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1. Introduction

It has realized since Pigou (1947) that if public goods are financed by distortionary taxation, the marginal social cost of providing the public good will exceed the actual resource cost by the marginal deadweight cost of taxation. Atkinson and Stern (1974) formally derived public goods decision rules for an economy financing the public goods using linear taxes. Subsequent work by Browning (1978), Wildasin (1984) and Usher (1986) has expanded our understanding of the way in which distortionary taxes influence the optimal size of the public sector. As the survey by Auerbach (1987) reveals, the direction of subsequent research on the marginal cost of public funds has been manifold, and has admitted, among other things, more complicated production structures, the effects of preexisting taxes, risk, and imperfect information. Recent has analyzed the marginal cost and marginal benefit of public funds when public goods are substitutes/complements to private goods (Mikami (1993)), when equity is an objective of taxation (Wilson (1991), when non-linear taxes are used to finance the public goods (Tuomala (1990), Boadway and Keen (1993)), and during a process of tax reform (Schob (1994))

An implicit assumption in this literature has been that the government institution charged with responsibility for taxation to finance the public good or service is the same as that which actually provides it. This assumption is at variance with the facts in a large number of cases. Typically, institutions that provide public goods and services to households are lower-level agencies whose financing comes from the revenues collected at a higher level. Welfare services are provided by local agencies, the care of children by day-care facilities, education by schools, health services by hospitals and doctors, and so on. The funds for the provision of such services come from the general revenues collected on behalf of a higher level of government which then disburses revenues to the many agencies under its control.

Given that the services are typically provided free of charge or with only nominal user fees, the usual disciplines of the market do not operate to ensure cost-effectiveness of the services provided. Moreover, in the absence of markets for such services, the costs of providing them are not readily observed. The higher level of government may not know the financing required either because it does not know the cost conditions of the agency providing the service, or because it cannot monitor the behavior of the agency to ensure that the efficient amount of effort is being expended in providing the service. The absence of full information on behalf of the government introduces the possibility of an additional source of excess burden in financing the provision of public services by the agency, over and above the standard excess burden arising from distortionary taxation. The government must incur a cost to elicit the required information from the agency in order to determine how much funds need to be provided. This extra cost of obtaining information is well-known from the mechanism design literature (e.g., Laffont and Tirole (1993)). In this paper we investigate the way in which the decision rules for the provision of public goods and services must be amended to account for the fact that the excess burden of financing them includes both the distortions due to taxation and the costs of inducing truthful revelation from the agencies supplying the public goods and services.

The model we use to analyze this issue is a straightforward extension of the standard principal-agent model that is typically used to analyze the regulation of private firms.[1] From the perspective of this paper, the literature on economic regulation has, among other things, been concerned with two crucial issues. First, it has tried to generalize the Boiteux(1956) rules for optimal pricing by regulated firms in a number of directions. Second, it has considered generalizations of the Baron-Myerson (1982) formulation where the regulator has to set optimal pricing rules for a monopolist whose cost structure is private information and is not observed by the regulator. The generalizations mainly involve reformulating the relation between price and marginal cost for the regulated firm. This literature has not so far concerned itself with the problem of production of public goods and consequent implications for the optimal rate of tax and the marginal cost of public funds, though in his careful survey of the regulation literature, Laffont (1994) has suggested this as a further direction for research.

The essential difference between our model and the standard regulation one is that the agency providing public goods and services is not a firm that sells its product on the relevant market. Our agencies provide public services free of charge to households and receive their financing from a central government. The agency receives a net surplus from providing the service, where the net surplus is the difference between the grant it receives from the government and the cost it incurs in providing the public service. It behaves so as to make the net surplus it receives as high as possible. The central government finances several agencies under its jurisdiction, each of which serves a given group of households. It taxes the households, using a simple linear distorting tax, and uses the proceeds to provide grants to the agencies. The grants each agency receives are contingent on the level of public services it provides. The government is interested in maximizing an objective function that depends upon the utility received by the households served by the various agencies in the economy. In our basic model, the government cannot observe the cost structure of each agency, although it has aggregate information on the distribution of costs across all agencies. It must choose its structure of grants to induce agencies to reveal their cost-types. In later sections, we also allow agency effort to vary. The grant structure then must not only induce truthful revelation; it must also induce optimal effort.

Other, more complex, ways of modeling the problem could be imagined. For example, one might permit horizontal communication between agencies, the central government

[1] There is a large literature on this. See, among others, Baron and Myerson (1982), Besanko and Sappington (1987), Laffont and Tirole (1993), and Laffont (1994).

being able to infer information about an agency's costs from the signals sent out by other agencies (referred to as yardstick competition and discussed by Laffont (1994)). Agencies might be allowed to have an interest in the in the welfare of the households they serve. Think of welfare case-workers deriving welfare recipients or teachers instructing students as examples. Households could also be allowed to choose among agencies so as to maximize their utilities, as in the literature on club goods or local public goods.[2] Such 'voting with their feet' could be an additional mechanism for disciplining local agencies, as recognized by Tiebout (1956). These extensions would involve more complicated modeling.

Recently,a series of papers have appeared in an area closely related to ours, grants from a central government to several regional governments under conditions of asymmetric information. The information involves incomes or preferences for local public goods of residents of the different regions or local costs of providing public goods. Some examples include the following. Cremer, Marchand, and Pestieau (1996) investigate the optimal structure of federal-regional grants and the rules for public goods provision for a social welfare maximizing central government when it is unable to observe either regional incomes or preferences for a local public good. Raff and Wilson (1995) investigate optimal redistributive tax policy of the Stiglitz (1982) sort when local governments are better informed about the abilities of their residents than is the federal government and when residents are mobile between regions. Cornes and Silva (1996) analyze federal-local grant policy in a federation when residents of different jurisdictions obtain differing levels of real income because of differences in costs of providing local public goods. The federal government is interested in redistributing to the jurisdiction with higher cost of production, which has the less well-off residents. But it cannot observe perfectly observe the costs of production, nor can it observe the effort level of the local jurisdiction. Lockwood (1996) analyzes the structure of grants between a federal government and a regional government when the regions are subject to shocks to their income, the cost of provision of public goods, and preferences.. The shocks are private information to the region, and the role of the grants is to ensure the regions, which are risk-averse, against adverse shocks. The model is unique in that decision making by both the federal and regional governments is determined by majority voting rather than by a benevolent government. All of these papers use a version of the principal-agent approach to study the structure of grants between a federal government and a local government when the local government is better informed about some relevant features of the local economy, be it incomes, cost conditions, or preferences. Whereas the focus in this literature is on the relationship between a federal government and local or regional governments, each of which has independent fiscal authority, our analysis considers the relationship between a government and the various agencies under its control whose only responsibility is to supply public goods or services to the households of the economy.

The paper proceeds as follows. In section 2, the features of the basic model used are outlined. Sections 3 and 4 use the basic model to derive the optimal grant structure and decision rules for the provision of public services by the agencies under two sets of

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[2] Useful surveys of club goods may be found in Oakland (1987) and of local public goods in Bewley (1981) and Wildasin (1986). They do not consider asymmetric information.

informational assumptions. In section 3, full information by the government is assumed. In contrast, section 4 considers how these rules change when the government cannot observe the cost conditions of each agency. In sections 5 and 6, the analysis is extended to allow for a choice of unobservable effort by the agencies. In section 5, effort is assumed to affect the probability of a given agency being high- or low-cost, while in section 6, agency effort affects its marginal cost. Section 7 concludes.

2. The Basic Model

In this section, we outline the model used to analyze the problem faced by a central government that does not know the cost characteristics of an agency supplying a public service. We proceed by describing household behavior, then the choices available to the local agency, and finally the central government's problem.

Household Behavior

Our analysis will involve the provision of a public service by agencies, each serving a given number of identical households. It will suffice to conduct the analysis in terms of the representative household served by a given agency. Consider a household facing a given set of policies implemented by the central government and by the agency responsible for providing the public service to this household.[3] As we shall see, the policies actually implemented by a given agency depend upon the cost function of the agency, and that is private information to the agency. Thus, *ex ante*, there will be some uncertainty about the policy parameters and the utility achieved by the representative household. We defer reference to the alternative states of the world until presenting the local agency's behavior.

The representative consumer has the following quasi-linear utility function:

$$U(L, X, G) = u(L) + X + b(G) \tag{1}$$

where L is labor, X is a private consumption good, G is the public service supplied by the agency, and both $u(L)$ and $b(G)$ are increasing and strictly concave. This is obviously a special form of utility function which has both additive separability and a constant marginal utility of consumption (and hence risk neutrality in the private good). It simplifies our presentation and brings out the intuition of the analysis most clearly. The consumer takes G as given and maximizes the utility function $U(\cdot)$ with respect to L and X subject to the budget constraint $X = (w - t)L$, where w is the wage rate and t the tax rate on labor supplied, assumed to be a per unit tax on labor for simplicity.

[3] We proceed as if the public service provided were purely private in nature. However, it is clear that it could easily be interpreted as being purely public, or more generally impurely public (congested) without affecting the basic results. In referring to a publicly provided private commodity as a public service, we are following the convention adopted by Bewley (1981).

The problem of the consumer can be written:

$$\max_{L} \quad u(L) + (w - t)L + b(G) \tag{2}$$

The solution to this problem yields the labor supply function:

$$L = L(w - t) \tag{3}$$

Because of the additive separability of the utility function, labor supply L does not depend upon the amount of the public service available, G. It is straightforward to show by differentiation of the first-order conditions for the consumer's problem that:

$$L'(w - t) = -\frac{1}{u''(L)} > 0 \tag{4}$$

Substituting (3) into (1), we obtain the household's indirect utility function:

$$v(w - t, G) \equiv u(L(w - t)) + (w - t)L(w - t) + b(G) \tag{5}$$

Application of the Envelope Theorem yields:

$$\frac{\partial v(\cdot)}{\partial t} = -L(w - t) \; ; \qquad \frac{\partial v(\cdot)}{\partial G} = b'(G) \tag{6}$$

The first of these is simply Roy's Theorem in this context. (Since utility is linear in the private good, the marginal utility of income is unity.) The second gives the marginal benefit of the public service to the household in terms of private consumption.

It is useful for analytical purposes to rewrite the household's indirect utility function in terms of total tax paid rather than the tax rate. Tax payments by the household are denoted R. They are determined by:

$$R = tL(w - t) \equiv R(t)$$

Inverting this, we obtain $t(R) = R^{-1}(t)$ where, by differentiation, $t(R)$ satisfies:

$$t'(R) = \frac{1}{L(w - t) - tL'(w - t)} \tag{7}$$

We assume that $L(w - t) - tL'(w - t) > 0$, which is equivalent to assuming that tax payments are increasing in the tax rate so that we are on the left-hand side of the Laffer curve. Therefore, $t'(R) > 0$: the government has to increase the tax rate to obtain more funds to transfer to the agency. Household indirect utility in terms of tax payments R and public services G is defined as:

$$V(R, G) \equiv v(w - t(R), G) \tag{8}$$

where

$$V_R = \frac{\partial v(\cdot)}{\partial t} t'(R) = \frac{-L}{L - tL'} < 0 \tag{9}$$

and

$$V_G = \frac{\partial v(\cdot)}{\partial G} = b'(G) > 0 \tag{10}$$

For future purposes, it is useful to investigate the sign of V_{RR}. Differentiating (9) with respect to R yields, after some manipulation:

$$V_{RR} = \frac{\partial V_R}{\partial t} t'(R) = \frac{-LL' - tL'L' + tL''}{(L - tL')^2} t'(R)$$

Given that $L' > 0$ and $t' > 0$, this is ambiguous in sign. But, if $L'' \leq 0$ or if tL'' is small enough in absolute value, then $V_{RR} < 0$. It will certainly be true for low values of t and thus R. In subsequent analysis, we shall assume that $V_{RR} < 0$ for illustrative purposes.

Writing the household indirect utility function in terms of R and G is useful since it allows us to treat these variables as government controls. The marginal rate of substitution of G for R is obtained by differentiating (8) holding utility constant to give:

$$\left.\frac{dR}{dG}\right|_V = -\frac{V_G}{V_R} = \frac{(L - tL')b'}{L} = (1 + \eta)b' > 0 \tag{11}$$

where η is the elasticity of labor supply with respect to the tax rate. Thus, indifference curves in (R, G) space have a positive slope. Furthermore, to avoid corner solutions, we assume that $b''(G)$ is sufficiently negative such that indifference curves show diminishing marginal rate of substitution of G for R, or, $d(dR/dG)/dG_{|V} < 0$. Note that if η is constant, a special case we shall sometimes use for expository purposes, indifference curves will be vertically parallel.

Behavior of a Local Agency

The public service is produced by agencies whose costs are known to themselves, but are unknown to the central government. Each agency serves a given set of households, determined perhaps by location. The amount of the public service supplied by a given agency is observable to all. The central government gives the agency a subsidy S related to the amount of the public service provided G, and the public service is then supplied by the agency free of cost to consumers. Effectively, the agency can choose the combination of subsidy and public service to provide from a schedule of possible combinations announced by the central government.

For simplicity, it is assumed that the agency can have two cost levels — high and low. We use superscripts h and ℓ to denote these two states, which are randomly determined. The agency of type i chooses the combination (S, G) from the menu offered by the central government that maximizes its profits defined as:

$$\Pi^i(S, G) \equiv S - C^i(G)$$

where $C^i(G)$ is the cost function of the agency, with $i = h, \ell$. The cost function of agency i is assumed to be increasing and convex, so $C^{i'}(G) > 0, C^{i''}(G) \geq 0$. Furthermore, at any given output level G, $C^h(G) > C^\ell(G)$, that is, the cost of producing any given amount of the public service is greater for the high-cost agency. Finally, the cost functions have the property that the high-cost agency has higher marginal costs than the low-cost agency at any level of output, $C^{h'}(G) > C^{\ell'}(G), \quad \forall G$. This is the well-known *single-crossing property.*[4] It implies that any pair of iso-profit curves for the high-cost and low-cost agencies drawn in (S, G)-space will intersect only once and those for the high-cost agency will be steeper, since the slope of an iso-profit curve is simply the marginal cost:

$$\left.\frac{dS}{dG}\right|_{\Pi} = C^{i'}(G) > 0$$

Central Government Behavior

The central government provides financing to each of the agencies in the economy, some of which are high-cost and others low-cost. It cannot observe the cost function of any given agency, but knows the probability of the agency being of type i, denoted ϕ^i. Naturally, the central government knows the agency profits associated with a given cost function and given transfer policies. Household welfare depends on the public service level provided by the agency where it resides, and that depends upon the cost function of the agency. In choosing its policies, the central government is assumed to be interested solely in the welfare of the two representative types of households in the jurisdictions served by the two types of agencies. It chooses its tax and transfer policies to maximize a weighted average of the two types of households, those served by a low-cost agency and those served by a high-cost agency.[5] This objective function can be given various interpretations. It can be interpreted as being the objective function for a government that seeks to achieve some unspecified Pareto optimum, where different values of the weights represent different points on the Pareto frontier. Using this interpretation yields rules for Pareto-efficient policies by the central government. Alternatively, the weighted-average objective function could be interpreted as representing a social-welfare maximizing central government where the weights are marginal social utilities of the two representative households. A special case of the latter that we shall highlight is that where the social welfare function is utilitarian. In this case, the relative weights placed on the two household types are the relative probabilities of the low-cost and high-cost outcomes, and the objective function is equivalent to maximizing the expected utility of the representative household. (Recall that all households are identical *ex ante.*)

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[4] In the case where only fixed costs vary across agencies, the single-crossing property does not hold. In this case, it can be readily shown that the optimum is characterized by a *pooling equilibrium* where the central government does not distinguish between high and low-cost types.

[5] For greater generality, we could assume that the planner also places a weight on the profits of the local agency. This would not affect the qualitative results of the analysis.

Transfers to the agencies must be financed by taxes on the households. The government knows which households are being serviced by which agencies so it can set different tax rates on households in different localities. In fact, it can set the local tax rate contingent on the observed behavior of the local agency. When an agency chooses a particular combination (S, G) from the menu offered by the central government, it is therefore implicitly also choosing a tax rate for the households it serves. An alternative would have been to assume that all transfers are financed out of general revenues to which all households contribute identical amounts. This will turn out to be the outcome in some particular cases.

Government policy consists of announcing a set of (R, S, G) combinations from which each agency is free to choose (S, G). The agency choice determines the tax payment R of the household it serves. The combinations (S, G) offered for the agencies are limited by the recognition that a generous offer to the high-cost agency designed to compensate it for its costs can induce the low-cost agency to mimic it, and vice versa. Let λ and $1 - \lambda$ be the weights put on the representative households served by the low-cost and high-cost agencies respectively. By the revelation principle, we can write the general problem of the central government in the following way:

$$\max_{\{R^i, S^i, G^i\}} \quad \lambda V\left(R^\ell, G^\ell\right) + (1-\lambda) V\left(R^h, G^h\right) \tag{12}$$

subject to

$$S^i - C^i(G^i) \geq 0 \qquad i = \ell, h \qquad (\theta^i)$$

$$S^i - C^i(G^i) - S^j + C^i(G^j) \geq 0 \qquad i, j = \ell, h \qquad (\theta^{ij})$$

$$\sum_{i=\ell,h} \phi^i \left(R^i - S^i\right) = 0 \qquad (\theta^S)$$

The first pair of constraints (θ^i) state that profits in either state must be non-negative. Local agencies have no resources of their own and cannot produce public services unless their costs are covered. The second pair of constraints (θ^{ij}) are the incentive or self-selection constraints: agency i can be no better off by mimicking an agency of type j. The last constraint is the budget constraint for the central government. The labels (θ^i), (θ^{ij}) (θ^S) refer to the Lagrangian multipliers that will be used in the next section for these constraints.

3. Optimal Policy with Full Information

Before analyzing this problem for the asymmetric information case, it is useful as a benchmark to characterize the full information optimal choices of R^i, S^i and G^i. With full information, the central government knows the cost conditions faced by the agency, and it can select the appropriate policy for those cost conditions.

In this case, the self-selection constraints are not binding. The Lagrangian expression for the fully informed government's problem is:

$$\mathcal{L}(R^i, S^i, G^i, \theta^i, \theta^S) = \lambda V(R^\ell, G^\ell) + (1-\lambda) V(R^h, G^h)$$

$$+\theta^\ell\left(S^\ell - C^\ell(G^\ell)\right) + \theta^h\left(S^h - C^h(G^h)\right)$$
$$+\theta^S\left(\phi^\ell(R^\ell - S^\ell) + \phi^h(R^h - S^h)\right)$$

The first-order conditions are:

$$\lambda V_R(\ell) + \theta^S\phi^\ell = 0 \tag{13}$$

$$(1-\lambda)V_R(h) + \theta^S\phi^h = 0 \tag{14}$$

$$\theta^\ell - \theta^S\phi^\ell = 0 \tag{15}$$

$$\theta^h - \theta^S\phi^h = 0 \tag{16}$$

$$\lambda V_G(\ell) - \theta^\ell C^{\ell\prime}(G^\ell) = 0 \tag{17}$$

$$(1-\lambda)V_G(h) - \theta^h C^{h\prime}(G^h) = 0 \tag{18}$$

where $V_R(\ell)$ refers to $V_R(R^\ell, G^\ell)$ and similarly for $V_R(h)$. Along with the three constraints, these equations determine the values of government policy variables (R^i, S^i, G^i) and the Lagrange multipliers (θ^i, θ^S).

Combining (13), (15) and (17), and (14), (16) and (18), we obtain:

$$-\frac{V_G(i)}{V_R(i)} = C^{i\prime}(G^i) \quad i = \ell, h$$

or using (11),

$$b'(G^i) = \frac{C^{i\prime}(G^i)}{(1+\eta^i)} \quad i = \ell, h \tag{19}$$

where $\eta^i = -t^i L^{i\prime}/L^i < 0$ is the elasticity of the labor supply with respect to the tax rate at the allocation achieved when the agency is of type i. Equation (19) says that the marginal benefit of supplying the public service, $b'(G^i)$, equals the marginal cost, $C^{i\prime}(G^i)$, multiplied by $(1+\eta^i)^{-1}$. The latter is a standard expression for the marginal cost of public funds (MCPF) when tax revenues are raised using a distortionary linear wage tax on the household served by agency i. Thus, the level of public services offered by an agency in the full-cost case is that at which the marginal benefit equals the marginal cost multiplied by the marginal cost of obtaining public funds from the household being served. The same applies for both agencies. Note that since the zero-profit constraints are binding for both agencies, neither will make any profit, as expected in this full information setting.

The relative tax applied on the two types of households can be obtained by combining (13) and (14) as follows:

$$\frac{V_R(\ell)}{V_R(h)} = \frac{(1-\lambda)}{\lambda}\frac{\phi^\ell}{\phi^h} \tag{20}$$

In the utilitarian case in which all households are given the same weight, $\lambda = \phi^\ell$ and $1-\lambda = \phi^h$ so $V_R(\ell) = V_R(h)$. Since, by (9), $V_R(\cdot)$ depends only upon R, this implies that $R^\ell = R^h = R$, so a common tax is applied on all households. The decision rule for public services then involves a common MCPF, denoted $(1+\eta)^{-1}$. This case, which corresponds with that in which expected utility is maximized will frequently be used as an illustrative example in what follows. If the weight on the household using the high-cost agency is higher than ϕ^h, for example because there is some aversion to inequality in the government's objective function, then $V_R(\ell)/V_R(h) > 1$. Then, under the assumption that $V_{RR} < 0$, $R^\ell > R^h$. The tax system serves to redistribute from the household served by the low-cost agency to the one served by the high-cost agency.

A solution to the utilitarian case is depicted in Figure 1. Since by (19), $C^{\ell'}(G^\ell)/b'(G^\ell) = C^{h'}(G^h)/b'(G^h)$, it must be the case that $G^\ell > G^h$, given the single-crossing property, the convexity of the cost functions and the concavity of the public service benefit function. However, the relative sizes of S^ℓ and S^h are ambiguous. That is, either agency could be a recipient of transfers larger than the tax paid by its representative user. This implies that the menu of (S, G) combinations facing the agencies could be upward or downward sloping. [6]

The allocations for the households are labelled E_R^h and E_R^ℓ for those served by the high-cost and low-cost agencies respectively. The point labelled E^h (above E_R^h) on the zero-profit curve for the high-cost agency $\pi^h = 0$ is that chosen by the high-cost agency. This point induces the optimal grant level S^h for the high-cost agency. In a parallel manner, we can find E^ℓ and S^ℓ for the low-cost agency on its zero-profit curve. In the case depicted, the low-cost agency is cross-subsidizing the high-cost one.

Note also that in the case shown in the figure, the allocation is not incentive-compatible. The low-cost agency would prefer the allocation E^h meant for the high-cost one since that would give it positive profits. Thus, if the central government could not observe the cost conditions of the two agencies, it could not implement this scheme. Depending on the conditions, the self-selection constraint may be binding in the other direction, or it may not be binding at all. We assume in the following section that the self-selection constraint is binding on the low-cost agency as in Figure 1.

4. Optimal Policy with Unobservable Costs

It is straightforward to prove that, given the single-crossing property, if the self-selection constraint on the low-cost agency is binding, the optimal policy will be a separating equilibrium. Furthermore, the profit constraint of the low-cost agency will not be binding $(\theta^\ell = 0)$ and the self-selection constraint on the high-cost agency will not be binding $(\theta^{h\ell} = 0)$. These are all demonstrated in a technical appendix. Based on problem (12),

[6] For values of λ lower than the utilitarian case, the tax paid by those served by high-cost agency is reduced and that for the other household increased so it is more likely that the low-cost agency cross-subsidizes the high-cost one. But it remains true that the relationship between the transfer to the agency and the amount of public good it produces could be positive or negative.

we can therefore write the central government's Lagrangian function as:

$$\mathcal{L}(R^i, S^i, G^i, \theta^h, \theta^{\ell h}, \theta^S) = \lambda V(R^\ell, G^\ell) + (1-\lambda)V(R^h, G^h)$$
$$+\theta^h(S^h - C^h(G^h)) + \theta^{\ell h}(S^\ell - C^\ell(G^\ell) - S^h + C^\ell(G^h))$$
$$+\theta^S(\phi^h(R^h - S^h) + \phi^\ell(R^\ell - S^\ell)) \tag{P}$$

The first order-conditions are:

$$\lambda V_R(\ell) + \theta^S\phi^\ell = 0 \tag{21}$$

$$(1-\lambda)V_R(h) + \theta^S\phi^h = 0 \tag{22}$$

$$\theta^{\ell h} - \theta^S\phi^\ell = 0 \tag{23}$$

$$\theta^h - \theta^{\ell h} - \theta^S\phi^h = 0 \tag{24}$$

$$\lambda V_G(\ell) - \theta^{\ell h}C^{\ell\prime}(G^\ell) = 0 \tag{25}$$

$$(1-\lambda)V_G(h) - \theta^h C^{h\prime}(G^h) + \theta^{\ell h}C^{\ell\prime}(G^h) = 0 \tag{26}$$

along with the three constraints satisfied with equality.

To interpret these conditions, note that combining (21), (23), and (25) yields:

$$-\frac{V_G(\ell)}{V_R(\ell)} = C^{\ell\prime}(G^\ell)$$

Using expression (11) for $V_R(\ell)$ and $V_G(\ell)$, this becomes:

$$b'(G^\ell) = \frac{C^{\ell\prime}(G^\ell)L^\ell}{L^\ell - t^\ell L^{\ell\prime}} = \frac{C^{\ell\prime}(G^\ell)}{1+\eta^\ell} \tag{27}$$

Thus, the presence of asymmetric information does not affect the form of the MCPF expression for the low-cost agency. It sets its output such that the marginal benefit to its representative user equals its marginal cost multiplied by the MCPF of obtaining revenues from its user.

For the high-cost agency, we obtain from (22), (24), and (26):

$$-\frac{V_G(h)}{V_R(h)} = \frac{\theta^h C^{h\prime}(G^h) - \theta^{\ell h}C^{\ell\prime}(G^h)}{\theta^h - \theta^{\ell h}} = C^{h\prime}(G^h) + \frac{\left(C^{h\prime}(G^h) - C^{\ell\prime}(G^h)\right)\theta^{\ell h}}{\theta^h - \theta^{\ell h}} \tag{28}$$

By (23), $\theta^{\ell h} > 0$, and by (24), $\theta^h - \theta^{\ell h} > 0$. Therefore, by the single-crossing property the second term on the right-hand side of (28) is positive, indicating that for the high-cost agency, the marginal rate of substitution of public services for transfers exceeds the marginal cost of provision. This induces a reduction in supply of public services and reflects the cost to the central government of eliciting truthful revelation of costs from the low-cost agency. The cost of information being private is further illustrated by the fact that, while the profit of the high-cost agency is zero, that of the low-cost agency is positive. The government is precluded by the self-selection constraint from reducing profits of the low-cost agency to zero.

The supply of public services by the high-cost agency can be further clarified by using (9), (10), (23), and (24) in (28) to give:

$$b'(G^h) = \frac{C^{h'}(G^h)}{1+\eta^h} + \frac{\phi^\ell}{\phi^h}\frac{C^{h'}(G^h) - C^{\ell'}(G^h)}{1+\eta^\ell} > \frac{C^{h'}(G^h)}{1+\eta^h} . \tag{29}$$

This clearly shows that the output for the high-cost agency is such that the marginal benefit exceeds the marginal cost by a proportion greater than the MCPF of its representative user. Thus, asymmetric information tends to require the high-cost agency to have a higher benefit-cost ratio than in the full-information case. The difference is greater the larger is the difference in marginal costs, the larger is the probability of the agency being low-cost, and the smaller is η^ℓ.

An important property of the separating equilibrium can be deduced immediately. Since the self-selection constraint on the low-cost agency is binding, the menus chosen by the low-cost and high-cost agencies $(S^\ell, G^\ell, S^h, G^h)$ must lie along an iso-profit curve for the low-cost agency. Therefore, unlike with the full-information case, the transfer schedule offered to the agency must be strictly increasing in G.

The pattern of optimal taxation of the two types of households is obtained by dividing (21) by (22) to yield (20), the same expression as in the full-information case. Again, in the utilitarian case where $\lambda = \phi^\ell$, $V_R(\ell) = V_R(h)$ so $R^\ell = R^h = \bar{R}$; all households pay the same tax. In this case $\eta^\ell = \eta^h$ so a common value for the MCPF, $(1+\eta)^{-1}$, appears in the decision rules for the two types of agencies, (27) and (29). If $\lambda < \phi^\ell$, so some inequality aversion is built into the government's objective function, households served by the low-cost agency will pay a higher tax $(R^\ell > R^h)$ to compensate for the fact that the size of their public services are higher $(G^\ell > G^h)$.

The solution for the case in which the government adopts a utilitarian objective function, or equivalently maximizes expected household utility, is also depicted in Figure 1. The common tax paid by all households is $\bar{R}$. The menus chosen by the two agencies are $\bar{E}^\ell$ and $\bar{E}^h$, leaving the households at $\bar{E}^\ell_R$ and $\bar{E}^h_R$. In this case, the high-cost agency must be cross-subsidizing the low-cost one in the sense that its transfer is less than the taxes paid by the households it serves. (If $\lambda > \phi^h$, the extent of the cross-subsidization will be less.) In the figure, we have supposed that the tax revenues are lower in the asymmetric information case that in the full information case $(\bar{R} < R)$. Intuitively, this is reasonable since the marginal cost of public funds is higher with asymmetric information. More formally, it is straightforward to show that as long as η is non-increasing in R, $\bar{R}$ will be less than R.[7]

[7] If η is constant, G^ℓ will be the same in the full-information and asymmetric information

And, η will be non-increasing in R whenever $V_R R < 0$, which we have argued above is reasonable.[8]

An interesting feature of the case where the government objective function is expected utility is that, given that $\bar{R} < R$, the households served by the low-cost agency are better off under asymmetric information than under full information. That follows from the fact that, since $\bar{R} < R$, and since η is non-increasing in R, η is higher under asymmetric information. Therefore, by (19) and (27), G^ℓ is higher under asymmetric information. Given that the households served by the high-cost agency will be worse off, government tax policy must be less redistributive under asymmetric information than under full information.

It is straightforward to infer the nature of the solution if there are more than two cost-types of agencies. Assume that there are n agencies and order them $(1, 2, 3, \cdots, n)$ in terms of descending costs of producing a given output G. Assume that the single-crossing property applies between all adjacent pairs (so marginal costs of producing a given G also descend with agency numbers.) The optimal outcome may be one which is fully separating where self-selection constraints are binding for all agencies $(2, 3, \cdots, n)$ with respect to their next lowest adjacent agency. All agencies except the highest-cost one (agency 1) will earn positive profits (information rents). The decision rule for all agencies $(1, 2, 3, \cdots, n-1)$ will be analogous to (29), while that for the lowest-cost agency is analogous to (27), the standard MCPF expression. The menu offered to all agencies will necessarily be strictly increasing in the level of public services provided. If the government chooses its tax policy according to a utilitarian objective function (or to maximize expected utility), all households will pay the same tax into general revenues. There will be a cut-off agency type such that all higher-cost ones are cross-subsidized by all lower-cost ones.

This is not the only type of equilibrium possible. The equilibrium might have partial pooling in the sense that some pairs of adjacent cost-types of agencies may choose the same menu combination (S^i, G^i).[9] The lower-cost of such pairs will obviously earn a higher profit, and households served by the two agencies will be equally well off. The menus offered to agencies will still have the property that transfers S are strictly increasing in public service levels G, and decision rules for all agencies but agency n will be like (29).

5. Agency's Effort Affects Probabilities

cases by (19) and (27). However, G^h will be lower with asymmetric information by (19) and (29), so $\bar{R} < R$. Suppose η is decreasing in R. If, to argue by contradiction, $\bar{R} > R$, η must be less in the asymmetric information case. By (19) and (27), G^ℓ must be then less in the asymmetric information case, and by (19) and (29), G^h must also be less in that case. But, this contradicts the assumption that $\bar{R} > R$. Therefore, $\bar{R} < R$ if η is non-increasing in R.

8 Since $\eta = -tL'/L$, $\partial\eta/\partial R = (\partial\eta/\partial t)t'(R) = t'(R)(-L'L - t(L')^2 + tL'')/L^2 < 0$ if $L'' \le 0$ or if the absolute value of tL'' is not too large.

9 The argument is analogous to partial pooling in the optimal non-linear income tax with more than two ability-types of households, as in Stiglitz (1987).

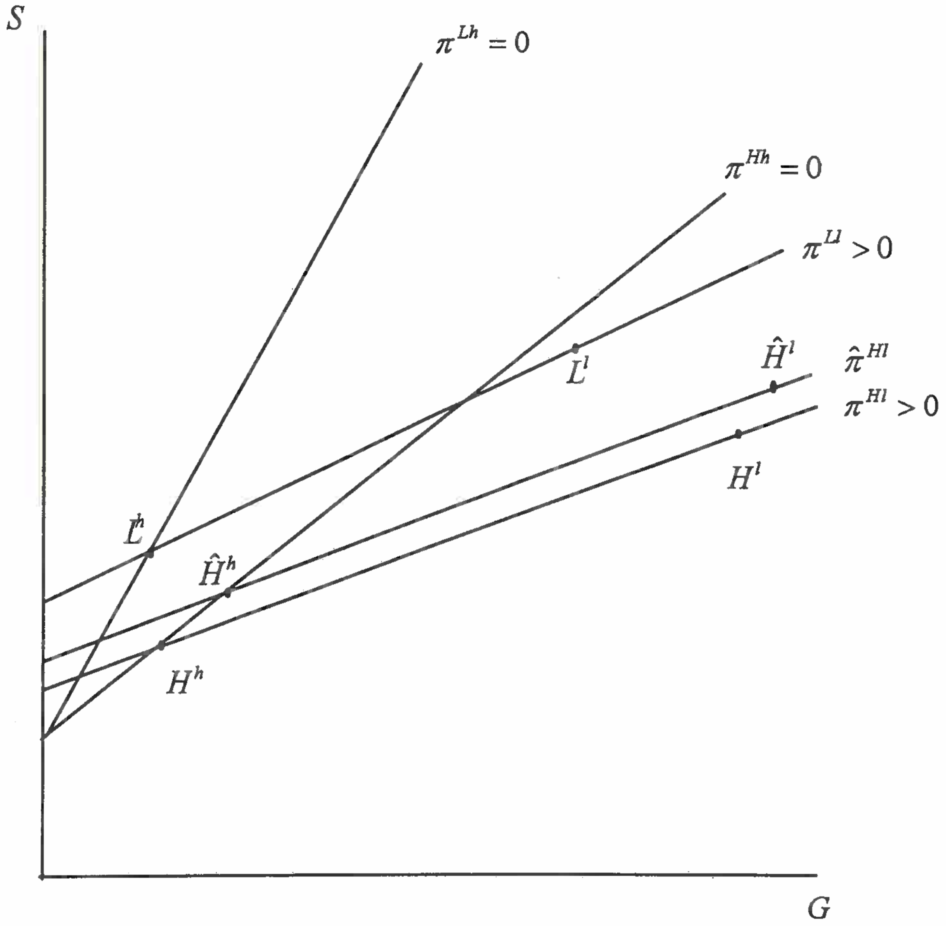

Figure 3

Suppose now that the agency, in addition to selecting a combination of S and G from those offered by the government, can also select a level of effort, and incurs a higher cost from exerting more effort. In this section we consider the case in which higher effort increases the probability of the low-cost state occurring. In the following section, we consider the case in which higher effort reduces the costs in each state. If the government cannot observe the effort level of the agency, we have a standard moral hazard problem along with the adverse selection problem outlined above. We investigate how the optimum achieved by the central government will be affected by the existence of moral hazard.

We assume for simplicity that there are only two possible effort levels, e^H (high effort) and e^L (low effort). The probabilities of the low-cost and high-cost states under these effort levels are $(\phi^{H\ell}, \phi^{L\ell}, \phi^{Hh}, \phi^{Lh})$, where $\phi^{H\ell} > \phi^{L\ell}$ (so $\phi^{Lh} > \phi^{Hh}$). The government offers a menu $(S^\ell, G^\ell, S^h, G^h)$ from which the agency must choose a pair. The agency, unlike the government, can observe the state of nature, but must choose an effort level before the state of nature is revealed; it will choose the effort level such that expected profits less costs of effort are the highest. Since cost functions are unaffected by effort, the choice of (S, G) by the agency in each state will be the same regardless of effort. As before, a separating equilibrium will be achieved. However, it is one that must take account of the moral hazard problem facing the government.

It is useful to proceed heuristically by showing the nature of the problem geometrically. Assume that the government maximizes a utilitarian objective function (or expected utility). And for simplicity, assume that the elasticity of labor supply with respect to the wage rate (η) is constant so that indifference curves are vertically parallel (cf. equation (11)) and that the cost functions are linear so that the iso-profit lines are also linear (since their slope is the marginal cost marginal costs are constant). To illustrate the nature of the moral hazard problem, Figure 2 depicts the outcomes that would be achieved if the government could observe the effort level. (Indifference curves are omitted for simplicity.) Given that η is constant, an increase in the probability of the high cost state has no effect on G^ℓ. Also, by totally differentiating (29), we obtain $dG^h/d\phi^h > 0$. Since lower effort increases the probability of the low-cost state, the low-effort outcomes are depicted by points (L^ℓ, L^h), and the high-effort outcomes by (H^ℓ, H^h).[10]

A notable point about these allocations is that expected utility is higher the higher is the probability of the low-cost outcome[11] This implies that the government will prefer

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[10] In both this section and the next one, we assume that the non-negative profit constraint takes the form $S^i - C^i(G^i) \geq 0$. This ensures that the agency if financially viable in each state. It could be argued that the profit constraint should apply to profits net of effort costs. This would complicate the presentation slightly without changing the results significantly.

[11] To show this, set $\lambda = \phi^\ell$ in problem (P) and differentiate the Lagrangian with respect to ϕ^h:

$$\frac{\partial \mathcal{L}}{\partial \phi^h} = V(h) - V(\ell) + \theta^S(R^h - S^h - R^\ell + S^\ell)$$

the high-effort outcome, and would like to implement it by offering the agency the menu $(S^{\ell}, G^{\ell}, S^h, G^h)$ associated with the points (H^{ℓ}, H^h). On the other hand, given this menu, the agency may prefer to take low effort. Suppose the effort level of the agency, e^H or e^L, is normalized to be its cost. The agency will take low effort if expected profits net-of-effort are higher with low effort, or:

$$E[\Pi^H] - E[\Pi^L] < e^H - e^L \tag{30}$$

where $E[\Pi^L] = \phi^{L\ell}\left(S^{\ell} - C^{\ell}(G^{\ell})\right)$ and $E[\Pi^H] = \phi^{H\ell}\left(S^{\ell} - C^{\ell}(G^{\ell})\right)$, since $S^h - C^h(G^h) = 0$ (i.e., the profit constraint in the high-cost state is binding).

Suppose this inequality is satisfied.[12] Then, the government cannot implement (H^{ℓ}, H^h). In effect, the government can only implement allocations that satisfy:

$$E[\Pi^H] - E[\Pi^L] \geq e^H - e^L \tag{30$'$}$$

We refer to this as the *forcing constraint* since it is what induces the agency to exert high effort. It can readily be seen that shifting the iso-profit line for the low-cost agency upward increases the left-hand side of (30). This involves increasing S^{ℓ}, holding G^{ℓ} constant. Since:

$$\frac{\partial(E[\Pi^H] - E[\Pi^L])}{\partial S^{\ell}} = \phi^{H\ell} - \phi^{L\ell} > 0$$

the constraint (30′) will be relaxed as the iso-profit line for the low-cost agency is shifted upward.

Since $R^h = R^{\ell}$ and using the self-selection constraint,this simplifies to:

$$\frac{\partial \mathcal{L}}{\partial \phi^h} = b(G^h) - b(G^{\ell}) - \theta^S(C^{\ell}(G^h) - C^{\ell}(G^{\ell}))$$

This can be rewritten as:

$$\frac{\partial \mathcal{L}}{\partial \phi^h} = \int_{G^{\ell}}^{G^h} b'(G)dG - \theta^S \int_{G^{\ell}}^{G^h} C^{\ell'}(G)dG$$

¿From the first-order conditions, we know that

$$b'(G^{\ell}) = \theta^S C^{\ell'}(G^{\ell})$$

Thus both integrals start at the same value at G^{ℓ}. Then, given that $b'' < 0$ and $C'' > 0$, the first term is larger negative than the second one is positive. Therefore, $\partial\mathcal{L}/\partial\phi^h < 0$ for this case.

12 If it is not, moral hazard is not an issue since both parties prefer high effort. Then the analysis of the previous sections applies.

As the iso-profit locus for the low-cost agency is shifted upward, one of two scenarios will occur. The first scenario is that the forcing constraint (30′) is satisfied before the iso-profit line for the low-cost case reaches $\Pi^{L\ell}$. This is the case shown by $\hat{\Pi}^{H\ell}$ in Figure 2. Here, the menu given by $(\hat{H}^{\ell}, \hat{H}^{h})$ would be that offered by the government. The expected utility level achieved by the household would be less than that for (H^{ℓ}, H^{h}), but still greater than that for the low-effort case.[13]

The second scenario is that the cost of high effort may be so great to the agency that even when the iso-profit line for the low-cost agency is raised to the same level as $\Pi^{L\ell}$, inequality (30) still applies. In this case, it would be welfare-improving for the government to raise the iso-profit line $\hat{\Pi}^{H\ell}$ even beyond that of $\Pi^{L\ell}$. This can cause qualitatively different results in two ways. First, the iso-profit line for the low-cost agency may be pushed up so far as to go beyond the point at which (H^{ℓ}, H^{h}) coincide. If (30′) applied at exactly that point, a pooling equilibrium would result. If it applied beyond it, the self-selection constraint for the high-cost agency would now become the binding one, and the MCPF conditions determining the optimal quantities of G^{ℓ} and G^{h} would be reversed. We do not consider that case any further. Second, it may be the case that when $\hat{\Pi}^{H\ell}$ is pushed far enough beyond $\Pi^{L\ell}$ so as to satisfy the forcing constraint (30′), expected utility may be less than it would be under low effort.[14] If so, the government would be best to announce (L^{ℓ}, L^{h}), and let the agency choose a low level of effort. In that case, there would be no forcing constraint, and the characterization of the outcome would be as in Section 4 above.

Algebraically, the first scenario where the agency is induced to choose high effort with $\hat{\Pi}^{H\ell} < \Pi^{L\ell}$ can be characterized as follows. The problem for the case of a non-linear cost function with preferences of the general form (8) is the same as problem (P) earlier with the addition of a forcing constraint of the form (30′) which, given that the profit constraint for the high-cost outcome will be binding, takes the form:

$$\left(\phi^{H\ell} - \phi^{L\ell}\right)\left(S^{\ell} - C^{\ell}\left(G^{\ell}\right)\right) \geq e^{H} - e^{L}$$

The central government's Lagrangian function can be written:

$$\mathcal{L}(\cdot) = \phi^{H\ell} V\left(R^{\ell}, G^{\ell}\right) + \phi^{Hh} V\left(R^{h}, G^{h}\right) + \theta^{h}\left(S^{h} - C^{h}\left(G^{h}\right)\right) +$$

$$\theta^{\ell h}\left(S^{\ell} - C^{\ell}\left(G^{\ell}\right) - S^{h} + C^{\ell}\left(G^{h}\right)\right) + \theta^{S}\left(\phi^{H\ell}(R^{\ell} - S^{\ell}) + \phi^{Hh}(R^{h} - S^{h})\right) +$$

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13 We deduce that as follows. As the iso-profit line for the low-cost agency rises, holding probabilities constant, expected utility falls because the central government becomes more and more constrained. When the iso-profit line reaches the level $\Pi^{L\ell}$, the menu is the same for the two cases. However, since the probability of the low-cost outcome is higher with more effort, we know that expected utility is higher if the agency chooses high effort than if it chooses low effort, when the two iso-profit lines coincide. That implies that expected utility must also be higher when the iso-profit line for the high-effort case is below that for low effort.

14 From the previous footnote, we know that that cannot occur unless $\hat{\Pi}^{H\ell} > \Pi^{L\ell}$.

$$\gamma\left[\left(\phi^{H\ell}-\phi^{L\ell}\right)\left(S^{\ell}-C^{\ell}\left(G^{\ell}\right)\right)-e^{H}+e^{L}\right]$$

The first order condition readily reduce to the following:

$$-\frac{V_G(\ell)}{V_R(\ell)}=C^{\ell'}\left(G^{\ell}\right)$$

$$-\frac{V_G(h)}{V_R(h)}=C^{h'}\left(G^{h}\right)+\frac{\left(C^{h'}\left(G^{h}\right)-C^{\ell'}\left(G^{h}\right)\right)\theta^{\ell h}}{\theta^{h}-\theta^{\ell h}}$$

These have precisely the same form as those obtained from problem (P) earlier. The same qualitative results apply. The only difference is that the levels of the Lagrangian multipliers will have changed because of the addition of the forcing constraint, and that will be reflected in the higher level of profits in the low-cost outcome. This higher profit level must be offered to the agency to induce higher effort. It results in lower expected utility for the household. The form of the decision rule given by the augmented MCPF rule is the same as in the adverse selection case of problem (P).

6. Agency's Effort Affects Costs

In this case, the level of effort affects the cost function of the agency in each state, $C^i(G^i, e)$, $i=\ell, h$. As above, we assume that there are two effort levels, e^H and e^L, so $C^i(G^i, e^H) < C^i(G^i, e^L)$. The nature of the problem is formally similar to that in the previous section except that effort now affects costs rather than probabilities. We proceed as before to develop the argument intuitively using geometric techniques.

For simplicity, assume again that that government maximizes expected utility, that the elasticity of supply of labor η is constant, and that cost functions are linear. Assume further that effort and the state of nature affect marginal costs, but not fixed costs. The cost functions for the two states and the two effort levels are assumed to be as follows:

$$C^i(G,e^j)=F+\frac{c^iG}{e^j}=F+\varepsilon^j c^i G \qquad i=\ell,h; \quad j=L,H \tag{31}$$

where $\varepsilon^j \equiv 1/e^j$. Proceeding as above, Figure 3 shows the outcomes that would be achieved for the two effort cases if the government could observe effort levels, but not costs. The low-effort level outcomes are depicted by (L^ℓ, L^h) and the high-effort outcomes by (H^ℓ, H^h). These are the outcomes that would be achieved when problem (P) is solved separately for the cases in which the cost function (31) is used for high and low effort. From the first-order conditions (27) and (29), it is easy to show by a comparative static analysis treating ε as a continuous variable that increases in effort (reductions in ε) will cause both G^ℓ and G^h to rise (as shown in the diagram for e^H and e^L). Moreover, applying the envelope theorem to the Lagrangian (P), increases in effort will cause expected utility to rise unambiguously. Therefore, if effort can be observed, the government would choose the allocation (H^ℓ, H^h).

If effort cannot be observed, the allocation (H^ℓ, H^h) may not be incentive-compatible. The agency will choose to supply low effort at the menu (H^ℓ, H^h) if inequality (30) applies, where now $E[\Pi^L] = \phi^h\left(S^h - F - \varepsilon^L c^h G^h\right) + \phi^\ell\left(S^\ell - F - \varepsilon^L c^\ell G^\ell\right)$ and $E[\Pi^H] = \phi^h\left(S^h - F - \varepsilon^H c^h G^h\right) + \phi^\ell\left(S^\ell - F - \varepsilon^H c^\ell G^\ell\right)$. Assuming that inequality (30) applies at the allocation (H^ℓ, H^h), the government must choose an alternative allocation that satisfies (30′). As in the previous section, it can be easily verified that shifting the iso-profit line for the low-cost agency upward increases the left-hand side of (30).[15]

The iso-profit line for the low-cost agency must be shifted upward until either (30′) is satisfied or expected utility is higher in the low-effort state. Figure 3 depicts an allocation $(\hat{H}^\ell, \hat{H}^h)$ for the former case. It turns out that in this case, unlike in the previous section, it is optimal to set $\hat{H}^\ell$ such that the slope of the indifference curve in the low-cost state (i.e., $-V_G(\ell)/V_R(\ell)$) is lower than the slope of the iso-profit line. That is, the MCPF in the low-cost state is *less* than in the standard expression.

To see this, consider the government's problem for the case in which the agency is induced to supply high effort. Using the general cost function and assuming the government maximizes expected utility, the Lagrangian expression for this problem may be written:

$$\mathcal{L}(\cdot) = \phi^\ell V\left(R^\ell, G^\ell\right) + \phi^h V\left(R^h, G^h\right) + \theta^h\left(S^h - C^h\left(G^h, e^H\right)\right)$$

$$+\theta^{\ell h}\left[S^\ell - C^\ell\left(G^\ell, e^H\right) - S^h + C^\ell\left(G^h, e^H\right)\right] + \theta^S\left(\phi^\ell(R^\ell - S^\ell) + \phi^h(R^h - S^h)\right)$$

$$+\gamma[\phi^\ell\left(C^\ell\left(G^\ell, e^L\right) - C^\ell\left(G^\ell, e^H\right)\right) + \phi^h\left(C^h\left(G^h, e^L\right) - C^h\left(G^h, e^H\right)\right) + e^L - e^H]$$

¿From the first-order conditions to this problem, we can derive the following equations:

$$-\frac{V_G(\ell)}{V_R(\ell)} = C_G^\ell\left(G^\ell, e^H\right) + \frac{\gamma\phi^\ell}{\theta^{\ell h}}[C_G^\ell\left(G^\ell, e^H\right) - C_G^\ell\left(G^\ell, e^L\right)] \tag{32}$$

$$-\frac{V_G(h)}{V_R(h)} = C_G^h(G^h, e^H) + \frac{\theta^{\ell h}}{\theta^h - \theta^{\ell h}}\left(C_G^h(G^h, e^H) - C_G^\ell(G^h, e^h)\right)$$

$$+\frac{\gamma\phi^h}{\theta^h - \theta^{\ell h}}\left(C_G^h(G^h, e^H) - C_G^h(G^h, e^L)\right) \tag{33}$$

[15] To see this, note that the left-hand side simplifies to $\phi^h c^h G^h(\varepsilon^L - \varepsilon^H) + \phi^\ell c^\ell G^\ell(\varepsilon^L - \varepsilon^H)$. Thus, increasing both G^h and G^ℓ will cause the left-hand side to increase since $\varepsilon^L > \varepsilon^H$. This will require the iso-profit curve of the low-cost state to be shifted upwards.

¿From (32), if high effort reduces marginal costs so $C_G^\ell\left(G^\ell, e^H\right) < C_G^\ell\left(G^\ell, e^L\right)$, then:

$$-\frac{V_G(\ell)}{V_R(\ell)} < C_G^\ell\left(G^\ell, e^H\right)$$

In this case, the marginal rate of substitution of G^ℓ for S^ℓ is *less* than the marginal cost of G^ℓ augmented by the MCPF of the users of the low-cost agency. That is:

$$b'(G^\ell) < \frac{C_G^\ell\left(G^\ell, e^H\right)}{1+\eta^\ell}$$

so the ratio of marginal benefits to marginal costs for the low-cost state is less than the standard case.

For the high-cost agency, equation (33) may be compared with (28). The first two terms on the right hand side of (33) are identical to those in (28) and have the same interpretation. Since $\theta^h - \theta^{\ell h} > 0$ from the first-order conditions, and using the single-crossing property that $C_G^h(G^h, e^H) > C_G^\ell(G^h, e^H)$, these terms tend to cause the MCPF for the high-cost agency to exceed that given by the standard expression, as in (29). However, the last term in (33) is negative, tending to reduce the MCPF. Thus, as with (32), the existence of unobserved effort that affects the cost of delivering public services tends to *reduce* the MCPF and to increase the amount of public services provided. This confirms what the diagrammatic analysis of Figure 3 suggested; to force the agency to supply high effort, the expected profit function for the high-effort outcomes must be shifted up, and that involves increasing G. The entire grant structure relating S to G is shifted upwards, as shown in Figure 3 by the points $(\hat{H}^h, \hat{H}^\ell)$. However, unlike with the low-cost agency, the MCPF for the high-cost agency could be higher or lower than for the standard case.

7. Conclusion

In this paper we have explored some of the implications of asymmetric information for the structure of grants between a central government and a local agency, and for the marginal cost of public funds. The agency is assumed to have information about its cost conditions that is not available to the government. As well, the government may not be able to observe the level of effort exerted by the agency, and hence may not know either the true probability distribution facing the agency or the state-contingent cost conditions. We have shown for each of these cases when and how the decision rule for public services and its relation to the standard MCPF is affected by the absence of full information. In most cases, the results are intuitive and readily understood geometrically.

Our analysis has been restricted to the cases of two states of the world and, where relevant, two effort levels. This was sufficient to yield the qualitative effects at work. We have also made some rather simple assumptions about the nature of the economy. Households are all identical and have quasi-linear preferences. The local agency is interested only in its own profit levels, and is able to use excess profits for its private benefit. Perhaps more important, the agency must select its effort level before it knows its cost type. Given the simple model we have adopted, the results we obtain must be viewed as exploratory. It remains to see how robust they are as different assumptions about the economy are adopted.

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TECHNICAL APPENDIX

This Appendix proves that if agency cost functions satisfy the single-crossing property, the equilibrium will be separating with $\theta^\ell = 0$, $\theta^h > 0$, $\theta^{h\ell} = 0$, and $\theta^{\ell h} > 0$. The proof proceeds in five steps.

1. The profit constraint on the low-cost agency is non-binding. From the self-selection constraint on agency ℓ and the profit constraint in agency h, using $C^h(G^h) > C^\ell(G^h)$, and assuming an interior solution so $G^\ell, G^h > 0$,

$$S^\ell - C^\ell(G^\ell) \geq S^h - C^\ell(G^h) > S^h - C^h(G^h) \geq 0$$

or,

$$S^\ell - C^\ell(G^\ell) > 0$$

Therefore, $\theta^\ell = 0$.

2. The Lagrangian of the central government with $\theta^\ell = 0$ is:

$$\mathcal{L} = \lambda V(R^\ell, G^\ell) + (1-\lambda)\, V(R^h, G^h) + \theta^h \left(S^h - C^h(G^h)\right)$$

$$+\theta^{\ell h}\left(S^\ell - C^\ell(G^\ell) - S^h + C^\ell(G^h)\right) + \theta^{h\ell}\left(S^h - C^h(G^h) - S^\ell + C^h(G^\ell)\right)$$

$$+\theta^S \left(\phi^\ell \left(R^\ell - S^\ell\right) + \phi^h \left(R^h - S^h\right)\right)$$

The first-order conditions are:

$$\lambda V_R^\ell + \theta^S \phi^\ell = 0 \qquad (A.1)$$

$$(1-\lambda) V_R^h + \theta^S \phi^h = 0 \qquad (A.2)$$

$$\theta^{\ell h} - \theta^{h\ell} - \theta^S \phi^\ell = 0 \qquad (A.3)$$

$$\theta^h - \theta^{\ell h} + \theta^{h\ell} - \theta^S \phi^h = 0 \qquad (A.4)$$

$$\lambda V_G^\ell - \theta^{\ell h} C^{\ell\prime}(G^\ell) + \theta^{h\ell} C^{h\prime}(G^\ell) = 0 \qquad (A.5)$$

$$(1-\lambda) V_G^h - \theta^h C^{h\prime}(G^h) + \theta^{\ell h} C^{\ell\prime}(G^h) - \theta^{h\ell} C^{h\prime}(G^h) = 0 \qquad (A.6)$$

By $(A.1)$, $(A.2)$, $(A.3)$ and $(A.4)$, $\theta^h = \theta^S$ and $\lambda V_R^\ell + (1-\lambda) V_R^h = -\theta^h < 0$. Therefore, $\theta^h > 0$, so

$$S^h - C^h(G^h) = 0 \qquad (A.7)$$

3. A pooling equilibrium is non-optimal. The best pooling equilibrium is the combination of (R^P, S^P, G^P) that maximizes $V(R, G)$ subject to non-negative profits for the high-cost agency, $S - C^h(G) \geq 0$ and the government budget constraint $S = R$. At an optimum, $-V_G^P / V_R^P = C^{h\prime}(G^P) > C^{\ell\prime}(G^P)$. Since $C^{\ell\prime} = dS/dG$ with profit π^ℓ constant, welfare can be increased by holding $R^h = R^P$ and $G^h = G^P$, but increasing R^ℓ and G^ℓ in the ratio $C^{\ell\prime}$

since $C^{\ell'} < -V_G^P/V_R^P$. (Welfare is also decreased by a small move in the other direction holding π^ℓ fixed.)

4. By $(A.1)$ and $(A.3)$, $\theta^{h\ell} - \theta^{\ell h} = \lambda V_R^\ell < 0$. Therefore, at least one of $\theta^{h\ell}$ or $\theta^{\ell h}$ must be non-zero. We can show that they cannot both be non-zero in a separating equilibrium. If they were:

$$S^\ell - C^\ell(G^\ell) = S^h - C^\ell(G^h)$$

and

$$S^\ell - C^h(G^\ell) = S^h - C^h(G^h)$$

Subtracting one from the other and rearranging,

$$C^h(G^\ell) - C^h(G^h) = C^\ell(G^\ell) - C^\ell(G^h)$$

so $C^{h'}(G) = C^{\ell'}(G)$, which is a contradiction. Therefore, either $\theta^{h\ell} = 0$ and $\theta^{\ell h} \neq 0$, or $\theta^{\ell h} = 0$ and $\theta^{h\ell} \neq 0$.

5. Finally, we can show that if $\theta^{\ell h} = 0$ and $\theta^{h\ell} \neq 0$, welfare is maximized at a pooling equilibrium. The first-order conditions reduce to:

$$\lambda V_R^\ell - \theta^{h\ell} = 0 \quad (A.1')$$

$$(1-\lambda)V_R^h + \theta^h + \theta^{h\ell} = 0 \quad (A.2')$$

$$\lambda V_G^\ell + \theta^{h\ell} C^{h'}(G^\ell) = 0 \quad (A.5')$$

$$(1-\lambda)V_G^h - \theta^h C^{h'}(G^h) - \theta^{h\ell} C^{h'}(G^h) = 0 \quad (A.6')$$

these yield:

$$-\frac{V_G^\ell}{V_R^\ell} = -\frac{V_G^h}{V_R^h} = C^{h'}(G^h)\ .$$

Thus, $R^\ell = R^h$ and $G^\ell = G^h$, so a pooling equilibrium is preferred if $\theta^{\ell h} = 0$ and $\theta^{h\ell} \neq 0$. Given that a pooling equilibrium cannot be optimal, it must be the case that $\theta^{\ell h} \neq 0$ and $\theta^{h\ell} = 0$. By $(A.1)$ and $(A.2)$, $\theta^{\ell h} > 0$.

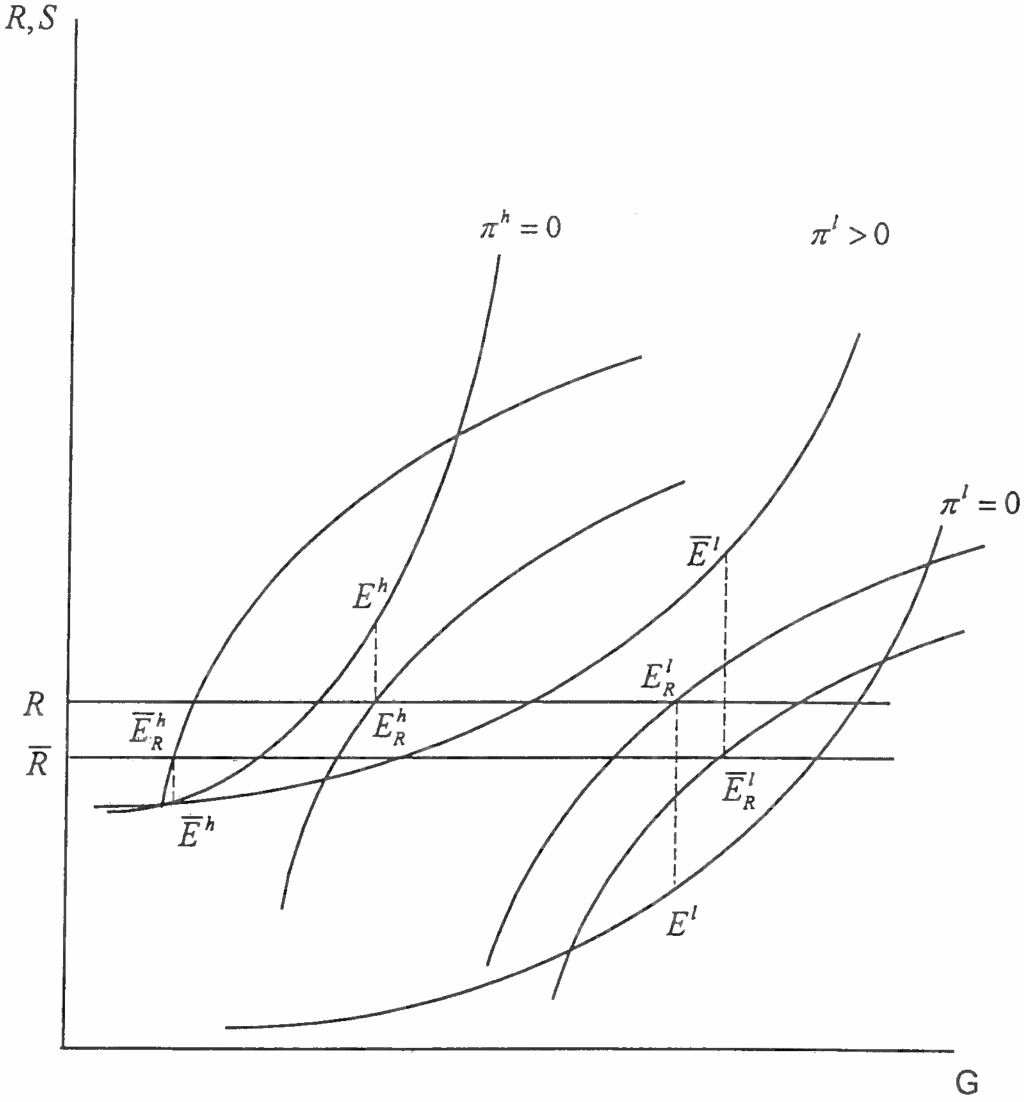

Figure 1

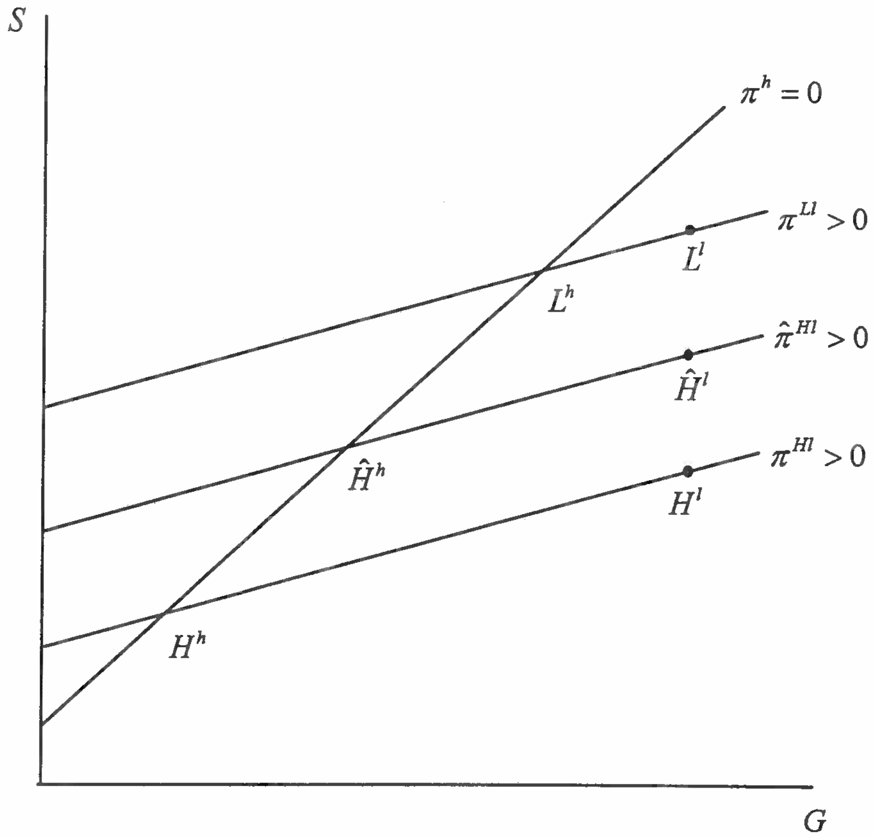

Figure 2