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Raghendra Jha* and Anandi P. Sahu**

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*Indira Gandhi Institute of Development Research
Research
General Vaidya Road, Goregaon (East)
Bombay 400 065
INDIA

**School of Business Administration
Oakland University
Rochester, MI48309
USA

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RAGHBENDRA JHA
Indira Gandhi Institute of Development Research
General Vaidya Road, Goregaon (East)
Bombay 400 065, INDIA

ANANDI P. SAHU
School of Business Administration
Oakland University
Rochester, MI 48309, USA

*Professor of Economics and Associate Professor of Economics, respectively. The former acknowledges the comments of colleagues at Queen’s University, Kingston, Canada where he was a Visiting Associate Professor during 1993-94. Both authors are grateful to two anonymous referees for useful comments on an earlier draft of the paper.
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Abstract

This paper examines the role tax policy can play in fostering human capital accumulation in a resource constrained dual economy whose population is growing. The study shows how human capital accumulation, in turn, affects the intersectoral terms of trade and the economic growth process of such an economy. The dual economy is assumed to consist of two sectors, agriculture and manufacturing. Production in agriculture requires unskilled labor, land and capital whereas production in the manufacturing sector requires skilled and unskilled labor and capital. Schooling facilities are limited and access is rationed by the government. Moreover, schooling requires an investment of time. The paper demonstrates the existence of a unique short run equilibrium. It also demonstrates that the steady state equilibrium is unique and locally stable. Comparative steady state analysis suggests that a balanced budget increase in public investment in education (financed by a tax increase on capital income and/or incomes of skilled workers), alters the terms trade between agriculture and manufacturing sectors and favorably affects the economic growth process.
I. INTRODUCTION

The process of economic development was first studied by economists at the aggregate level. It led to the development of the now-standard neoclassical model of Robert Solow (1956), Edward Denison (1961) and many others, first used to study twentieth century U.S. economic growth. These models were one-sector models, and did not incorporate human capital accumulation. Human capital has since been incorporated into models of economic growth in a number of papers (Arrow, 1962; Uzawa, 1965; Findlay and Kierzkowski, 1983; Romer, 1986; Lucas, 1988; Pecorino, 1992; Ljungqvist, 1995; among others). However, these don’t capture a key characteristic of underdeveloped economies — their dualistic economic structures. The main rationale for using dual economy models, to understand the growth process of developing countries, is based on the belief that the modern and the traditional sectors are so different from each other that aggregating them into a one sector model would tend to obfuscate some crucial aspects of the economic structure of developing countries. Recognizing this, Jorgenson (1961, 1967) developed neoclassical dual economy models to study the unique growth characteristics of an underdeveloped economy. At present, there exist several non-neoclassical versions of dual economy growth models — Harris-Todaro type (Harris and Todaro, 1970) models are well known examples of this literature.

Dual economy models have been used to study a number of issues and to analyze effects of some policies in developing countries. McIntosh (1986) utilizes a dual economy model to analyze the benefits of north-south trade to developing countries; Heady and Mitra (1986) use it to study the optimal mix of trade policy, tax policy and shadow prices; Jha and Lachler (1981), and Heady (1987) use it to examine the effect of tax policy on intersectoral migration; Gang and Gangopadhyay (1987) employ such a model to study choice of technique; McMohan (1990) utilize it to analyze the effects of tariff policy; Buffie (1993) employs such a model to study the impact of foreign investment on underemployment and domestic capital accumulation; Chao and Yu (1995) use it to study the shadow price of foreign exchange. The specific variants of the model used in the preceding studies differ. A discussion of the macro version of dual economy models can be found in Jha (1994).
The existing dual economy literature neglects three important issues. First, there is an almost complete neglect of the problem of human capital accumulation, a key to economic growth. Lucas (1988), for example, examines the role of human capital formation in developing countries. But, he does not use a dual economy model to address unique characteristics of a developing economy. Second, access to education is highly restricted in many overpopulated countries, unlike in developed countries where choice in education is more relevant. Endogenous growth models of Findlay and Kierzkowski (1983), Lucas (1988), Pecorino (1992), etc. are essentially for developed economies. Without much risk of oversimplification it may be stated that the endogenous growth literature has largely ignored the implications of human capital accumulation in developing countries where the issue of limited access to education is extremely important. In developing countries with large populations, the issue of choice of the amount of education to be undertaken is secondary to the constraint that there almost always exists an excess demand for education, and educational services have to be rationed among students. The constraint on resources introduces a role that the government tax policy can play in fostering human capital accumulation and economic growth in poor LDCs. Moreover, the role of the terms of trade between industry and agriculture has always been considered important in the development process of dual economies. While the endogenous economic growth literature has emphasized the role of acquisition of skills and research and development in the process of economic growth, it has not, so far, shed any light on the effects of human capital accumulation on the intersectoral terms of trade in dual economies. These terms of trade have, since long, been a crucial topic in the economics (and politics) of development of poor countries. Third, to a lesser extent, there is a paucity of literature on the role of taxation in the development process. As mentioned earlier, Jha and Lachler (1981), Heady and Mitra (1986), Heady (1987), and Mc Mohan (1990) are examples of studies that deal with some aspects of tax policy. However, since human capital has been neglected in the dual economy literature, it follows that the role of taxation in helping human capital accumulation in a dual economy has also been neglected. Consistent with the current theory and empirical evidence, it only seems desirable that we address the issue of financing of education in an appropriate dual economy growth model.

In this paper, we analyze the role of taxation in the development process of a resource constrained dual economy. Taxation would provide a better way of financing education than, say,
deficit financing which would raise interest rates and require additional taxation in the future to finance payments to bond holders, or money financing which would be inflationary. In the present paper, not only do we develop a model which allows education to play a central role in the development process of a resource constrained dual economy and government tax policy to influence human capital formation but also to have education rationed.

In the model employed in this paper, we focus attention and shed light on the behavior of the terms of trade between industry and agriculture -- the two sectors of the dual economy. Since an appropriate model that allows for rationing in education has not been built for dualistic developing economies, the question of discussing the behavior of the terms of trade in such models does not arise. The present paper attempts to fill this gap. The discussion of the terms of trade is important because of the well known debate over "price scissors" in the literature on Soviet industrialization, and which has dominated the discussion of dualistic growth models again (see, for example, Saah and Stiglitz, 1984; and Knight, 1995). As the terms of trade between industry and agriculture have been a central issue in development policy and politics for a very long time, our paper has addressed the effects of the tax policy on the terms of trade as well.

This paper blends the dual economy literature and the public finance literature through an examination of the role of taxation on human capital accumulation in dualistic economies. The focus on human capital accumulation is in line with the emphasis placed in endogenous growth models. We, however, employ a standard neoclassical dual economy model to accomplish this. To the best of our knowledge, such a study has not been attempted before.

We study a stylized version of a growing dual economy with human capital accumulation. Growth, however, is not endogenous. Some workers undertake human capital accumulation but the number of workers who can be trained at any point in time is limited by the existing capacity of schools. The capacity of the schools is, in turn, a result of government investment in education. Government expenditure is financed by a tax on capital income and/or the incomes of skilled workers. Moreover, education takes time and those who commit themselves to accumulating human capital must forswear earning any income during the period of their education. Skilled labor can work specifically in the manufacturing (secondary) sector whereas unskilled workers are mobile between the agricultural and manufacturing sectors. We examine short run equilibrium of this
economy and also study the dynamic and comparative static properties of steady state. We discover that, given the current investment in education, the steady state is unique, locally stable and is characterized by a unique ratio of skilled to unskilled workers. The analysis of the paper suggests that an increase in the investment in education (financed by an increase in the tax rate) would result in an increase in the ratio of skilled to unskilled workers and a drop in the relative price of manufactured goods and the capital intensity of agricultural operations. Government tax policy favorably affects the economic growth process by altering the terms of trade between agriculture and manufacturing sectors. Such effects cannot be captured in a traditional dual economy model (see, Ranis (1989)).

The analysis in this paper is the first to address the important issue of the effects of human capital accumulation on economic growth in LDCs in the early stages of their development process. The present analysis should, therefore, be viewed as an alternative to the endogenous growth literature. We are of the view that whereas the assumptions, analysis and concerns of the endogenous growth literature better reflect the stylized facts of the more developed economies, the analysis in this paper (which views access to schooling as being limited and necessarily rationed and which focuses on the importance of human capital on the terms of trade) is more suited to analyze the growth process in densely populated, dualistic, less developed economies. Our paper addresses some of the concerns of less developed economies more adequately than the endogenous growth literature, and explicitly incorporates role the tax policy can play in the development process of these resource constrained economies.

The remainder of this paper is organized as follows. In section II, we outline the model. Section III studies the short run equilibrium and comparative static properties of the model. Long run steady state and its comparative static and dynamic properties are studied in section IV. Section V discusses the policy implications of the model and the last section offers some concluding remarks.
II. THE MODEL

Consider a closed economy with two sectors, A (agriculture) and M (manufacturing). In sector A, capital \((K_A)\), land \((L)\) and unskilled labor \((N^u_A)\) produce output \((Q_A)\) according to a twice continuously differentiable, linear homogeneous production function \(H_A:\)

\[
Q_A = H_A(K_A, L, N^u_A) \tag{1'}
\]

If the supply of land is fixed, as is assumed in this paper, it is possible to choose a new origin for expression \((1')\). Consequently, \((1')\) can be rewritten in the simpler form \(F_A:\)

\[
Q_A = F_A(K_A, N^u_A) \tag{1}
\]

The production function \(F_A\) can be considered as the net production function of sector A, net of the contribution of the fixed factor, land.\(^6\)

In sector M, output \((Q_M)\) is produced by unskilled workers \((N^u_M)\), skilled workers \((N^s)\) and capital \((K_M)\) as per a twice continuously differentiable, linear homogeneous production function \(F_M:\)

\[
Q_M = F_M(K_M, N^u_M, N^s) \tag{2}
\]

Skills are acquired by accumulating human capital (through training) which takes time\(^7\), \(\tau\), during which people undergoing training do not work. To simplify the analysis we assume that it costs \(\eta\) units of commodity A to produce one skilled worker\(^8\) and that the government decides how much to spend on education.\(^9\) This expenditure is financed by taxing capitalists and/or skilled workers.\(^10\) The government's budget is always balanced.

Intersectoral migration of unskilled workers implies that their wage \((w_A)\) in sector A must equal their wage \((Pw_M)\) in sector M (where \(P\) is the price of commodity M in terms of commodity A). A further (purely simplifying) assumption is that sector A goods can only be consumed whereas sector M goods can be consumed or invested.

A worker in sector A is paid a fixed fraction\(^11\), \(\alpha\), of per capita output. Intersectoral migration of unskilled workers ensures that

\[
w_A = \alpha f_M, \text{ and} \tag{3}
\]
\[ \alpha f_A = Pf_M \cdot k_M \cdot f_M - \eta f_M \]

where \( f_A = F_A/N_A^u \), \( f_M = F_M/N_M^u \), \( k_M = K_M/N_M^u \), \( f_M = \partial F_M/\partial K_M \) = marginal product of capital in manufacturing, \( \eta = N^t/N_M^u \), \( f_M = \partial F_M/\partial N \) = marginal product of skilled labor. In equation (3'), we have assumed that all factors of production are paid their respective marginal products in this sector.

A further simplifying assumption is that workers consume all their income whereas capitalists save and invest all their income. Hence in sector A, market clearing requires that the following equation be satisfied:

\[ Q_A = C_A(P,w^u)N^u + C_A(P,f_M)N^t + \int_{-\tau}^{t} I_E(v)dv \]  

(4)

where \( C_A(.) \) is per worker demand for commodity A, \( w^u \) is the wage of unskilled workers, \( I_E(v) \) is the government's investment in education at time \( v \). Equation (4) says that the output of sector A is exhausted by the consumption of skilled and unskilled workers and those acquiring human capital. Clearly \( \partial C_A/\partial P > 0 \), and the sign of \( \partial C_A/\partial w^u \) = the sign of \( \partial C_A/\partial f_M > 0 \).

Capital accumulation in sector A is given as:

\[ \dot{K}_A = \dot{K}_A/K_A = (1-\alpha)f_A(I-r)/K_A \]

(5)

where it is implicit that all capital income in sector A is invested and that investment goods have to be purchased from sector M. \( r \) is the proportional tax rate on capital income. A hat (') refers to rate of growth and a dot (.) above a variable signifies its time derivative.

Capital accumulation in sector M is given by:

\[ \dot{K}_M = \dot{K}_M/K_M = f_M(I-r) \]

(6)

Total population, \( N \), consists of skilled and unskilled workers and those undergoing training. Hence we have

\[ N = N^u + N^t + \int_{-\tau}^{t} (I_E(v)/\eta)dv \]

(7)
where \( \eta \) is the cost (in terms of commodity A) of producing one skilled worker. Population grows at rate \( n \), i.e., \( \dot{N}/N = n \), whence

\[
\dot{N}^u = nN - I_E(t)/\eta
\]

Thus, population growth at time \( t \) consists of additions to unskilled labor and those beginning schooling.

### III. SHORT RUN EQUILIBRIUM

Define the following five variables \( l_A = N^u_A/N^u \), \( z_A = K_A/N^u \), \( z_M = K_M/N^u \), \( k_A = K_A/N^u \), and \( k_M = K_M/N^u \) so that

\[
z_A = k_A/l_A, \quad 0 < z_A \leq k_A \quad \text{and} \quad (9)
\]

\[
z_M = k_M(l - l_A), \quad 0 < z_M \leq k_M \quad \text{(10)}
\]

We can divide equation (4) by \( N^u_A \) to obtain

\[
Q_A/N^u_A = f_A = C_A(P, w^u)/l_A + C_A(P, f_M)N^u/(N^u/l_A) + \int_{i}^{t} I_E(v)dv/(N^u/l_A) \quad (11)
\]

In the short run \( z_A \) and \( z_M \) are fixed whereas \( k_A \) and \( k_M \) can vary because of changes in the allocation of unskilled labor between sectors A and M. The short run equilibrium can be defined by the 5-tuple \( (k_A, k_M, l_A, P, w^u) \) which satisfies equations (3), (3'), (9), (10), and (11) for given values of \( z_A, z_M, N^u/N^u, \) and \( \int_{i}^{t} I_E(v)dv \). We now proceed as follows. From equation (3') we write

\[
P = \alpha f_A/[f_M - k_M f_M] - n_M f_M \quad (12)
\]

Hence, using the fact that

\[
n^u = [N^u/N^u]/(1 - z_A/k_A) \quad (13)
\]

we have
\[
\frac{\partial P}{\partial k_A} = \frac{[f_M' k_M \alpha f_A' - \alpha f_A' (N^+/N^a)(z_A/k_A^2)]}{[f_M' k_M M_M1 - \{(N^+/N^a)/(1-z_A/k_A)\} f_M]^2} > 0
\]  
(14)

Further,

\[
\frac{\partial P}{\partial k_M} = \frac{[-\alpha k_A \{k_M^2 f_{M1} + n f_{M2}^2\}]}{[f_M' k_M M_M1 - \{(N^+/N^a)/(1-z_A/k_A)\} f_M]^2}
\]  
(15)

Here \(f_{M1} = \partial^2 f_{M1}/(\partial k_M^2)^2\), \(f_{M2} = \partial^2 f_{M1}/(\partial k_M \partial n^t)\). Equation (15) points to the fact that response of \(P\) to changes in \(k_M\) depends upon the structure of production in sector M, whereas we learn from equation (14) that the response of \(P\) to changes in \(k_A\) does not depend on the value of \(k_A\).

It is realistic to assume that \(f_{M2} > 0\), i.e., the marginal product of skilled labor increases with additions to capital.15 If we were to assume that \(f_{M2} = 0\), \(\partial P/\partial k_M\) is unambiguously negative.

To determine the sign of \(\partial P/\partial k_M\) under the assumption that \(f_{M2} > 0\), we have to examine the numerator of equation (15):

\[
\alpha k_A \{k_M^2 f_{M1} + n f_{M2}^2\}
\]

We should have \(|k_M f_{M1}| > |n f_{M2}^2|\) for the above expression to be negative. The symbol \(|\ |\) stands for absolute value. Note that \(n^t\) is a proper fraction whereas \(k_M\) may not be a proper fraction. Let us divide through the inequality by \(f_{M1}\) and rewrite it as follows:

\[
|k_M^2 f_{M1}/f_{M1}| > |n f_{M2}^2/f_{M1}|
\]

Now \(f_{M1}/f_{M1}\) is the rate of change of marginal product of capital consequent upon an increase in capital and \(f_{M2}/f_{M1}\) is the rate of change in marginal product of skilled labor consequent upon an increase in capital. Thus, our requirement boils down to saying that the direct effect (on marginal product of capital) of a change in capital is greater than the indirect effect (on marginal product of skilled labor), a reasonable assumption to make. Therefore, \(\partial P/\partial k_M < 0\) even if \(f_{M2} > 0\).

We totally differentiate equation (11) to get:

\[
\frac{d k_M}{d k_A} = \frac{(z_A/k_A^2)(k_M f_{M1}' \alpha - \alpha C_A)^2 \partial C_A/\partial P \partial P/\partial k_M \partial C_A/\partial w^u}{(\partial C_A/\partial P \partial P/\partial k_M + \alpha C_A)^2 (N^+/N^a) + \alpha C_A \partial C_f \partial f_{M2} \partial f_{M2} \partial k_M \partial N^+/N^a)}
\]  
(16)
With the above assumption \( dk_M/dk_A > 0 \).

We have depicted the short run equilibrium in Figure 1. We have plotted equations (10) and (11) and termed them "the manufacturing output" and "market clearing" curves respectively. These schedules uniquely determine the short run equilibrium \( k_A^* \) and \( k_M^* \).

---

**Insert Figure 1 about here.**

---

From the assumptions on \( C_A(.) \) and the well-behavedness of the production functions, it can be seen that the graph of equation (11) stretches from the line \( k_M = \bar{z}_M \) to \( (k_M, k_M) = (\infty, \infty) \). Equation (11) cannot intersect the line \( k_A = \bar{z}_A \). This is because if \( k_A = \bar{z}_A, N_M^* = 0 \) so that all output of sector A is consumed by sector A workers. But that cannot happen since their consumption is constrained to not exceed \( \alpha f_A \) (where \( \alpha < 1 \) by assumption).

**COMPARATIVE STATIC OF SHORT RUN EQUILIBRIUM**

We totally differentiate equations (10) and (11) taking equation (12) into account and write in matrix form:

\[
\begin{bmatrix}
Y_1 \\
Y_2
\end{bmatrix}
\begin{bmatrix}
dk_A \\
dk_M
\end{bmatrix}
= 
\begin{bmatrix}
-\frac{f_A}{k_A} dz_A + C_A(P, f_M) dN^* \\
\frac{z_A k_M/k_A^2}{1-z_A/k_A} dz_M + (k_M/k_A) dz
\end{bmatrix}
\]

(17)

where:

\[
Y_1 = f_A z_A / k_A - f_A / (k_A^2) z_A - (\partial C_A / \partial P) (\partial P / \partial k_A)
\]
\[
- (\partial C_A / \partial w') (\partial w' / \partial k_A) - (N^*/N^**) (\partial C_A / \partial P) (\partial P / \partial k_A) \text{ and}
\]
\[ Y_2 = -\left( \frac{\partial C_A}{\partial P} \right) \left( \frac{\partial P}{\partial k_M} \right) - \left( \frac{N^+}{N^+} \right) \left( \frac{\partial P}{\partial k_M} \right) - \left( \frac{\partial C_A}{\partial f_{M^2}} \right) \left( \frac{\partial f_{M^2}}{\partial k_M} \right). \]

We wish to calculate \( \frac{dk_A}{dz_A}, \frac{dk_A}{dN^1}, \frac{dk_M}{dz_A}, \frac{dk_M}{dN^1}, \frac{dk_A}{dz_M}, \frac{dk_M}{dz_M}, \) and \( \frac{dk_M}{dN^1} \). Let \( \Delta \) be the determinant of the matrix on the left hand side of equation (17). We have now

\[
\Delta = \left[ 1 - z_A/k_A \right] \left[ \left( z_A/k_A^2 \right) \left( k_A f_{M^2} \right) - \left( \frac{\partial C_A}{\partial P} \right) \left( \frac{\partial P}{\partial k_A} \right) - \left( \frac{\partial C_A}{\partial w^A} \right) \left( \frac{\partial w^A}{\partial k_A} \right) \right] - \left( \frac{N^1}{N^1} \right) \left( \frac{\partial C_A}{\partial P} \right) \left( \frac{\partial P}{\partial k_M} \right) - \left( \frac{N^1}{N^1} \right) \left( \frac{\partial P}{\partial k_M} \right).
\]

We can now prove the following proposition.

**Proposition 1:** Assume that the marginal product of capital in sector M does not depend upon the input of skilled labor \( f_{M^2} = 0 \), or, \( f_{M^2} > 0 \) but small.\(^1\) We then have (i) \( \frac{dk_A}{dz_A} > 0 \), (ii) \( \frac{dk_M}{dz_M} > 0 \), (iii) \( \frac{dk_A}{dN^1} < 0 \) (as more workers acquire skills, the capital intensity of agriculture drops), (iv) \( \frac{dk_M}{dz_A} < 0 \), (v) \( \frac{dk_M}{dz_M} > 0 \), (vi) \( \frac{dk_M}{dN^1} > 0 \) (capital labor ratio in sector M increases as more workers acquire skills).

**Proof:** See Appendix 1.

The above results may be compared to the standard results from the static models of human capital accumulation (e.g. Miyagiwa (1989)) and those of the dual economy literature (e.g. Amano (1980)). In Miyagiwa's model in the short run, \( N^1 \) is a parameter and comparative static results refer to a shock that increases the stock of capital. Our model permits a much richer set of comparative static results since we treat \( z_A, z_M \), and \( N^1 \) as parameters and allow the capital-labor ratios in the two sectors to adjust. Moreover, our results are fundamentally more general since we combine human capital accumulation with dualistic growth. Amano's model, for instance, is unable to decipher the effect of changes in education since he does not model human capital accumulation. We are now in a position to examine the long run properties of the model.

**IV. LONG RUN EQUILIBRIUM**

The long run steady state of the economy is defined by the following three equations:

\[
\hat{K}_A = \left[ 1/(k_A P) \right] \left[ (1-a) f_A (1-t_A) \right] = N^w_A
\]

(18)
\[
\tilde{K}_M = f_{M1}(1-t_s) = \tilde{N}_M^u
\]
\[
\tilde{n}^* = \tilde{N}_M^* - \tilde{N}_M^u = 0
\]

The growth of skilled labor today is governed entirely by investment in education made \(\tau\) periods earlier. Thus, using equation (20) we have
\[
\tilde{N}_M^* = \tilde{N}_M^u = \frac{1}{(\eta N^*)}[I_E(t-\tau)]
\]
We can then prove the following proposition.

**Proposition 2:** Given current investment in education, the steady state is characterized by an \textit{unique} ratio of skilled to unskilled workers. Further, there is a unique steady state \(k_A^*\).

**Proof:** At time \(t\), \(N^z\) is a datum. Using equations (8) and (21), we can define the unique steady state \(n^*\) consistent with current investment in education \(I_E(t)\) as:
\[
n^* = \frac{1}{(\eta N^*)}[I_E(t-\tau)]
\]
where the star (*) refers to steady state. Using equation (19) and the well-behavedness of the M sector production function we can solve for \(k_M^*\). We then have
\[
P^* = \frac{1}{(\eta N^*)}[I_E(t-\tau)]
\]

Using equations (23) and (24) it is possible to solve for the unique \(P^*\) and \(k_A^*\) consistent with steady state equilibrium.

We can depict \(P^*\) and \(k_A^*\) graphically as in Figure 2:

---

\textbf{Insert Figure 2 about here.}

---

Equation (23) describes a positive relation between steady state \(P\) and steady state \(k_A\) whereas equation (24) describes a negative relation between them. \(P^*\) and \(k_A^*\) are seen to exist uniquely. Current additions to the stock of skilled workers depends on expenditure on education \(\tau\) time periods ago whereas the current stock of skilled workers is a datum. Hence
the magnitude of the current investment in education determines the size of the unskilled labor force and, therefore, the steady state value of the capital-labor ratio in industry M. We can then solve for the steady state values of the price ratio and the capital labor ratio in sector A from two simultaneous equations. This completes the characterization of steady state as well as a proof of its existence and uniqueness.

**LOCAL STABILITY OF THE STEADY STATE**

We rewrite the steady state conditions as:

\[
\frac{1}{k_d P}[(1-\alpha)f_e(l-t_e) - (1/\eta N t)]r_{e(t-e)} = 0
\]

(25)

\[
f_M(l-t_e) - (1/\eta N t)[I_e(t-e)] = 0
\]

(26)

\[
[I_e(t-e)]/\eta - n^*[nN - I_e(t-e)/\eta] = 0
\]

(27)

We can then prove the following proposition:

**Proposition 3:** The long run steady state is a locally stable equilibrium.

**Proof:** See Appendix 2.

**COMPARATIVE STATICS OF STEADY STATE**

We finally examine the effects of an increase in current investment in education on the steady state properties of the model. Investment in education in the past is a datum but the current investment in education can be varied. From the government's budget constraint we know that a change in government investment in education implies a corresponding change in the tax rate on capital income. We can prove:

**Proposition 4:** An increase in current investment in education (an increase in \(t_e\)) raises the steady state ratio of skilled workers to unskilled workers in sector M, and lowers the steady-state values of \(P\) and \(k_d\).

**Proof:** See Appendix 3.

It is instructive to compare our results with the extant literature. The human capital literature associated with Findlay and Miyagawa is unable to analyze the dynamics of human capital accumulation. The dual economy literature associated with, among others, Marino and Amano, is unable to decipher the effects of human capital accumulation on the growth
characteristics of the economy. Thus, the synthesis presented in this paper is a generalization of both branches of the literature.

V. POLICY IMPLICATIONS

The model studied in this paper leads to interesting implications for role of tax policy in fostering economic growth in resource constrained overpopulated poor countries with dualistic economic structures.

The need for a tax policy to spur additional investment in education in a developing economy arises due to several factors. First, investment in education is a classic example of positive externalities. While a skilled worker benefits from increased income (as training increases productivity of a worker, and he/she can be paid more as a reward for the higher productivity), society too gains in different ways. Thus, gains from human capital formation are underestimated by unskilled workers. Moreover, recently, Ljungqvist (1995) has demonstrated that the investment in human capital is, in fact, riskier in underdeveloped countries, compared to developed countries. As a result, even if there were no constraints on borrowing, unskilled workers would underinvest in skill formation. Consequently, the government steps in to spur additional investment in education. This reasoning also applies to highly developed countries (such as, the United States) where governments at various levels finance education through taxation. Recently, Gary Becker (1992) argued that the US needs to upgrade its average labor force quality, and recommended that education be subsidized through a voucher plan and other means. Second, in an underdeveloped economy, the choice of training is further complicated by the existence of a rather rudimentary and imperfect capital market. That is, a worker, currently unskilled, cannot borrow from financial institutions to finance his or her training easily, and pay back in the future from increased income. This argument has been bolstered by a recent finding by Benabou (1994) that even minor differences in education technologies, preferences, wealth, or minor imperfections in capital markets can lead to low aggregate income or productivity growth. Thus, at least, for the preceding two reasons, the government of a developing economy has to step in to enhance investment in education. A recent cross-sectional empirical study by Romer and Barro (1990) suggests that
government policies play a significant role in influencing the rate of economic growth across countries.

In the case of a resource constrained economy, there are additional reasons for the government to step in. In such economies, there necessarily exists an excess demand for education as schooling facilities (itself a function of government investment) are limited and thus the access to education has to be rationed by the government. Moreover, since schooling requires an investment of time, current additions to the stock of skilled workers depends on investment in education made in the past ($\tau$ time periods ago). Thus, while the current stock of skilled workers is predetermined, government tax policy can influence the future stock of skilled workers. Finally, it must be emphasized that although the growth rate in the steady state is guided by the rate of growth of population, tax policy can certainly affect the characteristics of the steady state and the transitions to it. In particular, government tax policy can affect the split of the labor force (the stock of unskilled workers) into unskilled and skilled workers through investment in education. This human capital formation part is specially important since factors other than capital accumulation and population growth account for well over half of the growth in real national income. In our model, taxing skilled workers’ incomes to subsidize skill formation provides an alternative to other possible ways of financing, such as, deficit financing (which may raise interest rates) and increasing money supply (which may lead to inflation).

The Proposition 4 is of key importance in demonstrating how an increase in the tax rate ($t_\tau$) alters the characteristics of the growth model analyzed in this paper and fosters economic growth. Comparative steady state analysis suggest that a balanced budget increase in public investment in education (an increase in $t_\tau$) raises the steady state ratio of skilled workers to unskilled workers in sector $M$, and lowers the steady-state values of $P$ (price of commodity $M$ in terms of commodity $A$) and $k_A$ (the ratio of capital to unskilled workers in sector $A$).

An increase in the steady state value of the ratio of skilled to unskilled workers in the manufacturing sector suggests that this sector becomes more productive. Rising manufacturing output, consequently, leads to a decline in the relative price of manufacturing commodity (intersectoral terms of trade). On the other hand, capital to unskilled workers in the agricultural sector (the capital-labor ratio in this sector, as only unskilled workers are assumed to be used in
agriculture) falls. This suggests that the agricultural sector releases resources. The aforesaid results strengthen the conclusion that an increase in investment in education, financed by a tax increase, fosters economic growth by affecting the intersectoral terms of trade and characteristics of the steady state. Thus, government tax policy can play an important role in the development process of resource constrained LDCs whose economic structure is characteristically dual.

VI. CONCLUDING REMARKS

The existing literature on dualistic economic growth has neglected human capital formation. Our paper blends the dual economy literature and the public finance literature through an examination of the role of taxation on human capital accumulation in developing economies. In this paper, we have examined the dynamic long run and static short run properties of a dual economy with skilled and unskilled labor. Our analysis of the short run equilibrium revealed that the structure of production in the manufacturing sector is important. Moreover, for a whole class of production technology, it was shown that a unique short run equilibrium will exist. A comparative static analysis suggests that the capital-labor ratio in the agricultural sector would (i) rise when the ratio of capital to unskilled labor in agriculture or manufacturing rose, and (ii) fall when the stock of skilled workers rose. The capital-labor ratio in the manufacturing sector would (i) rise when the ratio of capital to unskilled workers in manufacturing or the stock of skilled workers rose, and (ii) fall when the ratio of capital to unskilled workers in the agricultural sector rose. We further discovered that given current investment in education, the ratio of skilled labor to unskilled labor and the capital-labor ratios in the two sectors are uniquely determined in the steady state. Moreover, if there is positive addition to the stock of skilled workers today, the steady state is locally stable.

The comparative steady state properties of the model for changes in balanced budget increases in current investment in education were also analyzed. An increase in tax rate was shown to lead to a rise in the steady state value of the ratio of skilled to unskilled workers in manufacturing, and a fall in the relative price of manufacturing commodity and the capital-labor ratio in agriculture. These resulted in an interesting implication for the role of government tax
policy in LDCs -- the government, through an increased investment in public education (financed by a tax on capital income and/or incomes of skilled workers), can foster economic growth in these countries. Taxing skilled workers' incomes to subsidize skill formation provides an alternative to other possible ways of financing, such as, deficit financing (which may raise interest rates) and increasing money supply (which may lead to inflation).

The analysis of this paper can be extended to cover the case when skilled workers are required in the agricultural sector as well. An obviously interesting extension is the determination of the optimal current investment in education. These constitute a partial agenda for future research in this area.
APPENDIX 1

COMPARATIVE STATICS OF SHORT RUN EQUILIBRIUM

Using (17) by Cramer’s rule we can solve:

(I) \[\frac{dk_A}{dz_A} = \frac{1}{\Delta} \left[ f_A'(k_A) (I-z_A/k_A) - (k_A^2/k_A) \left\{ \frac{\partial C_A}{\partial P} (\partial P/\partial k_M) - (N^*/N^*) (\partial P/\partial k_M) \right\} \right] > 0\]  \hspace{1cm} (A.1.1)

(ii) \[\frac{dk_M}{dz_M} = \frac{1}{\Delta} \left[ (\partial C_A/\partial P) (\partial P/\partial k_M) + (N^*/N^*) (\partial P/\partial k_M) \right] + \left( \frac{\partial f_M}{\partial f_M} \frac{f_M}{f_M} \right) > 0\]  \hspace{1cm} (A.1.2)

(iii) \[\frac{dk_A}{dN^*} = \frac{1}{\Delta} \left[ (1-z_A/k_A) (C_A(.)) \right] < 0\]  \hspace{1cm} (A.1.3)

It follows from this that the capital-labor ratio in sector A declines as more workers become skilled.

(iv) \[\frac{dk_M}{dz_A} = \frac{1}{\Delta} \left[ f_A'(k_A) \left\{ f_A'^2/k_A - (f_A'/k_A^2) z_A \right\} \right] - \left( \frac{\partial C_A}{\partial P} (\partial P/\partial k_M) (I + N^*/N^*) \right) + (f_A'/k_A) (z_A/k_A^2) < 0\]  \hspace{1cm} (A.1.4)

(v) \[\frac{dk_M}{dz_M} = \frac{1}{\Delta} \left[ f_A'^2 z_A/k_A - (f_A'/k_A^2) z_A \right] - \left( \frac{\partial C_A}{\partial P} (\partial P/\partial k_A) (1 + N^*/N^*) - (\partial C_A/\partial w^*) (\partial w^*/\partial k_A) \right] > 0\]  \hspace{1cm} (A.1.5)

(vi) \[\frac{dk_M}{dN^*} = \frac{1}{\Delta} \left[ -(z_A/k_A^2) (C_A(.)) \right] > 0\]  \hspace{1cm} (A.1.6)

Thus the capital-labor ratio in sector M increases as more workers acquire skills.
APPENDIX 2

LOCAL STABILITY OF STEADY STATE EQUILIBRIUM:

We can derive the Ruth-Hurwitz matrix of the system of equations (25), (26) and (27) using (23) as:

\[
J = \begin{bmatrix}
J_{AA} & J_{AM} & J_{AN} \\
J_{MA} & J_{MM} & J_{MN} \\
J_{NA} & J_{NM} & J_{NN}
\end{bmatrix}
\]

where:

\[
J_{AA} = \frac{1}{k_A^2} \left[ (1 - \alpha) (1 - t) \right] [f_M - k_M f_M - nt f_M] < 0,
\]

\[
J_{AM} = \frac{1}{k_A} \left[ (1 - \alpha) (1 - t) \right] [-k_M f_{MM}] > 0,
\]

\[
J_{AN} = \frac{1}{\alpha k_A} \left[ (1 - \alpha) (1 - t) f_M \right] + \frac{1}{(\eta N_M^w (n^2))} \left[ I_E (t - \tau) \right] > 0,
\]

\[
J_{MA} = 0,
\]

\[
J_{MM} = (1 - t) f_{MM} > 0,
\]

\[
J_{MN} = \frac{1}{(\eta N_M^w (n^2))} \left[ I_E (t - \tau) \right] > 0,
\]

\[
J_{NA} = 0,
\]

\[
J_{NM} = 0,
\]

\[
J_{NN} = -[nN - I_E(t)/\eta] < 0 \] (if total additions to population is greater than the current addition to the number of people undergoing training. In other words, there must be some increase in the size of the unskilled labor force).

A sufficient condition for trace \( J \) < 0 is that \( J_{NN} < 0 \). Moreover

\[
\begin{vmatrix}
J_{AA} & J_{AM} \\
J_{MA} & J_{MM}
\end{vmatrix} > 0.
\]

Further the determinant of \( J \) is negative when \( J_{NN} \) is negative. If this is the case the Ruth-Hurwitz condition for local stability are satisfied. (See Gandolfo (1980) pg. 276). From equation (27), however, \( [nN - I_E(t)/\eta] \) will be positive (and hence \( J_{NN} \) will be negative), so long as there was some positive investment in education at time \((t-\tau)\). Hence the dualistic growth model is seen to possess a unique steady state which is locally stable.
APPENDIX 3

COMPARATIVE STATIC EFFECTS OF AN INCREASE IN INVESTMENT IN EDUCATION:

Current investment in education is $P_{fMI, K_M^t}$. Substitute this into equation (22) and rewrite (22), (23), and (24) as:

\[ n^t = \left[ 1/(\eta nN - P_{fMI, K_M^t}) \right] [E(t-t)\eta] \]  
\[ P = [\alpha f_M][1/(f_M - k_{f_{M1}} - n^t f_{M1})] \]  
\[ P_{k_A} = [1/(I_E(t-t))][(1-\alpha)f_A(1-t)\eta N^t] \]

To examine the comparative steady state properties we recall that a change in current education expenditure is tantamount to an increase in the tax rate on capitalists $t_r$. Assuming that the cross products $f_{M1} = f_{M2} = 0$ (alternatively, one could assume that $f_{M1} = f_{M2} > 0$ but small), we totally differentiate equations (A.3.1) and rearrange to get:

\[ dn^t = \left[ 1/(\eta nN - P_{fMI, K_M^t}) \right] [E(t-t)\eta] [f_{MI}, K_M^t, dP \]
\[ + PK_{M}f_{M1,M2}dk_M + P_{fMI^t}dK_M + P_{fMI,K_M^t}dt_r] \]  
\[ (A.3.4) \]

Upon totally differentiating equation (A.3.2) we have

\[ dP = [1/(f_M - k_{f_{M1}} - n^t f_{M1})][\alpha f_M^t dk_M] \]
\[ + [1/(f_M - k_{f_{M1}} - n^t f_{M1})][\alpha f_A^t(k_{f_{M1}}dK_M + n^t f_{M2}^t dt_r)] \]  
\[ (A.3.5) \]

Upon totally differentiating (A.3.3) we have

\[ k_A^t dP + Pdk_A = [(1-\alpha)\eta/I_E(t-t)]f_A(1-t)N^t \]
\[ - f_A^t N^t dt_r + (1-t_r)N^t f_A^t dt_M^t \]  
\[ (A.3.6) \]

The equation system given by (A.3.4), (A.3.5), and (A.3.6) is solved for $dn^t/dt_r$, $dP/dt_r$, and $dk_A/dt_r$.

We now assume that all total derivatives except those associated with $dn^t$, $dP$, $dk_A$, and $dt_r$ are zero. (We cannot assume, however, that $dN^t = 0$ since $n^t = N^t/N_M^t$. For simplicity we assume that all changes in $n^t$, i.e., $dn^t$ comes from changes in $N_M^t$).

Using these assumptions we define the following notations:
\( z_{11} = (\eta nN - P_{f m} f_M d)^2 > 0, \)
\( z_{12} = -[I_E(t-\tau)\eta P_{f m} f_M d] < 0, \)
\( z_{13} = 0, \)
\( R = [I_E(t-\tau)\eta P_{f m} f_M d] > 0, \)
\( z_{21} = -[\alpha f_A n f_{m,2} d] > 0, \)
\( z_{22} = [f_M^2 k_M f_{m,1} - n f_{m,2} d]^2 > 0, \)
\( z_{23} = -[\alpha f_A^2 f_{m,1} k_M - n f_{m,2} d] < 0, \)
\( z_{31} = 0, \)
\( z_{32} = [k_A I_E(t-\tau)] > 0, \)
\( z_{33} = [(P I_E(t-\tau)-(1-\eta)\eta N)(1-t) d f_A^2] = ?, \)
\( S = -(1-\eta)\eta N d f_A^2 < 0. \)

We can now write equations (A.3.4) through (A.3.6) in matrix form as:

\[
\begin{bmatrix}
  z_{11} & z_{12} & z_{13} \\
  z_{21} & z_{22} & z_{23} \\
  z_{31} & z_{32} & z_{33}
\end{bmatrix}
\begin{bmatrix}
  dnt' \\
  dP \\
  dk_A
\end{bmatrix}
= \begin{bmatrix}
  R \\
  0 \\
  S
\end{bmatrix}
\quad (A.3.7)
\]

Let the determinant of the matrix on the left hand side of (A.3.7) be \( \delta \). Applying Cramer's rule:

\[
dnt'/dt = (1/\delta)
\begin{vmatrix}
  R & z_{12} & 0 \\
  0 & z_{22} & z_{23} \\
  S & z_{32} & z_{33}
\end{vmatrix} = \delta_1/\delta \quad (A.3.8)
\]

\[
dP/dt = (1/\delta)
\begin{vmatrix}
  z_{11} & R & 0 \\
  z_{21} & 0 & z_{23} \\
  0 & S & z_{33}
\end{vmatrix} = \delta_2/\delta \quad (A.3.9)
\]

and
\[ \frac{dk_A}{dt} = \frac{(I/\delta)}{\begin{vmatrix} z_{11} & z_{12} & R \\ z_{21} & z_{22} & 0 \\ z_{31} & z_{32} & S \end{vmatrix}} = \frac{\delta y}{\delta} \] (A.3.10)

Now \( \delta = z_{11}(z_{22}z_{33} - z_{23}z_{32}) - z_{21}(z_{12}z_{33}) \). The only sign that needs to be worked out is that of \( z_{33} \). Now, it can be checked that a sufficient condition for \( z_{33} \) to be positive is that \((1-\alpha)(1-\epsilon)\) is small in relation to \( f\sum_{j=1}^{J} (1/\eta N^j f^j_A) \sum_{j=1}^{J} (\alpha f^j_A \epsilon(t-\tau)) \). We make this assumption to get \( \delta > 0 \).

Now, \( \delta_t = R(z_{22}z_{33} - z_{23}z_{32}) + S(z_{12}z_{33}) > 0 \) (given that \( \alpha < 1, \epsilon < 1 \), and assuming \( k_A > f_A \)). Hence \( dP/dt, > 0 \).

Evaluating \( \delta_2 \), we have \( \delta_2 = -z_{11}Sz_{23} - z_{21}Rz_{33} < 0 \). Hence \( dP/dt, < 0 \).

Now evaluating \( \delta_3 \), we have \( \delta_3 = z_{11}z_{22}S - z_{21}(z_{12}S - z_{32}R) \). We can rewrite this as \( \delta_3 = z_{11}z_{22}S - z_{21}z_{12}S + z_{21}z_{12}R \). The first two terms seem to outweigh the third. If this is the case then \( \delta_3 < 0 \), yielding \( dk_A/dt, < 0 \). This establishes Proposition 4.
Figure 2

Equation (23)

Equation (24)

$k_A^*$

$P^*$

$P$

$k_A$
NOTES

1. Findlay (1984), Findlay and Kierzkowski (1983), and Miyagiwa (1989) have examined the role of human capital accumulation in developing countries, employing two sector models. These studies are, however, all static and can, therefore, shed very little light on the dynamics of human capital accumulation and economic growth. On the other hand, models of economic growth in developing dual economies have typically disregarded human capital accumulation. This is true both of the traditional dual economy literature (see, for instance, Marino (1975), and Amano (1980)) and its Harris-Todaro variant (see, for instance, Jha and Lachler (1981)). Since factors other than capital accumulation and population growth account for well over half the growth of real national income, this exclusion is especially significant. The literature on economic growth with human capital accumulation in both its endogenous growth and non-endogenous growth versions seems inapplicable to developing countries that exhibit the salient characteristics associated with dual economies.

2. In our view, the use of an endogenous growth model is not appropriate in the context of LDCs. This is because in LDCs, education is rationed and not a matter of choice. A recent work has cast some doubt on the validity of the endogenous growth model to LDCs -- a study by Allwyn Young (1995) shows that factor accumulation and total factor productivity growth along the lines of an exogenous growth model is more consistent with the growth experience of East Asian countries.

3. Past and current investment in education limit the production of skills. In an infinite horizon model like the present one, everyone would like to be a skilled worker since the wage of skilled workers is higher than that of unskilled workers. We imagine, therefore, some kind of rationing of available educational opportunities among applicants on the basis of a lottery.

4. The form of taxation plays no important role here. We work with proportional taxation and inelastic factor supplies. We may as well have worked with lump sum taxation without affecting the results.

5. The technology for producing skills in this paper is similar to that used by Miyagiwa (1989) and Findlay and Kierzkowski (1983). These authors have used a production of skills with time and commodity A (hence, indirectly, capital) as inputs. In a sense, the analysis contained in this paper is like the endogenous technical progress literature associated with, among others, Conlisk (1969), and Drandakis and Phelps (1966).

6. The treatment of land as a fixed factor in our model is analogous to its treatment by Jorgenson (1961; pages 313-314).

7. The time, $\tau$, for acquiring skills is assumed fixed and unalterable.

8. Relaxing this assumption and admitting a more general production function for skills clutters up the analysis without qualitatively affecting the results.

9. In Findlay and Kierzkowski (1983), the amount of time spent on education is a choice variable of the individual consumer. This stipulation is absent from Miyagiwa (1989). In the context of most developing countries where existing capacity severely restricts availability of education it is, perhaps, more appropriate to assume that education is rationed among the various applicants.

10. In what follows we work with a proportional tax on capitalists and assume inelastic supplies of factors of production. It is assumed that the government decides how much to spend on education and finances this expenditure by taxing capitalists. The government's budget is always balanced.

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11. It is easy to show that the following analysis would go through with only minor modifications even if we had assumed that workers in sector A get paid their marginal product, i.e., $w_A = f_A' - k_A f_A'$. We follow the stipulation in equation (3) in order to conform more closely to the traditional dual economy literature.

12. In sector A, market clearing requires that equation (4) be satisfied. Now write the equations of the disbursals of outputs of sectors A and M. Add them and subtract equation (4) of the text to realize that when the market for A clears so does the market for M. This is independent of the assumption made about investment.

13. It is not necessary to assume that all capital income is invested and all wage income consumed. This assumption is made purely to simplify the analysis.

14. Once again, the assumption being made here is that all capital income in sector M is invested.

15. We owe our gratitude to an anonymous referee for this assumption.

16. The cross product term $f_{xcm}$ is a second order term and thus relatively small in magnitude.
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