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**OPTIMISING AGENTS, STAGGERED WAGES AND  
PERSISTENCE IN THE REAL EFFECTS OF MONEY  
SHOCKS**

**Guido Ascari**

**No.486**

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THE REAL EFFECTS OF MONEY SHOCKS

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This paper is circulated for discussion purposes only and its contents  
should be considered preliminary.

# Optimising Agents, Staggered Wages and Persistence in the Real Effects of Money Shocks<sup>†</sup>

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## *Abstract*

*In this paper we incorporate staggered wage setting a` la Taylor (1979) into an optimising dynamic general equilibrium framework. The aim is to study whether staggered wages could induce a high degree of persistence in the real effects of money shocks, as some recent studies have suggested. Firstly, we are able to investigate how the parameters of Taylor's model depend upon the microeconomic fundamentals and the conduct of monetary policy. Secondly, we show that, once explicit microfoundations are taken into account: (i) the model is highly non-linear and consequently a log-linear approximation becomes a bad approximation for a staggered wage economy; (ii) the conduct of monetary policy affects the structural parameters of Taylor's wage setting equation, providing a clear example of the Lucas critique; (iii) the inertia of the system and the short-run output-inflation trade-off are inversely related to the level of average inflation. Thirdly, we conclude that high persistence of the real effects of money shocks in staggered wage models is an unlikely outcome. Sensible values of the microeconomic parameters and/or a moderate rate of underlying inflation as we observe in western economies cut down persistence not only far below a near random-walk behaviour, but also below any level notably different from zero.*

Keywords: Optimising agents, Staggered wages, Persistence

JEL classification: E31, E32

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## 1. Introduction

The evidence that the GDP process contains a unit root has been interpreted as rejecting traditional theories of economic fluctuations, which assume business cycle to be mainly driven by temporary nominal disturbances. In their seminal article, Nelson and Plosser (1982) claim that their finding “*gives an important role to real factors in output fluctuations and places limits on the importance of monetary theories of the business cycle*” (Nelson and Plosser, 1982, p. 161). The same conclusion was supported by the analysis of Campbell and Mankiw (1987), even though these authors were more cautious in drawing any definite conclusion.<sup>1</sup> Following the interpretation of Nelson and Plosser (1982), numerous scholars started to develop what is already a massive branch of the literature; i.e. the real business cycle literature.

Recent papers by West (1988) and Phaneuf (1990) try to challenge this view. They both use Taylor’s (1979, 1980a,b) staggered wage model with a feedback monetary policy rule and subject it to only monetary shocks. They show that, for plausible values of the parameters, the model is able to generate a high persistence of money shocks. It therefore appears that staggered wage models can induce a near-random walk behaviour in GDP, that is, an autoregressive root of about 0.8, statistically indistinguishable from a unit root in a finite sample.

However, Taylor’s (1979, 1980a,b) model is an *ad hoc* log-linear structural model. Wage setting rules are exogenously specified at the outset and the parameters are likely not to be policy-invariant (see Taylor, (1980b)). Taylor (1979) openly acknowledges the need for

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<sup>1</sup> “*A conclusion as extreme as that of Nelson and Plosser is of course not necessary.*” (Campbell and Mankiw, 1987, p. 876-877)

microfoundations.<sup>2</sup> In this paper we incorporate staggered wage setting à la Taylor (1979) into an optimising dynamic general equilibrium framework. The aim is to open the “black box” of the structural *ad hoc* parameters of the famous Taylor’s wage setting equation and subsequently, to investigate whether the model can confirm previous findings by West (1988) and Phaneuf (1990). Notwithstanding the apparent simplicity of Taylor’s model (1979), it turns out that a log-linearised version of our model coincides with that of Taylor. We are then able to show how the parameters of Taylor’s wage setting equation depend upon the microeconomic fundamentals and the conduct of monetary policy.

Our main finding is that high persistence of the real effects of money shocks in staggered wage models is an unlikely outcome. Consequently, our result rejects the earlier view of West (1988) and Phaneuf (1990) for almost any reasonable values of the microeconomic parameters. Blanchard (1990) stressed that a low responsiveness of nominal wages to the business cycle conditions was a key factor for generating high persistence of money shocks in staggered wage models. We are able to show how this responsiveness depends upon the structure of the preferences of the agents. In accordance with previous findings (see Blanchard and Fischer (1989), and Chari *et al.* (1996)), we show that a high degree of persistence can only arise from a low income effect on labour supply and a high intertemporal elasticity of labour. Moreover, new findings for the interrelation among the responsiveness of nominal wages to the business cycle, income effect and intertemporal elasticity of substitution of labour are provided.

Furthermore, it is shown that, once explicit microfoundations are taken into account: (i) the model is highly non-linear and hence a log-linear approximation becomes a bad

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<sup>2</sup> “Unfortunately, the assumed contract formation behavior is not explicitly derived from a utility maximization model. [...] the micro foundations of the staggered contract model presented here are far from complete” (Taylor, 1979, p. 111). “The microfoundations of such models need to be developed more rigorously” (Taylor, 1979, p. 112).

approximation for a staggered wage economy; (ii) the conduct of monetary policy affects the structural parameters of Taylor's wage setting equation, providing a clear example of the Lucas critique argument; (iii) the inertia of the system and the short-run output-inflation trade-off is inversely related to the level of average inflation as found by Ball *et al.* (1988).

We conclude that either sensible values of the microeconomic parameters, or a moderate rate of underlying inflation as observed in western economies, or both, cut down persistence not only far below a near random-walk behaviour, but also below any level notably different from zero.

The paper is organised as follows. Initially we briefly review the previous literature on Taylor's model and persistence of money shocks. Then, in section 3 we present the model, followed by a comparison with Taylor's model in section 4. Section 5 investigates how the degree of persistence varies as the microeconomic fundamentals vary, shows the importance of non-linearity, and how the Lucas critique does apply to Taylor's model. Section 6 concludes.

## 2. Taylor's Model and the Persistence of Money Shocks

Taylor's model consists of the following three equations:

$$x_t = b x_{t-1} + d E_{t-1}(x_{t+1}) + \gamma [b E_{t-1}(y_t) + d E_{t-1}(y_{t+1})] \quad (1)$$

$$p_t = \frac{1}{2} (x_{t-1} + x_t) \quad (2)$$

$$y_t = m_t - p_t \quad (3)$$

where  $p$  = price level,  $x$  = nominal wage,  $y$  = output,  $m$  = money supply and all the variables are expressed in terms of log-deviations from trend. Equation (2) is simply a mark-up equation, which implicitly implies constant return to scale, while equation (3) is a static

aggregate demand equation. The key equation is the wage setting rule (1). The new contract wage is assumed to depend upon the contract wage set in the previous period, the contract wage that will be set next period and the expected excess demands in the next two periods. *“The behavioral equations reflect a relative wage concern on the part of the workers”* (Taylor, 1983, p. 987-988). The parameters  $b$  and  $d$  represent the degree of backward and forward looking respectively. Besides, Taylor assumes that  $b + d = 1$ , that is homogeneity of degree one in nominal contracts. This avoids imposing any artificial money illusion behaviour. When  $b = d = 1/2$  *“contract decisions are unbiased. Wage setters look forward to the same degree they look backward”* (Taylor, 1979, p. 109). If  $b = 1$  then equation (1) is purely backward looking, alternatively if  $d = 1$  then it is purely forward looking.

For a given expected path of the money supply, the solution of the model is:

$$x_t = \lambda_s x_{t-1} + \sum_{i=0}^{\infty} \frac{(\alpha - 1) \lambda_s}{b} \left( \frac{1}{\lambda_u} \right)^i [bE_{t-1}(m_{t+i}) + dE_{t-1}(m_{t+1+i})] \quad (4)$$

where  $\lambda_s = \frac{\alpha - \sqrt{\alpha^2 - 4d(1-d)}}{2d}$ ;  $\lambda_u = \frac{\alpha + \sqrt{\alpha^2 - 4d(1-d)}}{2d}$ ;  $\alpha = \left( \frac{1+\gamma/2}{1-\gamma/2} \right)$ .  $\lambda_s$  e  $\lambda_u$  are respectively the stable and the unstable root of the saddle path equilibrium. Now suppose  $m_t$  follows a random walk. Then equation (4) becomes:

$$x_t = \lambda_s x_{t-1} + (1 - \lambda_s) m_{t-1} \quad (5)$$

and the dynamics of output are given by:

$$y_t = \lambda_s y_{t-1} + (m_t - m_{t-1}) + 1/2 (1 - \lambda_s)(m_{t-1} - m_{t-2}) \quad (6)$$

The model therefore exhibits persistence in the real effects of money shocks. Persistence of money shocks then basically depends on two parameters: the degree of forward looking



behaviour ( $d$ ) and the degree of sensitivity of the money wage to business cycle conditions ( $\gamma$ ). The higher  $d$ , the lower the inertia of the aggregate wages. However,  $d$  is thought to be relatively uninteresting and hence often put equal to  $\frac{1}{2}$  in the literature, which focuses on  $\gamma$  as the crucial parameter to determine the degree of persistence.<sup>3</sup> For the US, Taylor (1980b) estimated  $\gamma$  to be between 0.05 and 0.1, while Sachs (1980) estimated  $\gamma$  to be between 0.07 and 0.1. The intuition about the importance of  $\gamma$  is the following; given (3), money shocks can have significant and prolonged effects on output only if the price level adjusts slowly. Thus, given (1) and (2), the higher  $\gamma$ , the higher the sensitivity of the nominal variables to movements in output, the faster the adjustment of prices. It is straightforward to show that in fact  $\lambda_s$  is a decreasing function of  $\gamma$ .

Another implication of this model is that it exhibits a Phillips-curve-type output-inflation trade-off. Following a positive monetary shock, prices adjust sluggishly and output is temporary above its natural level. This trade-off would depend on  $\lambda_s$  (and hence on  $d$  and  $\gamma$ ): the higher  $\lambda_s$ , the flatter the Phillips curve.

West (1988) and Phaneuf (1990) both investigated whether this model could generate near random-walk behaviour in output. Phaneuf (1990) closed the model with the following equation, representing a feedback policy rule:<sup>4</sup>

$$m_t = a p_t + v y_t \quad (7)$$

Incorporating this equation, the solution of the system changes slightly, since now  $\alpha =$

$\left( \frac{1 + (1 - c)\gamma / 2}{1 - (1 - c)\gamma / 2} \right)$ , where  $c = (a - v)/(1 - v)$ . The parameter  $c$  is a policy parameter which

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<sup>3</sup> See, for instance, Driskill and Sheffrin (1986), De Long and Summers (1986), Ambler and Phaneuf (1989, 1992).

<sup>4</sup> Taylor (1979, 1980a,b) sets  $v = 0$ .

indicates the degree of accommodation of monetary policy to price changes; the higher  $c$ , the more accommodative is monetary policy. Analytically, the role of  $c$  parallels the one of  $\gamma$  and a lower value of  $c$  causes, *ceteris paribus*, a lower degree of persistence (as  $c$  approaches unity,  $\alpha$  approaches unity). In other words, the introduction of (7) into the model is analytically equivalent to multiplying  $\gamma$  by  $(1-c)$ . Phaneuf (1990) took estimated values of  $\gamma$  and  $c$  for Canada, Germany, Italy, United Kingdom and US from some previous studies in the literature. He found  $\gamma$  to lie between 0 and 0.32 and  $c$  to lie between 0.71 and 0.91. Despite the fact that his estimates of  $\gamma$  were on average bigger than those of Taylor (1980b) and of Sachs (1980), Phaneuf (1990) found that those countries associated with higher values of  $\gamma$ , also presented higher values of  $c$ . This observation led him to determine  $\lambda_s$  to be between 0.65 and 1 and to conclude : “...the evidence of a unit root in the real GNP process of many countries is not necessarily inconsistent with a contract-based approach to the business cycle. [...] the asynchronization of wage contracts can potentially play the role of an important dynamic propagation mechanism and can contribute substantially to the persistence of output fluctuations.” (Phaneuf, 1990, p. 590) Moreover, he acknowledged the potential role of  $d$  in generating persistence; the lower  $d$ , the higher the backward looking bias in the Taylor’s wage setting rule and consequently the higher is the persistence. However: “Given that no evidence is currently available on the direction of this bias, if any, it would seem that one important item on the agenda of future research should be to determine empirically the value [of  $d$ ].” (Phaneuf, 1990, pp. 590- 591)

West (1988) considered two different monetary policy rules; one targeted the interest rate and the other targeted the money supply. Then he simulated Taylor’s model in these two different cases, choosing values of  $\gamma$  between 0.01 and 0.1. He concluded that in an economy

characterised by overlapping nominal contracts money shocks could induce a near random-walk behaviour in output.

In the next section we develop a dynamic general equilibrium model with utility maximising agents and staggered wages to investigate the same issue as Phaneuf (1990) and West (1988). With respect to Phaneuf (1990), we reveal the micro determinants of  $d$  and hence of the forward versus backward looking bias. With respect to West (1988), we demonstrate that the parameters of Taylor's wage setting equation are likely to depend heavily on the monetary policy rule. Moreover, by the same token, we also provide an explanation of Phaneuf's (1990) empirical finding that countries in which the monetary policy is more loose exhibit a higher value of  $\gamma$ .

### 3. The Model

The model introduces staggering wage setting a` la Taylor (1979) in the framework of Rankin (1996). The economy is composed of a continuum of industries indexed by  $i \in [0,1]$  and of a continuum of industry-specific household-unions.<sup>5</sup> Every industry produces a single differentiated perishable product and the goods market in each industry is Walrasian. Since labour is not allowed to move across industries, the household-union has monopoly power in the labour market. Preferences are CES over consumption goods which are gross substitutes. All firms have the same technology and households have the same preferences. There is no uncertainty in the model and agents perfectly foresee the future. The introduction of staggered wages breaks the symmetry of the economy. In fact, the latter is divided into two sectors of equal size: sector A, industries  $i \in [0, \frac{1}{2}]$  and industry-specific household-unions  $j \in [0, \frac{1}{2}]$ ;

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<sup>5</sup> A continuum of industries means that no imperfectly competitive agent is "large" relative to the economy as a whole.

sector B, industries  $i \in (1/2, 1]$  and industry-specific household-unions  $j \in (1/2, 1]$ . In each sector nominal wages are negotiated every two periods and they are fixed within the two periods. Therefore, staggering is introduced supposing sector A fixes the wages in periods  $t, t+2, t+4...$  while sector B in period  $t-1, t+1, t+3...$

*Demands for output and labour in the two sectors*

All the households have the same utility function  $U_j = \sum_{t=0}^{\infty} \beta^t u_j(C_{jt}, (M/P)_{jt}, L_{jt})$ , where

$0 < \beta < 1$ .  $C_t$  is a consumption index defined by the CES function:

$$C_{jt} = \left[ \int_0^1 C_{jit}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \quad (8)$$

where the elasticity of substitution  $\theta$  is bigger than 1.  $M_t/P_t$  are the real balances held at the end of period  $t$ . Real money balances enter the utility function because of the liquidity services that money provides. The last term represents the quantity of labour supplied by the household during period  $t$ . The CES preferences give the usual demands for good  $i$ :

$$C_{jit} = \left[ \frac{P_{it}}{P_t} \right]^{-\theta} \frac{E_{jt}}{P_t} \quad (9)$$

where  $P_t$  is the price index defined as  $P_t = \left[ \int_0^1 P_{it}^{\frac{1}{1-\theta}} di \right]^{1-\theta}$ , and  $E_{jt}$  is total goods expenditure.

As a consequence, the maximised sub-utility  $C_{jt}$  is equal to  $E_{jt}/P_t$ .

All the firms have the following production function  $Y_{it} = \alpha L_{it}^{\sigma}$ ,  $0 < \sigma < 1$ , and they are price-takers both in the goods and in the labour markets. Firms maximise profits and therefore, given the nominal wage  $X_{it}$ , which is fixed by the monopolistic household-union, the demands for labour and the output are:

$$L_{it} = \left[ \frac{1}{\alpha\sigma} \frac{X_{it}}{P_{it}} \right]^{\frac{1}{\sigma-1}} ; \quad Y_{it} = \alpha \left[ \frac{1}{\alpha\sigma} \frac{X_{it}}{P_{it}} \right]^{\frac{\sigma}{\sigma-1}} \quad (10)$$

Each union realises that its behaviour influences the price of the output  $i$  and hence the demand for labour. Given the total demand for industry  $i$ , imposing the equilibrium condition on goods market,  $C_{it} = Y_{it}$ , yields the following relation between the labour demand and the nominal wage:

$$L_{it} = K_t X_{it}^{-\varepsilon} \quad \text{where} \quad \varepsilon = \frac{\theta}{\sigma + (1-\sigma)\theta} \quad \text{and} \quad K_t = (\sigma\alpha)^\varepsilon \left[ \frac{E_t}{\alpha P_t^{1-\theta}} \right]^{\frac{\varepsilon}{\theta}} \quad (11)$$

This represents the demand function faced by the monopoly union in industry  $i$  and it exhibits a constant money-wage elasticity. Since industry  $i$  has measure zero in the economy as a whole, aggregate expenditure and the price index are considered as given by the union and hence the term  $K_t$  is parametric to the union.

In equilibrium all the industries of the same sector produce the same level of output and charge the same price and we can easily aggregate within the two sectors. Supposing that sector A fixes the wage in period  $t, t+2, t+4...$  and sector B in period  $t-1, t+1, t+3...$ ,<sup>6</sup> the output levels of the two sectors in period  $t$  are:

$$Y_{At} = (1/2)\alpha \left[ \frac{1}{\alpha\sigma} \frac{X_t}{P_{At}} \right]^{\frac{\sigma}{\sigma-1}} ; \quad Y_{Bt} = (1/2)\alpha \left[ \frac{1}{\alpha\sigma} \frac{X_{t-1}}{P_{Bt}} \right]^{\frac{\sigma}{\sigma-1}} \quad (12)$$

In equilibrium the aggregate nominal output is equal to the aggregate nominal expenditure

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<sup>6</sup> In other words, at the beginning of period  $t$  sector A signs a new contract and fixes the nominal wage  $X_t$  for the two subsequent periods. At the same time sector B is locked in the contract it signed one period before that is  $X_{t-1}$ . At the beginning of period  $t+1$ , then sector B signs a new contract  $X_{t+1}$  for the next two period, while  $X_t$  remains valid for sector A and so on. Therefore  $X_t, X_{t+2}, X_{t+4}, ...$  are the nominal wages fixed by sector A, while  $X_{t-1}, X_{t+1}, X_{t+3}, ...$  are the ones fixed by sector B.

on consumption:

$$\frac{1}{2} P_{At} Y_{At} + \frac{1}{2} P_{Bt} Y_{Bt} = E_t = P_t C_t = \frac{1}{2} P_t C_{At} + \frac{1}{2} P_t C_{Bt} = \frac{1}{2} E_{At} + \frac{1}{2} E_{Bt}$$

The demands for the output of the two sectors in period  $t$  are:

$$Y_{At}^d = \frac{1}{2} \left( \frac{P_{At}}{P_t} \right)^{-\theta} \frac{E_t}{P_t} \quad ; \quad Y_{Bt}^d = \frac{1}{2} \left( \frac{P_{Bt}}{P_t} \right)^{-\theta} \frac{E_t}{P_t} \quad (13)$$

where in equilibrium  $P_t = \left( \frac{1}{2} P_{At}^{1-\theta} + \frac{1}{2} P_{Bt}^{1-\theta} \right)^{\frac{1}{1-\theta}}$ .

### *The intertemporal behaviour of the household-union<sup>7</sup>*

In each period the household chooses the level of consumption, the quantity of money and bonds it will transfer to next period and the supply of labour together with the level of money wage. Each household enters period  $t$  with a predetermined level of wealth, given by money balances  $M_{t-1}$  and by the gross interest on bonds  $i_{t-1} B_{t-1}$ . In other words households are permitted to trade in bonds. During period  $t$  households receive a nominal lump-sum transfer  $T_t$ , the wage income and profits distributed by the firms  $\Pi_t$ . The budget constraint is therefore given by:

$$P_t C_t + M_t + B_t = M_{t-1} + i_{t-1} B_{t-1} + W_t L_t + \Pi_t + T_t \quad (14)$$

Since the nominal wage is fixed for two periods, at the beginning of period  $t$ , the household-unions decide the nominal wage for the periods  $t$  and  $t+1$ . After two periods the problem faced by the households is again the same. Rearranging the first order conditions for this problem, we obtain:

$$u_C(t) = r_t \beta u_C(t+1) \quad (15)$$

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<sup>7</sup> In what follows index  $j$  is suppressed to lighten notation.

$$u_{M/P}(t) = (1 - 1/i_t) u_C(t) \quad (16)$$

$$u_{M/P}(t+1) = (1 - 1/i_{t+1}) u_C(t+1) \quad (17)$$

$$X_t = \left[ -\frac{\varepsilon}{\varepsilon - 1} \right] \left[ \frac{u_L(t) + \beta u_L(t+1) \frac{K_{t+1}}{K_t}}{u_C(t) \frac{1}{P_t} + \beta u_C(t+1) \frac{K_{t+1}}{K_t P_{t+1}}} \right] \quad (18)$$

where  $u_s(t) = \frac{\partial u(C_t, (M/P)_t, L_t)}{\partial s_t}$ , for  $s = C, M/P, L$ .

The first condition is a standard consumption Euler condition for the optimal intertemporal consumption choice. The second and the third one equate the marginal utility of real balances to the marginal consumption opportunity cost of holding money. The third one yields the optimal wage charged by the monopoly household-union. This is given by a fixed mark-up  $\varepsilon/(\varepsilon - 1)$  over the quantity in the square bracket. The latter is a ratio between weighted averages of the disutility from labour and of the utility from consumption over the two periods, that is, a weighted average between the optimal flexible wages of the two periods. These average values are weighted by the discount factor  $\beta$  and by the coefficient  $K$ . Note that, since  $\beta < 1$ , the second period optimal wage is discounted. Hence, it is given a lower weight in calculating the average value. The analysis of this third first-order condition and the comparison between it and Taylor's wage setting equation will be the focus of this paper.

#### 4. The Optimal Wage Setting Rule

A steady state in this model is characterised by a constant rate of money growth ( i.e., constant  $(1/\mu)-1$ , where  $\mu$  is defined as  $M_t/M_{t+1}$  ). In steady state the nominal variables of interest, i.e., money wages and aggregate price level, are growing at the same rate of the

money supply, while real aggregate output is constant. Without defining an explicit form for the utility function (see section 5) the model cannot be solved analytically, but we can log-linearise the model around a steady state. However, given the asymmetry in the economy – between the households belonging to different sectors, some aggregation problems must be solved. In particular: (i) we define aggregate output as in the national account, that is, the sum of nominal output of the sectors divided by the aggregate price level given by (9); (ii) given (i), the equilibrium condition on goods market is  $Y_t = C_t$ , where  $Y_t$  is aggregate output and  $C_t$  is household's consumption index; (iii) since households are allowed to exchange bonds, they can perfectly shield themselves from fluctuations in consumption. Therefore, the marginal utility of consumption is always the same in each period for all the households.<sup>8</sup>

Log-linearising (18) around a steady state, we get the following expression:

$$x_t = b_1 p_t + d_1 p_{t+1} + b_2 y_t + d_2 y_{t+1} + b_3 m_t + d_3 m_{t+1} \quad (19)$$

where lower case letters are used for variables expressed as log-deviations from trend and where  $b_i$  and  $d_i$ , for  $i = 1, 2, 3$ , are shown in Appendix I. These latters depend upon the parameters describing technology and preferences and upon the steady state rate of growth of money. From Taylor's model, substituting (2) in (1) yields:

$$x_t = b p_t + d E_{t-1}(p_{t+1}) + (\gamma/2) [b E_{t-1}(y_t) + d E_{t-1}(y_{t+1})] \quad (20)$$

which corresponds to (19). In fact, while Taylor justifies his wage setting rule advocating a “Keynesian” relative wage concern on the part of the workers, equation (20) shows his model to be analytically equivalent to one in which workers are “neoclassically” concerned about

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<sup>8</sup> I thank Giuseppe Bertola for pointing this out to me.



their real wage, as in the present paper.<sup>9</sup>

The following expression gives us the level of prices in the economy:

$$P_t = \alpha^{-1} \sigma^{-\sigma} \left( \frac{1}{2} \right)^{\frac{\sigma}{1-\varepsilon}} (E_t)^{1-\sigma} (X_t)^\sigma \left[ 1 + \left( \frac{X_{t-1}}{X_t} \right)^{1-\varepsilon} \right]^{\frac{\sigma}{1-\varepsilon}} \quad (21)$$

Imposing the equilibrium condition in the goods market,  $E_t = P_t Y_t$ , its log-linearisation is:

$$p_t = \left( \frac{1-\sigma}{\sigma} \right) y_t + q x_t + (1-q) x_{t-1} \quad q = \left( \frac{1}{1 + \mu^{1-\varepsilon}} \right) \quad (22)$$

The parallel between equation (22) and equation (2) is clear. Expression (22) is just more general since it allows both the returns to scale to be lower than one and the parameters to depend upon the rate of growth of the money supply.

Substituting (22) in (19), we get:

$$x_t = b_4 x_{t-1} + d_4 x_{t+1} + b_5 y_t + d_5 y_{t+1} + b_6 m_t + d_6 m_{t+1} \quad (23)$$

which corresponds to the Taylor's wage setting rule. Simply adding to the system (19), (22) and (23) from above, the static aggregate demand equation supposed by Taylor, i.e.,  $y_t = m_t - p_t$ , we can reproduce Taylor's result about the persistence of money shocks discussed in section 2. The comparison is then between:<sup>10</sup>

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$$(1) x_t = b x_{t-1} + d x_{t+1} + \gamma (b y_t + d y_{t+1}) \quad (23) x_t = b_4 x_{t-1} + d_4 x_{t+1} + b_5 y_t + d_5 y_{t+1} + b_6 m_t +$$

<sup>9</sup> This point is noted in Blanchard (1990, footnote 19, p. 805). "It is sometimes argued that the Taylor model depends on the assumption that workers care directly about their relative wages in comparison to other wages, an assumption which is thought by some to be unattractive. [...] this is not the case."

<sup>10</sup> For simplicity we drop the expectation operator in Taylor's model. Note that the expectation operator can straightforwardly be incorporated in the model of this paper, without affecting the results we are concerned with.

$$(2) p_t = \frac{1}{2} (x_t + x_{t-1}) \quad (22) p_t = \left( \frac{1-\sigma}{\sigma} \right) y_t + q x_t + (1-q) x_{t-1}$$

$$(20) x_t = b p_t + d p_{t+1} + (\gamma/2)(b y_t + d y_{t+1}) \quad (19) x_t = b_1 p_t + d_1 p_{t+1} + b_2 y_t + d_2 y_{t+1} + b_3 m_t + d_3$$

The following results are proved in Appendix I.<sup>11</sup>

**Proposition 1.** *If the utility function is additively separable in real money balances, that is, if  $u_{L,M/P} = \partial^2 u(t) / (\partial L_t \partial (M/P)_t) = 0$  and  $u_{C,M/P} = \partial^2 u(t) / (\partial C_t \partial (M/P)_t) = 0$ , then nominal balances do not appear in (23) and (19).*

Therefore, Taylor's wage setting rule could be justified only if the underlying utility function is additively separable in real money balances or if one is inclined to think that the above cross derivatives are of negligible magnitude. Otherwise, both (1) and (3) should include real money balances.

**Proposition 2.** *The "nominal-homogeneity" constraint  $(b + d) = 1$ , imposed ad hoc by Taylor, holds also in the microfounded model, since both  $(b_4 + d_4) = 1$  and  $(b_1 + d_1) = 1$ .*

*Moreover, the sums  $(b_2 + d_2) = g = \frac{\eta_{LC} - \eta_{CC} + (\varepsilon/\theta)(\eta_{LL} - \eta_{CL})}{1 + \varepsilon(\eta_{LL} - \eta_{CL})}$  and  $(b_3 + d_3) =$*

*$\frac{\eta_{L,M/P} - \eta_{C,M/P}}{1 + \varepsilon(\eta_{LL} - \eta_{CL})}$  are both independent of  $\mu$  (i.e., independent of the policy rule).*

We define  $\eta_{rs} = \frac{u_{rs} s}{u_r}$  which represents the elasticity of the marginal utility of  $r$  with

respect to  $s$ , for  $r, s = C, M/P, L$ . Both in (19) and in (23) the coefficient of the nominal

<sup>11</sup> It is probably worth noting that, despite being analytically equivalent, the numerical order of the equation reflects the different interpretation of the wage setting rule in the two models. In Taylor's, workers directly care about relative wages (see (1)) and indirectly about real wages through (2) (see (20)). In the microfounded model, instead, workers care directly about real wages (see (19)) and indirectly about relative wages through (22) (see (23)).

variables, respectively prices and wages, sum to 1, as supposed by Taylor. This would suggest that those parameters could be interpreted as  $d$  and  $b$  in Taylor's model, that is, the degree of backward and forward looking behaviour. However, since the expressions (19) and (23) cannot be factorised as can the corresponding Taylor's equations (20) and (1), we are not able to identify the crucial parameter  $\gamma$  in this general formulation.

**Proposition 3.** *All the parameters in (19), (22) and (23) depend upon the policy rule. In particular, as the rate of growth of money tends to infinity, the parameters  $b_i$ , for  $i = 1, \dots, 6$ , tend to zero and  $d_i$ , for  $i = 1, \dots, 6$ , tend to finite values, and viceversa as the rate of growth of money tends to -100%.*

It is immediately evident what the last two propositions imply for the wage setting rule (19). As the monetary trend increases, more weight is put on the future variables and less on present ones. In fact, we know from proposition 1 that the sum of a coefficient of a variable in  $t$  ( $b_i$ ) plus the coefficient of the same variable in  $t+1$  ( $d_i$ ) is constant. Then, as the rate of money growth increases,  $b_i$  decreases while  $d_i$  increases. As a matter of fact, the higher the inflation trend, the more forward-looking is the wage setting equation. This important issue, concerning the dependence of *ad hoc* Taylor's parameters on average inflation, will be thoroughly discussed in the next section.

**Proposition 4.** *If the utility function is additively separable in real money balances and if the money supply is constant in steady state, then the microfounded model exactly coincides with Taylor's model, where  $b = 1 - d = \left( \frac{1}{1 + \beta} \right)$  and  $\gamma = 2(g + (1 - \sigma)/\sigma)$ .*

As a result, Taylor's wage setting rule (1) can be interpreted as the log-linear

approximation of a monopolistic household-union optimal wage setting rule around a steady state with constant money supply. In fact, if  $\mu = 1$  in steady state and  $\eta_{L,M/P} = \eta_{C,M/P} = 0$ , (19) and (22) and (23) respectively become:

$$x_t = \left( \frac{1}{1+\beta} \right) [p_t + g y_t] + \left( \frac{\beta}{1+\beta} \right) [p_{t+1} + g y_{t+1}] \quad (24)$$

$$p_t = \left( \frac{1-\sigma}{\sigma} \right) y_t + \frac{1}{2} (x_t + x_{t-1}) \quad (25)$$

$$x_t = \left( \frac{1}{1+\beta} \right) [x_{t-1} + \gamma y_t] + \left( \frac{\beta}{1+\beta} \right) [x_{t+1} + \gamma y_{t+1}] \quad (26)$$

which precisely matches equations (20), (1) and (2).<sup>12</sup>

The striking feature of those three equations is not only that they perfectly parallel Taylor's assumptions, but that they provide an extremely natural interpretation of Taylor's *ad hoc* structural parameters. The backward and forward-looking parameters, respectively  $d$  and  $b$  in Taylor's model, simply depend only on the rate of time preferences. If the intertemporal rate of discount is zero then  $\beta = 1$  and  $b = d = \frac{1}{2}$  and "*contract decisions are unbiased*". As advocated by Phaneuf (1990), we are therefore able to provide an explanation of the direction of this bias. If agents naturally discount the future, then they are biased backward, in the sense that present variables have higher weight than future variables. Given that  $\beta \leq 1$ , it follows that  $d \leq b$ .<sup>13</sup> Recall that the degree of persistence is a decreasing function of the forward-looking parameter  $d$ . Then, the degree of persistence in Taylor's model is an increasing function of the intertemporal rate of discount.

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<sup>12</sup> Actually, despite the successive approximations used to get here, the model of this paper is still a little bit more general than Taylor's in that it allows  $\sigma$  to be different from 1 and therefore the mark-up equation (25) to incorporate a decreasing returns to scale effect.

<sup>13</sup> Proposition 3 should however have already warned the reader that the degree of backward and forward looking depends also on the steady state monetary policy.

The elasticity of the money wage to the business cycle conditions is given by  $\gamma = 2(g + (1 - \sigma)/\sigma)$  where, as in proposition 2,  $g = \frac{\eta_{LC} - \eta_{CC} + (\varepsilon/\theta)(\eta_{LL} - \eta_{CL})}{1 + \varepsilon(\eta_{LL} - \eta_{CL})}$ . Now suppose additive

– separability between consumption and labour. Then  $\eta_{LC} = \eta_{CL} = 0$  and  $g$  becomes:  $g = \frac{-\eta_{CC} + (\varepsilon/\theta)\eta_{LL}}{1 + \varepsilon\eta_{LL}}$ . It is standard to assume both an increasing marginal disutility of labour

and a decreasing marginal utility of consumption, hence  $g$  is always positive. Firstly,  $\gamma$  is a decreasing function of  $\theta$ , that is, the bigger  $\theta$ , the bigger the persistence. Following a positive money shock, the new money wage will be set higher than the one already fixed in the previous period by the other sector. However, the bigger  $\theta$ , the bigger the loss in demand a sector will face fixing the new level of money wage, and hence of price, bigger than the one of the other sector. Therefore, the unions will tend to fix the new wage close to the existing one inducing more price level inertia.<sup>14</sup>

Secondly,  $\gamma$  is decreasing in  $\sigma$ . Consistent with intuition, the case  $\sigma = 1$  corresponds to the maximum degree of nominal rigidities. As it is immediately evident from equation (18), if a positive money shock increases output, then, firms could satisfy the excess demand without changing their prices only if  $\sigma = 1$ . Therefore, Taylor's hypothesis of constant returns to scale, implicitly embedded into (2), actually favours persistence, while if  $\sigma < 1$  the degree of persistence will be lower. Moreover:

**Proposition 5.** (i) *The persistence is increasing (decreasing) in the intertemporal elasticity of substitution of labour,  $(-1/\eta_{LL})$ , if and only if the intertemporal elasticity of substitution of*

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<sup>14</sup> This result appears to be counterintuitive, since we might have expected a more competitive economy to exhibit a lower degree of price inertia. Note that the model presented in Blanchard and Fischer (1989), Ch. 8, which will be analysed in Appendix II, exhibits the same kind of effect. The result is mainly due to the institutional assumption about the fixed length of the contract. Presumably the more competitive the economy, the lower is the length of the contracts and the more flexible are the prices.

consumption,  $(-1/\eta_{CC})$ , is bigger (lower) than the elasticity of substitution in consumption goods,  $\theta$ ; (ii) A low  $(-\eta_{CC})$  together with a high intertemporal elasticity of substitution of labour causes a low value of  $\gamma$  and hence a high degree of persistence.

The second result is actually very intuitive and previously presented in Blanchard and Fischer (1989), Ch. 8.<sup>15</sup> Note that  $\gamma$  is an increasing function of  $-\eta_{CC}$ . The higher the elasticity of marginal utility of consumption, the more the marginal utility of consumption is going to fall for a given increase in output (= consumption, in equilibrium), the more wages are pushed up since households would prefer to exchange consumption for more leisure at the margin. In other words, the lower the intertemporal elasticity of substitution of consumption (that is just the inverse of the elasticity of marginal utility of consumption), the more households would like to substitute an increase in consumption with an increase in leisure within the period, leading to a rise in the wages. Note that if the utility over consumption is linear, then this income effect on labour supply is absent. With a zero income effect, wages are not responsive to changes in consumption. Hence the real wage just depends on the marginal utility of labour. Then, the less elastic is the latter (i.e., the lower is  $\eta_{LL}$ ) the lower is the pressure on the wages for a given increase in the labour demand. If the utility of consumption and labour is linear (i.e.,  $\eta_{CC} = \eta_{LL} = 0$ ) then  $g = 0$ . However,  $\gamma$  is still different from zero unless  $\sigma = 1$ .

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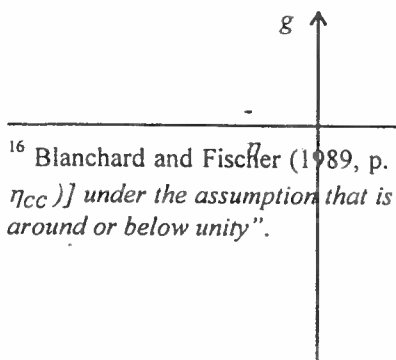
<sup>15</sup> Moreover, Chari *et al.* (1996), in contemporaneous and independent study, reproduced this result. Chari *et al.* (1996) build a dynamic general equilibrium model with staggered price setting. In spite of the similarities between the two analyses, the two studies have distinct features. (i) The model presented in this paper is closer in spirit to Taylor's original (1979) model in that it explicitly considers the labour market and the optimal wage setting rule. (ii) Our analysis is analytically oriented and tries to find explicit solutions and comparisons with previous results to explain the mechanism at work. On the other hand, Chari *et al.* (1996) heavily rely on calibration and simulation techniques, given the high complexity of their model with respect to the one here presented. This enables them to simulate quite a number of different versions of their basic model, providing robustness checks to their main finding. (iii) Our analysis focuses on the important issue of the relation between the policy behaviour and agents' response, while Chari *et al.* (1996) do not mention this point. Thus, the two works seem to be complementary.

The first result of proposition 5 is a novel and fairly general. It follows from simple algebra that:

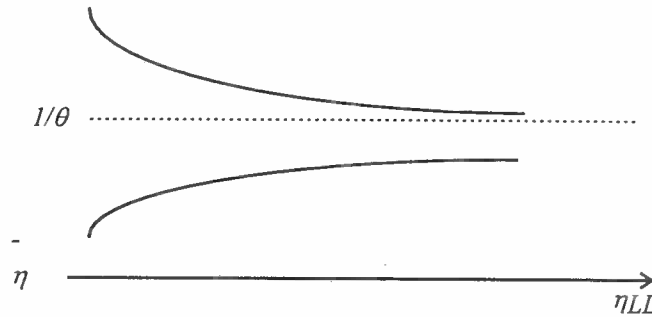
$$\frac{\partial g}{\partial \eta_{LL}} \begin{matrix} > \\ < \end{matrix} 0 \quad \Leftrightarrow \quad (-\eta_{CC}) \begin{matrix} < \\ > \end{matrix} 1/\theta$$

Therefore the direction of the effect of a higher elasticity of marginal utility of labour critically depends on the relative values of  $\eta_{CC}$  and  $\theta$ . As explained above, straightforward intuition would lead one to presume a positive relation between  $\gamma$  and  $\eta_{LL}$ ; the higher the intertemporal elasticity of substitution of labour (i.e., the lower  $\eta_{LL}$ ), the lower the sensitivity of wages to output, i.e.,  $\gamma$ , as found by Blanchard and Fischer (1989) and Chari *et al.* (1996). Instead, this result shows that this is true only if the intertemporal substitution of consumption is bigger than  $\theta$ . However, the intertemporal substitution of consumption is usually assumed to be low and generally around one, while  $\theta$  is, by definition bigger than one.<sup>16</sup> Therefore, the relation between  $\gamma$  and the intertemporal elasticity in labour is the opposite of what intuition would suggest, unless we are willing to assume unrealistically low  $(-\eta_{CC})$ . Figure 1 graphically shows the result summarised in the last proposition. Some of the results are consistent with the intuition and previous findings. Some others instead are not and show that previous findings do not hold in general. In Appendix II, we briefly compare the model of this paper and the one in Blanchard and Fischer (1989), Ch. 8. We think that comparison to be very useful to provide a better understanding of the above results.

Figure 1



<sup>16</sup> Blanchard and Fischer (1989, p. 44) wrote : "Substantial empirical work has been devoted to estimating  $[(-\eta_{CC})]$  under the assumption that is indeed constant [...] Estimates of  $[(-\eta_{CC})]$  vary substantially but usually lie around or below unity".



## 5. Persistence, Non-linearity and the Effects of Average Inflation

We are now ready to try to numerically evaluate the degree of persistence of money shocks implied by the model. Let us suppose the following explicit utility function:

$$U = \delta \frac{C_t^{1-a} - 1}{1-a} + (1-\delta) \ln (M/P)_t - d L_t^e \quad (27)$$

where  $a \geq 0$ ,  $e \geq 1$ . This utility function is additively separable and exhibits increasing marginal disutility of labour. Moreover, it allows the intertemporal elasticity of consumption to be different from one, since  $\eta_{CC} = -a$ .<sup>17</sup>

Table I

*Value of  $\gamma$  and  $\lambda_S$  as microeconomic parameters vary: 3 cases*  
( $\mu = 1$ ,  $\beta = 0.95$ )

	$\gamma$	$\lambda_S$
$\theta = 6; \sigma = 1; e = 1, a = 0$	0	1
$\theta = 6; \sigma = 1; e = 4.5, a = 1$	0.409	0.3868
$\theta = 15; \sigma = 1; e = 15, a = 1$	0.1422	0.5936

<sup>17</sup> It is probably worth noting that if  $a = 1$ , the demand side of the economy is given by:  $y_t = m_t - p_t - z_t$ , where  $z_t$  is a forward-looking variable whose dynamics depend upon the path of the money supply. That is, log-linearising (16) and (17) yields a money demand equation, similar to the static aggregate demand equation (3) of Taylor (1979), but now the log-linearised equations incorporate a Cagan-demand-for money effect. In fact, being derived by an intertemporal dynamic problem, the velocity of circulation of money (i.e.,  $1/z_t$ ) is proved to be a forward-looking variable whose steady state value is increasing in the rate of inflation. This Cagan-demand-for-money effect turns out to be very important for the dynamic response of this model following a disinflationary policy, see Ascari and Rankin (1997). However, the behaviour of  $z_t$  is not going to affect the degree of persistence, unless the money supply process is serially correlated.



In Table I we report three cases for a zero money growth policy rule. In the first case both consumption and labour enter linearly in the utility function, thus  $\eta_{CC} = \eta_{LL} = 0$ .<sup>18</sup> Then money wage is inelastic with respect to output changes and money shocks have permanent effects on the level of output. The second case is a kind of “base case”, given the values of microeconomic parameters used in the calibration literature.<sup>19</sup>  $\gamma$  is found to lie just outside the range of estimates in Phaneuf (1990) and to be much higher than the values used by Taylor (1980b) and West (1988).<sup>20</sup> The value of the stable root in this case is far from inducing a near-random walk behaviour in output following a monetary shock. However, even in the case of non-negligible income effects, the degree of persistence could be increased by raising the values of  $\theta$  and  $e$ , as the third case in Table I shows. Nonetheless, only for extreme and unrealistic values of these parameters ( $\theta = 42$ ,  $e = 20$ ) we can obtain Taylor’s estimate of  $\gamma$  ( $= 0.05$ ) and a near-random walk degree of persistence ( $= 0.745$ ).

In conclusion, it seems that the results of West (1988) and Phaneuf (1990) can only be supported for extreme values of the microeconomic fundamentals. In particular, either (i) a zero income effect and virtually infinite elasticity of substitution of labour supply<sup>21</sup>, or (ii)

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<sup>18</sup> If  $\eta_{CC} = \eta_{LL} = 0$  the model actually breaks down, since the labour demand and supply curves are both horizontal and they do not intersect. This case must then be interpreted as a limiting one.

<sup>19</sup> In this “base case” the value of  $\theta$  is set to 6 as in Hairault and Portier (1993). More problematic is to find a sensible value for  $e$ . Following Macurdy (1981)  $e$  should be put equal to 4.3. However, most Pencavel (1986) estimations place  $e$  between 3.2 and infinity. As a conclusion, the elasticity of intertemporal substitution of labour supply should be low. Therefore, in the “base case”  $e$  is put equal to 4.5, but there are virtually no reasons for not choosing a higher value of  $e$ . Following the remark in Blanchard and Fischer (1989, p. 44) quoted in note 13, we put  $a = 1$ . Then,  $\sigma = 1$ , as in Taylor’s model, and  $\beta = 0.95$ . (Moreover,  $d = 0.01$ ,  $\delta = 0.99$ , but these parameters are not important for the results).

<sup>20</sup> Chari *et al.* (1996) calibrate  $\gamma$  to be well above one. However, it should be stressed that ours is *not* a calibration exercise. We are more interested in a kind of robustness exercise to assess whether persistence can be a likely outcome and, if so, under which conditions.

<sup>21</sup> Note that in the case  $a = 0$  and  $e = 1.03$ , we get exactly  $\gamma = 0.05$ , as Taylor. However, a slight increase of  $e$  above one has a dramatic effect on persistence. Only for value of  $e$  lower than 1.05, we still get a degree of persistence bigger than 0.7. This suggests a highly non-linear relation between  $e$  and persistence in this case.

high values of  $\theta$  and  $e$ , for more realistic values of  $\eta_{CC}$ . A high degree of persistence could nevertheless occur in the *ad hoc* model because of a very accommodating feedback monetary policy rule (i.e., high value of  $c$ ).

However, as already noted in proposition 3, the structural parameters of Taylor's wage setting rule depend upon the steady state rate of money growth. We must then carefully consider this relation. Looking at Table II<sup>22</sup> it is evident that the previous conclusion is not robust when monetary policies other than a constant money are considered.

Firstly, look at the value of the stable root. For any case, the degree of persistence is the higher, the lower is the rate of growth of money (*rgm* in Table II). Besides, the sensitivity of  $\lambda_s$  with respect to changes in *rgm* is the higher, the bigger are the microeconomic parameters. If money *decreases* at the rate of 4% in steady state, then, with respect to the case of zero money growth, the persistence rises from 0.3868 to 0.5476 in the "base case" and from 0.5936 to 0.8335 in the third case. On the other hand, if money grows at a 10% rate, in both cases the persistence is virtually nil.

Table II

*Value of persistence and of the parameters of Taylor's wage setting rule  
as microeconomic fundamentals and rate of growth of money vary*

parameters ↓	<i>rgm</i> →	- 4%	0	+ 5%	+ 10%
$\theta = 6, \sigma = 1$ $e = 1, a = 0$		$\lambda_s = 1$ $b_4 = 0.575$ $d_4 = 0.425$ $b_5 = 0.0205$ $d_5 = - 0.0205$	$\lambda_s = 1$ $b_4 = 0.513$ $d_4 = 0.487$ $b_5 = 0$ $d_5 = 0$	$\lambda_s = 0.822$ $b_4 = 0.439$ $d_4 = 0.561$ $b_5 = - 0.025$ $d_5 = 0.025$	$\lambda_s = 0.6343$ $b_4 = 0.3614$ $d_4 = 0.639$ $b_5 = - 0.051$ $d_5 = 0.051$
$\theta = 6, \sigma = 1$		$\lambda_s = 0.5476$ $b_4 = 0.771$ $d_4 = 0.229$ $b_5 = 0.332$ $d_5 = 0.105$	$\lambda_s = 0.3868$ $b_4 = 0.513$ $d_4 = 0.487$ $b_5 = 0.210$ $d_5 = 0.199$	$\lambda_s = 0.1291$ $b_4 = 0.204$ $d_4 = 0.796$ $b_5 = 0.098$ $d_5 = 0.347$	$\lambda_s = - 0.0167$ $b_4 = 0.002$ $d_4 = 0.998$ $b_5 = 0.040$ $d_5 = 0.494$

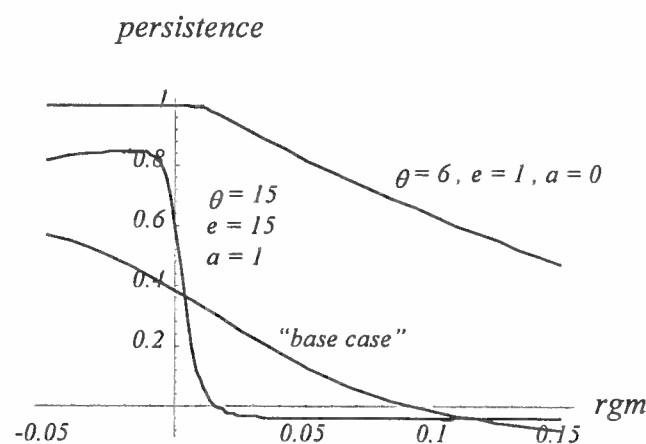
<sup>22</sup> The rate of growth of money (*rgm* in Table II) is defined over one period. Further, if *rgm*  $\neq 0$  there is no single measure for  $\gamma$  and that is why we report all the parameters of the wage setting rule in Table II.

$e = 4.5, a = 1$				
$\theta = 15, \sigma = 1$	$\lambda_S = 0.8335$ $b_4 = 1.042$ $d_4 = -0.042$ $b_5 = 0.201$ $d_5 = 1.9 \cdot 10^{-5}$	$\lambda_S = 0.5936$ $b_4 = 0.513$ $d_4 = 0.487$ $b_5 = 0.073$ $d_5 = 0.069$	$\lambda_S = -0.0394$ $b_4 = -0.047$ $d_4 = 1.047$ $b_5 = 3.9 \cdot 10^{-6}$ $d_5 = 0.217$	$\lambda_S = -0.043$ $b_4 = -0.057$ $d_4 = 1.057$ $b_5 = 1.82 \cdot 10^{-10}$ $d_5 = 0.356$
$e = 15, a = 1$				

The degree of sensitivity of  $\lambda_S$  to  $rgm$  is indeed somewhat impressive. Further even in the first case, the persistence strongly diminishes as the rate of growth of money rises and at a 10%  $rgm$ ,  $\lambda_S$  is far from a near-random walk behaviour. Figure 3 visualises these effects.

To understand this result we now look at the other parameters of Taylor's wage setting rule (23).  $b_i$  refers to the backward-looking variables, while  $d_i$  to the forward-looking ones; the subscript 4 and 5 respectively refer to the nominal wage and to output. As the  $rgm$  increases, the  $b$ 's decrease and the  $d$ 's increase; the higher the  $rgm$ , the higher the weights on the forward-looking variables and the lower the persistence. In other words, a high rate of underlying inflation causes Taylor's wage rule to collapse into a pure forward-looking equation where only the future variables are taken into account. Consequently, the inertia due to the backward-looking behaviour in Taylor's rule vanishes, and so does persistence.

Figure 3



This striking finding raises a number of interesting points.

(i) It demonstrates that, once microfoundations are taken into account, the importance of non-linearity becomes evident and can no longer be neglected. A linear approximation is just a bad approximation and we need to study the dynamics of the full non-linear model.<sup>23</sup> The response of the system to a money shock heavily depends on the starting point, that is on the underlying rate of inflation, in a non-linear manner. The *ad hoc* log-linear models are therefore quite misleading.

(ii) The previous point is very much related to the famous Lucas critique. As Sargent puts it: *“Robert E. Lucas (1976) criticised a range of econometric policy evaluation procedures because they used models that assumed private agents’ decision rules to be invariant with respect to the laws of motion that they faced. Those models took as structural [...] private agents’ decision [...] violated the principle that an optimal decision rule  $h(x)$  is a function of the law of motion  $g(x_t, u_t, \varepsilon_t)$ .”* (Sargent, 1987, pp. 40-41) Once the model is microfounded and the fully optimising decision process of the agents is explicitly taken into account, the policy behaviour enters the *ad hoc* structural parameters. And this occurs in a clear and intuitive way in this model. A high underlying rate of inflation would demolish the inertia in the system, reducing Taylor’s wage setting equation to a purely forward looking one. Quite a large number of scholars suggested that fixed staggered contracts could represent a good approximation of reality only in economies displaying fairly stable prices. On the basis of Lucas critique, they observed that contracts would not survive in an environment without stable prices simply because agents would take that into account and adjust their behaviour. This is exactly what the present analysis suggests. Parameters are policy-dependent such that

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<sup>23</sup> Since in this paper we are dealing with the comparison with the Taylor’s (1979) log-linear model and given the complexity of investigating the dynamics of the full non-linear, this latter is left to further research.

a high rate of underlying inflation would actually make the contracts irrelevant, dramatically changing their structure. This point demonstrates once more the pervasive importance of Lucas critique.

- (iii) Following another Lucas critique argument, one would expect that a higher level of average inflation would shorten the length of the contracts. Since the length is fixed by hypothesis, this is not possible within this model. However intuition strongly suggests that this would happen if it was not precluded. This is consistent with the intuition of Ball *et al.* (1988). Ball *et al.* (1988) suggested that high inflation lubricates the frictions in price adjustment. The higher the inflation rate, the more often firms adjust their prices to keep up with the price level, the faster the adjustment in the aggregate price level and the smaller the real effects following an aggregate demand disturbance. This would imply a negative relation between the real effects of aggregate demand disturbances and the average level of inflation. Ball *et al.* (1988) tested this implication and concluded: “*A robust finding is that this trade-off is affected by the average rate of inflation. In countries with low inflation, the short-run Phillips curve is relatively flat - fluctuations in nominal aggregate demand have large effects on output. In countries with high inflation, the Phillips curve is steep*”. (Ball *et al.*, 1988, p. 59) The model presented in this paper has the same empirical implication and, as previously explained, the intuition is somewhat similar. Recall that Taylor’s model implies that the slope of the Phillips curve is inversely related to  $\lambda_S$ . Then, the higher the rate of average inflation, the more forward looking Taylor’s wage setting rule becomes, hence the lower  $\lambda_S$  and the steeper the Phillips curve. “*Traditional Keynesian models, such as textbook models of price adjustment or the staggered contracts models of Fischer and Taylor, do not share the key predictions of our model. These older theories treat the degree of nominal rigidity (for example, the length of labor contracts or the adjustment speed of the price level) as fixed*

*parameters; thus they rule out the channel through which average inflation affects the output-inflation trade-off.*" (Ball *et al.*, 1988, p. 29) Instead, we were able to show that, once microfoundations are explicitly considered, the adjustment speed of labour contracts do

— depend upon average inflation. We are however aware that a more satisfactory model should allow for changes in the length of the contracts.

(iv) West (1988) simulates Taylor's model under a different monetary policy rule for given values of the structural parameters of the wage equation. Given the above results, one may question the theoretical validity of analysis such as those carried out in West (1988). The structural parameters are not policy invariant. It would be very interesting to examine how these parameters and persistence vary as different feedback monetary policy rules are implemented.

(v) By the same token, the model provides an explanation for Phaneuf's (1990) empirical finding that, across country, higher values of  $\gamma$  are associated with a higher degree of monetary accommodation. It is, in fact, probably reasonable to think that countries displaying a relatively higher underlying inflation are also those more likely to be prone to accommodate monetary shocks.

(vi) Moreover, this suggests a further interpretation. There exists a widely confirmed positive relationship between average inflation and the variance of nominal output. It seems that countries exhibiting low inflation rates are strictly controlling their monetary policy and will seldom be affected by substantial monetary shocks. Therefore, staggered contracts could potentially induce a fairly high degree of persistence only when a monetary shock is unlikely to occur. In other words, staggered contracts could generate high persistence of monetary shocks only when the staggered structure does not matter, since monetary policy is tightly controlled.

Before concluding a final remark follows. From proposition 4, we know that, when  $rgm = 0$ ,  $\gamma = 2(g + (1-\sigma)/\sigma)$ . The hypothesis  $\sigma = 1$ , implicit in Taylor's analysis, actually induces, *ceteris paribus*, the maximum degree of persistence. Is this hypothesis the best one? Probably not. In fact, the analysis is concerned with short-run adjustment. Labour is the only input in the production function. It is therefore sensible to interpret the production function as a reduced form of a short-run Cobb-Douglas one, in which capital is simply fixed and embodied in the constant term. Following this interpretation, then calibrated studies would suggest the labour's share of national income, i.e., 0.7, to be a good approximation for  $\sigma$ . But as the formula at the beginning of the paragraph shows,  $\gamma$  and hence  $\lambda_S$  tends to be particularly sensitive to  $\sigma$ . In fact, with constant steady state money supply (i.e., Table I), if  $\sigma = 0.7$ , in the "base case"  $\gamma$  jumps to 1.37 and persistence falls to 0.19. In the case  $a = 0$  and  $e = 1$ ,  $\gamma$  rises to 0.86 and persistence drops to 0.3. Moreover, the lower  $\sigma$ , the lower the sensitivity of the structural parameters of Taylor's equation to the underlying rate of inflation (see Figure 5a and 5b). If someone was puzzled about the excessive sensitivity of these parameters to average inflation, then he will be inclined to think  $\sigma = 0.7$  to be the relevant case.

Figure 5a

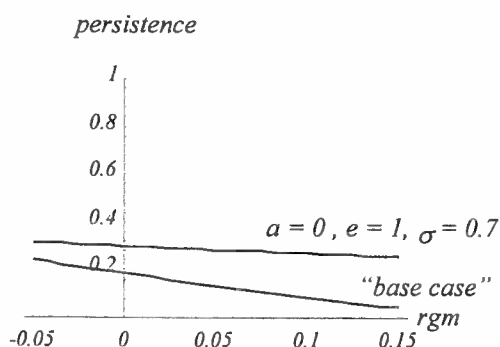
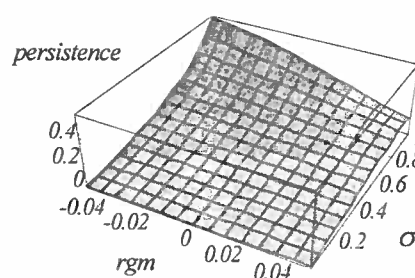


Figure 5b



To conclude, whatever sensible values one assigns to the parameters, either a moderate rate

of underlying inflation as observed in western economies, or a value of  $\sigma$  equals to the share of labour of output, or both, cut down persistence not only far below a near random-walk behaviour, but also below any level notably different from zero.

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## 6. Conclusions

In this paper we build a dynamic general equilibrium model with optimising agents and staggered wages a` la Taylor (1979). If, as West (1988) and Phaneuf (1990), we had been looking for results to corroborate the view that staggered wage models could induce a high degree of persistence of money shocks, the microfounded model does not seem to provide them. On the contrary, it confutes that view. The model demonstrates that for a large range of reasonable parameter values a notable degree of persistence is an unlikely outcome, and when it is, it probably does not matter. Moreover, investigating the microeconomic fundamentals of the *ad hoc* Taylor wage rule, the model emphasises the role of non-linearity and of the Lucas critique. In brief, the model shows that staggered wages are not able to explain a notable degree of persistence of the real effects of money shocks.



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## Appendix I

Substitute in (18)  $K_t$ , as given by (11) and impose the equilibrium condition  $Y_t = C_t$ . Then,

log-linearising (18) around a steady state we obtain:

$$x_t = a_1 y_t + a_2 y_{t+1} + a_3 l_t + a_4 l_{t+1} + a_5 m_t + a_6 m_{t+1} + a_7 p_t + a_8 p_{t+1} \quad (A1)$$

where now the variables are expressed as log-deviations from steady state and where:

$$a_1 = \left( \frac{\eta_{LC} + \varepsilon/\theta}{1 + \beta \frac{u_L(t+1)}{u_L(t)} \mu^{-\varepsilon}} \right) - \left( \frac{\eta_{CC} + \varepsilon/\theta}{1 + \beta \mu^{1-\varepsilon}} \right) \quad (A2)$$

$$a_2 = \beta \left\{ \left( \frac{\eta_{LC} + \varepsilon/\theta}{\frac{u_L(t)}{u_L(t+1)} \mu^\varepsilon + \beta} \right) - \left( \frac{\eta_{CC} + \varepsilon/\theta}{\mu^{\varepsilon-1} + \beta} \right) \right\} \quad (A3)$$

$$a_3 = \left( \frac{\eta_{LL}}{1 + \beta \frac{u_L(t+1)}{u_L(t)} \mu^{-\varepsilon}} \right) - \left( \frac{\eta_{CL}}{1 + \beta \mu^{1-\varepsilon}} \right) \quad (A4)$$

$$a_4 = \beta \left\{ \left( \frac{\eta_{LL}}{\frac{u_L(t)}{u_L(t+1)} \mu^\varepsilon + \beta} \right) - \left( \frac{\eta_{CL}}{\mu^{\varepsilon-1} + \beta} \right) \right\} \quad (A5)$$

$$a_5 = \left( \frac{\eta_{L,M/P}}{1 + \beta \frac{u_L(t+1)}{u_L(t)} \mu^{-\varepsilon}} \right) - \left( \frac{\eta_{C,M/P}}{1 + \beta \mu^{1-\varepsilon}} \right) \quad (A6)$$

$$a_6 = \beta \left\{ \left( \frac{\eta_{L,M/P}}{\frac{u_L(t)}{u_L(t+1)} \mu^\varepsilon + \beta} \right) - \left( \frac{\eta_{C,M/P}}{\mu^{\varepsilon-1} + \beta} \right) \right\} \quad (A7)$$

$$a_7 = \left( \frac{\varepsilon}{1 + \beta \frac{u_L(t+1)}{u_L(t)} \mu^{-\varepsilon}} \right) - \left( \frac{\varepsilon - 1}{1 + \beta \mu^{1-\varepsilon}} \right) \quad (\text{A8})$$

$$a_8 = \beta \left\{ \left( \frac{\varepsilon}{\frac{u_L(t)}{u_L(t+1)} \mu^\varepsilon + \beta} \right) - \left( \frac{\varepsilon - 1}{\mu^{\varepsilon-1} + \beta} \right) \right\} \quad (\text{A9})$$

Note that  $(a_1 + a_2)$ ,  $(a_3 + a_4)$ ,  $(a_5 + a_6)$ ,  $(a_7 + a_8)$  do not depend on  $\mu$ .

Then from (11) we know that the amount of labour in each of the two periods of the contract is given by:  $l_{it} = K_t X_{it}^{-\varepsilon}$  and  $l_{it+1} = K_{t+1} X_{it+1}^{-\varepsilon}$ . Substitute for  $K$  and express the variables as log-deviations, to obtain:

$$l_t = (\varepsilon/\theta) y_t + \varepsilon (p_t - x_t) \quad \text{and} \quad l_{t+1} = (\varepsilon/\theta) y_{t+1} + \varepsilon (p_{t+1} - x_t) \quad (\text{A10})$$

Use these two expressions to substitute out labour in (A1) and get:

$$x_t = b_1 p_t + d_1 p_{t+1} + b_2 y_t + d_2 y_{t+1} + b_3 m_t + d_3 m_{t+1} \quad (19 \text{ in the main text})$$

where:

$$b_1 = \frac{a_7 + a_3 \varepsilon}{1 + \varepsilon (a_3 + a_4)} \quad (\text{A11})$$

$$d_1 = \frac{a_8 + a_4 \varepsilon}{1 + \varepsilon (a_3 + a_4)} \quad (\text{A12})$$

$$b_2 = \frac{a_1 + a_3 \varepsilon/\theta}{1 + \varepsilon (a_3 + a_4)} \quad (\text{A13})$$

$$d_2 = \frac{a_2 + a_4 \varepsilon/\theta}{1 + \varepsilon (a_3 + a_4)} \quad (\text{A14})$$

$$b_3 = \frac{a_5}{1 + \varepsilon (a_3 + a_4)} \quad (\text{A15})$$

two sectors is the same and constant over time. Then  $\frac{u_L(t+1)}{u_L(t)} = 1$ . Given that, substitute  $\mu =$

$\frac{u_L(t+1)}{u_L(t)} = 1$  in the above expressions and the result follows.

## Appendix II: A Useful Comparison

Here we briefly compare the model of this paper and the one in Blanchard and Fischer (1989), (B/F in what follows), Ch. 8, to provide a better understanding of the results in section 4 and more precisely in proposition 5. The B/F model derives from a simplified version of Blanchard and Kiyotaki's (1987) static model. In particular, the B/F analysis removes the labour market assuming that each household is at the same time producer and consumer. On this simplified version they superimpose dynamics due to Taylor's staggered structure of price decisions. There are two basic differences between the setting of their model and of the one presented here: (i) staggering in wages versus staggering in prices; (ii) while in their model the dynamics are superimposed on a static model in a somewhat *ad hoc* way, the presented model is truly intrinsically dynamic, since it is derived from an explicit intertemporal optimisation process. Nonetheless, as we will see, both these differences turn out not to be very important with respect to the analysis of the persistence of money shocks. The crucial difference comes from the utility function specification. B/F's utility function of agent  $i$  is the following:  $U_i = (C_i/f)^f [(M_i/P)/(1-f)]^{1-f} - (1/e)Y_i^e$ , where  $e \geq 1$ . This specification implies no income effect on labour supply.<sup>1</sup> Then, the degree of persistence of money shocks is given by:

---

<sup>1</sup> The utility function is borrowed from Blanchard and Kiyotaki (1987). They wrote: "The assumption that utility is homogenous of degree one in consumption and real money balances, as well as additively separable in consumption and real money balances on the one hand, and leisure, on the other, eliminates income effects on labor supply. Under these assumptions, competitive labor supply would just be a function of the real wage..." (Blanchard and Kiyotaki, 1987, p. 650, footnote 7)

$$\lambda_s = \frac{e + \theta (e - 1) - 2 \sqrt{[(e - 1) (1 + \theta (e - 1))]} }{1 + (e - 1)(\theta - 1)} \quad (\text{A17})$$

where  $\theta$  is again the elasticity of substitution among goods. The degree of persistence is a decreasing function of  $(e-1)$ , that is, an increasing function of the intertemporal substitution of effort in production (which corresponds to the labour supply in a more general model).<sup>2</sup> If  $e = 1$ , then  $\lambda_s = 1$  and money shocks have permanent effects on production. It is easy to see why.

The price setting rule in B/F is:

$$x_t = \frac{1}{2} \{h x_{t-1} + (1-h) E[m_t|t]\} + \frac{1}{2} \{h E[x_{t+1}|t] + (1-h) E[m_{t+1}|t]\} \quad (\text{A18})$$

where  $h = \frac{1 + (\theta - 1)(e - 1)}{1 + (\theta + 1)(e - 1)}$ . Using the log-linear static aggregate demand equation  $y_t = m_t -$

$p_t$  to substitute out  $m_t$  and  $m_{t+1}$  in (A18) we can get Taylor's price setting rule:

$$x_t = \frac{1}{2} \{x_{t-1} + 2 \left( \frac{1-h}{1+h} \right) E[y_t|t]\} + \frac{1}{2} \{E[x_{t+1}|t] + 2 \left( \frac{1-h}{1+h} \right) E[y_{t+1}|t]\} \quad (\text{A19})$$

where  $\gamma_{B/F/2} = \left( \frac{1-h}{1+h} \right) = \frac{e-1}{1+\theta(e-1)}$  has the same role as  $\gamma$  in Taylor's model.<sup>3</sup> Given that

there are no income effects on labour supply ( $-\eta_{CC} = 0$ ), then we are in the bottom part of

<sup>2</sup> If instead of the farmer hypothesis, households supply labour in a competitive market, then  $(1/(e-1))$  would be the intertemporal elasticity of labour supply. Moreover, as anticipated in footnote 14 in the main text, note that the effect of  $\theta$  on  $\lambda_s$  is the same as in the model presented here.

<sup>3</sup> In fact the stable root is:  $\lambda_s = \frac{1 - \sqrt{1-h^2}}{h} = \frac{1}{h} - \sqrt{\frac{1}{h^2} - 1}$  which delivers equation (A17) (Blanchard and Fischer, p. 395). Then  $1/h$  corresponds to  $\alpha$  in (4), since B/F suppose  $b = d = \frac{1}{2}$  (that is,  $\beta = 1$  in the model of the paper). Given the definition of  $\alpha = \frac{1+\gamma/2}{1-\gamma/2}$ , then putting  $\alpha = 1/h$  we get exactly  $\gamma = 2 \left( \frac{1-h}{1+h} \right)$ .

Alternatively, putting  $\alpha = 1/h$ , the stable root in (4) can be written as:  $\lambda_s = \frac{1 - \sqrt{\frac{1-h}{1+h}}}{1 + \sqrt{\frac{1-h}{1+h}}} = \frac{1 - \sqrt{\gamma_{B/F}/2}}{1 + \sqrt{\gamma_{B/F}/2}}$ .

There is therefore perfect correspondence between Taylor's model and B/F's one.

Figure 1. In fact, the bigger is the intertemporal elasticity of substitution of labour (i.e.,  $(1/\eta_{LL}) = 1/(e-1)$ ), the lower is  $\gamma_{B/F}/2$ , then the bigger is the stable root and the persistence.

Let us put  $a = 1$  in (27), so that the utility function becomes:

$$U = \delta \ln C_t + (1 - \delta) \ln (M/P)_t - d L_t^e \quad (\text{A20})$$

Moreover, in order to make the model more similar to the one in Blanchard and Fischer

(1989), let us suppose  $\sigma = \beta = 1$ . In this case,  $\lambda_s = \frac{1 - \sqrt{g}}{1 + \sqrt{g}}$  where  $g = \gamma = \frac{e}{1 + \theta(e-1)}$ .

The simple comparison between  $g$  and  $\gamma_{B/F}$  can be summarised in the following proposition.

**Proposition 6.** *Comparing the model of this paper with the one in B/F we can conclude:*

(i) *while in the model illustrated in the paper the degree of inertia of nominal variables is a decreasing function of the intertemporal elasticity of substitution of labour, the contrary is true in B/F's model;*

(ii) *the elasticity of nominal wages to fluctuations in output, i.e.,  $\gamma$ , is however always lower in B/F and hence the degree of inertia of nominal variables is always bigger in B/F's model, with respect to the one here presented.*

Proposition 6 just basically restates proposition 5 applying it to the two proposed examples. Let's develop the intuition. Recall that the optimal wage setting rule of the model, given by (18), is strictly linked to the optimal rule in the flexible wage case:

$$\frac{X_t}{P_t} = \left[ -\frac{\varepsilon}{\varepsilon - 1} \right] \left[ \frac{u_L(t)}{u_C(t)} \right] = (\text{constant}) Y_t^g \quad (\text{A21})$$

where  $g$  is exactly the elasticity of the optimal wage with respect to output. In fact, as we saw above, the simplifying assumptions  $\mu = \beta = 1$ , basically remove any asymmetry among the weights of the log-linearised version of (18) and not surprisingly  $g$  appears as the elasticity of

the nominal wage in (19) with respect to both  $y_t$  and  $y_{t+1}$ . (A21) can be rewritten as:

$$X_t = (\text{constant}) \left[ \frac{P_t^{1+\theta(e-1)} Y_t^{e-1} X_t^{-\theta(e-1)}}{I/Y_t} \right] \quad (\text{A22})$$

An increase in  $Y_t$  has two effects: (i)  $K_t$  increases shifting the demand for labour curve upwards; (ii) the marginal utility of wealth (that, given (A20) is equal to that of consumption) decreases since an increase in consumption is expected. Both these effects go in the same direction and the money wage has to rise. Then, there is a third effect: the union realises that an increase in the money wage causes the demand for labour, and hence the marginal disutility of labour, to decrease proportionally to the parameter  $\theta$  (which equals  $\varepsilon$  if  $\sigma = 1$ ). How much should the money wage increase? Taking into account the three effects then the elasticity of  $X_t$  with respect to  $Y_t$  is exactly  $g$ . In B/F's case the second effect is absent, since the marginal utility of wealth is constant. Therefore the elasticity of  $X_t$  with respect to  $Y_t$  is exactly  $\gamma_{B/F} / 2$ . The absence of the second effect makes  $\gamma_{B/F} / 2$  lower than  $g$ .

Alternatively, we can reason in the following way. The optimal price choice in B/F can be thought as the optimal decision of a monopolist which maximises profits given the demand curve. The profits are given by the following indirect utility function:

$$U_i = (P_i/P) Y_i - (d/e) Y_i^e - (M_i/P) \quad (\text{A23})$$

where the last term is given for the agent. The demand function is:

$$Y_i = (P_i/P)^{-\theta} (M/P) \quad \text{where} \quad (M/P) = (\text{const.}) Y \quad (\text{A24})$$

The optimal rule is then simply found by equating the marginal cost and the marginal revenue of the monopolist, that is:<sup>4</sup>

$$MR_{B/F} = (\text{const.}) P_i^{-\theta} P^{\theta-1} Y = MC_{B/F} = (\text{const.}) P_i^{-1-\theta e} P^{\theta e} Y^e \quad (\text{A25})$$

<sup>4</sup> Note that MR and MC are negative since they do not correspond to the usual textbook definitions. In fact, MR and MC are the partial derivatives of revenues and costs respectively, with respect to price rather than quantity.

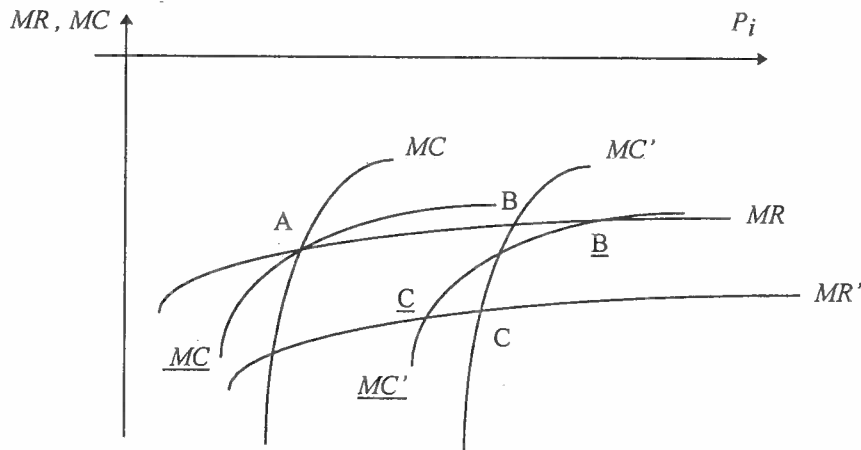


In the model of this paper the comparable condition for the monopoly union is:

$$MR = (const.) X_t^{-\theta} P_t^{\theta-1} = MC = (const.) X_t^{-1-\theta e} P_t^{\theta e} Y_t^e \quad (A26)$$

Comparing (A25) with (A26) again the above argument is reproduced. While the marginal cost is exactly the same, the marginal revenue is different. In particular,  $MR < MR_{B/F}$  (since the constant term is negative). One unit of income spent in consumption and real balances produce a constant level of utility in B/F. In the model of this paper, instead, the utility produced by one unit of income decreases with the level of consumption (or income, since in equilibrium  $C = Y$ ). Hence,  $MR = MR_{B/F}/Y$ , where  $1/Y$  is just the marginal utility of wealth.

Figure A1



What happens when output rises? Look at Figure A1. Both  $MC$  and  $MR$  are increasing function of  $P_i$  and  $MC$  is steeper than  $MR$ . Moreover, the higher  $e$ , the steeper  $MC$ . In Figure A1 we have drawn two marginal cost curves:  $MC$  corresponds to a high value of  $e$ , while  $\underline{MC}$  to a low one. Firstly note the difference between the two models. Suppose we are at point A. If  $Y$  increases then in B/F both  $MC$  and  $MR$  shifts to the right, while in the model presented here only  $MC$  shifts. Therefore, for a given increase in  $Y$ ,  $P_i$  is bigger in the model of this paper than in B/F's one (compare B with C, and/or  $\underline{B}$  with  $\underline{C}$ ). That is, the elasticity of  $P_i$  with

respect to  $Y$  is always lower in B/F's model. Moreover, the flatter  $MC$ , the bigger the difference between the level of the new price/wage in the two models (compare the difference between  $B$  and  $C$ , with the one between  $\underline{B}$  and  $\underline{C}$ ). In other words, the lower is  $e$ , the bigger is — the difference between the elasticity of  $P_i$  with respect to  $Y$  in the two models, exactly as in Figure 1. Secondly, consider the effect of  $e$  on this elasticity in the two models. In B/F, the lower is  $e$ , the flatter the  $MC$  curve, the lower is the elasticity of  $P_i$  with respect to  $Y$  (compare  $C$  with  $\underline{C}$ ). In the model illustrated in this paper the opposite is true (compare  $B$  with  $\underline{B}$ ).

To conclude, in this Appendix we presented a comparison between the model of the paper and the one in Blanchard and Fischer (1989), Ch. 8, in order to illustrate the relation between income effects on labour supply, intertemporal substitution of labour and persistence of money shocks. This enabled us to explain in a very plain and intuitive way the apparent puzzle outlined in Figure 1 of section 4.<sup>5</sup>

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<sup>5</sup> Moreover note that the results of this section about the elasticity of price/wage to output are generally valid, in the sense that they do not depend on the assumption of staggered wages/prices. Figure A1 has been in fact drawn looking at the optimal flexible price/wage rule.