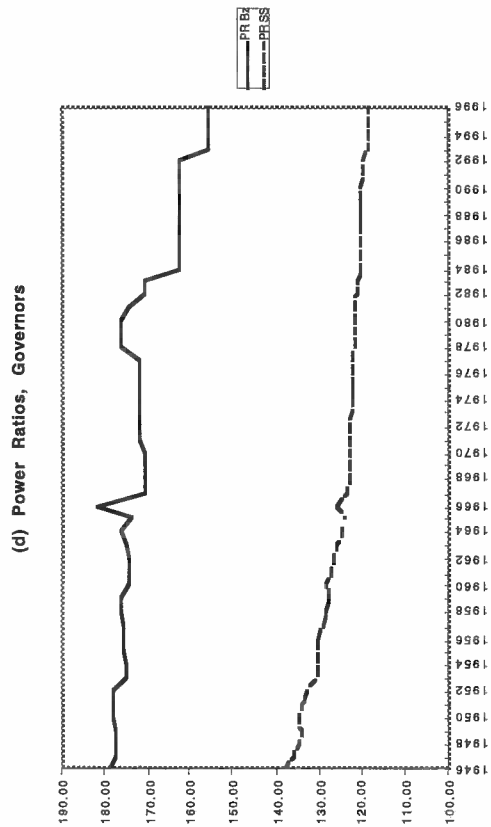
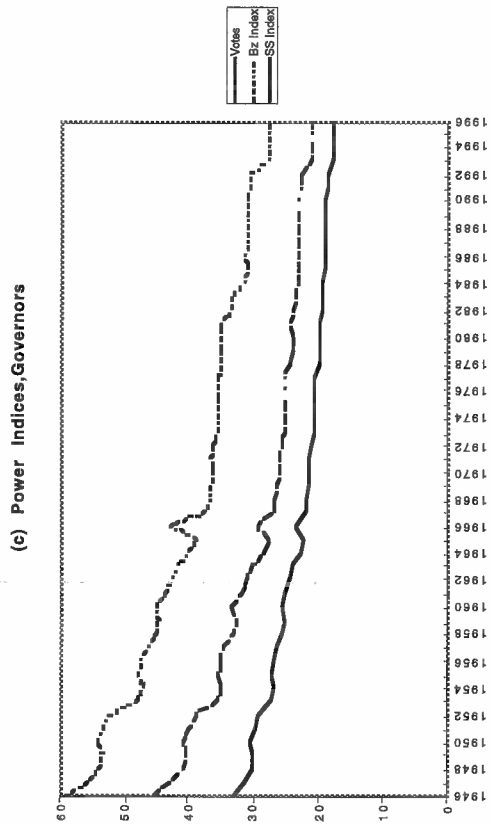
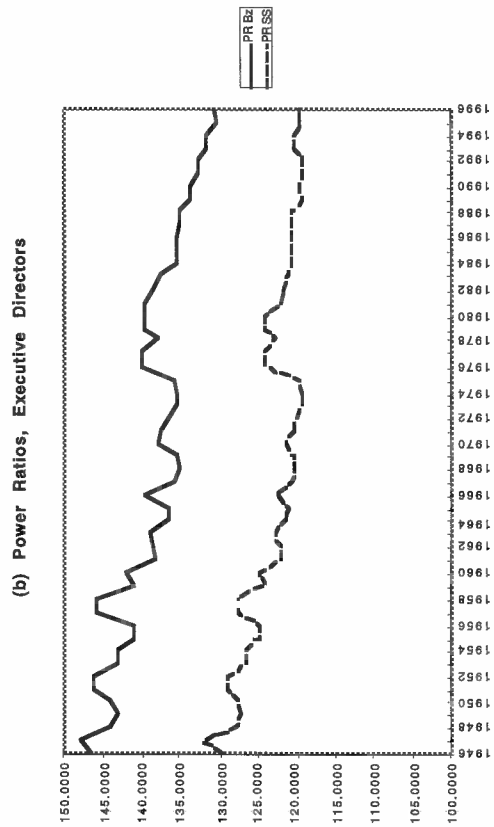
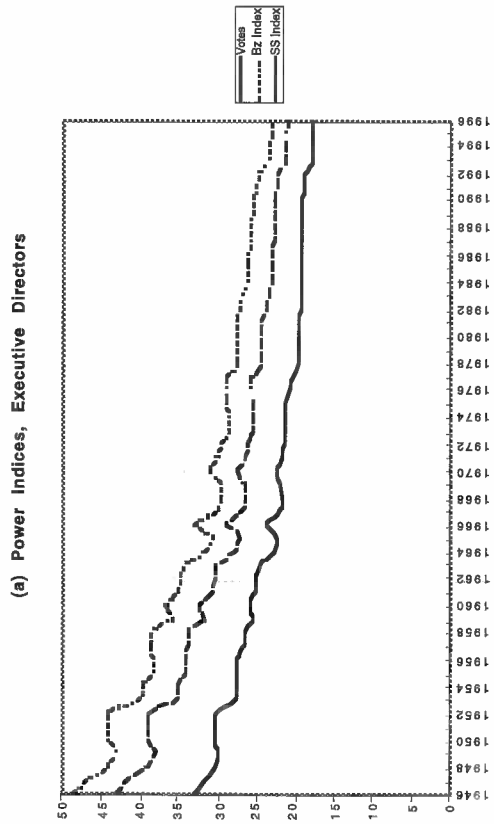
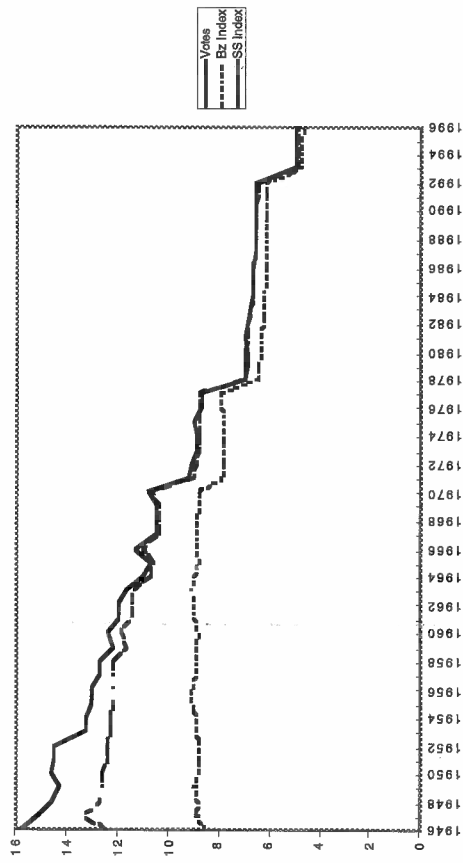
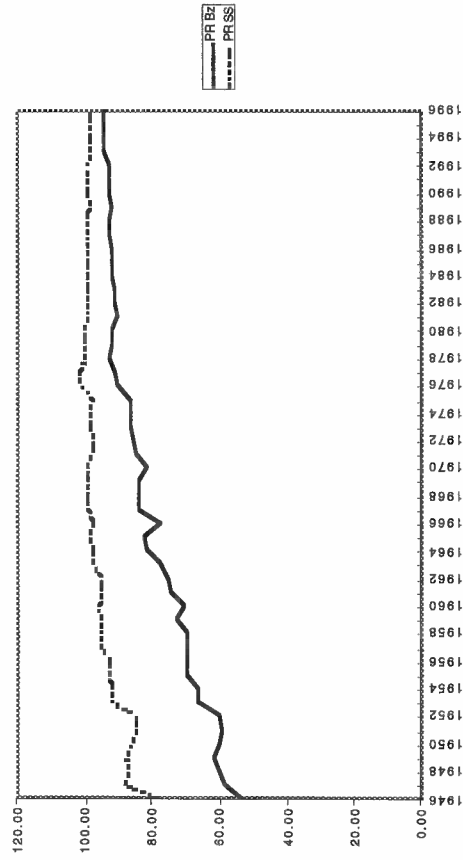


Figure 2: Historical Trends (Ordinary Decisions) UK

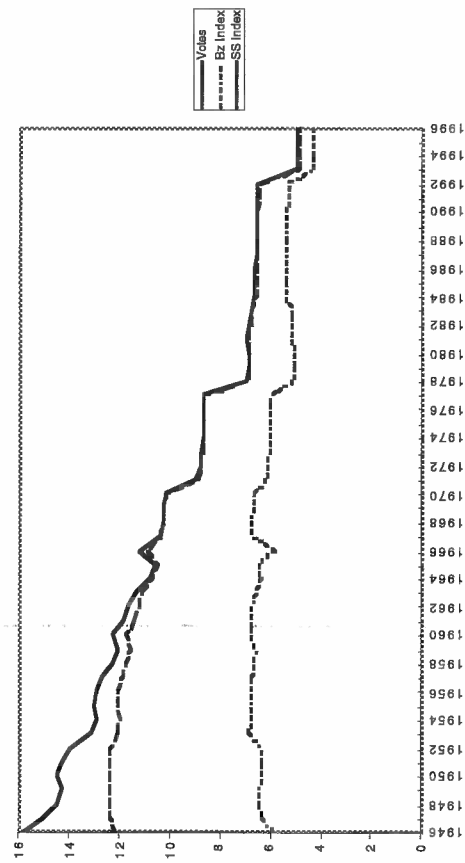
(a) Power Indices, Executive Directors



(b) Power Ratios, Executive Directors



(c) Power Indices, Governors



(d) Power Ratios, Governors

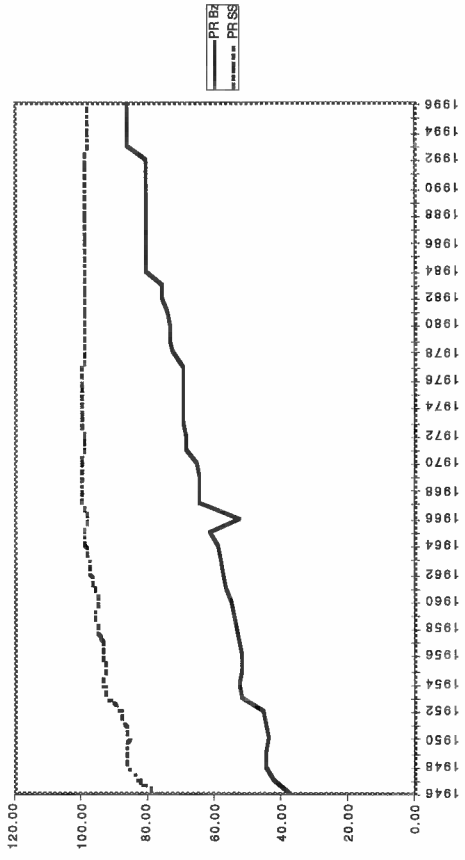
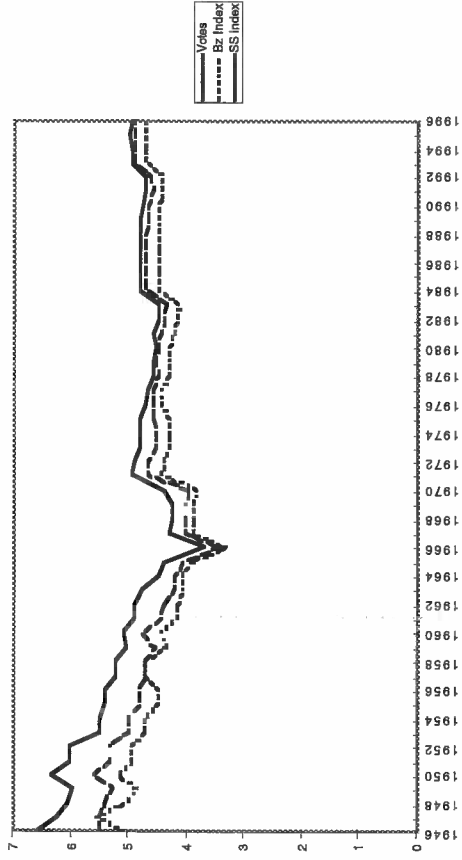
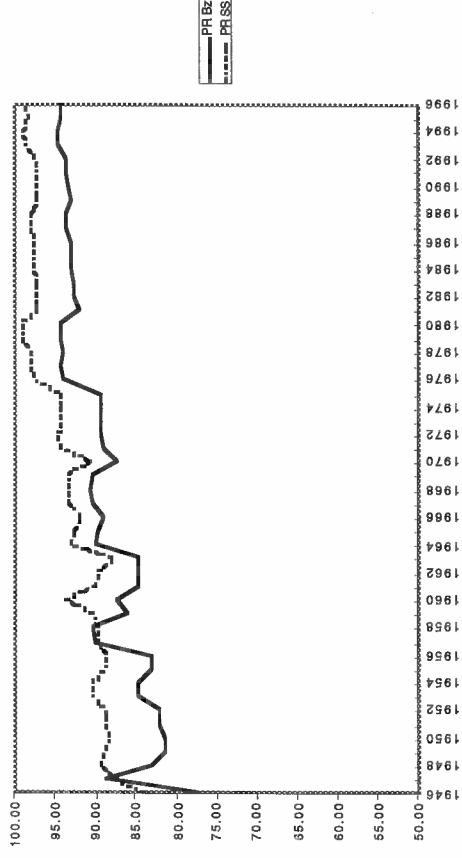


Figure 3: Historical Trends (Ordinary Decisions) France

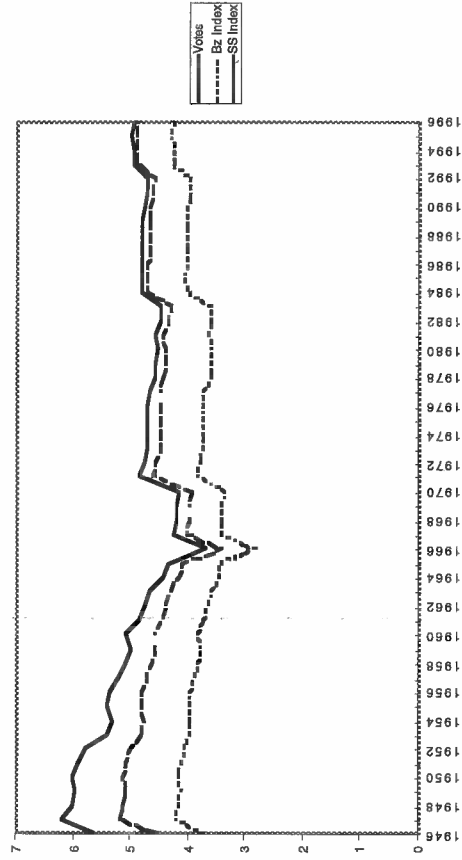
(a) Power Indices, Executive Directors



(b) Power Ratios, Executive Directors



(c) Power Indices, Governors



(d) Power Ratios, Governors

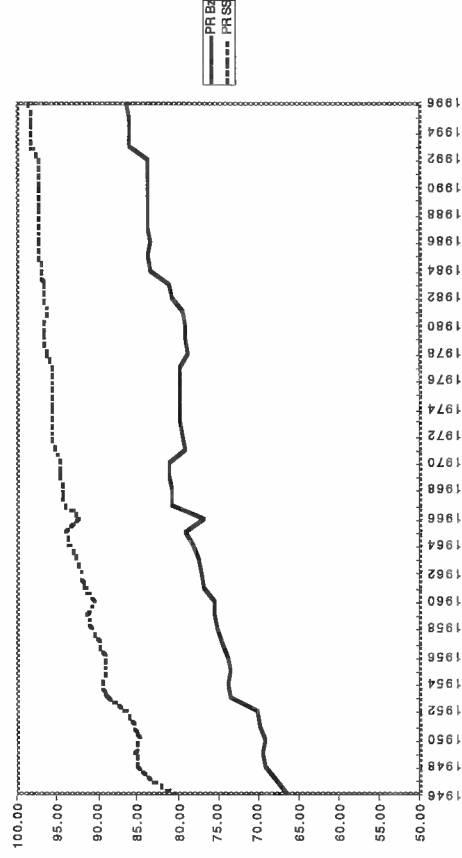
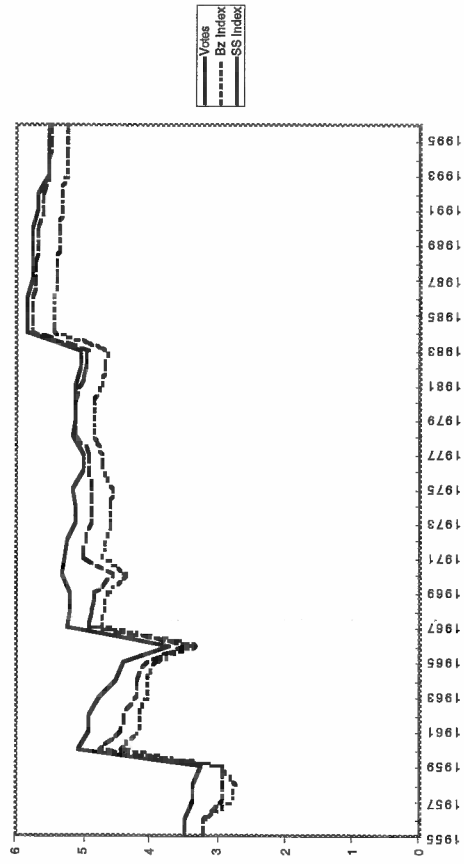
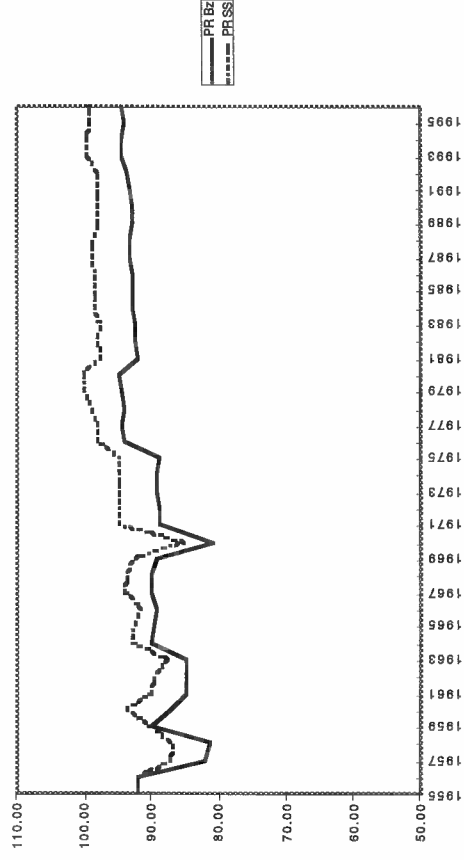


Fig. 4: Historical Trends (Ordinary Decisions) Germany

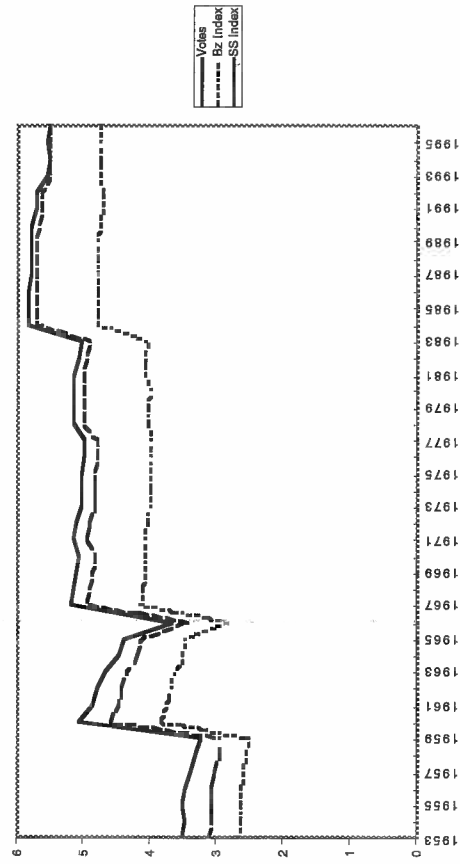
(a) Power Indices, Executive Directors



(b) Power Ratios, Executive Directors



(c) Power Indices, Governors



(d) Power Ratios, Governors

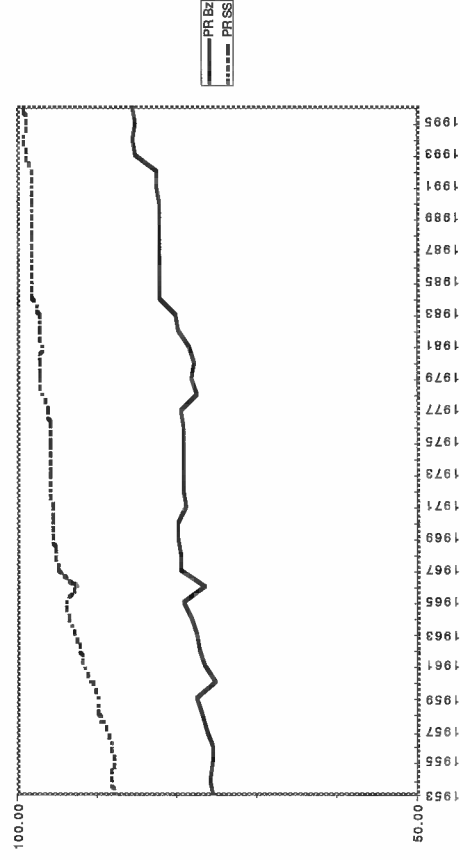
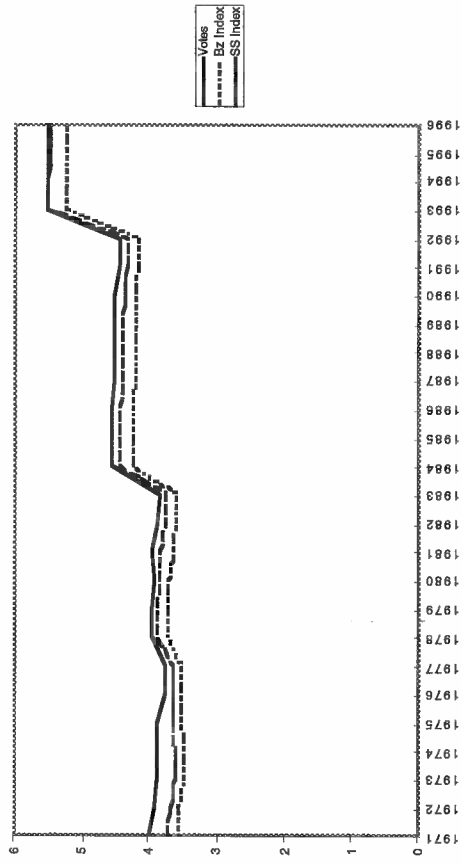
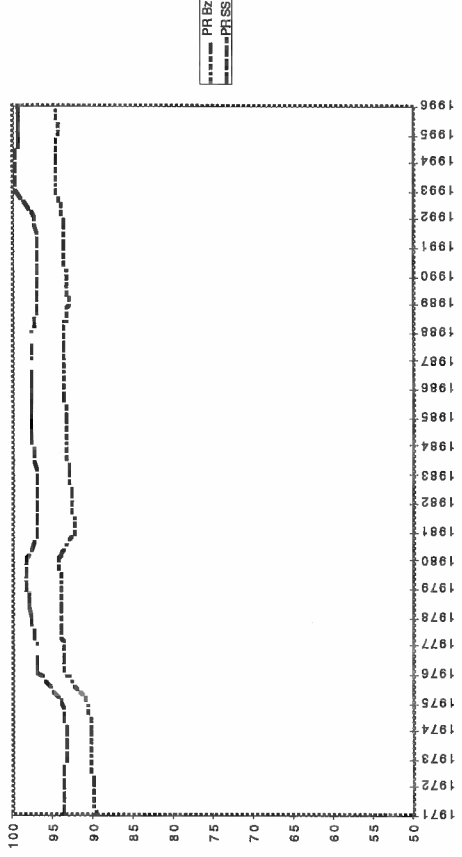


Figure 5: Historical Trends (Ordinary Decisions) Japan

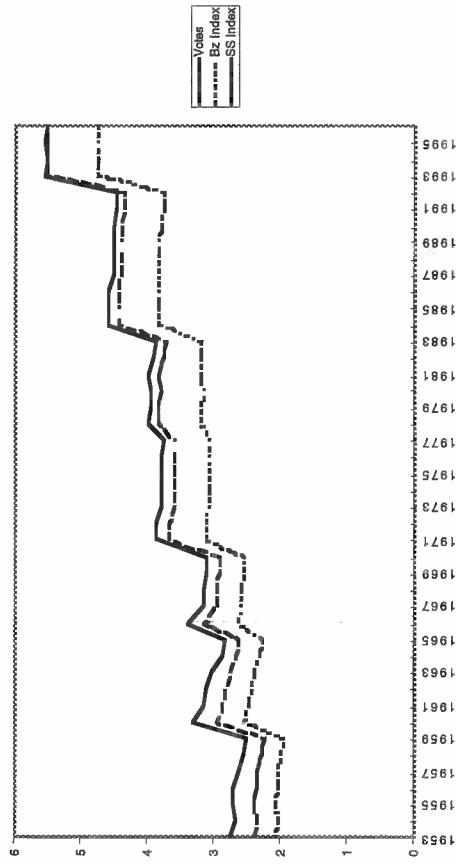
(a) Power Indices, Executive Directors



(b) Power Ratios, Executive Directors



(c) Power Indices, Governors



(d) Power Ratios, Governors

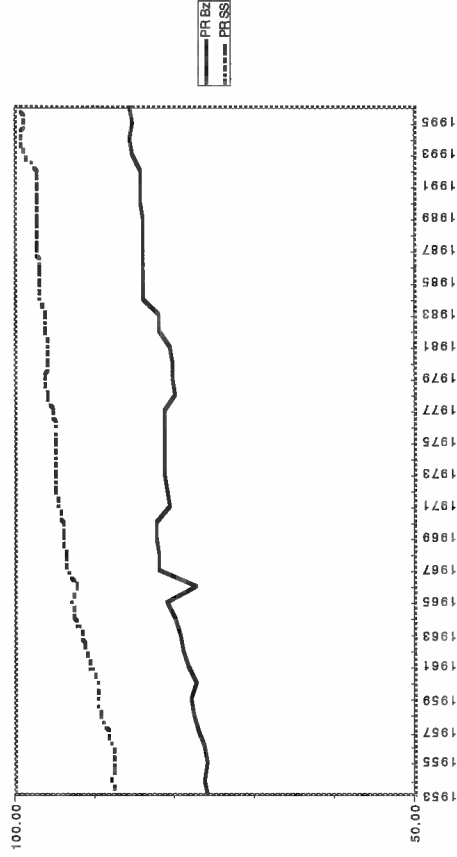


Fig. 6 Trends in Power Ratios (Ordinary Decisions)

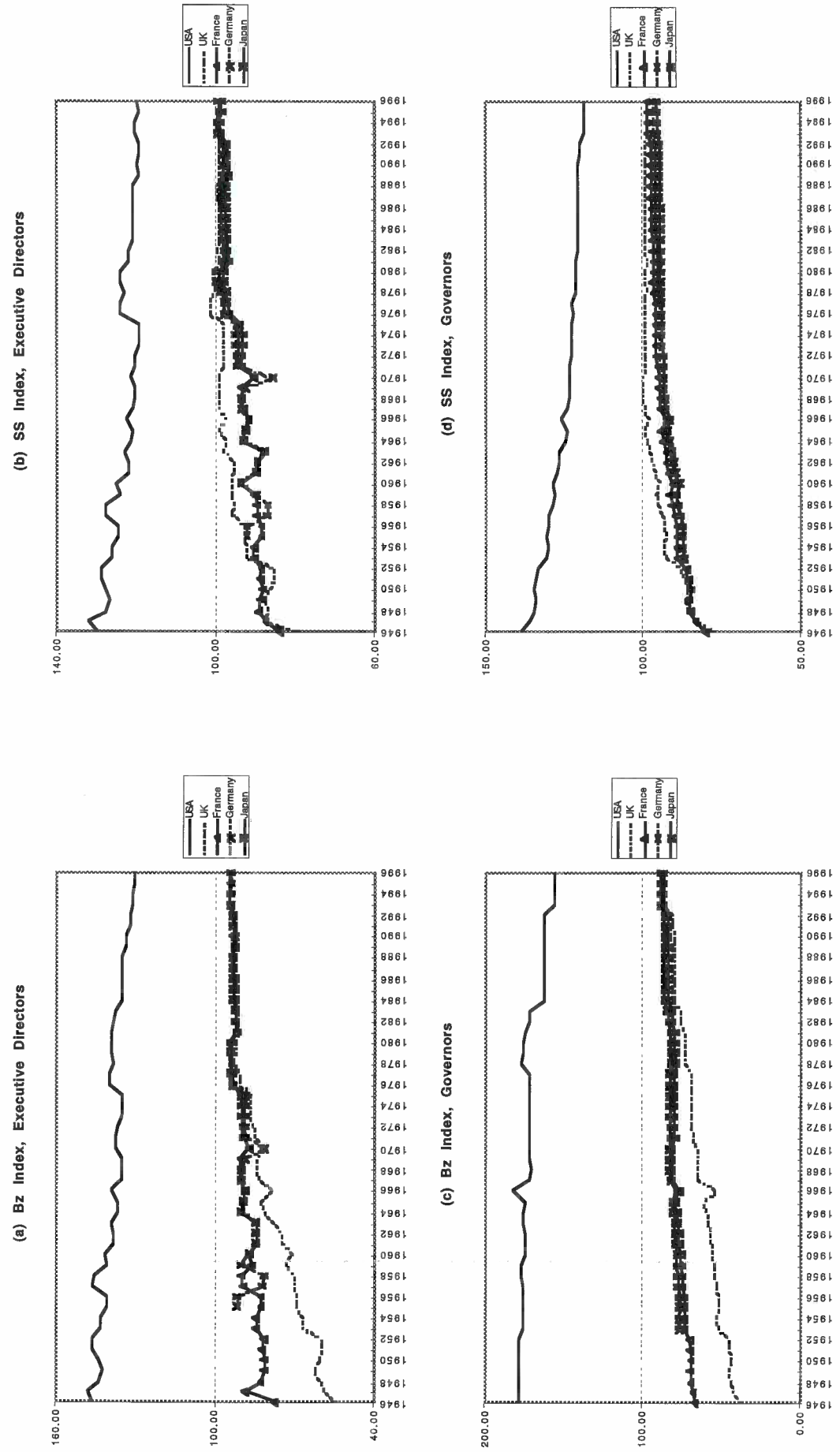
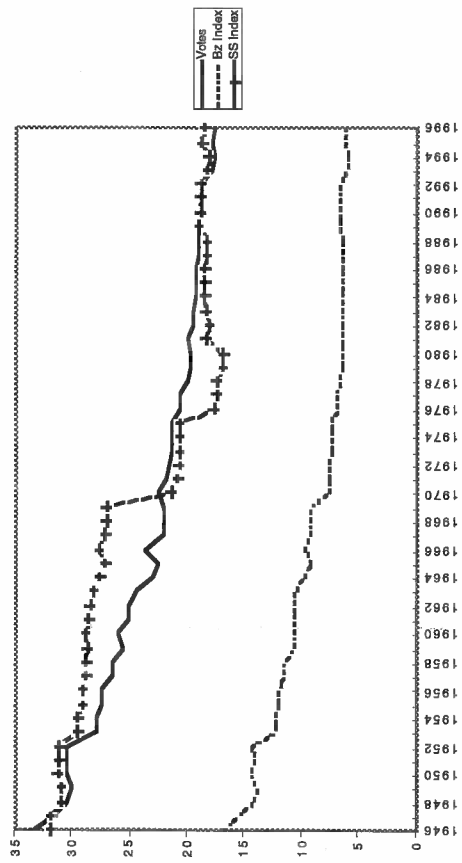


Figure 7: Historical Trends (Special Majorities) USA

(a) Power Indices, Executive Directors



(b) Power Ratios, Executive Directors

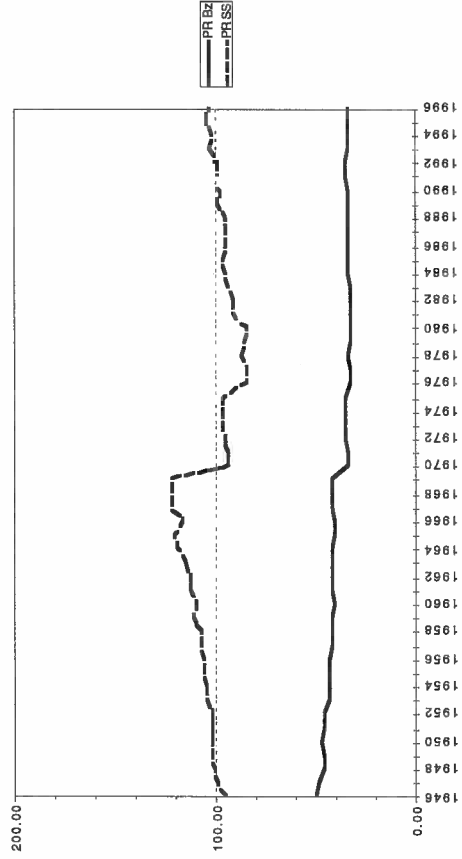
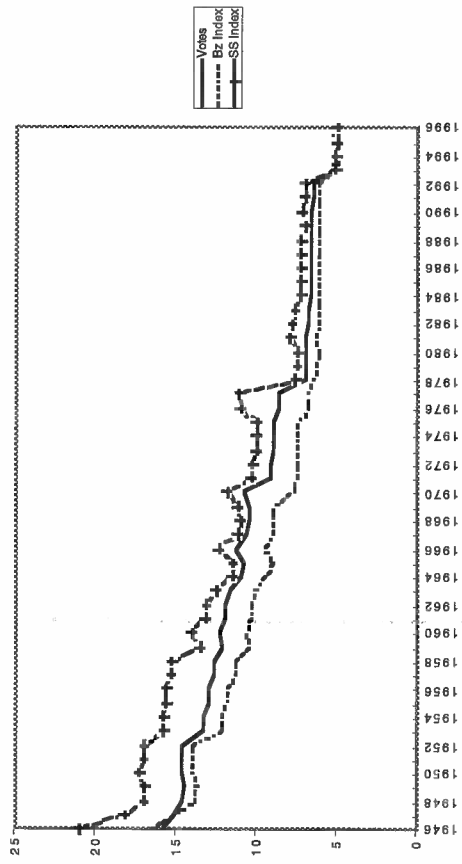


Figure 8: Historical Trends (Special Majorities) UK

(a) Power Indices, Executive Directors



(b) Power Ratios, Executive Directors

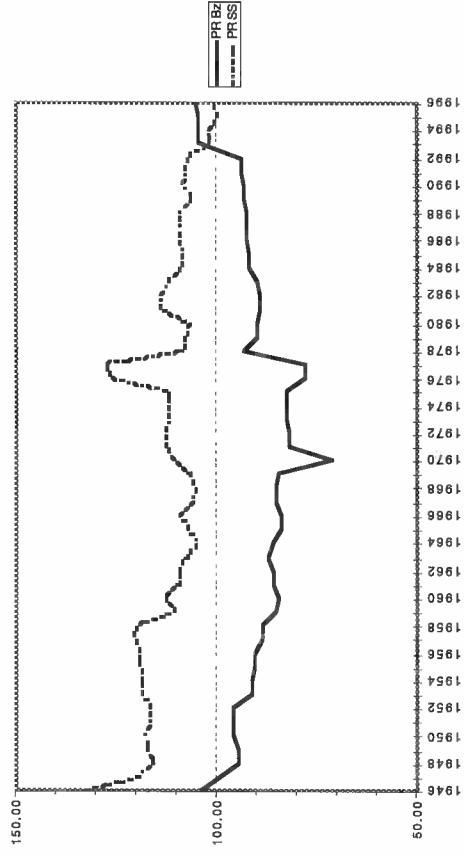
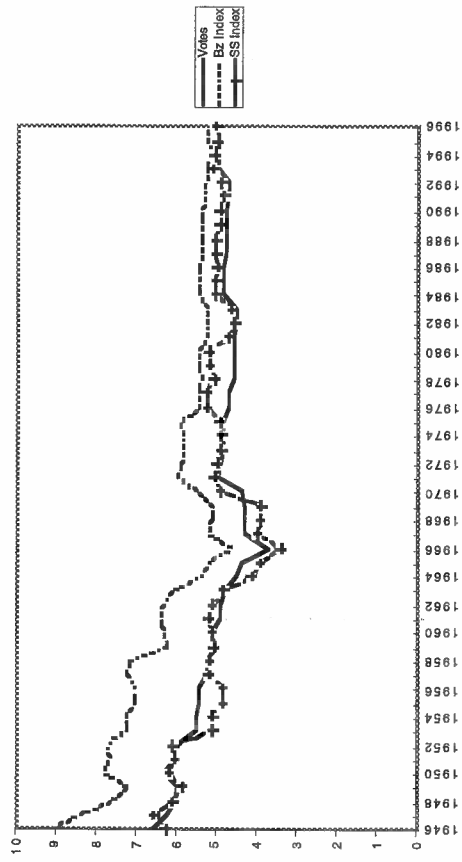


Figure 9: Historical Trends (Special Majorities) France

(a) Power Indices, Executive Directors



(b) Power Ratios, Executive Directors

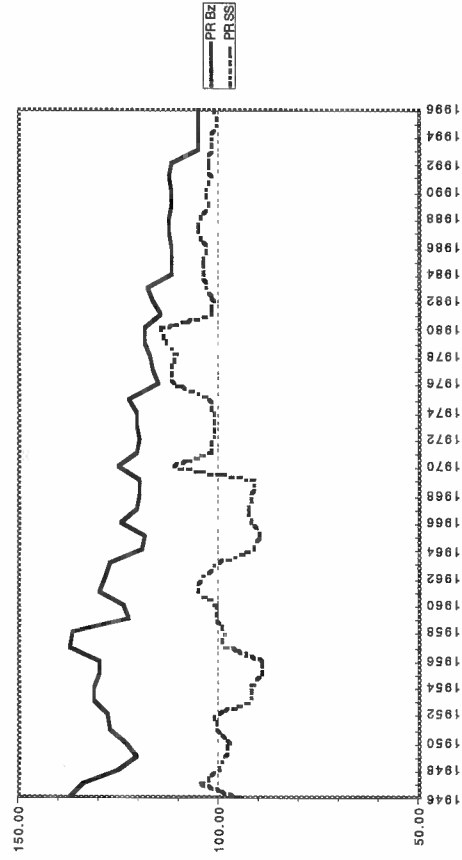
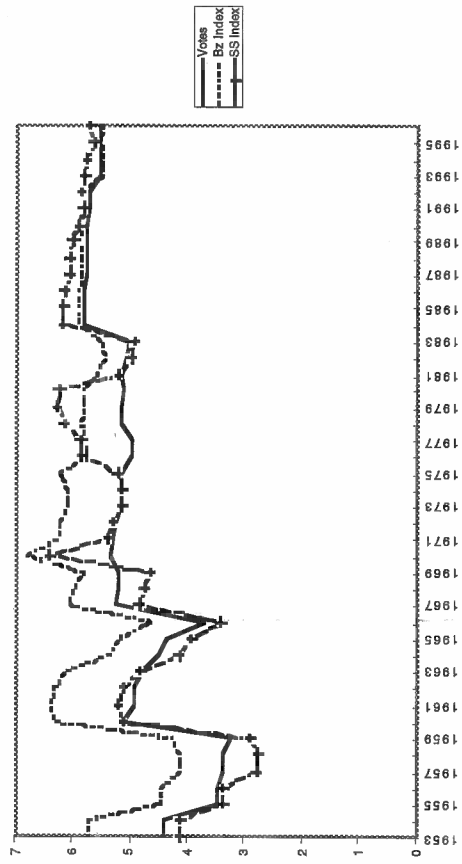


Figure 10: Hist. Trends (Special Majorities) Germany

(a) Power Indices, Executive Directors



(b) Power Ratios, Executive Directors

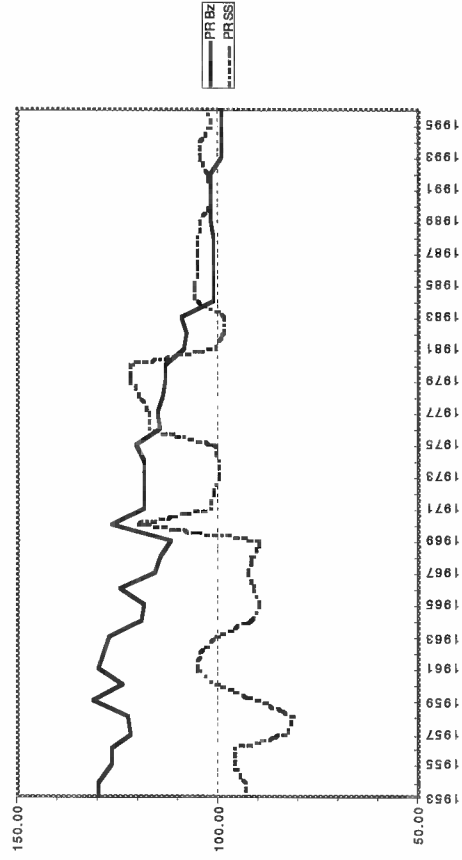
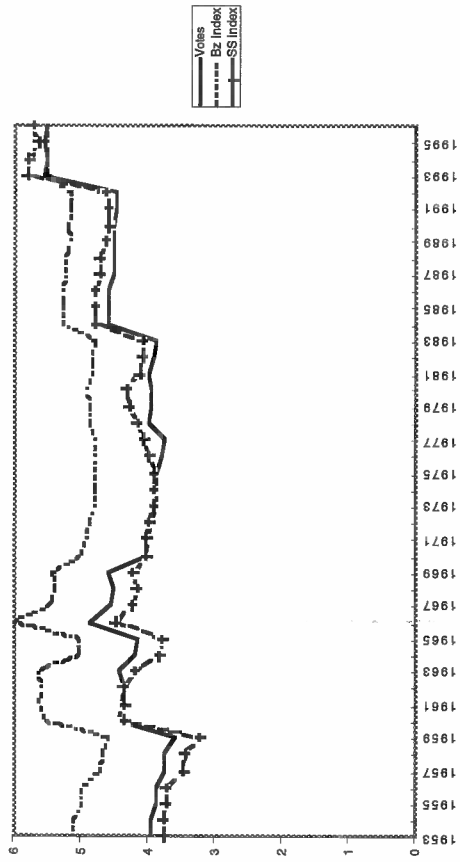


Figure 11: Hist. Trends (Special Majorities) Japan

(a) Power Indices, Executive Directors



(b) Power Ratios, Executive Directors

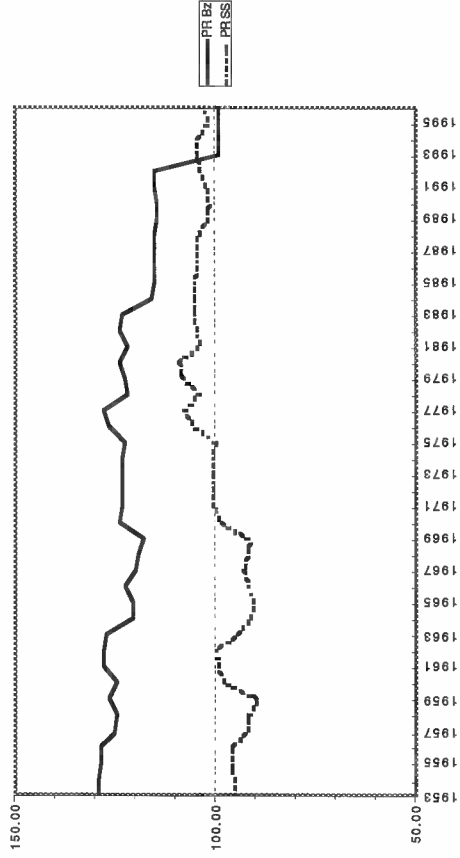
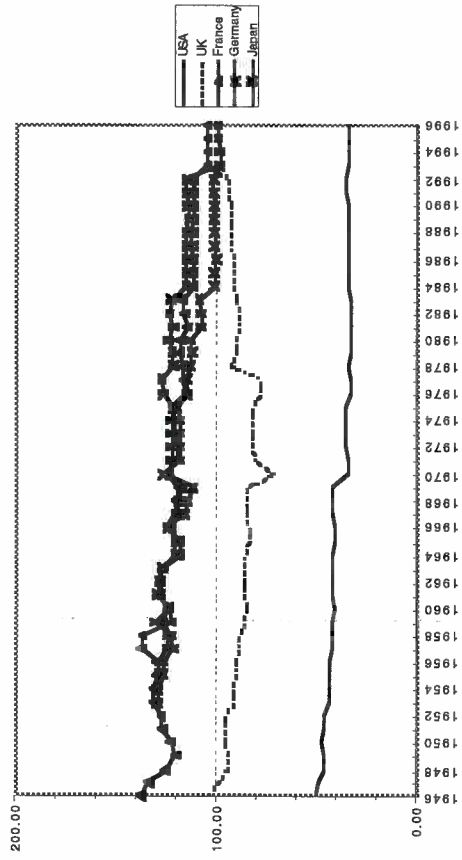
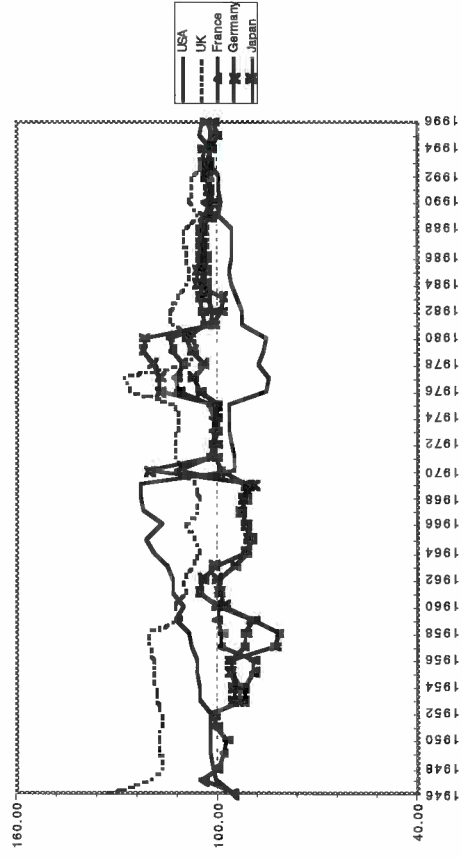


Figure 12: Trends in Power Ratios (Special Majorities)

(a) Bz Index, Executive Directors



(b) SS Index, Executive Directors



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Appendix

IMF EXECUTIVE DIRECTORS AND VOTING POWER ON APRIL 30, 1996¹

Director ² (Alternate)	Casting Votes of	Votes by Country	Total Votes ³	Percent of Fund Total ⁴
Appointed:				
United States (United States)	United States	265,518	265,518	17.78
Germany (Germany)	Germany	82,665	82,665	5.54
Japan (Japan)	Japan	82,668	82,665	5.54
France (France)	France	74,396	74,396	4.98
United Kingdom (United Kingdom)	United Kingdom	74,396	74,396	4.98
Elected:				
Belgium (Austria)	Austria	12,133		
	Belarus	3,054		
	Belgium	31,273		
	Czech Republic	6,146		
	Hungary	7,798		
	Kazakhstan	2,725		
	Luxembourg	1,605		
	Slovak Republic	2,824		
	Slovenia	1,755		
	Turkey	<u>6,670</u>	75,983	5.09
Netherlands (Canada)	Armenia	925		
	Bulgaria	4,899		
	Croatia	2,866		
	Cyprus	1,250		
	Georgia	1,360		
	Israel	6,912		
	Macedonia, former Yugoslav Republic of	746		
	Moldova	1,150		
	Netherlands	34,692		
	Romania	7,791		
	Ukraine	<u>10,223</u>	72,814	4.88
	Venezuela (Spain)	Costa Rica	1,440	
El Salvador		1,506		
Guatemala		1,788		
Honduras		1,200		
Mexico		17,783		
Nicaragua		1,211		
Spain		19,604		
Venezuela		<u>19,763</u>	64,295	4.31
Italy (Greece)	Albania	603		
	Greece	6,126		
	Italy	46,157		
	Malta	925		

¹ From the IMF Annual Report 1996.

² Nationality of Director or Alternate.

³ Voting power varies on certain matters pertaining to the General Department with use of the Fund's resources in that Department.

⁴ Percentages of total votes (1,493,331) in the General Department and the Special Drawing rights Department.

	Portugal	5,826		
	San Marino	<u>350</u>	59,987	4.02
Canada (Ireland)	Antigua and Barbuda	335		
	Bahamas, The	1,199		
	Barbados	739		
	Bruce	385		
	Canada	43,453		
	Dominica	310		
	Grenada	<u>335</u>		
	Ireland	5,500		
	Jamaica	2,259		
	St. Kitts and Nevis	315		
	St. Lucia	360		
	St. Vincent and the Grenadines	<u>310</u>	58,800	3.72
Sweden (Denmark)	Denmark	10,949		
	Estonia	715		
	Finland	8,868		
	Iceland	1,103		
	Latvia	1,165		
	Lithuania	1485		
	Norway	11,196		
	Sweden	<u>16,390</u>	51,771	3.47
Saudi Arabia (Saudi Arabia)	Saudi Arabia	51,556	51,556	3.45
Australia (Korea)	Australia	23,582		
	Kiribati	290		
	Korea	8,246		
	Marshall Islands	275		
	Micronesia, Federated States of	285		
	Mongolia	621		
	New Zealand	6,751		
	Papua New Guinea	1,203		
	Philippines	6,584		
	Seychelles	310		
	Solomon Islands	325		
	Vanuatu	375		
	Western Samoa	<u>335</u>	49,182	3.29
Egypt (Bahrain)	Bahrain	1,078		
	Egypt	7,034		
	Iraq	5,290		
	Jordan	1,467		
	Kuwait	10,202		
	Lebanon	1,710		
	Libya	8,426		
	Maldives	305		
	Oman	1,444		
	Qatar	2,155		
	Syrian Arab Republic	2,349		
	United Arab Emirates	4,171		
	Yemen, Republic of	<u>2,015</u>	47,646	3.19
Russia (Russia)	Russia	43,381	43,381	2.90
Indonesia (Malaysia)	Cambodia	900		
	Fiji	761		
	Indonesia	15,226		
	Lao People's Democratic			

	Republic	641		
	Malaysia	8,577		
	Myanmar	2,099		
	Nepal	770		
	Singapore	3,826		
	Thailand	5,989		
	Tonga	300		
	Vietnam	<u>2,666</u>	41,755	2.80
Switzerland (Poland)	Azerbaijan	1,420		
	Kyrgyz Republic	895		
	Poland	10,135		
	Switzerland	24,954		
	Tajikistan	850		
	Turkmenistan	730		
	Uzbekistan	<u>2,145</u>	41,229	2.76
Iran (Morocco)	Afghanistan, Islamic State of	1,454		
	Algeria	9,394		
	Ghana	2,990		
	Iran, Islamic Republic of	11,035		
	Morocco	4,527		
	Pakistan	7,832		
	Tunisia	<u>2,310</u>	39,542	2.65
Brazil (Colombia)	Brazil	21,958		
	Colombia	5,863		
	Dominican Republic	1,838		
	Ecuador	2,442		
	Guyana	922		
	Haiti	857		
	Panama	1,746		
	Suriname	926		
	Trinidad and Tobago	<u>2,718</u>	39,470	2.63
India (Sri Lanka)	Bangladesh	4,175		
	Bhutan	295		
	India	30,805		
	Sri Lanka	<u>3,286</u>	38,561	2.58
Swaziland (Zimbabwe)	Angola	2,323		
	Botswana	616		
	Burundi	822		
	Eritrea	365		
	Albania	1,233		
	Gambia, The	479		
	Kenya	2,144		
	Lesotho	489		
	Liberia	963		
	Malawi	759		
	Mozambique	1,090		
	Namibia	1,246		
	Nigeria	13,066		
	Sierra Leone	1,022		
	Swaziland	615		
	Tanzania	1,719		
	Uganda	1,589		
	Zambia	3,885		
	Zimbabwe	<u>2,863</u>	37,388	2.50
China (China)	China	34,102	34,102	2.28
Peru	Argentina	18,621		

<i>(Argentina)</i>	Bolivia	1,512		
	Chile	6,467		
	Paraguay	971		
	Peru	4,911		
	Uruguay	<u>2,503</u>	31,985	2.14
Cote d'Ivoire	Benin	703		
<i>(Gabon)</i>	Burkina Paso	692		
	Cameroon	1,601		
	Cape Verde	320		
	Central African Republic	662		
	Chad	663		
	Comoros	315		
	Congo	829		
	Cote d'Ivoire	2,632		
	Djibouti	368		
	Equatorial Guinea	493		
	Gabon	1,383		
	Guinea	1,037		
	Guinea-Bissau	355		
	Madagascar	1,154		
	Mali	939		
	Mauritania	725		
	Maldives	983		
	Niger	733		
	Rwanda	848		
	Sao Tome and Principe	305		
Senegal	1,439			
Togo	<u>793</u>	19,936	1.34	
		1,475,533 ⁵⁶	98.81 ⁷	

⁵ This total does not include the votes of Bosnia and Herzegovina, Brunei Darussalam, Somalia, and South Africa, which did not participate in the 1994 Regular Election of Executive Directors. The combined votes of those members total 17,808-1.19 percent of those in the General Department and Special Drawing Rights Department.

⁶ This total does not include the votes of Sudan and Zaire, which were suspended effective August 9, 1993 and June 2, 1994, respectively, pursuant to Article XXVI, Section 2(b) of the Articles of Agreement.

⁷ This figure may differ from the sum of the percentages shown for individual Directors because of rounding.