THE RESEARCH ASSESSMENT EXERCISE AND TRANSFER OF ACADEMICS AMONG DEPARTMENTS

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Introduction

In these times of budget cutting, government departments are looking into performance based funding schemes for public institutions, such as universities and hospitals, to obtain more output (research or patient care) for the same amount of total funds. A key difference from other applications of relative performance schemes lies in the fact that here the performance of institutions depends on their personnel who can move from one institution to another. Thus the incentives that are meant for individual researchers or doctors are provided via institutions and labor markets. This paper is a first attempt in the literature to look at such schemes.

The British government launched the research assessment exercise (RAE) to rank all academic departments of universities according to the research output of their faculty. These rankings are used to determine the funding of universities. Other considerations in the funding decision are based on the number of students and factors that are unrelated to the quality of research staff. In determining the research quality in the latest RAE, the government looks at the publications of the current members of staff regardless of where they have conducted the research. Thus the research capability of a department, as measured by the RAE, depends on the number and quality of researchers currently on its payroll. As the third RAE, scheduled for March 1996, approaches, departments are transferring researchers with successful publication records in their quest to boost or defend their RAE rankings.

Departments are of course motivated by the prestige of higher rankings, but an additional motivation is the extra money from a higher ranking that can be used to promote faculty members more rapidly, increase their pay, or provide non-pecuniary benefits (eg, travel and research funds or more teaching assistants). In this respect, most departments behave like labor-managed firms, while academics are trying to find a department that best balances their concerns about being in a good department and getting higher pay. My main interest in this paper is the impact of the RAE on the quality differences among the departments. In other words, does competition through RAE increase existing inequalities among departments?

Outline

In answering this question, I first ignore any effects of the RAE on the productivity of academics. By its design, RAE considers only the existing publications of the members of a department. In almost any branch of science, the publications of an academic that will be admissible in the RAE are known six months or one year ahead of the RAE. So every department can accurately judge the possible impact on its research ranking of transferring a researcher. Therefore, the transfers simply reallocate the academics and their assessable output among the departments. It turns out that departments can follow different strategies depending on their initial research capacity: those with a very high or a very low number of researchers will lose some to departments that are trying to upgrade. I also found that the variation among top-ranked departments will be higher than the variation among lower ranked departments. I then look at the comparative statics of the model and how the competitiveness of the scheme affect the final number of top-ranked departments.
In the next section, I incorporate the production of research and analyze how the RAE may impact research productivity in different departments. The results indicate that incumbents in expanding and inactive departments work less and get more out of the scheme. In the conclusion I discuss my results in relation to the actual implementation of the RAE.

Related Literature

Here I briefly discuss different strands of literature and discuss why the analysis of the RAE cannot be conducted within the existing frameworks. The literature on the economics of sports leagues considers profit maximizing clubs making offers to athletes of different abilities (El Hodiri and Quirk 1971). The reasons why the sports league framework cannot readily be applied to academic departments under the RAE are as follows: First, most academic departments behave more like labor managed firms than profits maximizers. Second, as the teams in a sports leagues teams are in direct competition with one another, there are very high rewards for being marginally better than others. These high rewards result in high transfer fees for good players and in some cases lead to destructive competition (Whitney 1993). In the RAE, there is no competition among the departments in the same category. There is however competition among the marginal departments to make the cut for the higher category, yet a few good researchers can always make up for one big superstar and thus there are no superstars with very high the marginal contributions. Thirdly, the main controversy of the sports literature is about whether winning percentages or championships generate fan interest (Whithney 1988) which is completely irrelevant for the RAE. Finally, the sports literature is not concerned with the asymmetries among clubs resulting from where players currently are (they can be hired at the spot labor market), whereas in academia the costs of reallocation are quite high relative to the pay and there are marked differences between places that already have researchers and others that do not.

My paper is of course closely related to the literature on tournaments and prizes (Nalebuff and Stiglitz 1983), piece rates (Lazear 1986), and also incentives in basic research (Lazear 1996). In all these models, utility maximizing agents directly react to the incentive schemes offered by principles who are incapable of perfect monitoring. In schemes like RAE, although the main purpose is to induce higher research output, the incentives to the agents are intermediated through the institutions. The competition among the institutions determine expected pay and agents react to it. The institutions may use internal incentive schemes, but their usefulness in the presence of labor markets is rather doubtful.

My decision to treat academic department as labor managed firms is based on casual empiricism. On the question of how do labor-managed academic departments hire new researchers, two possibilities are considered in the literature: accepting them as new members or employing them with no membership rights (see Ireland 1987). I have chosen the middle road: departments extend full membership privileges to new researchers, but can also offer them extra incentives. In the conclusion I discuss how the administrators in universities may conflict the decisions of the labor managed departments.

A Model of Transfers

Each academic department has m staff members; m is fixed and the same for all departments. There are two types of staff members:

- "teachers" only do teaching, they have no research output and they can be hired at the spot labor market for a fixed wage of W;
- "researchers" do both teaching and research. I assume that all researchers are identical.

Let n denote the number of researchers in a department, $n \leq m$. I assume that each department is like a labor managed firm, run by the researchers to maximise their own average utility. A department with n researchers hires $m - n$ teachers from the spot labor market and pays them a total of $(m - n)W$. I assume that the only source of income for the universities is government funds allocated through the RAE which classifies departments into 3 categories:

- an A-department has a total research output of a or higher (ie, has a or more researchers among its m members) and gets funds equal to $Y = A + mW$ from the RAE;
- a B-department has a total research output of less than a but more than b and gets $Y = B + mW\ (A > B)$;
- a C-department has a total research output of less than b and is reimbursed only for the salaries of its teachers: $Y = mW$.

To sum up, a department with n researchers gets

$$Y(n) = \left\{ \begin{array}{ll} A + mW, & n \geq a\,, \\ B + mW, & a > n \geq b\,, \\ mW, & n < b\,. \end{array} \right\}$$

Since A and B are positive, researchers in A- and B-departments always get a premium over the teachers. More specifically, the income per researcher in department with n researchers would then be

$$y(n) = \frac{Y(n) - (m-n)W}{n}$$

Furthermore, I assume that a researcher transferring from one department to another incurs a private cost of relocation of C.

I assume that each department is atomistic and there is a continuum of departments indexed by the initial number of researchers they have: $n \in [b, m]$. Let $F(n)$ be the cumulative density. For simplicity, I normalise the total mass of departments to 1. The government allocates a fixed amount of funds G for all departments (above the teaching allowance of mW for each department). As the total mass in normalised to 1, G is also the average funding. The government also fixes $\{A, B\}$ and the minimum research quality b. The total funds are, of course, not sufficient to pay A to all departments: $A > G > B$. The minimum amount of research output to qualify to be an A-department (denoted by a) endogenously adjusts to satisfy the budget balance and labor market equilibrium conditions. This is consistent with the actual implementation of the scheme; in classifying departments, assessors rely on comparisons rather than absolute criteria.

Equilibrium of the Model of Transfers

Let me sum up. The exogenous parameters of the model are:

m: fixed number of staff each department must retain (number of researchers plus the teachers);

W: wage rate for teachers;

C: a researcher's cost of reallocating from one department to another.

The government announces the RAE scheme by specifying G, A, B, and b (average funding, funding for an A-department, funding for a B-department, and the minimum amount of research output to qualify as a B-department). If $C = 0$, the initial distribution of researchers among the departments would not have mattered, and the equilibrium would be achieved by equalizing the income per member in B-departments to that in A-departments. With $C > 0$, departments may follow five different strategies to adjust the number (percentage) of researchers. In Figure 1, I show the departments pursuing different strategies depending on the initial number of researchers they have. Before deriving formal conditions, I briefly describe each strategy and the kind of departments that follow it:

- The top departments (those with high n) have too many researchers and they are not awarded any additional funds for those in excess of a. Some of their researchers will leave and they will shrink (in terms of research output) to size α as shown in Figure 1.
- Some good departments will have sufficient funds so that the pay in the job market will not attract their researchers and they will retain all their original researchers. These department will be inactive in the labor market.
- Some departments do not have enough research output to be A-departments. They can attract new researchers to expand just enough to qualify as A-departments. (These are the departments with initial number of researchers $n \in [\beta, a]$ in Figure 1.)
- Some "bad" departments cannot benefit from the expansion strategy. They will therefore shrink their research output to the minimum b to qualify as B-departments.
- Furthermore, if the market for researchers is tight, some of the shrinking B-departments may lose all their researchers. Let ϕ be the percentage of B-departments that are relegated to C-classification. ϕ is an endogenous variable determined by the budget balance and labor market equilibrium.

[Figure 1]

I now analyze formally the ranges of departments that follow these strategies. For the researchers who leave their departments, the law of one-price holds: a transferring researcher gets V, which is to be determined by the equilibrium conditions. Departments that have more researchers than necessary to remain in their category will shrink to the point that the remaining members get $V - C$ and are indifferent between leaving and staying. A-class departments with more than α researchers will shrink to α, where α is determined by the following equality:

$$\frac{A + \alpha W}{\alpha} = V - C \ . \tag{1}$$

Note that as researchers double up as teachers, the department saves αW from the allowance for teaching. Similarly, a B-class department with more than b researchers, but not enough to warrant an expansion into A-class, will shrink to increase the remaining members' income to their outside opportunity: $V - C$. But if V is too high, ϕ percent of the shrinking B-departments will be completely deserted by their researchers. This extra supply of researchers from the C-departments will lower V and ensure that the income of B-class departments with b researchers is just enough to keep them:

$$\frac{B + bW}{b} = V - C \ . \tag{2}$$

The other option for a B-department is to expand to be an A-department. One problem is that after expansion, income per member may be above $V - C$, thus implying no outflows, but not high enough to attract new members. To make up for this shortfall, the department pays new transferees a lump sum "transfer fee." This transfer fee (a department with n existing members pays a transfer fee of $r(n)$) supplements the transferee's share of department income so that his final income becomes V:

$$\frac{A + aW - (a-n)r(n)}{a} + r(n) = V$$

The first term is the income per member: the department gets $A + aW$, net of teaching expenses, and pays out the transfer fee to $(a - n)$ new researchers. This amount is shared by all researchers. The transferees get $r(n)$ and their final income is V. The cost of attracting new researchers will be lower when a department is already close to the cutoff level a. As income of existing members from expansion is increasing in n, there exists a marginal department with β researchers that is indifferent between expansion and shrinking to size b:

$$\frac{A - (m-a)\,W - (a-\beta)\,V}{\beta} = \frac{B - (m-b)\,W}{b} \qquad (3)$$

We have thus formally defined each one of the marginal types (β, a, and α) in Figure 1. To determine the income level of transferees, V, and the percentage of B-departments relegated to C-classification, we use the budget balance:

$$B\,(1-\phi)\,F(\beta) + A\,(1-F(\beta)) = G \qquad (4)$$

and the labor market equilibrium (supply equals demand):

$$\varphi\, b\, F(\beta) + \int_b^{\beta} (n-b)\,f(n)\,dn + \int_{\alpha}^{m} (n-\alpha)\,f(n)\,dn = \int_{\beta}^{a} (a-n)\,f(n)\,dn \qquad (5)$$

The last equation reflects the three sources of supply of researchers (all researchers who used to be in C-departments, shrinking B-departments and shrinking A-departments) all go to expanding B-class departments. We can solve Equations (1)- (5) to determine β, a, α, V, and ϕ. The resulting distribution of researchers among the departments is shown in Figure 2. There is some variety among the top departments: perviously better ones are still better than the new A-departments. However, there is no such differentiation among the ranks of the B-departments. Consequently, the first prediction of the model is that there will be more variation among A-departments than B- or C-departments.

[Figure 2]

The final income of the remaining members of departments is shown in Figure 3, which suggests the following conclusions: First, the researchers are as well off in the top A-departments as in B-departments. Second, existing members of expanding and inactive departments do better than the others. Finally, the closer a department to the cutoff point a, the better off are its researchers.

[Figure 3]

Comparative Static Analysis

In conducting the comparative static analysis of the model, I am mainly concerned with the effects of using more competitive schemes (ie, schemes with higher A). The first observation is that given the stock of bottom departments that may dissolve to release more researchers, the income of transferees is fixed by Equation (2):

$$V = \frac{B}{b} + W + C$$

Substituting this in Equation (1), we get the following expression for α:

$$\alpha = b\frac{A}{B}$$

Increasing A (or lowering B) will increase α: fewer A-departments will shrink and the top quality in the final distribution will be higher. Moreover, substituting the above expression for V into the indifference condition for the marginal B-class department with β members (Equation (3)), we obtain

$$a - \beta = \frac{A - B}{\left[\frac{B - mW}{b} + C\right]}$$

Thus increasing A will increase the range of departments that are expanding to A-class. Note that this range may increase because a increases faster than β (departments with $a > n \geq \beta$ expand to become A-departments). In that case, increasing A will lower the number of A-departments, but more researchers will be transferred. To figure out what actually happens to the number of A-class departments, we have to look at the comparative statics for β (see the Appendix for the details). The first unambiguous result is that increasing A will increase a, and thus raise the standard of A-class departments. Consider the mass of all expanding and no-action departments (given by $F(\alpha)$ - $F(\beta)$) that must be sorted out by the RAE. The higher this mass, the higher will be a (the minimum standard to be an A-department). If the cutoff point a is around the mode of the distribution, further increases in A will have a bigger impact on a. This property follows from the budget balance equation: the higher the number of departments in contention for an A-class berth, the higher will be the standard.

When a goes up faster than A, expansion will no longer be attractive to marginal B-departments and they will shrink rather than expand. Thus when the current scheme divides the departments somewhere close to the mode of the distribution, increasing A will lower the number of A-class departments.

To be more specific, increasing A will increase β only if the mass of inactive departments is bigger than the mass of non-A departments (ie, $F(\alpha)$ - $F(a) > F(\beta)$). Increasing β will also shrink the range of A-departments (but as this range will be shifting, the impact on the mass depends on the distribution). The relative masses of inactive and non-A departments depend on how stringent the budget constraint is: a relaxed budget constraint (A is not much higher than G) would create a larger mass of inactive departments. Note that the stringency of the budget depends on A/G, whereas the competitiveness of the scheme depends on A/B. Increasing A will make the budget constraint more stringent. If we start with a relaxed budget constraint and keep on increasing A, we will first observe increasing β and then, beyond a certain value of A, this will be reversed. See Figure 4 for an illustration of this point. If an objective of the scheme is to have more A-departments, there must either be a relaxed budget with little competition or a strict budget and considerable competition (in this case there will be many C-departments). This objective is also consistent with the objective of having a wider range of A-departments.

[Figure 4]

The Model with Research Output

What is the impact of the RAE on the research productivity? To answer this question, I will augment the previous model by an initial period in which researchers conduct research before the transfer market opens. For the sake of simplicity, I will set $W = 0$. Only research conducted in this initial period (like the five year limit in the RAE) counts for the RAE. The sequence of events is as follows:

- The government announces the RAE scheme $\{A, B, G, \Gamma_b\}$ at the beginning of period 0. Note that the criterion to be a B-department is now the total amount of research conducted by the staff of the department (including any transferees) in period 0 (denoted by Γ_b).
- Researchers, anticipating the transfer fees and the strategies that will be followed by their current departments, decide on how much research to conduct. Let γ_i denote the amount of research conducted by researcher i during this period. I assume the cost of conducting research to be $\gamma^2/2\theta$ and the same for all researchers.
- In period 1, the transfer market opens and the minimum amount of total research required from an A-class department (Γ_a) is determined.

In period 1 there may be researchers with different γs on the market. Equilibrium requires that a department's cost of acquiring a fixed research capacity be independent of the γs of the researchers it hires. Therefore a transferee with γ will receive $V\gamma$ where V is the income of a researcher with unit research output. Given the expected V (with perfect foresight, as in my model, it is the actual V that will prevail) all researchers who are going to the job market will maximize:

$$\underset{\gamma}{\text{Max}} \left[V\gamma - \frac{\gamma^2}{2\theta} \right] \quad \Rightarrow \quad \gamma = \theta V$$

For the ease of comparison with the previous model, I fix $\Gamma_b = 1$ and $\theta = 1/V$ so that each researcher on the job market has $\gamma = 1$ and gets a final payoff of $V/2 - C$. I now look at the equilibrium in each type of department by specifying a two-period game with the outside option to go to the job market, which is open to all researchers.

Shrinking B-Departments

As the RAE scheme fixes the minimum amount of research output to qualify for the B-class berth at $\Gamma_b = b$, the equilibrium for the researchers in any non-expanding B-class department is to set $\gamma = 1$ and shrink to b researchers. Those who leave and those who stay get the same amount $V - C$:

$$\frac{B}{b} = V - C \tag{1*}$$

Shrinking A-Departments

Researchers in a shrinking A-department engage in a game the payoffs of which are determined by the job market equilibrium (which they take as given). In the first stage, each researcher chooses his research output. In the second stage, he decides whether to stay or leave. In the subgame perfect Nash Equilibrium, each staying researchers' income must be the same or lower than the leaving researchers' income. Otherwise some of the leaving researchers would rather stay. Any researcher that leaves the department sets his research output at the utility maximising level $\gamma = 1$. The total research output of staying researchers must exactly be equal to Γ_a. Otherwise, at least one of them can lower his research output without affecting his or others' income. If the equilibrium number of staying members is α, they will share among themselves the research output necessary to get an A-class berth (denoted by Γ_a) and set $\gamma = \Gamma_a/\alpha$. Finally, in the first stage of the game, each member must be indifferent between staying (setting $\gamma = \Gamma_a/\alpha$) and leaving (setting $\gamma = 1$):

$$\frac{A}{\alpha} - \frac{V}{2}\left[\frac{\Gamma_a}{\alpha}\right]^2 = \frac{V}{2} - C \tag{2*}$$

The first term is the income per member and the second term is the cost of research output of Γ_a/α. For departments with fewer than α researchers, no one will have the incentive to leave for the job market. But, the fewer the number of researchers, the higher will be the research load per researcher necessary to reach Γ_a. This gives us a lower bound on the number of researchers that can be kept in an A-department. Departments with fewer members will have to attract researchers to get an A-class berth. Thus the lower bound on number of researchers that can be kept in an A-department corresponds to the cutoff point a of the previous model and given by the following equation:

$$\frac{A}{a} - \frac{V}{2}\left[\frac{\Gamma_a}{a}\right]^2 = \frac{V}{2} - C \qquad (3^*)$$

This equation is identical to the one for α. It is a quadratic equation: the smaller root gives us a and the larger root α.

Expanding Departments

Consider the game played among the members of an expanding department. Researcher i's best response to everyone else producing γ units of research is to conduct only $1/n$ units of research:

$$\underset{\gamma}{\text{Max}} \; \frac{A - (\Gamma_a - (n-1)\gamma - \gamma_i)V}{n} - \frac{V\gamma_i^2}{2} \Rightarrow \gamma_i = \frac{1}{n}$$

The total research output of the n-original members of the department is therefore $(n - 1)\gamma + \gamma_i$. They obtain the extra research output at V per unit to become an A-class department and get A. The resulting utility from the Nash Equilibrium of the expansion game must be higher than or equal to what a researcher can get by going to the job market. But with only $1/n$th the effort, an expanding department will never reach that level in a symmetric Nash Equilibrium.[1] There may be a symmetric mixed strategy equilibrium that achieves the utility level of the outside option. With mixed strategies, there is always a positive probability that the department will find itself classified as B and some researchers will be stuck with too low levels of research to benefit from the job market. Considering the substantially long period over which research is conducted and the ability of researchers in the same department to observe each other's output, the following asymmetric pure strategy equilibrium is more plausible than a mixed strategy equilibrium: in a department with n researchers, μ percent

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[1] Neither is everyone in the department setting $\gamma = 1$ a Nash Equilibrium: in that case the income per member will exceed V - C (by the definition of β, as long as $n \geq \beta$, expansion will generate strictly higher income than V - C). A researcher may deviate from this by cutting down his output: the average income falls by a factor of V/n while his cost savings are first order.

of them set $\gamma = 1$ and the remaining $(1 - \mu)$ percent set $\gamma = 1/n$, and μ is such that the final income is sufficient to compensate the high output researchers:

$$\frac{A - (\Gamma_a - \mu n - (1-\mu) n (1/n)) V}{n} = V - C$$

Let me show that this is an equilibrium: low-output researchers are on their best response function, thus they have no profitable deviations. If a high-output researcher reduces his output, the department will not be able to generate enough income to keep the other high-output researchers and the final income for all remaining members will be much lower and he will get less than $V/2 - C$ in the job market. Hence there is no profitable deviation for the high-output researchers.

Note that the equilibrium μ is decreasing in n:

$$\mu = 1 + \frac{\Gamma_a V - A - n C}{(n-1) V}$$

Thus the department that is indifferent between expansion and shrinking to ϕ_b must have $\mu = 1$. The final utility for the high-output researchers is independent of n. At the cutoff point $n = a$, the utility from sharing Γ_a equally among the researchers must provide the same level of utility for the high-output researchers. This is consistent with the above equation for a. Finally, the marginal B-department that will be indifferent between shrinking and expanding satisfies:

$$\frac{A - (\Gamma_a - \beta) V}{\beta} = V - C \quad \Rightarrow \quad \beta = \frac{A - \Gamma_a V}{C} \qquad (4^*)$$

The model can be closed by using the budget balance and labor market equilibrium conditions. There are several important conclusions to be drawn:

- The average output of an incumbent member of an expanding department is less than the output of a transferring researcher.

- The minimum level of total research output required to get an A-class berth is less than a: a member of a non-shrinking and -expanding department conducts less research than a transferring academic.
- The higher the number of transferring researchers, the higher is the average research output of the whole university system. There is however an associated welfare loss from higher total costs of reallocation.

Conclusions

The apparent reason for using schemes like Research Assessment Exercise (RAE), rather than incentive schemes for individual researchers, is that the government agencies can measure the research quality of a department more accurately than an individual researcher. My two models show that there will be more variation among A-departments than B-departments (A-departments can be thought of as 5- or 4-star departments in the actual RAE and B-departments as 2- or 3-star ones). The main beneficiaries of the transfers will not so much be the transferred researchers but the incumbent researchers in expanding or unaffected departments. This is consistent with the casual observation that many incumbent researchers used the threat of outside offers to increase their wages. Furthermore, departments that were active in the researcher market offered better deals to their incumbents as well as transferees.

More normative conclusions concern the possible effects of using more competitive schemes (eg, those that give relatively higher funds to higher ranked departments). In my first model with no research production, more competitive schemes unambiguously increase the average quality of top-ranked departments. A reasonable objective of such schemes could be to increase the number of top ranked departments. Interestingly, this objective can be achieved by using two diametrically opposed schemes:

- A moderately competitive scheme with a stringent budget constraints forces middling departments to make a choice between becoming A-departments or completely withdrawing from research. This scheme will increase the number of top ranked departments by polarizing the university system.

- A mildly competitive scheme with a relaxed budget constraint induces very few middling departments to make the effort to be top ranked. But the relaxed budget constraint allows more departments to be classified as A-departments.

The first scheme seems better as it creates a comparable number of top ranked department of higher quality. Yet the welfare gains from agglomeration of researchers is not immediately clear. The answer depends on the incentives provided to researchers in the process of going to the job market. I have looked at the impact of the RAE on the research output in my second model. The main observation is that the increase in research activity comes from the transferring researchers. However, the incumbent researchers in expanding and already high ranked departments reach higher levels of utility than the transferees, and they do so with lower levels of research output.

Unlike the sports leagues, the RAE does not create destructive competition. How could we then explain the well-publicized efforts of some universities to attract researchers? I must point out that the researcher-managed departments in my models do not share the benefits of higher rankings with either the teachers or the administrators (who are not in my models). Although a department finds it unnecessary to attempt to get a higher ranking, the interests of the administrators and other departments within the same university may be hurt (conflicts between business-minded administrators and the academics in other areas are rather commonplace). They may then force a department that would otherwise shrink to expand. This may cause some universities to spend extra effort and money to boost their research ranking (eg, using research fellowship programs).

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Appendix

The parameters of the model are m, W, and C; the government determines A, B, G, and b; and the endogenous variables of the model are a, β, α, V, and ϕ. For these five unknowns we have Equations (1) to (5) in the text. We can directly solve for V:

$$V = \frac{B}{b} + W + C$$

Substituting this in Equations (1), (3), (4), and (5), we get the following system of equations for ϕ, β, a, and α:

$$\phi\, bF(\beta) + \int_b^a n dF(n) + \int_\alpha^m n dF(n) - a\big(F(a) - F(\beta)\big) - bF(\beta)\alpha\big(1 - F(\alpha)\big) = 0 \qquad \textbf{(A-1)}$$

$$\big[A - (1-\phi)B\big]F(\beta) - A + G = 0 \qquad \textbf{(A-2)}$$

$$\alpha B - bA = 0 \qquad \textbf{(A-3)}$$

$$(a-\beta)\left[\frac{B}{b} + C\right] - A + B = 0 \qquad \textbf{(A-4)}$$

Three quick observations:

- W has no affect on the equilibrium.
- Equation (A-3) implies that α is increasing in A.
- Equation (A-4) implies that (a - β) is increasing in A.

We want to obtain the comparative statics of β and a with respect to A. The system to be solved is

$$\begin{bmatrix} b\,F(\beta) & (\phi b + a - b) f(\beta) & -F(a) & -(1-F(\alpha)) \\ B\,F(\beta) & [A - (1-\phi)B] f(\beta) & 0 & 0 \\ 0 & 0 & 0 & B \\ 0 & -\left[\dfrac{B}{b} + C\right] & \left[\dfrac{B}{b} + C\right] & 0 \end{bmatrix} \begin{bmatrix} d\phi \\ d\beta \\ da \\ d\alpha \end{bmatrix} = \begin{bmatrix} 0 \\ (1-F(\beta))\,dA \\ b\,dA \\ dA \end{bmatrix}$$

The determinant of the matrix of coefficients is

$$\Delta = -B \left[\frac{B}{b} + C \right] F(\beta) \left(f(\beta)(bA - aB) + F(a) B \right) < 0$$

It is positive because from Equation (A-3) implies that $bA - aB > 0$.

Using the Kramer's rule, we can solve for da/dA:

$$\frac{da}{dA} = \frac{B\,F(\beta)}{\Delta} \left[-b \left[\frac{B}{b} + C \right] \left[F(\alpha) - F(\beta) \right] - f(\beta)(bA - aB) \right] > 0$$

We can sign this derivative as we know that $F(\alpha) > F(\beta)$. The sign of the comparative statics of β with respect to A cannot be determined unambiguously:

$$\frac{d\beta}{dA} = \frac{B\,F(\beta)}{\Delta} \left[-b \left[\frac{B}{b} + C \right] \left[F(\alpha) - F(\beta) \right] + B\,F(a) \right]$$

If we ignore the bC term that multiplies both $F(\alpha)$ and $F(\beta)$, the sign of this comparative statics depends on the relative masses of inactive departments ($F(\alpha)$ - $F(a)$) and non-A departments ($F(\beta)$). Similarly, we cannot unambiguously determine whether α - a is increasing or decreasing in A:

$$\frac{d(\alpha - a)}{dA} = \frac{b\,F(\beta)}{\Delta}\left[- C\,f(\beta)\,\left(bA - aB\right) + B\left[\frac{B}{b} + C\right]\left[F(\alpha) - F(\beta) - F(a)\right]\right]$$

Note that A or B are very large numbers compared to others. The second term dominates the first one as it is of magnitude B^2, while the first one is only of magnitude B. The signs of both comparative statics depend on the same criterion:

$$\text{If } F(\alpha) - F(a) > F(\beta) \quad \Rightarrow \quad \frac{d\beta}{dA} > 0 \text{ and } \frac{d(\alpha - a)}{dA} < 0 .$$

Figure A-1 shows an example. Note that α and $(a - \beta)$ are fixed by the relative sizes of A and B, and other parameters. Equilibrium is achieved by shifting the range $[a, \beta]$ between b and α.

[Figure A - 1]

If the budget constraint is relatively generous and enough researchers are released by the shrinking A-departments, this range will be closer to b and the mass of inactive departments will outweigh the mass of non-A departments. In this case, using higher A will result in higher β and shrink the range of inactive departments. On the other hand, if the budget constraint is stringent, a will be very close to α and the mass of inactive departments will be smaller than the mass of non-A departments. In this case, using higher A will have the opposite impact: β will come down and the range of inactive firms will expand.

Figure 1

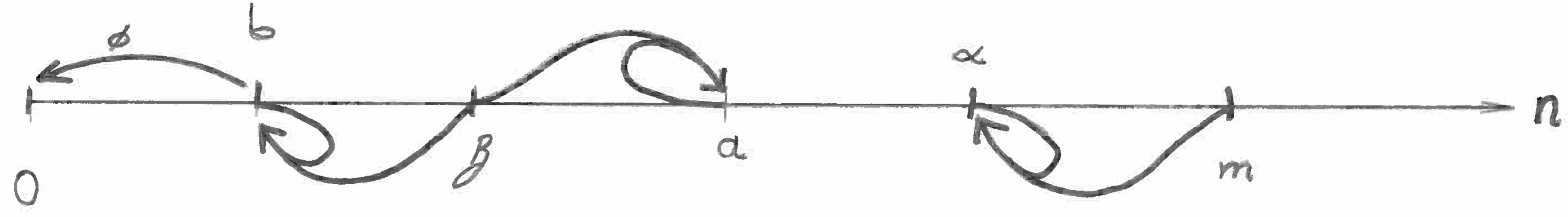


Figure 2

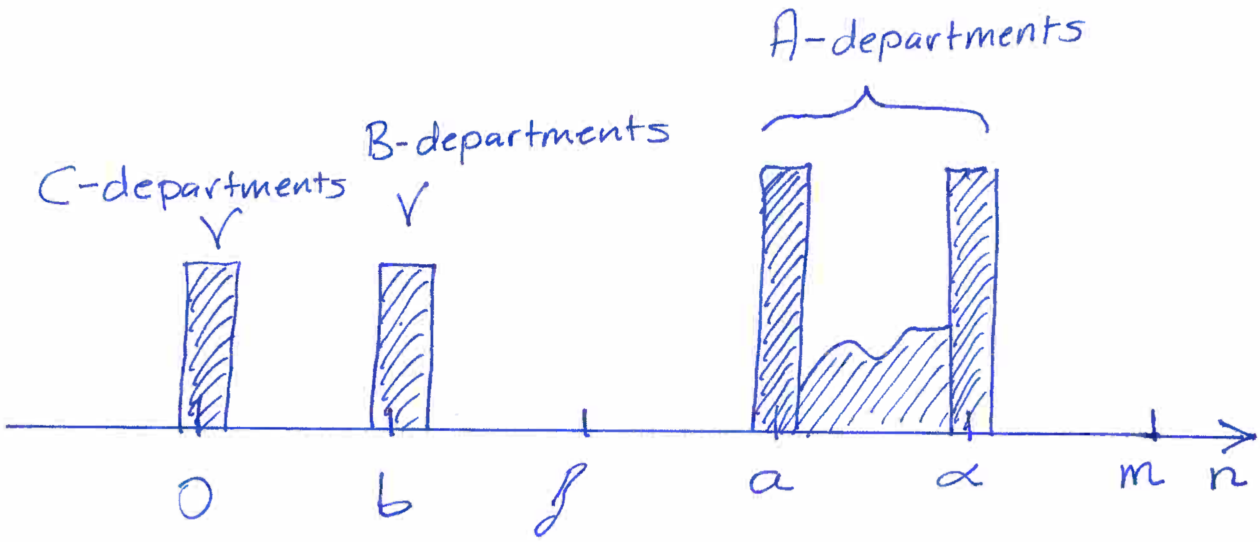


Figure 3

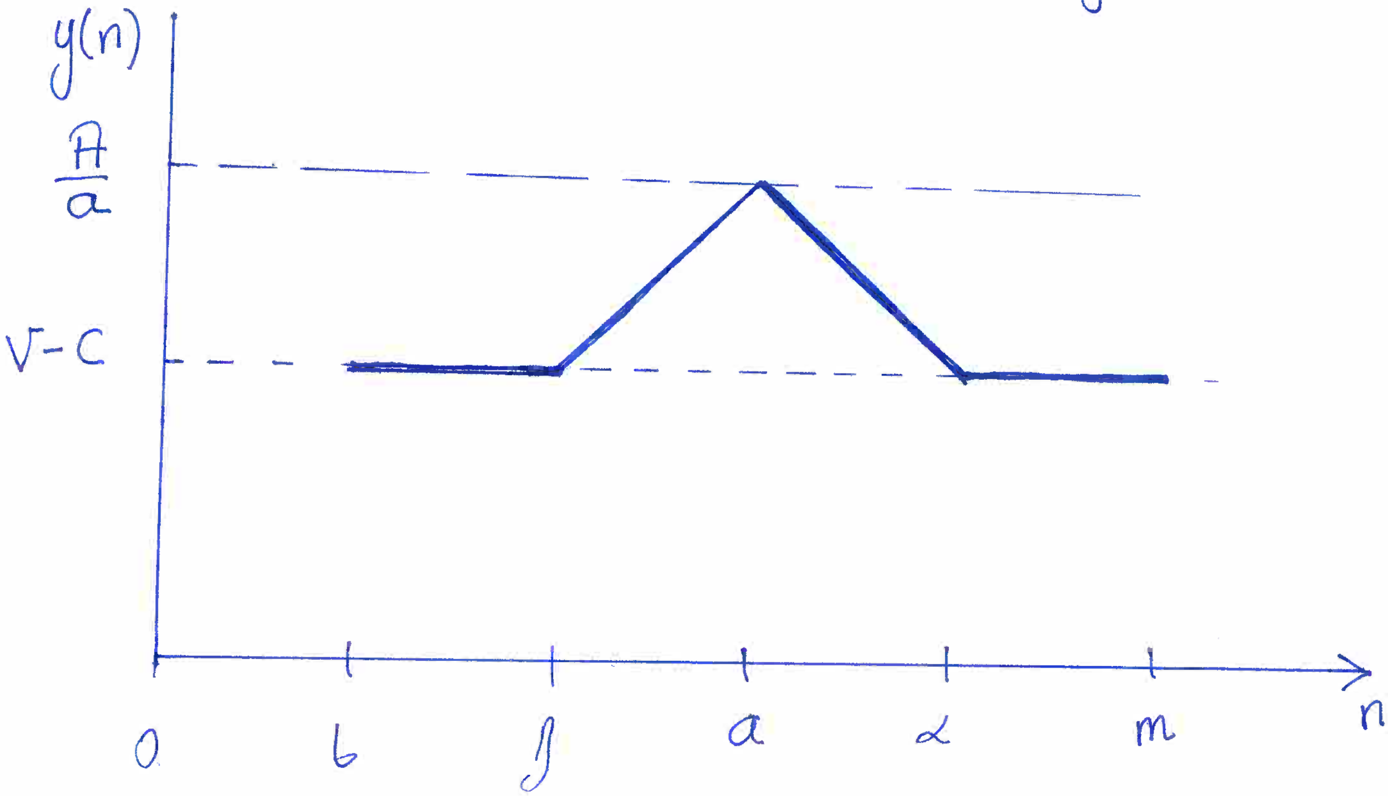


Figure 4

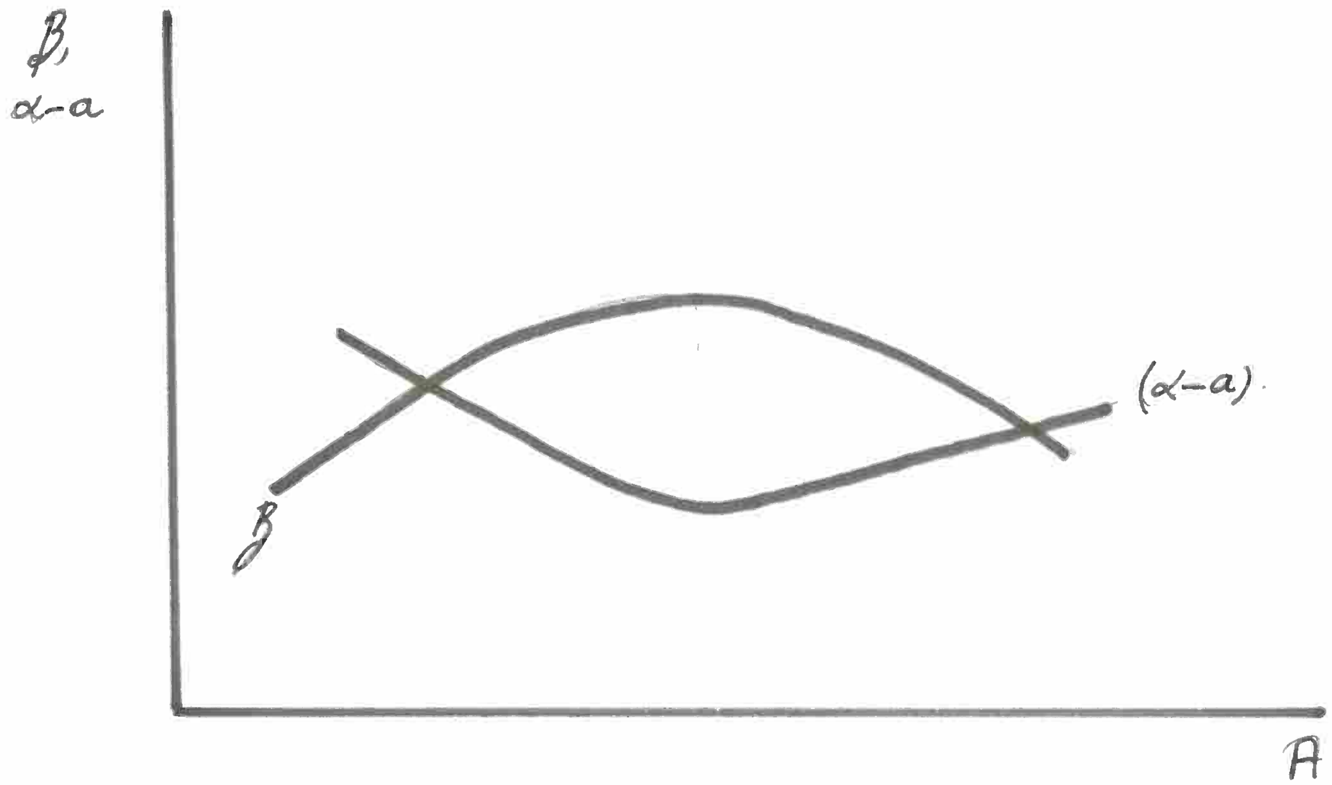
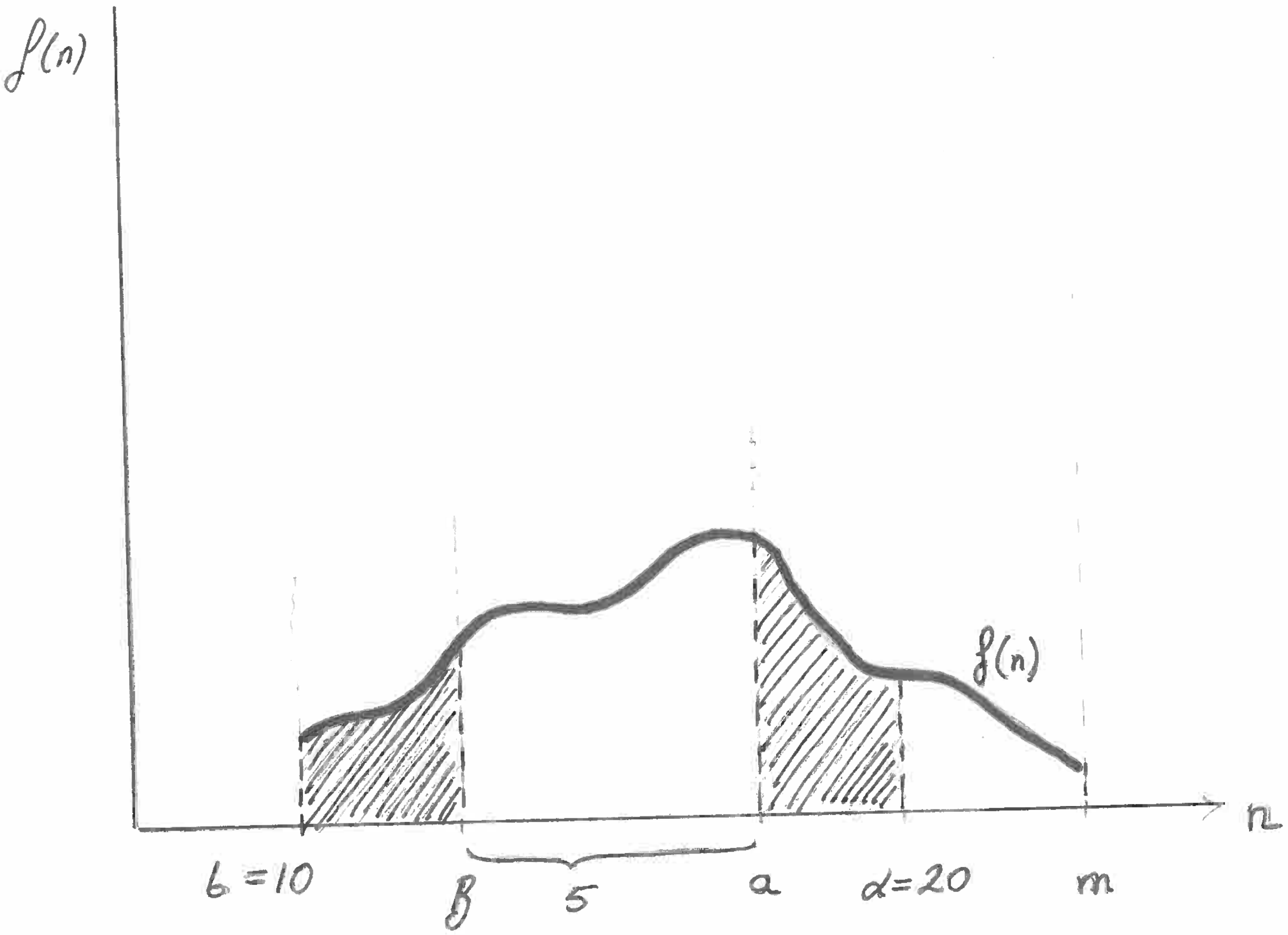


Figure A-1



$A = 2B$

$10C = B$

$b = 10$