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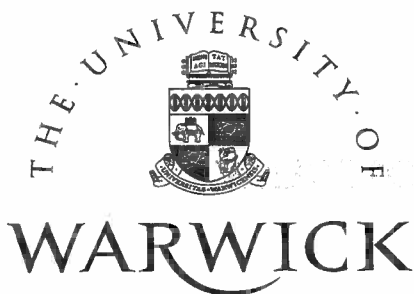
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**TAXING ECONOMIC RENTS IN OIL PRODUCTION:  
AN ASSESSMENT OF UK PRT**

Lei Zhang

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AN ASSESSMENT OF UK PRT

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.

# Taxing economic rents in oil production: an assessment of UK PRT\*

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## Abstract

Using an irreversible investment model of oil development, this paper shows how a fiscal regime can be *neutral* in that the decision to develop is not affected by tax and *efficient* in recouping economic rents where cumulated operating profits are taxed if and only if they surpass an appropriate level of tax deductible allowances. For a simplified version of the Petroleum Revenue Tax (PRT) applied to the United Kingdom Continental Shelf until 1993, numerical calculations suggest that PRT was both neutral and relatively efficient. Why then was it substantially removed in 1993? One explanation is that the tax regime may be responding to the oil price so the fiscal change may be reversible, another is that it had disincentive effects not captured in our analysis.

Key words: Irreversible investment, Oil field development, Tax neutrality, PRT.

JEL Classification: D81, G31, Q32, Q38.

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# 1 Introduction

The extraction of oil from the North Sea has added, on average, more than 2% to UK GDP for almost two decades: yet the licences to develop the UK Continental Shelf (UKCS) and extract oil are allocated without charge. Applicants for permits to develop do have to submit detailed plans for development, indicating expected outgoings and planned production: but, if approved, development proceeds without any licensing fee. One key aim of the tax regime in the North Sea is that of recouping some of the resources transferred to the private sector in this way; and this must surely account for the very high effective rates of tax which prevailed up until 1993 (see Lawson, chapter 17, 1992 and Energy Committee, 1987).

The fiscal regime in the UKCS prior to March 1993 consisted of three principal taxes: Petroleum Revenue Tax (PRT), Corporation Tax (CT) and Royalty. But while Royalty payments provided — and continue to provide — a substantial flow of tax revenue, no Royalty has been charged on any field given development consent since March 1982; so we omit them from the analysis that follows. The combination of PRT and CT implied an effective marginal rate of about 83% on oil revenues in the UKCS in the earlier 1990s<sup>1</sup>.

Imposing high tax rates clearly runs the risk of deferring development, for sunk costs are substantial and the oil price highly variable. But the granting of tax deductibility for development costs plus an uplift will tend to correct this distortionary effect of high marginal tax rates. (The up-lift allowance — set at about 1/3 of development costs — was designed to take account of the substantial delay between capital expenditures and receipts from the sale of oil, given that interest charges cannot be set against PRT.) In this paper we determine analytically whether the key parameters of a simplified PRT can be found so that the tax is both non-distortionary and tax-efficient in extracting the economic rent arising from oil production.

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<sup>1</sup>Since 1991, CT has been levied at a rate of 33% on profits after deductions for Royalty and PRT. The latter, a field-based tax on cumulated operating profits when they surpass allowances had been levied at a rate of 75% since 1982. The principal deductibles for PRT are capital expenditure and its supplement (so-called up-lift), an oil allowance, exploration and appraisal relief together with cross-field allowances, and a safeguard clause (see, for example, Tax Acts (1988)). Note that in the interests of tractability, we drop all PRT deductions other than that for capital expenditure and its supplement (up-lift). So the simplified PRT considered in this paper is essentially a “resource rent tax”, in the terminology of Neher (1990).

We study the implications of various tax regimes in a stochastic model of irreversible investment based on earlier work by Brekke and Øksendal (1992). What we find is that there is a unique rate of up-lift which ensures what we refer to as tax neutrality, i.e., that the threshold price for developing an oil field under PRT is the same as it would have been in the absence of any taxes. Further, under such a neutral up-lift rate, varying the actual tax rate does not have any effect on the threshold entry price. Given stylised parameter values based on oil price variability etc., our results imply that the neutral up-lift rate is about half, i.e., total allowances will be  $1\frac{1}{2}$  times the actual sunk cost. Up-lift rates less than this will of course tend to defer investment, although this can be offset by lowering the effective tax rate.

Our analysis deals only with a simplified version of PRT and omits Royalties; but it is nevertheless worth considering what the results imply for the actual tax rates and allowances applied in the North Sea. As indicated earlier, the tax rate was high (about 5/6); and the level of allowances substantially greater than the capital costs — with an up-lift rate of 35% of sunk costs. Using what we regard as realistic parameters, we find that the resulting tax regime was quite close to being neutral despite the high marginal rates of tax on oil revenues when all allowances are exhausted. This means of course that resources were being recouped from the private sector without substantially affecting the incentives to invest in the North Sea. This conclusion is in line with the evidence which the Government supplied to the Energy Committee in 1987. Their Memorandum noted that “it is no purpose of the Government’s fiscal policies to transform projects that are uneconomic in pre-tax terms into projects that are economic in post-tax terms”; and it provided specific evidence to show that “the fiscal regime does not generally have the effect of making uneconomic after tax those new oilfields that are economic before tax”, Energy Committee (1987, Memorandum 1, p5). See also the view expressed by Philip Verleger (1994) where he commends the economic efficiency of the UK tax regime applied to the North Sea.

In 1993, however, when real oil prices had fallen substantially without any prospect of recovery, PRT was abolished for new fields. There was no PRT and no up-lift — but the corporate tax of 33% remained in place<sup>2</sup>. Why, one might ask, did the Government wish to jettison what appeared to be an efficient tax system, in favour of another regime which promised less revenues to the Government (though the effects on incentives to invest do

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<sup>2</sup>For existing fields, the rate of PRT was cut to 50%, implying an effective marginal rate of about 67%.

not appear much different from the PRT regime)? One answer is that the switch is *not* permanent; but is contingent on the real price of oil; so if and when oil prices go up again, PRT will return. Another, which we return to in conclusion, is that the high marginal rates had disincentive effects not captured in our analysis (see the Speech of the Financial Secretary, 1994).

Although our basic assumptions about the fiscal regime are broadly in line with those of earlier studies by Kemp *et al* (1988) and Kemp and MacDonald (1992), we assume, unlike them, that oil prices are reasonably persistent, however, so that the option to develop has economic value<sup>3</sup>. The stochastic model of development adopted here is close to that of Brekke and Øksendal (1992) but differs in the structure of operating cost and the explicit treatment of taxation. Under simplifying assumptions, it has been shown elsewhere (Miller and Zhang (1992)) that their analysis can be reduced to the more familiar model of Dixit (1992) and Pindyck (1992), in which the trigger for investment is determined by the requirement that the expected value of oil revenues matches the costs of development *plus* the option value of postponing investment till later. It is this simpler model which we use here to embody tax provisions like the PRT applied in the UKCS. To quantify the value of tax allowances we have used techniques to be found in Brennan and Schwartz (1985) and Bartolini and Dixit (1991).

The plan of the paper is as follows, section 2 outlines the basic model and section 3 analyses the irreversible development decision under certainty. In section 4, we assess the impact of tax policy on optimal development decisions under future oil price uncertainty. Section 5 discusses the neutrality and efficiency of PRT; and the effects of varying volatility of oil prices on development decisions are calibrated using numerical examples in section 6. Section 7 looks at the case when the fiscal regime change is not permanent. Finally, section 8 concludes the paper.

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<sup>3</sup>In the other studies cited, it is postulated that the real oil price is independently and normally distributed around a drift with a time-increasing variance, which implies that the real oil price is *not* persistent and that waiting has *no* value (since the accumulated information does not in any way change the conditional expectation of future oil prices): all future risks can be immediately evaluated without waiting to see what happens to prices.

## 2 The Model

As is standard in models of irreversible investment (Brennan and Schwartz 1985, Paddock *et al* 1989, Brekke and Øksendal 1992), we assume that the real price of oil follows a geometric Brownian motion with drift,

$$\frac{dP_t}{P_t} = \alpha dt + \sigma dW_t, \quad P_0 = P, \quad (1)$$

where  $P_t$  is the real price of oil,  $\alpha$  is the percentage trend in the real price of oil,  $\sigma$  is its instantaneous variance, and  $W_t$  is a standard Brownian motion. (See Favero *et al* (1992) for econometric evidence in support of this assumption.)

Suppose that the available reserves follow an exponentially declining path (see McCray 1975, Paddock *et al* 1989, Brekke and Øksendal 1992), then

$$\frac{dQ_t}{Q_t} = \begin{cases} 0, & \text{idle} \\ -\gamma dt, & \text{with extraction} \end{cases}, \quad Q_0 = Q, \quad (2)$$

where  $Q_t$  is the current reserve level whose volume has been established by exploration, and  $\gamma$  is the exogenous extraction rate which is assumed to be fixed when extraction starts. From (1) and (2), we define  $R_t = P_t Q_t$  which can be shown to follow a geometric Brownian motion

$$\frac{dR_t}{R_t} = \begin{cases} \alpha dt + \sigma dW_t, & \text{idle} \\ (\alpha - \gamma)dt + \sigma dW_t, & \text{with extraction} \end{cases}, \quad R_0 = R = PQ. \quad (3)$$

For simplicity, we assume that operating costs in extraction are proportional to revenues, so the flow of operating profits from extraction at time  $t$  is given by

$$\pi_t = \gamma(1 - \delta)R_t,$$

where  $\delta$  denotes the proportion of operating costs.

Let the tax-deductible allowances at time  $t$  be  $A_t$ , with initial value  $A$ . As long as  $A_t > 0$ , no tax is paid; but  $A_t$  is gradually used up as operating profits accumulate. Specifically,  $A_t$  evolves as

$$dA_t = -\pi_t dt = -\gamma(1 - \delta)R_t dt. \quad (4)$$



Once  $A_t = 0$ ,

$$dA_t = 0. \quad (5)$$

At this time, firm starts paying PRT with the tax rate at  $\lambda$ . So the flow of net operating profits becomes

$$\pi_t = (1 - \lambda)\gamma(1 - \delta)R_t.$$

Define the first hitting time  $\tau$  when the allowances are used up

$$\tau = \inf\{t : t > 0, A_t = 0\}, \quad (6)$$

then for  $t < \tau$ , equation (4) holds; and for  $t \geq \tau$ ,  $A_t = 0$ , since it is easily seen from (4) and (5) that  $A_t$  is a non-decreasing process as  $\pi_t \geq 0$ . Thus, the stopping time  $\tau$  in (6) effectively decomposes the problem into two distinct regimes, where PRT is payable only in the regime where  $A_t = 0$ . As  $A_t$  is a non-increasing process, the switch between regimes is *irreversible*. In what follows, we first analyse how such tax regime affects development decisions when future oil price is certain and then we assess the impact of incorporating price uncertainty.

### 3 Optimal Development under Certainty

Before considering the effects of PRT on the development decision when the oil price is volatile, we show briefly that, under certainty, there exist a unique up-lift rate  $\eta_N$  which does not distort the entry decision irrespective of the tax rate,  $\lambda$ . This up-lift rate produces just enough returns to induce the development of the oil field; but, as for any further profits being economic rent, they can be taxed away without affecting the supply of oil.

If no taxes are paid, the value of the field is the present discounted value of the operating profits,

$$V_N(R) = \int_0^\infty \gamma(1 - \delta)R_t e^{-\rho t} dt = \frac{\gamma(1 - \delta)R}{\rho + \gamma - \alpha}, \quad (7)$$

where  $\rho$  is the discount rate. Without taxes and uncertainty, the so-called Marshallian trigger for entry is determined by the condition that the value of the field just covers the development costs,

$$V_N(R_M) = \frac{\gamma(1 - \delta)R_M}{\rho + \gamma - \alpha} = D, \quad (8)$$

where  $R_M$  denotes Marshallian trigger. Equation (8) characterises the optimal development decision, namely, entry is optimal only if current  $R$  is at least the same as  $R_M$ .

It can be seen from (7) and (8) that an up-lift rate  $\eta_N$  which does not distort the entry trigger  $R_M$  is equivalent to an initial allowance  $A_N$  which shields the field from taxes at least for  $R \leq R_M$ . Technically, this implies that, at  $R_M$ , the first time when the field starts paying PRT is  $+\infty$ . If  $\gamma - \alpha > 0$ , then the integral form of (4) becomes

$$E^{-(\gamma-\alpha)\tau} = 1 - \frac{\gamma - \alpha}{\rho + \gamma - \alpha} \cdot \frac{A_N}{V_N(R_M)},$$

where  $\tau$  is the first time the field pays PRT. Letting  $\tau \rightarrow +\infty$  implies that the non-distorting tax shield  $A_N$  is defined as

$$A_N = \left(1 + \frac{\rho}{\gamma - \alpha}\right) V_N(R_M) = \left(1 + \frac{\rho}{\gamma - \alpha}\right) D. \quad (9)$$

It is obvious from (9) that the appropriate up-lift rate is

$$\eta_N = \frac{A_N - D}{D} = \frac{\rho}{\gamma - \alpha}. \quad (10)$$

How far  $A_N$  exceeds  $D$  depends on two factors: the interest effect and the depletion effect. The former reflects the fact that expenditure comes at the beginning of the field's life while the allowances only take effect later as oil is recovered. In this particular case, the allowances last as long as the field life itself. Since interest costs are not deductible against PRT,  $A_N$  has to be raised to make its present discounted value match the initial development costs paid by the firm. The depletion effect measures how quickly firm can recover the assigned allowance. The larger the extraction rate, the less time it takes to use up a given allowance and so the less the interest rate matters. In the limit as  $\gamma \rightarrow \infty$ ,  $A_N \rightarrow D$ , irrespective of the interest rate.

For the initial allowance given in (9), it can be shown that the value of the field under PRT is

$$V_T(R) = \begin{cases} \frac{\gamma(1-\delta)R}{\rho+\gamma-\alpha} & \text{for } R \leq R_M, \\ \frac{\gamma(1-\delta)R}{\rho+\gamma-\alpha} \left[ 1 - \lambda \left( \frac{\alpha-\gamma}{\gamma(1-\delta)} \frac{A_N}{R} + 1 \right)^{\frac{\rho+\gamma-\alpha}{\gamma-\alpha}} \right] & \text{for } R > R_M. \end{cases} \quad (11)$$

As the tax rate  $\lambda$  only affects the value function for  $R > R_M$ , changing  $\lambda$  does not have any effect on the entry trigger.

Figure 1 shows the entry behaviour under certainty, where the horizontal axis represents  $R$  and vertical the value of the field. If no taxes are levied, the value of the field is the schedule labelled  $V_N(R)$  and the Marshallian entry trigger (where  $V_N(R)$  just matches  $D$ ) is given by point  $R_M$ . For an initial allowance given in (9) and  $\lambda < 1$ , the value function is the schedule  $V_T(R; \lambda < 1)$  which has a kink at  $R_M$ : for  $R \leq R_M$  the schedule is identical to that without taxes ( $V_N(R)$ ), and for  $R > R_M$ , it asymptotically tends to  $EE$ . Changing tax rate to  $\lambda = 1$  does not affect the value function for  $R \leq R_M$  but reduces it to  $V_T(R; \lambda = 1)$  for  $R > R_M$  as the schedule  $V_T(R; \lambda = 1)$  asymptotically tends to  $AA$ . As can be seen from the figure that changing tax rate  $\lambda$  does not effect entry trigger  $R_M$ , so PRT with such selected up-lift rate is neutral. Furthermore, as  $\lambda$  can be set at 100%, any additional value above  $A_N$  can be collected by the authority without distorting entry decision, so PRT with such  $A_N$  is also a device of collecting economic rents. In what follows, we will illustrate that such tax neutrality can also be achieved with an appropriate selection of up-lift rate even under oil price uncertainty.

## 4 Optimal Development under Uncertainty

For simplicity, we assume that the firm will never exit once it has entered production. Under such conditions, the present discounted value of an operating field yields the following value function,

$$\begin{aligned} V(R, A) &= E_0 \left\{ \int_0^\tau \gamma(1-\delta)R_t e^{-\rho t} dt + \int_\tau^\infty (1-\lambda)\gamma(1-\delta)R_t e^{-\rho t} dt \right\} \\ &= E_0 \left\{ \int_0^\tau \gamma(1-\delta)R_t e^{-\rho t} dt + \frac{(1-\lambda)\gamma(1-\delta)}{\rho + \gamma - \alpha} R_\tau e^{-\rho \tau} \right\}, \end{aligned} \quad (12)$$

where  $E_0$  is an expectation operator conditional on the information available at  $t = 0$ , and the condition  $\rho + \gamma - \alpha > 0$  ensures the convergence of the above integral.

Using Itô's lemma, the arbitrage condition for (12) under (3) and (4) gives rise to the following partial differential equation (PDE) for  $A > 0$

$$\frac{1}{2}\sigma^2 R^2 V_{RR}(R, A) + (\alpha - \gamma)R V_R(R, A) - \gamma(1-\delta)R V_A(R, A) + \gamma(1-\delta)R = \rho V(R, A), \quad (13)$$

where subscripts denote partial derivatives. The boundary conditions are given as

$$V(0, A) = 0, \quad (14)$$

$$V(R, 0) = \frac{(1 - \lambda)\gamma(1 - \delta)R}{\rho + \gamma - \alpha}. \quad (15)$$

where (14) follows from the fact that  $R = 0$  is an absorbing state, see (3); and equation (15) gives the value of an operating field when initial allowances are zero.

The solution to (13)–(15) is found to be

$$V(R, A) = \frac{(1 - \lambda)\gamma(1 - \delta)R}{\rho + \gamma - \alpha} + \lambda A f(y), \quad (16)$$

where

$$f(y) = \frac{\sigma^2}{2} \frac{1}{\rho + \gamma - \alpha} y \left\{ 1 - y^{\theta_+ - 1} \frac{\Gamma(\omega(\theta_+) - \theta_+ + 1)}{\Gamma(\omega(\theta_+))} {}_1F_1[\theta_+ - 1; \omega(\theta_+); -y] \right\}, \quad (17)$$

$$y = \frac{2\gamma(1 - \delta)R}{\sigma^2} \frac{R}{A}, \quad (18)$$

and  $\Gamma(\cdot)$  is a Gamma function,  ${}_1F_1[\cdot; \cdot; \cdot]$  is a Kummer's function, and  $\theta_+$  and  $\omega(\theta_+)$  are two positive constants. (See Appendix A for a detailed derivation.)

From (16), we note that the value of an operating field can be decomposed into two parts: the first term on the right hand side represents the present discounted value of *after tax* operating profits given no deductibility, and the second term gives the expected value of the *tax shield* under the specified fiscal regime. The first term requires little elaboration. Suffice it to say that the value of the field (without deductibility) is clearly sensitive to the rate of extraction: for very large  $\gamma$ , the value tends to  $(1 - \lambda)(1 - \delta)R$  as reserves can be instantly lifted, but it falls as  $\gamma$  decreases. (If, for example,  $\gamma = \rho$  and  $\alpha = 0$ , the value of the field falls to  $(1 - \lambda)(1 - \delta)R/2$ .)

It may be useful to characterise the properties of the tax shield in the following proposition (where the proof can be found in Appendix B):

**Proposition 1**  *$f(y)$  is a strictly increasing and concave function of  $y$  with*

$$\begin{aligned} \lim_{y \downarrow 0} f(y) &= 0, \\ \lim_{y \uparrow \infty} f(y) &= 1. \end{aligned}$$

For any given initial allowances  $A$ ,  $\lambda A$  is the maximum amount of recoverable development costs (maximum tax shield); while  $f(y)$  can be interpreted as the probability of recovering the maximum tax shield conditional on  $R$ .

The *ex ante* value functions under different fiscal regimes are drawn in Figure 2, where horizontal axis represents  $R$  and vertical axis the value of a field.  $OT$  is the schedule with full tax and no deductibility and so corresponds to the first term on the right hand side of (16). By setting  $\lambda = 0$  this term represents the expected present value of the field when *no* tax is levied; this is drawn as line  $ON$  in the figure. When there is no uncertainty, and the allowance is chosen such that the value of the field is shielded from taxes for  $R \leq R_M$ , then the value of the field is the schedule  $OSV_T$ , which has a kink at  $S$ . Under uncertainty, the value of the field is shown as the curve  $OV$  which corresponds to equation (16). Near the origin it lies close to  $ON$ , but it asymptotically tends to  $SV_T$  as the probability of recovering the full tax shield becomes 1. As can be seen from the figure, the schedule  $OV$  is smoother and lower than  $OSV_T$ , which are essentially caused by the Ito's lemma.

Let development expenditure be  $D$  and up-lift rate be  $\eta$ , then initial allowances are given by

$$A = (1 + \eta)D. \quad (19)$$

So, under the PRT, the government has two instruments in its disposal: the tax rate  $\lambda$  and the up-lift rate  $\eta$ . In what follows, we first derive the entry trigger  $R_e$  above which the firm starts development; and then we assess how changing  $\lambda$  and  $\eta$  affects this trigger.

If development costs are sunk costs and development can be completed immediately by paying a lump-sum  $D$ , the entry option can be written as

$$U(R) = \sup_{\tau'} E_0\{(V(R_{\tau'}, A) - D)e^{-\rho\tau'}\}, \quad (20)$$

where  $\tau'$  is the first time development occurs.

From (20) and (3), and using Itô's lemma, the arbitrage condition for  $U(R)$  becomes

$$\rho U(R) = \alpha U'(R) + \frac{1}{2}\sigma^2 U''(R)$$

which permits the following general solution

$$U(R) = C_+ R^{\xi_+} + C_- R^{\xi_-}, \quad (21)$$

where  $C_{\pm}$  are two arbitrary constants and  $\xi_{\pm}$  are positive and negative roots of

$$\frac{1}{2}\sigma^2\xi(\xi - 1) + \alpha\xi - \rho = 0.$$

As  $R \rightarrow 0$  the entry option becomes worthless, so  $C_- = 0$ .

Denote the optimal entry trigger by  $R_e$ , the optimality conditions are the following value matching and smooth pasting conditions,

$$U(R_e) = V(R_e, A) - D, \quad (22)$$

$$\frac{\partial U(R_e)}{\partial R_e} = \frac{\partial V(R_e, A)}{\partial R_e}. \quad (23)$$

The proof of existence and uniqueness of  $R_e$  generated from (22) and (23) is provided in Appendix C.

It is straightforward to show how to obtain such trigger in Figure 3, where the horizontal axis represent  $R$  and the vertical the value functions. The curve labelled  $V$  is the present discounted value of an operating field under PRT and  $U$  represents the value of waiting option. The point of entry ( $R_e$ ) is determined by the conditions that the value of the field equals the sum of waiting option *plus* the capital cost  $D$  (“value matching”), and the slope of the value function  $V$  is the same as that of  $U$  (“smooth pasting”). Subtracting the capital cost  $D$  from the value of the field  $V$ , we obtain the net value of the field  $V - D$  shown as the dashed line in the figure. At the point of entry,  $R_e$ , the value of waiting  $U$  both value matches and smooth pastes the net value of the field  $V - D$ . The entry decision is made by comparing the option value of waiting with the net value of the field. As can be seen from the figure, for  $R < R_e$ ,  $U > V - D$ , so waiting is optimal; for  $R \geq R_e$ , the net value of the field is at least the same as  $U$ , so entry is optimal.

How does changing the tax instruments affect entry trigger? First we note that, as in the case of certain prices, there is a unique up-lift rate  $\tilde{\eta}$ , which does not distort entry irrespective of the tax rate  $\lambda$ . Second, the effect of changing the tax rate  $\lambda$  on the entry decision crucially depends on this critical up-lift rate  $\tilde{\eta}$ : when the up-lift rate is above this critical level, raising the tax rate will encourage early entry; when the up-lift rate is below this critical level, raising the tax rate discourages entry. These results are summarised in the following proposition (and the proof can be found in Appendix D):

**Proposition 2** *There exists a critical up-lift rate ( $\tilde{\eta}$ ) such that*

$$\frac{\partial R_e}{\partial \lambda} \begin{cases} > 0 & \text{if } \eta < \tilde{\eta} \\ = 0 & \text{if } \eta = \tilde{\eta} \\ < 0 & \text{if } \eta > \tilde{\eta}. \end{cases} \quad (24)$$

Furthermore, for  $\lambda = 0$ ,

$$\frac{\gamma(1-\delta)R_e}{\rho + \gamma - \alpha} = \frac{\xi_+}{\xi_+ - 1} D. \quad (25)$$

Equation (25) defines the tax neutral trigger since this is the trigger when no taxes are levied. When  $\eta = \tilde{\eta}$ , it can be seen from (24) that changing the tax rate does not affect entry trigger, so this entry trigger must be the same as that without any tax at all.

How is the neutral up-lift rate affected under price uncertainty? Apart from the two effects (interest effect and depletion effect) discussed earlier, the volatility in the oil prices will also influence the determination of the neutral up-lift rate. In the numerical examples below, we will show that the interest effect and depletion effect on the neutral up-lift rate are in line with those under certainty. To illustrate the effect of uncertainty, we use Figure 2. For any given tax rate and initial allowance, the introduction of price uncertainty has two effects: first it increases the neutral entry trigger; second it reduces the value of the field under taxes and increases its relative waiting option value, so increases the entry trigger under taxes. As the increase in these two triggers may not be of equal proportion, so how price volatility alters the neutral up-lift rate depends on which of these two effects dominates. For the case we drawn in the figure, the oil price volatility pushes up the neutral trigger from  $R_M$  to  $R_e^N$  while it pushes the entry trigger under taxes from  $R_M$  to  $R_e^T$  which is to the left of  $R_e^N$ . So to move  $R_e^T$  to  $R_e^N$ , a smaller up-lift rate is required. As this is not the only possible scenario, the effect of price uncertainty on the neutral up-lift rate is ambiguous. We will use numerical examples to assess this effect.

## 5 Tax Neutrality and Efficiency

Tax neutrality is one aspect of a fiscal regime, another is how efficient the regime is in collecting the economic rent available. We discuss both of these with the aid of Figure 4.

Figure 4 illustrates various value functions and related options in different fiscal regimes, where horizontal axis represents  $R$  and vertical the asset value. The horizontal lines indicated by  $D$  and  $(1 + \eta)D$  are the development cost and initial allowance respectively.  $V_N$  is the schedule of the present discounted value of an oil field without any taxes and  $U_N$  is the cost of development plus the waiting option. The optimal entry trigger is characterised by a point  $R_N$  at which total costs (capital cost plus the opportunity cost of waiting) equals the value of the field. If PRT is neutral, varying the tax rate  $\lambda$  must not affect the entry trigger, so for simplicity, we choose the limiting case where  $\lambda = 1$ . As shown in the figure, schedule  $V_T$  is the value of the field under PRT with  $\lambda = 1$ . This concave curve asymptotically tends to  $A$  which is the maximum tax shield. The related option of waiting plus development cost of  $D$  is drawn as schedule  $U_T$ . As varying  $\eta$  will affect both  $V_T$  and  $U_T$ , by selecting an appropriate  $\tilde{\eta}$ , one can make  $V_T$  and  $U_T$  value match and smooth paste at  $R_N$ . This unique up-lift rate  $\tilde{\eta}$  generates a neutral tax regime.

Suppose that the government can sell the rights of development in a competitive auction, then at  $R_N$ , the maximum price which could be achieved at the bidding is the distance shown as  $XZ$ , namely, the value of the waiting option. This is the maximum economic rent which can be recouped by the government. The selection of  $\lambda = 1$  clearly gives rise to the maximum tax revenue which under this regime is the distance  $XY$ , the difference between value of the field without taxes and that under PRT. The largest tax revenue extracted in this case is obviously less than the full economic rent, and the shortfall  $YZ$  is in fact equal to the value of waiting option under PRT. In this case, we define the efficiency ( $E$ ) of PRT as the ratio between  $XY$  and  $XZ$ . So the larger is  $E$ , the more efficient is the PRT in collecting economic rents. In the numerical examples below, we assess how  $E$  is affected by changes in the parameters.

Can government recoup additional tax revenue under PRT without disturbing entry trigger? In principle, it is possible, as the government could auction the rights to develop oil *subject to PRT* (and making such rights transferable). The sum which the government could obtain at auction would of course be  $YZ$ , the value of option to wait, which is the shortfall to be collected.



## 6 Numerical Simulations

In what follows, we use numerical simulations to assess first how the neutral up-lift rate is affected by other parameters and how efficient such tax regime would be in collecting economic rents, and second how quantitatively significant the tax regimes adopted in the UKCS impact on firms' development decisions. Specifically, we look at how development triggers are affected in two permanent fiscal regimes: PRT with corporate tax and corporate tax only.

In the PRT regime, the rate of PRT levied on a particular field was 75% with the rate of up-lift set at 35%. Assuming there are no other deductible costs emerging at the corporate level and PRT is deductible against corporate tax, then 33% corporate tax rate adds an additional  $33\%(1-75\%) = 8.25\%$  to the PRT rate levied at the field level. So the effective rate of tax on an operating field is 83.25%. To characterise this fiscal regime, we therefore set  $\lambda=83.25\%$  and  $\eta=35\%$ .

Abolishing PRT removes both the PRT rate of 75% and the up-lift rate of 35%. What remain is the rate of corporate tax at 33% and no up-lift at all. In simulating the post PRT regime, we therefore set  $\lambda=33\%$  and  $\eta=0$ .

For simulation purposes, following Paddock *et al* (1989), we choose the stylised standard deviation ( $\sigma$ ) of real oil price to be 0.2 and vary it between 0.1 and 0.4. For the base case, we choose other parameter values as follows:  $\rho = 0.05$ ,  $\alpha = 0$  and  $\gamma = 0.07$  (which corresponds to half life of 10 years). To see the properties of comparative statics, we also vary these parameter values. So  $\rho$  varies between 0.03 and 0.07,  $\alpha$  from -0.02 to +0.02, and  $\gamma$  from 0.05 to 0.1. In what follows, we first compute how the critical up-lift rate  $\tilde{\eta}$  respond to changes in the parameter values. Second, we compare these  $\tilde{\eta}$  with the actual up-lift rate adopted in UKCS so that the effect of permanent shift of fiscal regime can be assessed.

Tables 1 and 2 show how the critical level of up-lift rate ( $\tilde{\eta}$ ) varies with changes in the discount rates, the extraction rates and the variability of real oil prices. It is seen that  $\tilde{\eta}$  varies positively with increasing discount rates ( $\rho$ ) and inversely will the extraction rates. These results resemble those in the certainty case. The effect of changing oil price volatility is ambiguous, as we discussed before. But  $\tilde{\eta}$  varies inversely with  $\sigma^2$  in most cases indicating that the effect on increasing neutral trigger dominates. Tables 3 and 4 show the efficiency

of PRT at the neutral entry trigger. The efficiency coefficient varies inversely with the discount rate, and positively with the extraction rate and the price volatility. For the base case we have chosen,  $E$  is a little over 80%, showing that the PRT can extract at most 80% of the profits.

How do these simulations imply about the fiscal regime operated in the UKCS? The model is, of course, very much simplified but we can at least see how the theoretical tax neutral up-lift rates compared with those applied in practice. Consider the parameters for the base case, i.e.,  $\rho = 0.05$ ,  $\alpha = 0$  and  $\gamma = 0.07$ . In the absence of oil price uncertainty, the tax neutral up-lift rate would be

$$\eta_N = \frac{\rho}{\gamma - \alpha} = 0.71.$$

but under oil price uncertainty with  $\sigma = 0.2$ , we find

$$\tilde{\eta} = 0.46.$$

When PRT was first introduced in the early 80s, the uplift rate was in fact 75%; this was during the period when OPEC was still an effective cartel and BNOC set the price of North Sea oil (i.e.,  $\sigma$  close to zero). So perhaps it is not surprising to find that what we calculated theoretically is close to what was applied. When OPEC collapsed and oil prices fell, the uplift rate was cut to 35%. Interestingly enough, this is not far from the theoretical value of 46% we have computed for a volatile free market price. The fact that the theoretical value is 11% higher than the actual up-lift rate used in the North Sea implies that the fiscal regime applied to the North Sea was to some degree non-neutral and discouraged entry relative to the case where no taxes are levied (see Proposition 2).

Comparing the actual up-lift rate with the neutral up-lift rate only indicates the *qualitative* nature of the fiscal regime in place. To assess quantitatively how much the actual PRT defers development, we compute the ratio ( $r$ ) of the development trigger under PRT to that under neutral tax regime. Table 5 gives the simulation results for various parameters. In the base case ( $\alpha = 0$ ,  $\rho = 0.05$ ,  $\gamma = 0.07$  and  $\sigma = 0.2$ ), the trigger under PRT is only 5% higher than that under neutral tax regime. As we have not incorporated other forms of allowances into our simulation (such as oil allowances etc.), the actual difference may even be narrower. We may conclude that the optimal development trigger is very close to that under no taxes, i.e., *PRT is basically neutral*. Table 6 shows how much of the economic

rent will be extracted, assuming that the tax is neutral and the marginal rate is 100%. Fore the base case, about 73% of the rent arising at the entry price will be transferred to the Government. The comparable transfer calculated using the actual PRT parameters ( $\lambda = 83.25\%$  and  $\eta = 35\%$ ) implies a rent transfer of about 66%. These tax parameters may fail to recoup all the economic rent, but they nevertheless succeed in collecting about 90% of maximum that a neutral PRT could have yielded. Our calculations imply therefore that the PRT regime was relatively *efficient* in collecting economic rents.

Suppose that the Government abolishes PRT permanently, this leaves only Corporation Tax in place. How does this affect the development of the new fields? In Table 7, we provide the numerical results for the ratio of development trigger without PRT (and without its related allowances) to that with PRT. For the base case, abolishing PRT results in a development trigger which is about 2% lower than that with PRT permanently in place and about 4% higher than the development trigger under no taxes. Not surprisingly, we conclude that the post PRT tax regime is even closer to being neutral.

While the shift of tax may represent a small move towards tax neutrality, the Government will doubtless lose a large amount of oil revenues when oil prices recover (or new fields come on stream). Table 8 shows that, for the base case, the tax efficiency coefficient falls to 37%. This is only about 50% of what an ideal PRT would have yielded (see Table 6)—as compared to the 90% take attributed to the previous PRT regime. Why did the Government abandon such basically efficient tax system? One answer discussed in next section is that the shift in the tax regime may be “oil price contingent”.

## 7 Reversible Tax Regime Shift

PRT was abolished in the 1993 budget after a period of persistently low real oil prices. But if oil prices were to rise, the government would fail to collect economic rents. Assume, alternatively, that as and when oil prices rise to some higher level, the government will re-introduce a PRT so as to collect the economic rents. If such scenario is fully anticipated by the oil companies, how would this “reversible” tax regime shift affect the development decisions? And what would be the implications for the expected tax revenues to the Government? In what follows, we graphically illustrate the effects of this scenario.

Suppose the introduction of PRT is state contingent and reversible, i.e., the government will put PRT in place as long as real oil prices are higher than a threshold and withdraw it otherwise; then, for a given field, such threshold on the real oil prices corresponds to a threshold of  $\bar{R}$  above which PRT is in place. Assuming further that such threshold ( $\bar{R}$ ) is very small compared with the entry trigger ( $R_e$ ). In Figure 5, we draw the *ex ante* value functions under reversible tax regime shift, where horizontal axis represents  $R$  and vertical axis the *ex ante* value of the field. As shown in the figure,  $ON$  is the value of the field where no taxes are levied,  $OD$  is the value of the field where PRT is permanently in place,  $OP$  is the value of the field where PRT is abolished for ever. If PRT is temporary and in force only when  $R$  is greater than  $\bar{R}$ , the value of the field is drawn as the darker curve  $OT$ . As  $R$  goes to zero,  $OT$  is zero due to absorption; as  $R$  becomes very large,  $OT$  asymptotically tends to  $OD$  as the prospect of switching to the no tax regime is remote. At  $\bar{R}$ ,  $OT$  is smoothly joined because of value matching and smooth posting (see Whittle, 1983). As  $\bar{R} \ll R_e$ , for  $R \gg \bar{R}$  (but  $R < R_e$ ,  $OT$  is almost identical to  $OD$  due to the asymptotic condition. This is because the diffusion process characterising  $R$  makes it very persistent, when  $R$  is large (compared with  $\bar{R}$ ), the expected effects of being in the no tax regime is almost negligible.

The values of waiting options for  $OD$  and  $OT$  are shown as  $OW_D$  and darker curve  $OW_T$  respectively. As  $OT$  is nearly identical to  $OD$  for larger  $R$ , its corresponding option of waiting  $OW_T$  is almost identical to  $OW_D$ . This implies that the development trigger under reversible tax regime  $R_e^T$  is close to  $R_e^D$ , i.e., the tax neutrality is almost retained.

How will the introduction of the reversible tax regime affect tax revenues accruing to the government? We first look at the case under PRT. As  $ON$  is the expected value of an oil field without any taxes and  $OD$  is that under PRT, the difference between  $ON$  and  $OD$  gives the expected tax revenues to the government. Likewise, the distance between  $ON$  and  $OP$  gives the expected tax revenues without PRT. It is evident from the figure that the tax revenues in PRT regime is higher than that without PRT since the difference between  $ON$  and  $OD$  is larger than that between  $ON$  and  $OP$  especially when  $R$  is large. However, if the tax regime is reversible as we assumed above, the government can recoup similar amount of tax revenues to that under PRT as from the figure that  $OT$  is almost identical to  $OD$  when  $R$  is relatively large. As the oil-price-contingent regime has little effect on the entry trigger and lose little tax revenue, it would appear to be a reasonably efficient way for the

government to collect economic rents.

## 8 Conclusions

This paper has studied the impact of various tax regimes on the decision to develop an oil field using the real options approach to irreversible investment. It is clear that, even with no taxes, the option of waiting will have value if oil prices are volatile; so delaying investment (relative to the certainty equivalent case) is economically efficient. The issue analysed here is how the key parameters of the tax regime may “distort” this development decision; whether, indeed, it is possible to achieve tax “neutrality” (so that the price of oil which triggers investment remains unaffected); and how much of the economic rents of oil production are being recouped.

We show specifically how the two key parameters which broadly characterise the UK Petroleum Revenue Tax regime operated in the North Sea (the tax rate  $\lambda$  and the up-lift rate  $\eta$ ) alter the trigger for development. In particular, it was found that there exists a unique up-lift rate which ensures neutrality, irrespective of the tax rate levied on the revenues after the allowance is exhausted. The sensitivity of the development trigger to variations of these tax instruments was assessed numerically where three salient characteristics of the economic environment (oil price volatility, the discount rate and the rate of extraction) are varied over plausible ranges. Using central values for these characteristics, we went on to address the key issue: was the PRT regime as it existed before 1993 basically neutral or not? The answer implied by our analysis is that the regime was very close to being neutral; as the combination of an up-lift rate of 35% and a tax rate of over 80% only affect the trigger marginally (raising it by about 5%).

If this is correct the question immediately arises, why was such an efficient tax regime essentially dismantled in 1993 (where tax rate was cut substantially on existing fields and the PRT for the new fields was abolished). An explanation could be that the tax cuts were induced by the low price of oil and will be raised again if and when real oil prices recover. So the regime was not dismantled; rather the tax structure is contingent on the oil price. Our analysis confirms that such a price contingent regime needs not much affect either the development trigger or tax revenues.

Another explanation is that the high marginal rates do in fact have incentive effects which are not captured in this simple model. Two such effects, stressed by the Financial Secretary in his speech at the Petroleum Economist Conference, are the *disincentive* effect on incremental developments which extend the size and life of a field at a cost which is not deductible under PRT; and second the incentive to indulge in excessive drilling because Exploration and Appraisal costs for a new field can be transferred to existing fields and so almost full deductibility for PRT. These are important issues which could be explained more fully in a model with incremental development and exploration.

But what if the regime shift is not price contingent, and PRT really has been abolished permanently? In this case, one might consider the use of "bonus bidding" (where the rights to extract oil are sold at competitive auction, see Neher (1990), p321) as a mechanism for capturing the economic rents to be found in the North Sea. Alternatively, one might wish to reintroduce royalties on new fields. Or one could have bonus bidding for new fields subject to Royalty as in the U.S.

## Appendices

### A Value of an operating field

It can be seen from (3), (4), (12) and boundary conditions (14) and (15) that  $V(R, A)$  is homogeneous of degree 1 in  $(R, A)$ , so the transformation

$$x = \frac{R}{A}, \quad v(x) = \frac{V(R, A)}{A} \quad (\text{A1})$$

makes (12) an ordinary differential equation (ODE). Specifically, we derive

$$\frac{\partial V(R, A)}{\partial R} = \frac{\partial(V/A)}{\partial(R/A)} = v'(x) \quad (\text{A2})$$

$$\frac{\partial V(R, A)}{\partial A} = \frac{\partial(V/A)(A/R)}{\partial(A/R)} = v(x) - xv'(x) \quad (\text{A3})$$

$$\frac{\partial^2 V(R, A)}{\partial R^2} = \frac{\partial v'(x)}{\partial R} = \frac{1}{A}v''(x). \quad (\text{A4})$$

Substitution of (A2)–(A4) into (13)–(15), we obtain

$$\frac{1}{2}\sigma^2 x^2 v''(x) + (\alpha - \gamma + \gamma(1 - \delta)x)xv'(x) + \gamma(1 - \delta)x = (\rho + \gamma(1 - \delta)x)v(x), \quad (\text{A5})$$

and the boundary conditions

$$\lim_{x \downarrow 0} v(x) = \lim_{x \downarrow 0} \frac{\gamma(1 - \delta)x}{\rho + \gamma - \alpha} = 0, \quad (\text{A6})$$

$$\lim_{x \uparrow \infty} v(x) = \lim_{x \uparrow \infty} \frac{(1 - \lambda)\gamma(1 - \delta)x}{\rho + \gamma - \alpha}. \quad (\text{A7})$$

As the ODE given in (A5) is linear, we first solve for its particular solution which turns out to be the following

$$v^p(x) = \frac{\gamma(1 - \delta)x}{\rho + \gamma - \alpha}. \quad (\text{A8})$$

And its homogeneous solution  $v^h(x)$  satisfies

$$\frac{1}{2}\sigma^2 x^2 v_{xx}^h(x) + (\alpha - \gamma + \gamma(1 - \delta)x)xv_x^h(x) = (\rho + \gamma(1 - \delta)x)v^h(x). \quad (\text{A9})$$

Let  $v^h(x) = x^\theta n(z)$  where

$$\frac{1}{2}\sigma^2 \theta(\theta - 1) + (\alpha - \gamma)\theta - \rho = 0, \quad (\text{A10})$$

and  $z = -2\gamma(1 - \delta)x/\sigma^2$ , then  $n(z)$  is a solution to the Kummer's equation specified as

$$zn''(z) + (\omega(\theta) - z)n'(z) - (\theta - 1)n(z) = 0, \quad (\text{A11})$$

where

$$\omega(\theta) = 2\left(\theta + \frac{\alpha - \gamma}{\sigma^2}\right). \quad (\text{A12})$$

Equation (A11) has two linearly independent solutions which can be expressed by two Kummer's functions as  ${}_1F_1[\theta_+ - 1; \omega(\theta_+); z]$  and  ${}_1F_1[\theta_- - 1; \omega(\theta_-); z]$ . So the general solution to (A5) can be written as

$$v(x) = \frac{\gamma(1 - \delta)x}{\rho + \gamma - \alpha} + B_1 x^{\theta_+} {}_1F_1[\theta_+ - 1; \omega(\theta_+); z] + B_2 x^{\theta_-} {}_1F_1[\theta_- - 1; \omega(\theta_-); z]. \quad (\text{A13})$$

where  $\theta_+$  and  $\theta_-$  are positive and negative roots to (A10), and  $B_1$  and  $B_2$  are two arbitrary constants.

Applying boundary condition (A6), we set  $B_2 = 0$ . To determine  $B_1$ , we use the asymptotic condition for  $x \rightarrow +\infty$ . From Slater (1960, p59), the asymptotic expansion of a Kummer's function shows

$${}_1F_1[a; b; -x] = x^{-a} \frac{\Gamma(b)}{\Gamma(b-a)} (1 + O|x|^{-1}). \quad (\text{A14})$$

Substituting (A14) into (A13) and using the boundary condition (A7), one finds

$$B_1 = -\frac{\lambda\gamma(1 - \delta)}{\rho + \gamma - \alpha} \left(\frac{2\gamma(1 - \delta)}{\sigma^2}\right)^{\theta_+ - 1} \frac{\Gamma(\omega(\theta_+) - \theta_+ + 1)}{\Gamma(\omega(\theta_+))}, \quad (\text{A15})$$

where  $\Gamma(\cdot)$  is a Gamma function. Substituting (A15) into (A13) and making simply rearrangement, we obtain (16) in the text.

## B Proof of Proposition 1

The limits of  $f(y)$  in Proposition 1 is straightforward to show. As  ${}_1F_1[a; b; 0] = 0$ , so

$$\lim_{y \downarrow 0} f(y) = 0.$$

Using asymptotic expansion of the Kummer's function in (A14), one can show

$$\lim_{y \uparrow +\infty} f(y) = -\frac{\sigma^2}{2} \frac{1}{\rho + \gamma - \alpha} (\theta_+ - 1) (\omega(\theta_+) - \theta_+) = 1.$$

To show that  $f(y)$  is concave, we simply differentiate  $f(y)$  twice with respect to  $y$ . Using differential properties of Kummer's function (Slater 1960, p15, equation (2.1.10)) and the first Kummer's theorem (Slater 1960, p6, equation (1.4.1)), we derive

$$f''(y) = -\frac{\Gamma(\omega(\theta_+) - \theta_+)}{\Gamma(\omega(\theta_+))} \theta_+ e^{-y} y^{\theta_+ - 2} {}_1F_1[\omega(\theta_+) - \theta_+ - 1; \omega(\theta_+); y]. \quad (\text{B1})$$



Since  $\omega(\theta_+) - \theta_+ - 1 > 0$ , so  ${}_1F_1[\omega(\theta_+) - \theta_+ - 1; \omega(\theta_+); y] \geq 1$  for  $y \geq 0$  (Slater 1960, p103). Therefore,

$$f''(y) < 0 \quad \text{for } y \geq 0. \quad (\text{B2})$$

Finally, we prove that  $f(y)$  is a increasing function of  $y$ . Differentiating  $f(y)$  with respect to  $y$  yields

$$\begin{aligned} f'(y) &= \frac{\sigma^2}{2} \frac{1}{\rho + \gamma - \alpha} \left\{ 1 - \frac{\Gamma(\omega(\theta_+) - \theta_+ + 1)}{\Gamma(\omega(\theta_+))} y^{\theta_+ - 1} \right. \\ &\quad \left. \times ({}_1F_1[\theta_+ - 1; \omega(\theta_+); -y] + (\theta_+ - 1) {}_1F_1[\theta_+; \omega(\theta_+); -y]) \right\}. \end{aligned} \quad (\text{B3})$$

Using asymptotic expansion for Kummer's function from (A14), we obtain

$$\lim_{y \uparrow +\infty} f'(y) = 0. \quad (\text{B4})$$

Combining it with (B2), we conclude  $f'(y) > 0$  for  $y \geq 0$ . QED

## C Proof of existence and uniqueness of the optimal development trigger

Substituting (21) and (16) into (22) and (23), and eliminating  $C_+$  yields

$$g(y_e) = \left(\frac{1}{\lambda} - 1\right)c\left(y_e - \frac{1}{c(1+\eta)}\right) + \frac{\eta}{1+\eta} \equiv \phi(y_e), \quad (\text{C1})$$

where

$$g(y_e) = 1 + \frac{1}{\xi_+} y_e f'(y_e) - f(y_e), \quad (\text{C2})$$

$$c = \left(1 - \frac{1}{\xi_+}\right) \frac{\sigma^2}{2} \frac{1}{\rho + \gamma - \alpha}, \quad (\text{C3})$$

$$y_e = \frac{2\gamma(1-\delta)}{\sigma^2} \cdot \frac{R_e}{(1+\eta)D}. \quad (\text{C4})$$

As (22) and (23) give the sufficient conditions for the optimality (cf Brekke and Øksendal 1992), the existence and uniqueness of the optimal solution to (20) only depends on the existence and uniqueness of the solution to (C1). The lemma below provides such results.

**Lemma C1** *There exists a unique  $y_e$  to (C1).*

PROOF: From (C1) and (C2), differentiating  $g(y)$  with respect to  $y$  yields

$$g'(y) = \frac{1}{\xi_+} y f''(y) - \left(1 - \frac{1}{\xi_+}\right) f'(y) < 0, \quad (\text{C5})$$

as  $\xi_+ > 1$ ,  $f''(y) < 0$  and  $f'(y) > 0$ . Differentiating  $\phi(y)$  with respect to  $y$  yields

$$\phi'(y) = \left(\frac{1}{\lambda} - 1\right) c \geq 0, \quad \text{if } 0 < \lambda \leq 1. \quad (\text{C6})$$

As

$$\lim_{y \downarrow 0} g(y) = 1 > \lim_{y \downarrow 0} \phi(y) = -\left(\frac{1}{\lambda} - 1\right) \frac{1}{1 + \eta},$$

and

$$\lim_{y \uparrow +\infty} g(y) = 0 < \lim_{y \uparrow +\infty} \phi(y) \rightarrow +\infty.$$

So (C1) has a unique solution for  $0 < \lambda \leq 1$ .

If  $\lambda = 1$ , then equation (C1) becomes

$$y_e = \frac{1}{1 + \eta},$$

or

$$\frac{\gamma(1 - \delta)R_e}{\rho + \gamma - \alpha} = \frac{\xi_+}{\xi_+ - 1} D, \quad (\text{C7})$$

which is the entry trigger in the absence of PRT. So, we complete the proof. QED

## D Proof of Proposition 2

Before proceeding to prove proposition 2, we first provide the following lemma.

**Lemma D1** *For  $a > 0$ ,  $d > 0$ ,  $b > 1$  and  $y > 0$ , the following equation has a unique root;*

$$y \frac{{}_1F_1'[a; b; y]}{{}_1F_1[a; b; y]} = d. \quad (\text{D1})$$

PROOF

**Existence:** For  $y \rightarrow 0$ ,

$$\lim_{y \rightarrow 0} y \frac{{}_1F_1'[a; b; y]}{{}_1F_1[a; b; y]} = 0 < d;$$

for  $y \rightarrow +\infty$

$$\begin{aligned} \lim_{y \rightarrow +\infty} y \frac{{}_1F_1'[a; b; y]}{{}_1F_1[a; b; y]} &= \lim_{y \rightarrow +\infty} \frac{a}{{}_1F_1[b - a; b; -y]} \frac{{}_1F_1'[b - a; b + 1; -y]}{{}_1F_1[b - a; b; -y]} \\ &= \lim_{y \rightarrow +\infty} y \rightarrow +\infty > d. \end{aligned}$$

Since the left hand side of (D1) is continuous for  $y \geq 0$ , (D1) has at least one solution.

**Uniqueness:** Denote  ${}_1F_1[a; b; y]$  by  ${}_1F_1$  and let

$$h(y) = y \frac{{}_1F_1'[a; b; y]}{{}_1F_1[a; b; y]}. \quad (\text{D2})$$

Differentiating (D2) with respect to  $y$  yields

$$h'(y) = \frac{{}_1F_1'}{{}_1F_1} + y \left[ \frac{{}_1F_1''}{{}_1F_1} - \left( \frac{{}_1F_1'}{{}_1F_1} \right)^2 \right]. \quad (\text{D3})$$

From Slater (1960, p2), it can be shown that  ${}_1F_1[a; b; y]$  is the solution to the following ODE:

$$y {}_1F_1'' + (b - y) {}_1F_1' - a {}_1F_1 = 0. \quad (\text{D4})$$

So

$$\frac{{}_1F_1''}{{}_1F_1} = \frac{a}{y} - \frac{b - y}{{}_1F_1} \frac{{}_1F_1'}{{}_1F_1}. \quad (\text{D5})$$

Substituting (D5) into (D2), we derive

$$h'(y) = a + (1 - b + y) \frac{{}_1F_1'}{{}_1F_1} - y \left( \frac{{}_1F_1'}{{}_1F_1} \right)^2. \quad (\text{D6})$$

Let the solution to (D1) be  $\tilde{y}$ , then at  $\tilde{y}$ , equation (D1) is satisfied. Substitution of (D1) into (D6) gives the slope of  $h(y)$  at  $\tilde{y}$ :

$$h'(\tilde{y}) = \frac{a + d}{\tilde{y}} \left( \tilde{y} - \frac{(b + d - 1)d}{a + d} \right). \quad (\text{D7})$$

For  $\tilde{y} \geq 0$ ,  $h'(\tilde{y}) \geq 0$  if and only if  $\tilde{y} \geq (b + d - 1)d/(a + d)$ . Since  $h(0) < d$ , so at the smallest root to (D1),  $h(\tilde{y})$  must have a non-negative slope, then there is no solution for  $y < (b + d - 1)d/(a + d)$ . For any root to (D1) such that  $\tilde{y} > (b + d - 1)d/(a + d)$ , as the slope of  $h(y)$  is positive, there is at most one root for  $y > (b + d - 1)d/(a + d)$ . To prove the uniqueness, we only have to show that  $\tilde{y} = (b + d - 1)d/(a + d)$  is not the solution to (D1).

Differentiating (D6) with respect to  $y$  and using (D5) yields

$$\begin{aligned} h''(y) &= \frac{a}{y} (1 - b + y) + (1 + 2a - \frac{b - y}{y} (1 - b + y)) \frac{{}_1F_1'}{{}_1F_1} \\ &\quad - (2 - b + y) \left( \frac{{}_1F_1'}{{}_1F_1} \right)^2 + 2y \left( \frac{{}_1F_1'}{{}_1F_1} \right)^3. \end{aligned} \quad (\text{D8})$$

If  $\tilde{y}$  is the root to (D1), then by substituting (D1) into (D8), we obtain

$$h''(\tilde{y}) = \{(a+d)\tilde{y}^2 + [(a+2d)(1-b-d) - c(a+d)]\tilde{y} + d(2d+b)(b+d-1)\}/\tilde{y}^2, \quad (\text{D9})$$

as  $\tilde{y} = (b+d-1)d/(a+d)$ , then

$$h''(\tilde{y}) = \frac{(a+d)^2}{(b+d-1)d} > 0. \quad (\text{D10})$$

Since  $h'(\tilde{y}) = 0$ ,  $h''(\tilde{y}) > 0$  at  $\tilde{y} = (b+d-1)d/(a+d)$  and  $h(0) < d$ , there must be another smaller root which contradicts that there is no solution for  $y < (b+d-1)d/(a+d)$ . So  $\tilde{y} = (b+d-1)d/(a+d)$  is not the solution to (D1). Thus we complete the proof.

For any given up-lift rate  $\eta$ , we notice that the right hand side function (denoted by  $\phi(y_e)$ ) of equation (C1) is a straight line, where the slope depends on the tax rate  $\lambda$  but not the up-lift rate  $\eta$ . For any given  $\eta$ , these straight lines go through a fixed point  $(1/c(1+\eta), \eta/(1+\eta))$  which is independent of the tax rate. So the effect of changing  $\lambda$  only depends on whether that fixed point lies below or above the function  $g(y_e)$  on the left hand side of equation (C1). Notice further that, by varying the up-lift rate  $\eta$  ( $0 \leq \eta < +\infty$ ), these fixed points are actually lying on a straight line as

$$\varphi(y_e) = 1 - cy_e, \quad y_e = \frac{1}{c(1+\eta)}, \quad 0 < y_e \leq c^{-1}. \quad (\text{D11})$$

Before proceeding to discuss the tax rate effect on entry, we first provide a lemma which establishes the relationship between  $\varphi(y_e)$  and  $g(y_e)$ .

**Lemma D2** *There exists a unique solution  $\tilde{y}_e > 0$  to*

$$g(y_e) = \varphi(y_e). \quad (\text{D12})$$

And

$$\begin{cases} g(y_e) < \varphi(y_e) & \text{if } 0 < y_e < \tilde{y}_e \\ g(y_e) = \varphi(y_e) & \text{if } y_e = \tilde{y}_e \\ g(y_e) > \varphi(y_e) & \text{if } y_e > \tilde{y}_e. \end{cases}$$

**PROOF:** To prove lemma D2, we first simplify equation (C1). Using Kummer's first theorem (Slater 1960, p6, equation (1.4.1)) and differential property of Kummer's function (Slater, p15, equation (2.1.14)), we can show

$$\begin{aligned} \text{Sign}(g(y_e) - \phi(y_e)) &= \text{Sign} y_e^{\theta_+} \{ (\xi_+ - \theta_+) {}_1F_1[\omega(\theta_+) - \theta_+; \omega(\theta_+); y_e] \\ &\quad + \frac{\xi_+ - 1}{\omega(\theta_+) - \theta_+} y_1 F_1'[\omega(\theta_+) - \theta_+; \omega(\theta_+); y_e] \}. \end{aligned} \quad (\text{D13})$$

So equation (C1) has a root  $y_e = 0$  which corresponds to  $\eta \rightarrow +\infty$ . For  $\eta < +\infty$ ,  $y_e > 0$ , therefore (C2) is reduced to

$$(\xi_+ - \theta_+) {}_1F_1[\omega(\theta_+) - \theta_+; \omega(\theta_+); y_e] + \frac{\xi_+ - 1}{\omega(\theta_+) - \theta_+} y_e {}_1F_1'[\omega(\theta_+) - \theta_+; \omega(\theta_+); y_e] = 0. \quad (\text{D14})$$

By simple rearrangement, the above equation can be transformed into the form as in (D1). Because  $\omega(\theta_+) - \theta_+ > 0$  and  $\omega(\theta_+) > 1$ , (D14) has a unique root  $\tilde{y}^e > 0$ . As the right hand side of (D13) is negative when  $y_e \rightarrow 0$ , positive when  $y_e \rightarrow +\infty$ , so we obtain Lemma D2. QED

To complete the proof of Proposition 2, we proceed graphically. In Figure 6  $g(y_e)$  and  $\varphi(y_e)$  are shown as  $GK$  and  $GH$  where horizontal axis represents  $y_e$  and vertical its related functions. Suppose we choose a particular  $\eta$  such that the fixed point locates at  $L$  which is above curve  $GK$  in Figure 6. As increasing  $\lambda$  decreases the slope of straight line  $\varphi(y_e)$ , so  $\varphi(y_e)$  rotates to the left and intersects  $GK$  at a lower  $y_e$ . Because

$$\text{Sign} \frac{\partial R_e}{\partial \lambda} = \text{Sign} \frac{\partial y_e}{\partial \lambda},$$

then increasing  $\lambda$  decreases the entry trigger  $R_e$ .

If the up-lift rate  $\eta$  is chosen such that the fixed point locates at  $M$  which is below  $GK$  in Figure 6, applying similar argument as from above we can show that increasing  $\lambda$  increases the entry trigger  $R_e$ . If the fixed point is chosen at the intersection of  $GK$  and  $GH$  at  $N$  in the figure, changing  $\lambda$  does not have any effect on the entry trigger. Specifically at  $N$ , it can be shown that

$$\frac{\gamma(1 - \delta)R_e^{(N)}}{\rho + \gamma - \alpha} = \frac{\xi_+}{\xi_+ - 1} D,$$

so  $R_e^{(N)}$  is the entry trigger as if the field is not taxed. Thus, a particular  $\eta$  corresponds to  $N$  generates a *neutral tax regime*.

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# Figures

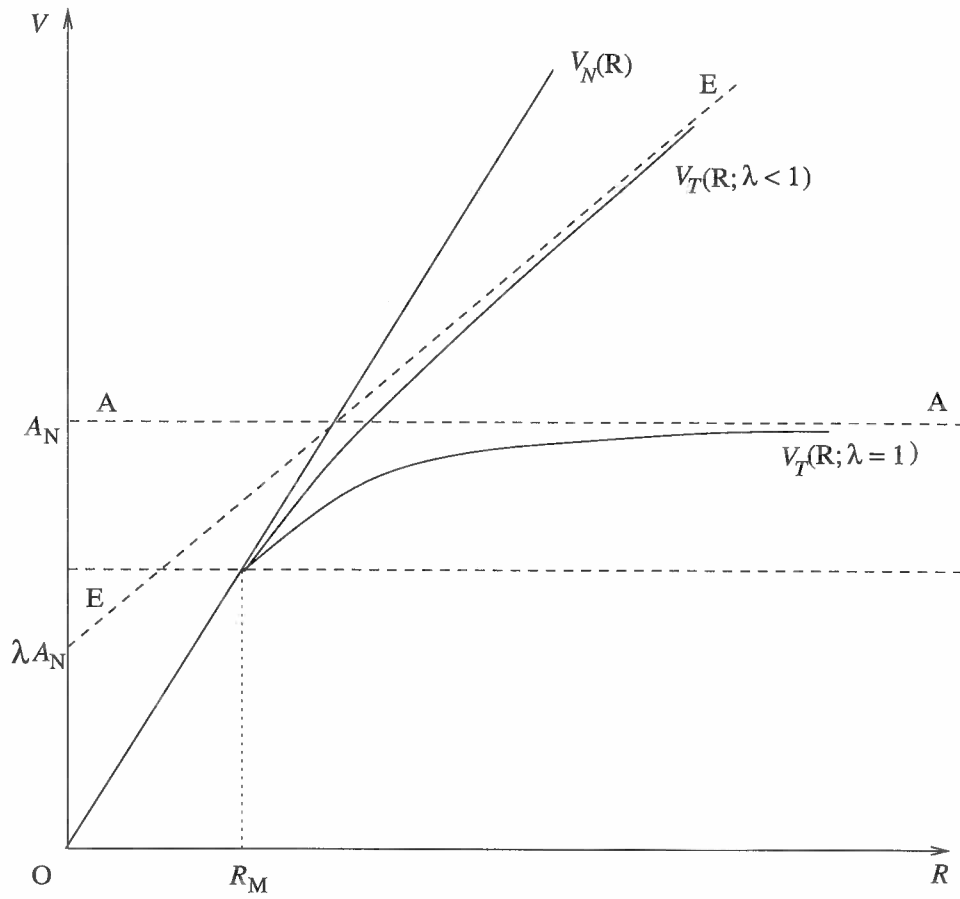


Figure 1: Neutral allowances and the taxing of economic rents.

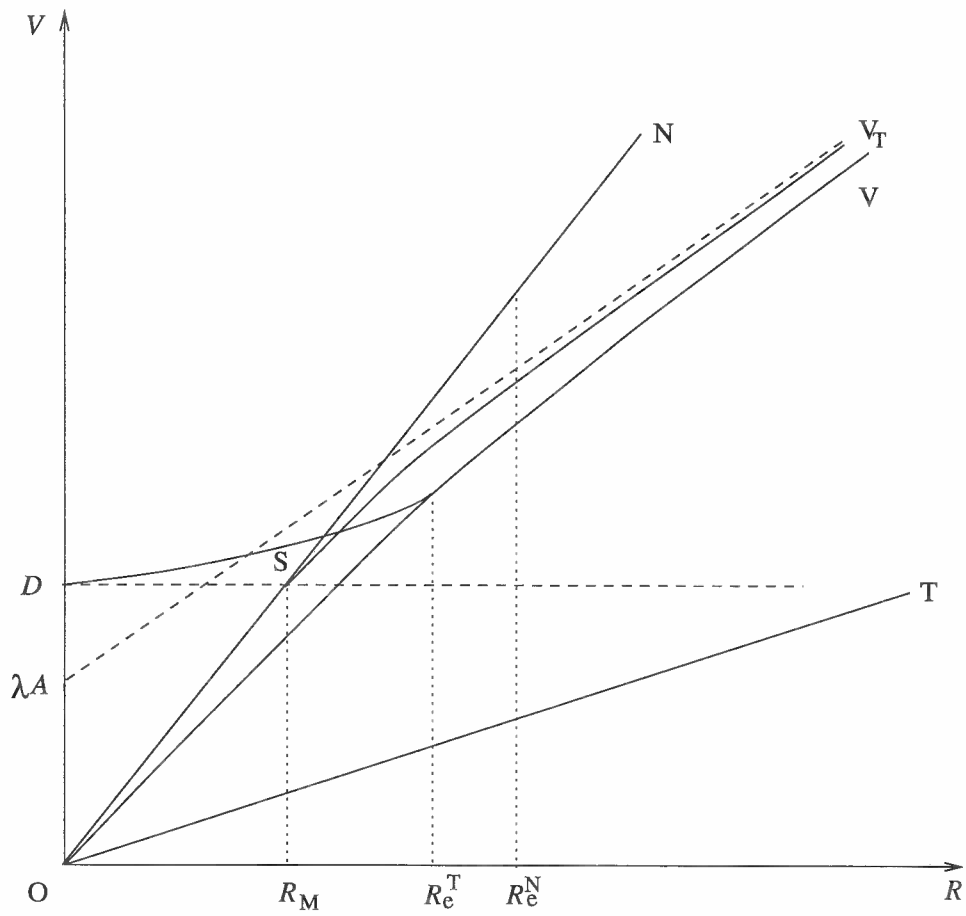


Figure 2: *Ex ante* value of the field subject to PRT.



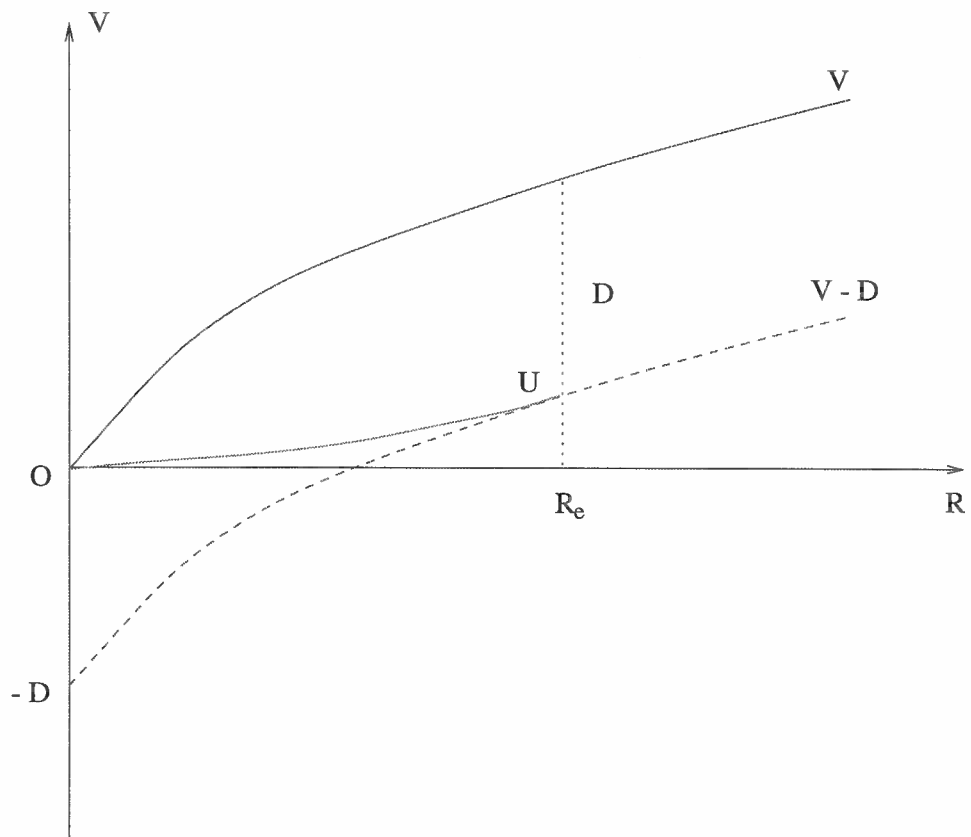


Figure 3: Entry trigger under PRT.

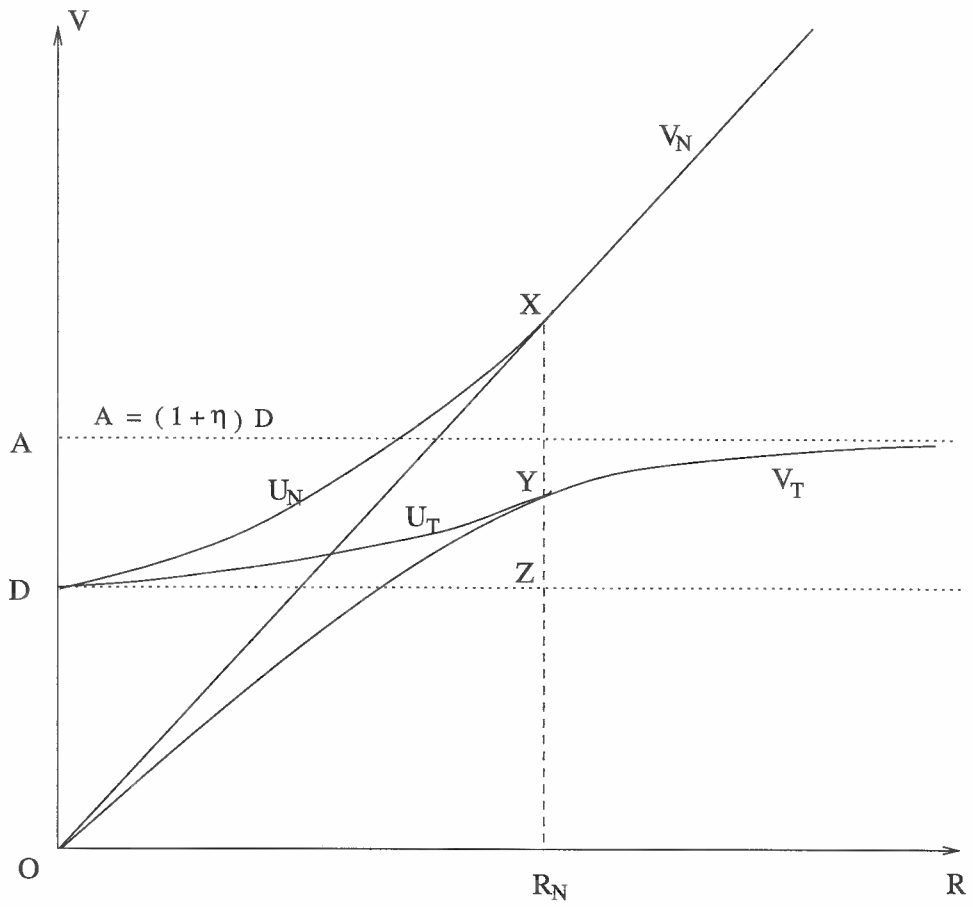


Figure 4: Entry under neutral PRT.

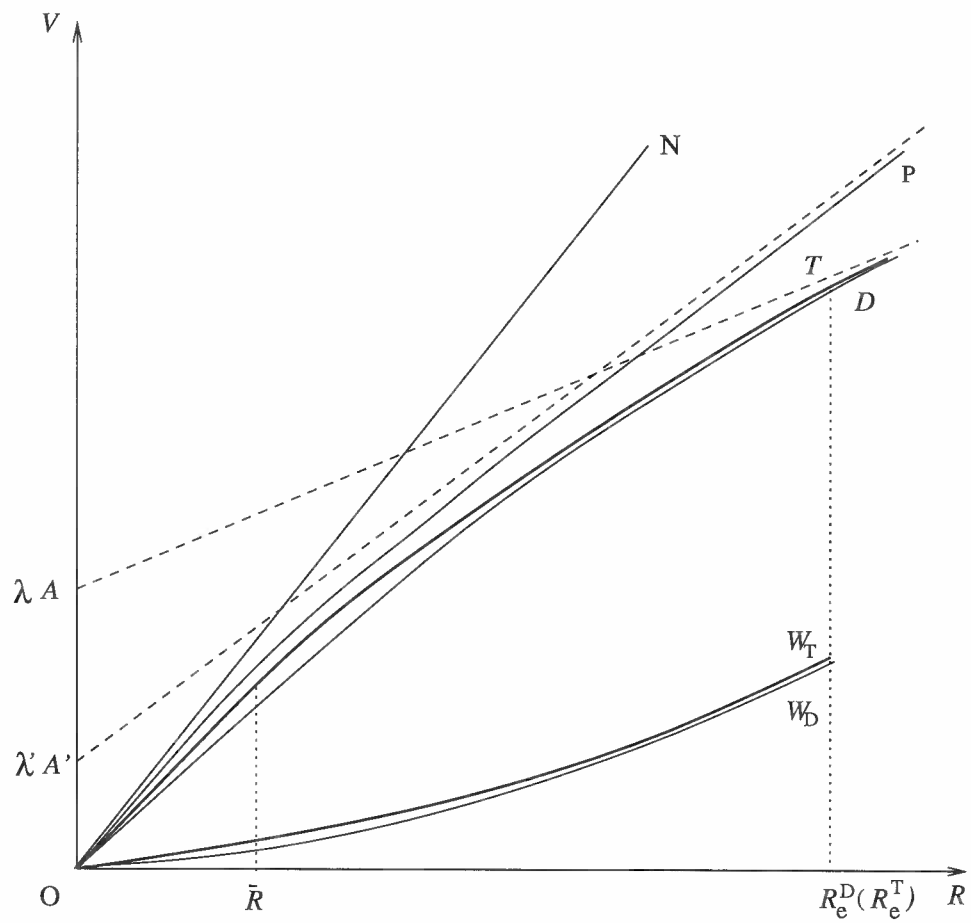


Figure 5: Effect of temporary fiscal regime shift on the development trigger.

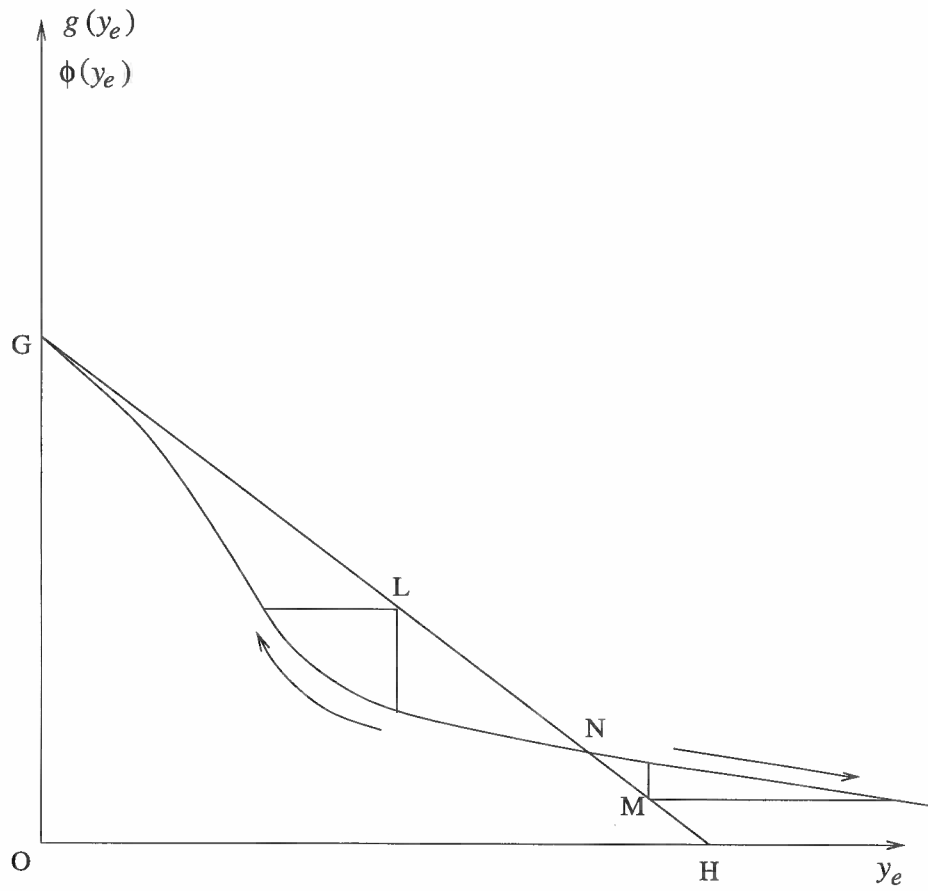


Figure 6: Response of  $y_e$  to changing tax rate  $\lambda$  for given up-lift rate.

## Tables

Table 1: Neutral up-lift rate  $\tilde{\eta}$ : response to changing  $\sigma$  and  $\rho$ .

$\tilde{\eta}$	$\sigma = 0.0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.4$
$\rho = 0.03$	0.43	0.44	0.34	0.22
$\rho = 0.05$	0.71	0.57	0.46	0.33
$\rho = 0.07$	1.00	0.80	1.02	1.19
$\gamma = 0.07, \alpha = 0.0$				

Table 2: Neutral up-lift rate  $\tilde{\eta}$ : response to changing  $\sigma$  and  $\gamma$ .

$\tilde{\eta}$	$\sigma = 0.0$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.4$
$\gamma = 0.05$	1.00	0.81	0.67	0.50
$\gamma = 0.07$	0.71	0.57	0.46	0.33
$\gamma = 0.10$	0.50	0.39	0.31	0.21
$\alpha = 0, \rho = 0.05$				

Table 3: Efficiency of PRT: the effects of varying  $\sigma$  and  $\rho$ .

$E$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.4$
$\rho = 0.03$	80.63%	91.83%	97.75%
$\rho = 0.05$	62.28%	80.27%	92.83%
$\rho = 0.07$	46.87%	68.41%	86.52%
$\gamma = 0.07, \alpha = 0.0$			

Table 4: Efficiency of PRT: the effects of varying  $\sigma$  and  $\gamma$ .

$E$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.4$
$\gamma = 0.05$	52.00%	73.41%	89.66%
$\gamma = 0.07$	62.28%	80.27%	92.83%
$\gamma = 0.10$	71.92%	86.16%	95.35%
$\rho = 0.05, \alpha = 0.0$			

Table 5: Response of  $r$  to changing  $\sigma$  and  $\gamma$ : PRT in place.

$r$	$\sigma = 0.2$		$\sigma = 0.2$
$\gamma = 0.05$	1.125	$\rho = 0.03$	0.921
$\gamma = 0.07$	1.053	$\rho = 0.05$	1.053
$\gamma = 0.10$	0.978	$\rho = 0.07$	1.135
$\alpha = 0, \rho = 0.05$		$\alpha = 0, \gamma = 0.07$	

Table 6: Tax efficiency: PRT in place.

$E$	$\sigma = 0.2$		$\sigma = 0.2$
$\gamma = 0.05$	75.28%	$\rho = 0.03$	69.94%
$\gamma = 0.07$	72.82%	$\rho = 0.05$	72.82%
$\gamma = 0.10$	69.21%	$\rho = 0.07$	75.01%
$\alpha = 0, \rho = 0.05$		$\alpha = 0, \gamma = 0.07$	

Table 7: Response of  $r$  to changing  $\sigma$  and  $\gamma$ : with and without PRT.

$r$	$\sigma = 0.2$		$\sigma = 0.2$
$\gamma = 0.05$	1.024	$\rho = 0.03$	0.868
$\gamma = 0.07$	0.986	$\rho = 0.05$	0.986
$\gamma = 0.10$	0.912	$\rho = 0.07$	1.031
$\alpha = 0, \rho = 0.05$		$\alpha = 0, \gamma = 0.07$	

Table 8: Tax efficiency: without PRT.

$E$	$\sigma = 0.2$		$\sigma = 0.2$
$\gamma = 0.05$	37.18%	$\rho = 0.03$	34.82%
$\gamma = 0.07$	36.69%	$\rho = 0.05$	36.39%
$\gamma = 0.10$	36.14%	$\rho = 0.07$	38.42%
$\alpha = 0, \rho = 0.05$		$\alpha = 0, \gamma = 0.07$	