COMPARING THE BIAS AND MISSPECIFICATION IN ARFIMA MODELS

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Comparing the Bias and Misspecification in ARFIMA Models

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Abstract

A number of papers have looked at the bias in the fractional integration parameter, d, using a variety of alternative estimation techniques. This paper supplements that literature by investigating the bias in both the short-term and long-term parameters for a range of ARFIMA models using a more comprehensive range of estimation techniques. The results suggest that all estimation procedures yield slightly biased estimates of the long-run parameter, and that these biases become larger with the introduction of short-term AR or MA parameters. The bias in the short-run parameters mirrors that in the long-run parameters. These biases often causes model selection criteria to select an incorrect ARMA specification, having filtered out the long-run parameter. Incorrect specification of the short-run parameters in the ARFIMA model can accentuate the bias in the long-run parameter.

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1. Introduction

Papers by Ray (1993) and Smith and Yadav (1994) investigate the forecasting accuracy of AutoRegressive Fractional Integrated Moving Average (ARFIMA) models, compared with high ordered Autoregressive (AR) models. Both papers find only a limited potential gain from using the correct ARFIMA model, even when the fractional integration parameter, d, is assumed to be known. Estimating the fractional parameter, d, makes the use of the ARFIMA model even less attractive. Part of the explanation for this is the bias that can arise from estimating the fractional parameter. Sowell (1992), Agiakloglou (1993), Cheung (1993) and Reisen (1994), show that, across a variety of alternative estimation techniques, there can be considerable bias in estimating the fractional integration (long-run) parameter, d, in the presence of non-zero Autoregressive (AR) and/or Moving Average (MA) (short-run) parameters in ARFMA models.

This bias in the long-run parameter can lead to problems of identifying the short-run parameter. This issue has been investigated by Schmidt and Tschernig (1993) and Crato and Ray (1995), who calculate the probability of choosing the correct ARFIMA specification, using a number of alternative selection criteria.

This paper looks at issues of bias in the long-run and short-run parameters, as well as problems of misspecification of the ARFIMA models according to a number of selection criteria. The next section considers both the bias in the long-run and short-run parameters from estimating the correct ARFIMA model using a variety of alternative estimation methods. Section 3 analyses the likelihood of selecting the correct model using the Akaike Information Criterion (AIC), Schwarz Criterion (SC) and the Hannan-Quinn Criterion (HQ), as well as problem of potential misspecification of the short-run model in the maximum likelihood procedure. Section 4 offers some concluding remarks.

2.1 Bias in the Long-run Parameter in ARFIMA Models

Consider the simple ARFIMA model of the form:

$$\Phi(L)(1-L)^d y_t = \Theta(L)\varepsilon_t \qquad (1)$$

where, $\Phi(L) = 1-\phi_1 L-\phi_2 L^2-..-\phi_p L^p$, and $\Theta(L) = 1+\theta_1 L+\theta_2 L^2+..+\theta_q L^q$ are polynomials in the lag operator, L, $\Phi(z)$ and $\Theta(z)$ have roots outside the unit circle and $d\in(-0.5, 0.5)$. A number of alternative estimators have been developed for estimating the fractionally integrated model (1). This paper looks into the properties of three estimators which estimate the long-run parameter, d, irrespective of the short-run parameters; (i) the Geweke Poter-Hudak (GPH) (1983) procedure using the trimming approach of Robinson (1992), (ii) the average periodogram method of Robinson (1995) and Lobato and Robinson (1995) (APER), (iii) the rescale-range (R/S) statistic, where $d = \frac{\log(RS(q))}{\log(n)} - 0.5$, and $RS(q)$ is the modified R/S statistic. These estimators are then compared with the performance of the maximum likelihood (ML) method of Sowell (1992), which estimates the long-run and short-run parameters simultaneously.

Of the range of alternative models nested in equation (1), only two models are considered; the ARFIMA(1,d,0) and the ARFIMA(0,d,1).[1] The results for the GPH, APER and R/S statistics are based upon 1000 replications. However, due to the time consuming nature of the ML procedure, results for this procedure are based upon just 100 replications. In all cases the errors are drawn from a random normal distribution using the NAG library (routine G05DDF) and the sample size is 256.

[1] Only a limited range of AR and MA parameter values are reported due to the large bias observed for some of the short-run parameter values

The results for the bias in the fractional integration parameter, d, and the mean square error (MSE) of these parameter estimates are reported in Tables 1-4. The results are similar to those seen elsewhere and will only be briefly discussed here. The modified R/S statistic (Table 1) consistently under-estimates the fractional integration parameter for $d \neq 0$, suggesting a greater degree of stationarity in the series than is truly the case. The bias in d is related to the short-run AR and MA parameters. In particular, the R/S statistic has a small bias when the short-run and long-run effects are not off-setting one another. Consequently, positive (negative) AR or MA parameters and positive (negative) values of d tend to yield smaller biases than when the AR or MA parameters and the fractional parameter are of different signs. The biases appear to be similar between ARFIMA(0,d,1) models and ARFIMA(1,d,0) models.

Table 2 reports the bias in the estimate of the fractional integration parameter, d, for ARFIMA(1,d,0) and ARFIMA(0,d,1) models from using the GPH procedure. The results are similar to those shown by Cheung (1993) and demonstrate that the bias in d is negligible for all values of d and for values of the short-run parameter $\phi < 0.5$ $(\theta > -0.5)$. However, the MSE values are much larger than those associated with the R/S statistics, even though the bias is generally smaller.[2] For APER (Table 3) the bias in the fractional integration parameter is generally small, but like the R/S procedure it tends to under-estimate the value of d. However, for the APER procedure, this bias appears unrelated to the AR or MA parameter for $\phi < 0.5$ and $\theta > -0.5$.

The bias in the fractional integration parameter from using the ML procedure tends to be negative for all values of d and for all AR or MA parameters (Table 4), this contrasts slightly with the results of Sowell (1992). The results for the ARFIMA(1,d,0) model with $\phi > 0.0$

[2] The large MSE is due to the small choice of $n^{0.5}$=16 observations are in the regression. Use of $n^{0.75}$=64 reduces the MSE by a factor of approximately 5.

contrasts substantially with the results derived from the other estimation procedures. Despite the evidence of Table 4, the bias in d is not particularly associated with the AR or MA coefficient. However, for the ARFIMA(0,d,1) model with d > 0.0, bias does increase as the MA coefficient becomes more negative. While for the other three procedures considered in this paper, for an ARFIMA(1,d,0) model large positive values of the short-run AR parameter produces substantial bias in the fractional integration parameter, this is not true for the ML procedure. However, all procedures have substantial bias associated with large negative values for the MA parameter in the ARFIMA(0,d,1) model.

2.2 Bias in the Short-run Parameters in ARFIMA Models

For GPH, R/S and APER the AR and MA parameter estimates are obtained in a two-stage process, which first estimates the fractional integration parameter, d, then filters the data using this parameter estimate and finally estimates the appropriate short-run parameters.[3] THE ML procedures estimates the short-run and long-run parameters simultaneously. In all models and across all estimation procedures the bias in the short-term parameters (Tables 5-8) is basically the mirror image of that in the fractional parameter, with negative bias in the short-term parameters associated with positive bias in the fractional integrated parameter. Overall, there appears little to choose between the ML procedure and the GPH or APER procedures in terms of the estimates of either the long-run or short-run parameter estimates. The exception to this is for ARFIMA(1,d,0) models with a large positive AR parameter, in this case the ML procedure does not exhibit the same evidence of over-estimation of the long-run parameter. The

[3] Iterating the process so that given the short term parameter we filter out the short-run dynamics and estimate the fractional integration term, d, which we then use to filter the original data and estimate the ARMA parameters and so on, until convergence yielded inferior estimates compared with the non-iterated procedure.

MSE for the GPH procedure is again markedly larger than those observed for the other three procedures.

3.1 Model Misspecification

The previous section demonstrated that all estimation procedures could incur reasonable biases in both the short-run and long-run parameters, even when the correctly specified ARFIMA model is estimated. In this section, we investigate the probability of specifying the correct model, given the bias in the long-run and short-run parameters. This idea has been examined previously by Schmidt and Tschernig (1993) and Crato and Ray (1995), who used the ML procedure of Whittle (1951) as well as the GPH procedure and found that there was only a relatively low probability of selecting the correct model. Tables 9-16 report the percentage of times an ARFIMA(0,d,0), (0,d,1), (1,d,0) or (1,d,1) model was selected by the AIC, SC and HQ criteria when the DGP was an ARFIMA(1,d,0) and an ARFIMA(0,d,1) model.[4]

The results for the GPH procedure, when the DGP is an ARFIMA(1,d,0) model, are reported in Table 9 and are similar for all values of d. For $\phi = \pm 0.3$ the ARFIMA(1,d,0) model is selected most often, although even in this case there is a non-negligible probability of selecting the ARFIMA(0,d,1) model. For $\phi = \pm 0.2$ or smaller, the probability of selecting the ARFIMA(0,d,0) increases relative to both the ARFIMA(1,d,0) and the ARFIMA(0,d,1). The ARFIMA(1,d,1) is rarely selected. The results for the case when the DGP is an ARFIMA(0,d,1) model (Table 10) are again similar for all values of d. In this case there is a high probability of incorrectly selecting either the ARFIMA(1,d,0) or ARFIMA(1,d,1) model. There is a clear non-

[4] Including up to AR(4) models as potential alternatives to the correct model does not seriously change the results as these were selected on only 5%-15% of occasions in total using the SIC or HQ criteria. The AIC criteria selected these models far more frequently.

symmetry in the results around $\phi = 0.0$, with a higher probability of selecting the correct model for $\phi < 0.0$. In both these tables the probability of selecting the correct model is 50% at best.

Tables 11 and 12 report the selection probabilities for the APER procedure for an ARFIMA (1,d,0) and ARFIMA(0,d,1), respectively. The probability of selecting the correct model for an ARFIMA(1,d,0) is approximately equal to 50% for d > 0.0 and greater than 50% for d < 0.0 (mainly at the expense of the probability of selecting an ARFIMA(1,d,1) model). There is approximate symmetry in the probabilities around $\phi = 0.0$. The results for the ARFIMA(0,d,1) (Table 12) show that for d < 0.0 and $\theta < 0.0$ there is the greatest probability of correctly selecting an ARFIMA(0,d,1), while for d > 0.0 there is symmetry around $\theta < 0.0$.

The results for the R/S statistics (Tables 13 and 14) are more complicated and strongly reflect the nature of the bias in the estimate of d. Consequently, in Table 13 for d > 0.0 (< 0.0) and $\phi > 0.0$ (< 0.0), the probability of selecting the correct model is very high as the bias in d is low, however for d > 0.0 (< 0.0) and $\phi < 0.0$ (> 0.0) the bias in d is large and the probability of selecting the correct model is small. The same result hold for the ARFIMA(0,d,1) model in Table 14.

The results for the ML procedure are shown in Tables 15 and 16, in both cases the results are similar for all values of d. For the ARFIMA(1,d,0) the results show that the probability of selecting the correct model is approximately 70% for $\phi = -0.3$ and 60% for $\phi = 0.3$, although these probabilities fall to around 30% for $\phi = \pm 0.1$. For the ARFIMA(0,d,1) the probability of selecting the correct model are much smaller, with the incorrect ARFIMA(1,d,0) often selected in preference to the correct ARFIMA(0,d,1) model.

GPH, R/S and APER estimate the fractional parameter irrespective of the short-run parameters, consequently, misspecification of the correct ARMA process will only lead to bias in the short-run parameters. In comparison the ML procedure estimates the short-run and long-run parameters simultaneously, therefore the problems of identifying the correct ARFIMA model using a selection criterion (seen in Tables 15 and 16), can lead to bias in both the long-run and short-run parameters. Therefore, in talking about the bias in the fractional integration parameter associated with the ML procedure, it is necessary to calculate the bias in the ML model associated with the three types of misspecification we have looked at; (i) the data generating process (DGP) is an ARFIMA(1,d,0) (ARFIMA(0,d,1)) model and an ARFIMA(0,d,0) model is fitted, (ii) the DGP is an ARFIMA(1,d,0) (ARFIMA(0,d,1)) model and an ARFIMA(0,d,1) (ARFIMA(1,d,0)) model is fitted, (iii) the DGP is an ARFIMA(1,d,0) (ARFIMA(0,d,1)) and an ARFIMA(1,d,1) is fitted.

The results for the case when the DGP is either an ARFIMA(1,d,0) or an ARFIMA(0,d,1) and a misspecified ARFIMA(0,d,0) is estimated are similar, consequently only the results from the case when the DGP is an ARFIMA(1,d,0) are reported. Table 17 shows that the bias in d is approximately constant for a given value of the short-run parameter across all values of d considered. However, this bias is positively related to the size of the short-run parameter. The bias induced from estimating an ARFIMA(1,d,0) when the DGP is an ARFIMA(0,d,1) is negligible, see Table 18. In contrast, when the DGP is an ARFIMA(1,d,0) and an ARFIMA(0,d,1) is estimated the bias in d can be quite marked, compare Table 19 with Table 4. Finally when an over-specified ARFIMA(1,d,1) model is estimated, d is always negatively biased regardless of the underlying DGP (see Tables 20 and 21). This bias arises because the estimate of the AR parameter is too large, although when the DGP is an ARFIMA(1,d,0) and $\phi > 0.0$ the bias in the AR parameter is negligible. The upward bias in the AR parameter, in these models, does not seem to adversely affect the MA parameter.

For the ML procedure, the expected bias in the estimate of the fractional integration parameter is calculated for both the ARFIMA(1,d,0) model and the ARFIMA(0,d,1) model using both the AIC and the SC selection procedures.[5] These results are reported in Tables 22-25, for both models it is no-longer the case that the bias in the parameter estimate is always negative, due to the positive bias in the estimate of d associated with omitting short-run parameters, for $\phi(\theta) > 0.0$. For the ARFIMA(0,d,1) model, the expected bias in d is often larger than that reported in Table 4, whereas for the ARFIMA(1,d,0) model the bias in d is only larger than that reported in Table 4 for $\phi < 0.0$.

3. Concluding Remarks

This paper looks at the bias in the fractional integration parameter, d, when short-run AR or MA parameters are included in an ARFIMA model. The results suggest that, of the procedures considered, providing the correct ARFIMA model is fitted then ML, GPH and APER perform creditably. Bias in the short-run parameters mirrors that in the long-run parameters. The probability of specifying the correct model is relatively low. This is problematic as incorrect specification of the AR or MA short-run parameters can adversely affect the performance of the ML procedure. In particular, estimating ARFIMA(1,d,1) models when the underlying DGP is either an ARFIMA(1,d,0) or an ARFIMA(0,d,1) can lead to relatively large negative bias in the long-run parameter, d, as can estimating an ARFIMA(0,d,1) when the DGP is an ARFIMA(1,d,0).

[5] The expected bias for the ARFIMA(1,d,0) model is calculated as:
Pr(ARFIMA(1,d,0))* BT4+Pr(ARFIMA(0,d,0))*BT17+Pr(ARFIMA(0,d,1))*BT19+Pr(ARFIMA(1,d,1)*BT20
where, for example, BT4 is the bias in Table 4.

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Table 1: Bias in the Fractional Integration Parameter for Alternative ARFIMA Models : R/S

	d							d						
	ARFIMA(1,d,0)							ARFIMA(0,d,1)						
$\phi(\theta)$	0.3	0.2	0.1	0	-0.1	-0.2	-0.3	0.3	0.2	0.1	0	-0.1	-0.2	-0.3
-0.8	-.286	-.241	-.195	-.142	-.086	-.017	.056	-.383	-.326	-.264	-.196	-.119	-.034	.054
	(.084)	(.061)	(.040)	(.022)	(.009)	(.001)	(.004)	(.149)	(.107)	(.070)	(.039)	(.015)	(.002)	(.003)
-0.5	-.208	-.160	-.112	-.060	-.008	.054	.119	-.245	-.196	-.146	-.091	-.034	.033	.103
	(.046)	(.028)	(.015)	(.005)	(.001)	(.004)	(.015)	(.062)	(.041)	(.023)	(.010)	(.002)	(.002)	(.011)
-0.3	-.177	-.126	-.075	-.025	.030	.091	.153	-.190	-.139	-.088	-.038	.019	.081	.145
	(.034)	(.018)	(.008)	(.002)	(.002)	(.009)	(.024)	(.038)	(.022)	(.010)	(.003)	(.002)	(.008)	(.022)
-0.2	-.164	-.111	-.059	-.006	.048	.108	.170	-.170	-.117	-.065	-.012	.042	.103	.166
	(.030)	(.015)	(.006)	(.002)	(.004)	(.013)	(.030)	(.031)	(.016)	(.006)	(.002)	(.003)	(.012)	(.028)
-0.1	-.153	-.097	-.044	.012	.065	.125	.186	-.154	-.099	-.045	.010	.063	.124	.185
	(.026)	(.012)	(.004)	(.002)	(.006)	(.017)	(.036)	(.026)	(.012)	(.004)	(.002)	(.005)	(.016)	(.035)
0	-.142	-.084	-.029	.028	.081	.142	.203	-.142	-.084	-.029	.028	.081	.142	.203
	(.023)	(.009)	(.003)	(.002)	(.008)	(.021)	(.042)	(.023)	(.009)	(.003)	(.002)	(.008)	(.021)	(.042)
0.1	-.132	-.072	-.014	.043	.098	.159	.220	-.133	-.073	-.016	.042	.097	.158	.219
	(.020)	(.008)	(.002)	(.004)	(.011)	(.026)	(.050)	(.020)	(.008)	(.002)	(.003)	(.011)	(.026)	(.049)
0.2	-.121	-.059	.000	.058	.116	.177	.239	-.126	-.064	-.005	.052	.110	.171	.233
	(.017)	(.006)	(.002)	(.005)	(.015)	(.032)	(.058)	(.018)	(.006)	(.002)	(.004)	(.013)	(.030)	(.055)
0.3	-.111	-.047	.015	.075	.134	.196	.258	-.121	-.058	.003	.061	.120	.182	.245
	(.015)	(.004)	(.002)	(.007)	(.019)	(.039)	(.067)	(.017)	(.006)	(.002)	(.005)	(.016)	(.034)	(.061)
0.5	-.090	-.020	.048	.113	.174	.239	.302	-.114	-.050	.013	.075	.134	.197	.261
	(.010)	(.003)	(.004)	(.015)	(.032)	(.058)	(.092)	(.015)	(.005)	(.002)	(.007)	(.019)	(.040)	(.069)
0.8	-.047	.035	.115	.190	.263	.334	.402	-.111	-.045	.019	.081	.143	.207	.272
	(.004)	(.003)	(.015)	(.038)	(.071)	(.113)	(.163)	(.014)	(.004)	(.003)	(.008)	(.022)	(.044)	(.075)

MSE figures reported in parentheses

Table 2: Bias in the Fractional Integration Parameter for Alternative ARFIMA Models : GPH

	ARFIMA(1,d,0)							ARFIMA(0,d,1)						
	d							d						
$\phi(\theta)$	0.3	0.2	0.1	0	-0.1	-0.2	-0.3	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3
-0.8	-.017	-.016	.006	-.015	-.030	.004	.021	-.404	-.397	-.367	-.388	-.372	-.315	-.260
	(.121)	(.120)	(.119)	(.114)	(.128)	(.113)	(.136)	(.288)	(.275)	(.251)	(.266)	(.258)	(.229)	(.211)
-0.5	-.013	-.014	.008	-.008	-.032	-.002	.008	-.080	-.079	-.058	-.073	-.094	-.061	-.049
	(.122)	(.122)	(.121)	(.118)	(.130)	(.113)	(.139)	(.128)	(.128)	(.124)	(.124)	(.135)	(.114)	(.143)
-0.3	-.012	-.012	.010	-.008	-.030	-.001	.007	-.030	-.029	-.008	-.025	-.047	-.017	-.008
	(.121)	(.121)	(.120)	(.120)	(.131)	(.114)	(.139)	(.123)	(.122)	(.121)	(.121)	(.133)	(.113)	(.139)
-0.2	-.010	-.010	.011	-.008	-.028	.001	.008	-.017	-.017	.004	-.015	-.035	-.006	.002
	(.122)	(.121)	(.119)	(.118)	(.131)	(.114)	(.140)	(.122)	(.121)	(.120)	(.118)	(.131)	(.114)	(.139)
-0.1	-.008	-.007	.013	.000	-.025	.002	.009	-.010	-.009	.012	-.002	-.027	.001	.008
	(.123)	(.121)	(.119)	(.118)	(.130)	(.115)	(.140)	(.123)	(.121)	(.119)	(.118)	(.130)	(.115)	(.140)
0.0	-.004	-.004	.017	-.001	-.021	.005	.012	-.004	-.004	.017	-.001	-.021	.005	.012
	(.123)	(.121)	(.120)	(.117)	(.132)	(.114)	(.141)	(.122)	(.121)	(.119)	(.120)	(.130)	(.117)	(.140)
0.1	.001	.001	.022	.001	-.016	.010	.017	-.001	-.001	.020	-.001	-.018	.009	.015
	(.122)	(.121)	(.119)	(.121)	(.130)	(.116)	(.141)	(.122)	(.121)	(.119)	(.121)	(.130)	(.116)	(.141)
0.2	.009	.009	.030	.011	-.008	.018	.024	.002	.002	.023	.004	-.015	.011	.017
	(.123)	(.120)	(.119)	(.119)	(.130)	(.118)	(.142)	(.123)	(.121)	(.119)	(.119)	(.130)	(.117)	(.141)
0.3	.021	.021	.042	.028	.004	.029	.035	.004	.004	.025	.011	-.013	.012	.018
	(.124)	(.120)	(.120)	(.117)	(.130)	(.119)	(.143)	(.120)	(.120)	(.119)	(.116)	(.130)	(.118)	(.141)
0.5	.071	.071	.092	.073	.054	.078	.084	.006	.006	.027	.008	-.011	.014	.020
	(.129)	(.125)	(.127)	(.126)	(.132)	(.123)	(.150)	(.123)	(.120)	(.119)	(.121)	(.130)	(.118)	(.141)
0.8	.387	.390	.413	.395	.378	.397	.407	.007	.007	.028	.011	-.010	.015	.021
	(.279)	(.274)	(.291)	(.271)	(.268)	(.274)	(.304)	(.123)	(.120)	(.119)	(.115)	(.130)	(.117)	(.141)

MSE figures reported in parentheses

Table 3: Bias in the Fractional Integration Parameter for Alternative ARFIMA Models : APER

	d							d						
	ARFIMA(1,d,0)							ARFIMA(0,d,1)						
$\phi(\theta)$	0.3	0.2	0.1	0	-0.1	-0.2	-0.3	0.3	0.2	0.1	0	-0.1	-0.2	-0.3
-0.8	-.094	-.073	-.037	-.031	-.025	.002	.019	-.334	-.348	-.328	-.344	-.338	-.302	-.266
	(.026)	(.029)	(.031)	(.036)	(.051)	(.054)	(.067)	(.151)	(.169)	(.163)	(.183)	(.197)	(.176)	(.177)
-0.5	-.093	-.071	-.036	-.030	-.024	-.004	.008	-.124	-.109	-.078	-.077	-.075	-.055	-.042
	(.026)	(.029)	(.031)	(.038)	(.050)	(.055)	(.069)	(.035)	(.039)	(.039)	(.047)	(.060)	(.062)	(.076)
-0.3	-.092	-.070	-.035	-.027	-.023	-.005	.007	-.099	-.080	-.046	-.039	-.036	-.018	-.007
	(.025)	(.028)	(.031)	(.037)	(.049)	(.055)	(.069)	(.027)	(.031)	(.033)	(.038)	(.052)	(.056)	(.070)
-0.2	-.091	-.069	-.034	-.027	-.021	-.004	.008	-.094	-.073	-.038	-.032	-.027	-.010	.002
	(.025)	(.028)	(.031)	(.037)	(.049)	(.055)	(.069)	(.026)	(.029)	(.031)	(.038)	(.050)	(.055)	(.070)
-0.1	-.090	-.068	-.033	-.024	-.019	-.003	.009	-.090	-.069	-.034	-.025	-.021	-.004	.007
	(.025)	(.028)	(.030)	(.037)	(.049)	(.055)	(.069)	(.025)	(.028)	(.030)	(.037)	(.049)	(.055)	(.069)
0.0	-.088	-.066	-.031	-.023	-.017	.000	.011	-.088	-.066	-.031	-.023	-.017	.000	.011
	(.012)	(.011)	(.015)	(.036)	(.022)	(.026)	(.030)	(.012)	(.011)	(.014)	(.036)	(.022)	(.026)	(.030)
0.1	-.086	-.064	-.027	-.021	-.013	.004	.015	-.086	-.065	-.028	-.022	-.014	.002	.014
	(.024)	(.027)	(.030)	(.036)	(.048)	(.054)	(.068)	(.024)	(.027)	(.030)	(.036)	(.048)	(.055)	(.069)
0.2	-.082	-.060	-.023	-.016	-.007	.009	.021	-.085	-.063	-.027	.-.020	-.012	.004	.015
	(.023)	(.026)	(.029)	(.035)	(.047)	(.054)	(.068)	(.024)	(.027)	(.029)	(.035)	(.047)	(.054)	(.068)
0.3	-.077	-.053	-.016	-.006	.002	.019	.031	-.085	-.062	-.026	-.017	-.011	.005	.016
	(.022)	(.025)	(.028)	(.034)	(.046)	(.053)	(.067)	(.024)	(.027)	(.029)	(.035)	(.047)	(.054)	(.068)
0.5	-.057	-.029	.013	.025	.038	.057	.072	-.084	-.061	-.025	-.018	-.009	.006	.018
	(.017)	(.021)	(.025)	(.033)	(.044)	(.053)	(.067)	(.023)	(.027)	(.029)	(.037)	(.047)	(.054)	(.068)
0.8	.053	.106	.167	.205	.239	.276	.307	-.083	-.061	-.024	-.016	-.008	.007	.018
	(.009)	(.020)	(.040)	(.058)	(.080)	(.105)	(.132)	(.023)	(.026)	(.029)	(.034)	(.047)	(.054)	(.068)

MSE figures reported in parentheses

Table 4: Bias in the Fractional Integration Parameter for Alternative ARFIMA Models : ML

	d							d						
	ARFIMA(1,d,0)							ARFIMA(0,d,1)						
$\phi(\theta)$	0.3	0.2	0.1	0	-0.1	-0.2	-0.3	0.3	0.2	0.1	0	-0.1	-0.2	-0.3
-0.8	-.032	-.018	-.022	-.002	-.023	-.021	-.014	-.164	-.122	-.105	-.224	-.069	-.020	-.002
	(.004)	(.003)	(.004)	(.002)	(.004)	(.003)	(.003)	(.067)	(.066)	(.053)	(.156)	(.041)	(.029)	(.020)
-0.5	-.036	-.024	-.029	-.012	-.027	-.030	-.020	-.112	-.102	-.063	-.089	-.083	-.081	-.029
	(.005)	(.004)	(.005)	(.004)	(.005)	(.005)	(.004)	(.026)	(.027)	(.025)	(.037)	(.033)	(.033)	(.022)
-0.3	-.045	-.029	-.033	-.009	-.030	-.038	-.022	-.056	-.037	-.032	-.070	-.053	-.063	-.024
	(.006)	(.005)	(.006)	(.005)	(.007)	(.007)	(.005)	(.010)	(.008)	(.011)	(.022)	(.019)	(.021)	(.013)
-0.2	-.048	-.031	-.045	-.004	-.027	-.044	-.024	-.043	-.025	-.017	-.048	-.045	-.061	-.022
	(.007)	(.006)	(.020)	(.005)	(.007)	(.009)	(.006)	(.007)	(.006)	(.007)	(.016)	(.015)	(.013)	(.011)
-0.1	-.053	-.034	-.040	-.011	-.031	-.049	-.027	-.041	-.026	-.019	-.011	-.032	-.052	-.022
	(.009)	(.006)	(.009)	(.005)	(.009)	(.011)	(.007)	(.007)	(.006)	(.006)	(.007)	(.008)	(.011)	(.009)
0.0	-.079	-.060	-.058	-.005	-.054	-.062	-.033	-.043	-.026	-.011	-.013	-.021	-.044	-.014
	(.002)	(.001)	(.001)	(.003)	(.002)	(.003)	(.003)	(.002)	(.001)	(.001)	(.004)	(.002)	(.003)	(.003)
0.1	-.096	-.090	-.072	-.027	-.071	-.088	-.052	-.044	-.027	-.015	-.007	-.017	-.031	-.020
	(.039)	(.041)	(.040)	(.022)	(.035)	(.036)	(.022)	(.007)	(.005)	(.005)	(.005)	(.005)	(.006)	(.007)
0.2	-.124	-.123	-.087	-.024	-.087	-.127	-.074	-.044	-.031	-.015	-.031	-.015	-.024	-.019
	(.055)	(.056)	(.046)	(.016)	(.037)	(.062)	(.038)	(.006)	(.005)	(.004)	(.007)	(.004)	(.005)	(.006)
0.3	-.153	-.159	-.096	-.048	-.139	-.138	-.086	-.041	-.029	-.023	-.038	-.023	-.025	-.016
	(.068)	(.072)	(.053)	(.030)	(.064)	(.060)	(.042)	(.006)	(.005)	(.005)	(.009)	(.005)	(.005)	(.005)
0.5	-.174	-.142	-.116	-.076	-.153	-.138	-.121	-.036	-.023	-.025	-.019	-.023	-.026	-.019
	(.064)	(.047)	(.047)	(.038)	(.053)	(.046)	(.055)	(.005)	(.004)	(.004)	(.005)	(.004)	(.005)	(.004)
0.8	-.077	-.035	-.035	-.007	-.018	-.005	-.025	-.031	-.017	-.021	-.027	-.022	-.021	-.017
	(.015)	(.011)	(.017)	(.007)	(.015)	(.015)	(.016)	(.004)	(.003)	(.004)	(.004)	(.004)	(.003)	(.003)

MSE figures reported in parentheses

Table 5: Bias in the Short-term Parameters for Alternative ARFIMA Models: R/S

	d							d						
	ARFIMA(1,d,0)							ARFIMA(0,d,1)						
$\phi(\theta)$	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3	0.3	0.2	0.1	0	-0.1	-0.2	-0.3
-0.8	.133	.103	.074	.051	.029	.010	-.006	.353	.289	.222	.151	.081	.015	-.044
	(.021)	(.013)	(.008)	(.005)	(.002)	(.001)	(.001)	(.127)	(.086)	(.052)	(.025)	(.009)	(.002)	(.004)
-0.5	.166	.118	.076	.003	.003	-.028	-.057	.250	.196	.141	.084	.022	-.043	-.111
	(.018)	(.009)	(.005)	(.003)	(.003)	(.006)	(.013)	(.066)	(.042)	(.023)	(.010)	(.003)	(.004)	(.015)
-0.3	.165	.109	.060	.016	-.027	-.063	-.099	.193	.138	.084	.028	-.031	-.093	-.161
	(.033)	(.016)	(.008)	(.004)	(.004)	(.007)	(.033)	(.041)	(.023)	(.011)	(.005)	(.004)	(.012)	(.029)
-0.2	.161	.101	.048	.002	-.044	-.082	-.122	.169	.114	.059	.004	-.054	-.116	-.183
	(.032)	(.015)	(.007)	(.004)	(.005)	(.010)	(.018)	(.033)	(.018)	(.008)	(.004)	(.007)	(.017)	(.037)
-0.1	.154	.092	.035	-.013	-.062	-.102	-.145	.144	.093	.039	-.016	-.073	-.134	-.200
	(.030)	(.013)	(.006)	(.005)	(.007)	(.014)	(.024)	(.026)	(.014)	(.006)	(.005)	(.010)	(.022)	(.044)
0.0	.145	.080	.021	-.030	-.081	-.124	-.169	.131	.076	.002	.032	-.088	-.147	-.212
	(.027)	(.011)	(.005)	(.005)	(.010)	(.019)	(.032)	(.022)	(.011)	(.005)	(.006)	(.013)	(.027)	(.050)
0.1	.135	.068	.006	-.047	-.101	-.145	-.193	.116	.062	.010	-.042	-.070	-.155	-.218
	(.024)	(.009)	(.005)	(.007)	(.014)	(.025)	(.041)	(.018)	(.009)	(.005)	(.007)	(.010)	(.029)	(.053)
0.2	.122	.054	-.010	-.065	-.121	-.168	-.218	.103	.051	.002	-.049	-.101	-.156	-.217
	(.020)	(.007)	(.005)	(.009)	(.019)	(.032)	(.051)	(.015)	(.008)	(.005)	(.008)	(.016)	(.030)	(.053)
0.3	.108	.039	-.027	-.083	-.142	-.190	-.243	.091	.043	-.004	-.051	-.100	-.152	-.209
	(.017)	(.006)	(.005)	(.012)	(.024)	(.040)	(.063)	(.013)	(.007)	(.005)	(.008)	(.016)	(.029)	(.050)
0.5	.075	.008	-.062	-.121	-.184	-.237	-.295	.068	.030	-.007	-.044	-.084	-.126	-.172
	(.009)	(.004)	(.008)	(.019)	(.038)	(.060)	(.091)	(.008)	(.005)	(.004)	(.007)	(.012)	(.022)	(.036)
0.8	.012	-.041	-.108	-.171	-.242	-.308	-.378	.034	.017	.001	-.017	-.035	-.054	-.076
	(.002)	(.004)	(.014)	(.033)	(.062)	(.099)	(.148)	(.003)	(.002)	(.002)	(.003)	(.004)	(.006)	(.010)

MSE figures reported in parentheses

Table 6: Bias in the Short-term Parameters for Alternative ARFIMA Models: GPH

	d							d						
	ARFIMA(1,d,0)							ARFIMA(0,d,1)						
$\phi(\theta)$	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3
-0.8	.095	.091	.082	.093	.109	.079	.089	.434	.427	.417	.414	.383	.356	.313
	(.083)	(.077)	(.076)	(.076)	(.107)	(.065)	(.082)	(.345)	(.332)	(.317)	(.313)	(.289)	(.273)	(.246)
-0.5	.112	.110	.093	.111	.128	.098	.110	.110	.111	.111	.110	.106	.100	.089
	(.121)	(.117)	(.111)	(.118)	(.146)	(.110)	(.128)	(.136)	(.136)	(.135)	(.132)	(.130)	(.128)	(.125)
-0.3	.102	.100	.081	.102	.115	.090	.098	.039	.040	.041	.040	.037	.034	.028
	(.127)	(.123)	(.117)	(.124)	(.146)	(.118)	(.134)	(.132)	(.130)	(.129)	(.128)	(.126)	(.127)	(.127)
-0.2	.092	.090	.071	.092	.104	.081	.087	.015	.017	.018	.017	.014	.011	.007
	(.125)	(.122)	(.116)	(.123)	(.141)	(.118)	(.134)	(.132)	(.130)	(.129)	(.128)	(.127)	(.127)	(.130)
-0.1	.079	.077	.057	.080	.089	.069	.073	-.003	.000	.001	.000	-.003	-.006	-.010
	(.121)	(.118)	(.114)	(.119)	(.135)	(.117)	(.131)	(.132)	(.129)	(.128)	(.127)	(.126)	(.127)	(.130)
0.0	.016	.008	.019	.064	.017	.016	.012	-.016	-.014	-.013	-.014	-.016	-.019	-.022
	(.116)	(.112)	(.109)	(.114)	(.127)	(.113)	(.126)	(.129)	(.126)	(.125)	(.125)	(.124)	(.125)	(.127)
0.1	.043	.042	.022	.045	.052	.036	.035	-.026	-.024	-.023	-.024	-.026	-.029	-.032
	(.110)	(.108)	(.105)	(.109)	(.118)	(.107)	(.120)	(.123)	(.122)	(.120)	(.120)	(.120)	(.120)	(.122)
0.2	.020	.019	-.001	.023	.028	.013	.011	-.034	-.032	-.030	-.031	-.033	-.036	-.039
	(.104)	(.101)	(.100)	(.102)	(.109)	(.100)	(.113)	(.115)	(.114)	(.113)	(.113)	(.113)	(.114)	(.116)
0.3	-.008	-.008	-.029	-.004	-.001	-.013	-.018	-.038	-.036	-.035	-.036	-.038	-.041	-.043
	(.098)	(.096)	(.096)	(.096)	(.101)	(.095)	(.107)	(.105)	(.104)	(.103)	(.104)	(.104)	(.105)	(.106)
0.5	-.085	-.085	-.105	-.081	-.079	-.088	-.097	-.039	-.037	-.036	-.037	-.039	-.041	-.043
	(.094)	(.092)	(.095)	(.091)	(.093)	(.090)	(.105)	(.079)	(.078)	(.077)	(.077)	(.078)	(.079)	(.079)
0.8	-.386	-.382	-.405	-.379	-.378	-.385	-.397	-.025	-.024	-.023	-.023	-.027	-.025	-.028
	(.242)	(.235)	(.252)	(.232)	(.232)	(.234)	(.257)	(.037)	(.037)	(.037)	(.036)	(.040)	(.040)	(.041)

MSE figures reported in parentheses

Table 7: Bias in the Short-term Parameters for Alternative ARFIMA Models: APER

	d							d						
	ARFIMA(1,d,0)							ARFIMA(0,d,1)						
$\phi(\theta)$	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3
-0.8	.044	.040	.030	.032	.042	.039	.045	.337	.312	.317	.328	.321	.300	.275
	(.008)	(.010)	(.009)	(.015)	(.024)	(.026)	(.032)	(.163)	(.157)	(.172)	(.191)	(.196)	(.193)	(.190)
-0.5	.078	.067	.046	.044	.056	.052	.058	.118	.094	.078	.066	.057	.057	.048
	(.023)	(.025)	(.025)	(.033)	(.047)	(.051)	(.065)	(.037)	(.039)	(.044)	(.050)	(.058)	(.067)	(.075)
-0.3	.089	.075	.048	.042	.052	.049	.051	.093	.063	.040	.022	.008	-.004	-.015
	(.030)	(.032)	(.032)	(.040)	(.054)	(.060)	(.073)	(.031)	(.032)	(.038)	(.045)	(.054)	(.063)	(.074)
-0.2	.092	.076	.046	.039	.048	.044	.044	.086	.055	.030	.011	-.004	-.018	-.030
	(.032)	(.034)	(.034)	(.042)	(.056)	(.061)	(.075)	(.028)	(.030)	(.036)	(.044)	(.053)	(.063)	(.074)
-0.1	.092	.075	.042	.034	.041	.037	.035	.080	.049	.024	.004	-.013	-.027	-.040
	(.032)	(.034)	(.034)	(.043)	(.056)	(.061)	(.074)	(.026)	(.028)	(.034)	(.041)	(.051)	(.061)	(.073)
0.0	.059	.034	.025	.027	.007	.008	.003	.075	.044	.020	-.001	-.017	-.032	-.046
	(.014)	(.014)	(.017)	(.042)	(.024)	(.028)	(.033)	(.024)	(.026)	(.031)	(.038)	(.048)	(.058)	(.070)
0.1	.086	.066	.029	.018	.021	.016	.009	.070	.041	.017	-.003	-.020	-.034	-.048
	(.030)	(.032)	(.033)	(.041)	(.052)	(.058)	(.069)	(.021)	(.023)	(.028)	(.035)	(.043)	(.053)	(.064)
0.2	.079	.057	.019	.007	.008	.001	-.008	.065	.037	.015	-.004	-.020	-.034	-.048
	(.027)	(.029)	(.031)	(.039)	(.049)	(.054)	(.066)	(.018)	(.020)	(.024)	(.030)	(.038)	(.047)	(.057)
0.3	.069	.046	.006	-.008	-.010	-.018	-.028	.059	.034	.013	-.004	-.019	-.032	-.045
	(.023)	(.025)	(.028)	(.036)	(.045)	(.051)	(.062)	(.016)	(.017)	(.021)	(.026)	(.032)	(.040)	(.049)
0.5	.036	.009	-.034	-.052	-.060	-.072	-.088	.047	.027	.011	-.002	-.014	-.025	-.036
	(.014)	(.017)	(.023)	(.031)	(.039)	(.046)	(.057)	(.010)	(.011)	(.013)	(.016)	(.020)	(.025)	(.031)
0.8	-.062	-.103	-.161	-.199	-.232	-.264	-.299	.025	.017	.010	.003	-.002	-.007	-.012
	(.008)	(.016)	(.035)	(.053)	(.072)	(.093)	(.120)	(.003)	(.003)	(.004)	(.004)	(.005)	(.006)	(.007)

MSE figures reported in parentheses

Table 8: Bias in the Short-term Parameters for Alternative ARFIMA Models: ML

	d							d						
	ARFIMA(1,d,0)							ARFIMA(0,d,1)						
$\phi(\theta)$	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3	0.3	0.2	0.1	0.0	-0.1	-0.2	-0.3
-0.8	.010	.009	.016	.006	.011	.009	.009	.132	.107	.086	.215	.059	.021	.016
	(.002)	(.002)	(.002)	(.002)	(.002)	(.001)	(.002)	(.053)	(.059)	(.040)	(.104)	(.034)	(.022)	(.012)
-0.5	.017	.020	.025	.011	.015	.020	.012	.102	.103	.058	.135	.079	.078	.025
	(.004	(.005)	(.006)	(.004)	(.005)	(.005)	(.005)	(.028)	(.030)	(.029)	(.057)	(.035)	(.035)	(.026)
-0.3	.027	.026	.029	.006	.019	.030	.012	.045	.042	.031	.131	.045	.060	.015
	(.007)	(.007)	(.009)	(.007)	(.008)	(.008)	(.007)	(.013)	(.013)	(.017)	(.037)	(.023)	(.024)	(.018)
-0.2	.032	.029	.040	.006	.017	.037	.013	.033	.033	.018	.075	.036	.057	.012
	(.009)	(.009)	(.025)	(.008)	(.009)	(.011)	(.008)	(.011)	(.011)	(.014)	(.019)	(.020)	(.015)	(.016)
-0.1	.037	.033	.034	.012	.022	.043	.015	.035	.034	.022	.049	.026	.048	.011
	(.011)	(.010)	(.014)	(.009)	(.012)	(.013)	(.009)	(.011)	(.009)	(.010)	(.011)	(.011)	(.013)	(.013)
0.0	.062	.059	.050	-.008	.043	.055	.020	.031	.028	.012	.009	.017	.039	.002
	(.032)	(.024)	(.032)	(.007)	(.019)	(.021)	(.013)	(.011)	(.010)	.(011)	(.004)	(.009)	(.012)	(.019)
0.1	.079	.088	.061	.016	.059	.081	.038	.030	.030	.023	-.005	.017	.030	.009
	(.039)	(.043)	(.042)	(.021)	(.037)	(.038)	(.027)	(.009)	(.008)	(.010)	(.007)	(.007)	(.008)	(.009)
0.2	.104	.117	.072	.012	.074	.116	.058	.025	.030	.020	.021	.015	.024	.009
	(.053)	(.056)	(.048)	(.017)	(.036)	(.061)	(.041)	(.007)	(.008)	(.010)	(.007)	(.006)	(.007)	(.008)
0.3	.129	.146	.078	.041	.119	.121	.066	.021	.025	.025	.030	.018	.024	.009
	(.061)	(.068)	(.048)	(.029)	(.059)	(.056)	(.041)	(.006)	(.007)	(.008)	(.007)	(.007)	(.006)	(.007)
0.5	.133	.116	.086	.064	.119	.106	.087	.015	.016	.018	.011	.019	.014	.007
	(.043)	(.036)	(.038)	(.027)	(.041)	(.034)	(.042)	(.004)	(.005)	(.005)	(.005)	(.010)	(.004)	(.004)
0.8	.032	.010	.006	-.013	-.010	-.029	-.002	.007	.008	.014	.065	.014	.006	.008
	(.004)	(.004)	(.009)	(.005)	(.010)	(.012)	(.009)	(.002)	(.003)	(.002)	(.007)	(.005)	(.001)	(.002)

MSE figures reported in parentheses

Table 9: Model Selection for ARFIMA(1,d,0) by GPH

		ϕ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
0.3	0,0	11.0 27.6 21.8	23.8 42.8 33.4	36.0 57.0 47.2	34.6 53.2 44.2	19.4 33.4 26.8	8.0 17.8 11.4
	0,1	28.4 22.4 24.2	22.0 17.2 19.4	15.8 11.0 13.8	26.8 26.0 26.8	34.8 34.0 34.6	26.2 27.2 27.2
	1,0	47.6 46.0 46.4	37.2 35.2 37.2	26.0 24.8 26.2	16.0 12.6 14.4	29.4 23.8 27.8	46.6 46.8 49.0
	1,1	13.0 4.0 7.6	17.0 4.8 10.0	22.2 7.2 12.8	22.6 8.2 14.6	16.4 8.8 10.8	19.2 6.2 12.4
0.2	0,0	13.0 28.6 21.0	23.0 44.4 33.4	35.6 57.8 47.6	33.4 53.6 43.8	19.0 34.6 26.8	8.2 17.2 11.4
	0,1	27.6 22.6 25.6	22.8 16.6 20.2	15.2 10.8 13.4	27.0 25.4 27.2	35.6 34.2 35.8	26.2 27.2 27.4
	1,0	47.2 45.0 46.2	37.6 34.2 36.4	26.4 25.6 26.0	17.8 13.0 15.6	29.0 23.6 27.0	47.0 47.8 49.2
	1,1	12.2 3.8 7.2	16.6 4.8 10.0	22.8 5.8 13.0	21.8 8.0 13.4	16.4 7.6 10.4	18.6 7.8 12.0
0.1	0,0	13.4 29.2 20.8	23.8 44.2 34.4	35.2 58.8 47.8	34.2 53.2 43.0	18.2 35.0 26.0	8.4 17.6 12.4
	0,1	27.8 22.2 25.6	22.8 16.6 20.2	14.6 10.6 13.4	27.2 25.4 27.2	35.6 33.8 35.6	27.6 29.0 28.8
	1,0	46.6 45.0 46.6	37.4 35.0 36.2	26.8 24.8 27.2	17.6 13.6 16.0	30.2 24.0 27.6	45.8 46.2 47.6
	1,1	12.2 3.6 7.0	16.0 4.2 9.2	23.4 5.8 11.6	21.0 7.8 13.8	16.0 7.2 10.8	18.2 7.2 11.2

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 9 (cont'd): Model Selection for ARFIMA(1,d,0) by GPH

		φ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.1	0,0	12.4 28.6 21.0	23.8 45.0 32.2	35.4 59.0 48.0	34.0 54.2 44.2	18.6 34.8 25.6	7.8 17.0 11.4
	0,1	28.2 22.4 25.2	22.0 15.6 20.0	15.2 11.2 14.0	28.0 25.6 27.4	36.4 35.0 36.6	29.2 30.6 30.4
	1,0	46.8 45.2 46.4	38.4 35.0 37.8	26.8 23.4 25.8	16.8 13.4 15.4	30.4 23.8 27.8	46.2 46.2 47.2
	1,1	12.6 3.8 7.4	15.8 4.4 10.0	22.6 6.4 12.2	21.2 6.8 13.0	14.6 6.4 10.0	16.8 6.2 11.0
-0.2	0,0	13.2 28.2 20.8	24.4 45.6 32.4	35.4 58.0 48.8	34.0 54.6 44.8	19.6 34.8 26.6	7.8 17.4 11.0
	0,1	28.0 23.2 26.2	22.6 15.0 20.2	15.4 12.0 13.6	28.0 25.6 27.0	36.6 36.4 37.4	29.6 31.0 30.4
	1,0	45.8 44.6 45.4	37.0 34.6 37.2	26.2 23.4 25.6	16.6 13.2 15.4	30.4 22.4 27.2	45.6 45.6 46.8
	1,1	13.0 4.0 7.6	16.0 4.8 10.2	23.0 6.6 12.0	21.4 6.6 12.8	13.4 6.4 8.8	17.0 6.0 11.8
-0.3	0,0	13.2 29.6 21.4	23.6 44.8 32.8	35.4 59.4 48.0	35.0 54.2 43.8	19.4 33.8 25.0	6.0 16.4 10.8
	0,1	28.6 23.8 26.6	24.4 16.2 21.0	15.2 12.4 14.6	28.6 26.8 28.2	36.8 35.8 37.4	30.0 31.2 30.8
	1,0	45.0 42.8 44.4	36.2 33.2 36.0	25.0 21.0 24.0	16.0 12.8 14.6	30.6 23.8 28.0	46.6 45.8 47.2
	1,1	13.2 3.8 7.6	15.8 5.8 10.2	24.4 7.2 13.4	20.4 6.2 13.4	13.2 6.6 9.6	17.4 6.6 11.2

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 10: Model Selection for ARFIMA(0,d,1) by GPH

		θ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
0.3	0,0	2.8 17.2 8.0	11.8 25.8 20.0	17.6 32.8 24.4	20.0 31.8 25.2	16.2 26.4 21.2	11.6 21.0 15.2
	0,1	29.0 28.8 30.4	17.0 13.4 15.6	10.4 9.8 10.2	26.6 34.8 30.8	39.0 45.0 42.8	49.0 52.0 50.8
	1,0	33.8 37.2 35.8	34.0 37.8 36.2	25.6 30.8 28.4	9.0 10.0 9.4	9.6 7.8 8.6	11.8 9.0 11.6
	1,1	34.4 16.8 25.8	37.2 23.0 28.2	46.4 26.6 37.0	44.4 23.4 34.6	35.2 20.8 27.4	27.6 18.0 22.4
0.2	0,0	5.4 16.6 9.2	9.2 27.0 16.6	17.2 32.8 23.8	19.6 31.6 24.0	14.8 26.8 19.2	10.0 18.6 13.2
	0,1	30.0 30.2 31.8	19.2 14.6 18.0	12.6 13.0 12.2	28.8 32.0 31.6	39.0 42.2 40.8	46.4 49.8 49.2
	1,0	29.6 33.4 31.6	37.0 41.0 40.0	25.6 31.0 29.0	8.0 8.8 9.4	10.6 9.0 10.4	13.0 12.0 13.0
	1,1	35.0 19.8 27.4	34.6 17.4 25.4	44.6 23.2 35.0	43.6 27.6 35.0	35.6 22.0 29.6	30.6 19.6 24.6
0.1	0,0	3.6 15.8 8.6	11.4 26.6 18.4	20.0 34.0 26.4	20.2 32.4 26.4	15.2 27.2 20.8	11.8 22.0 16.0
	0,1	27.6 28.8 29.4	18.4 15.0 16.8	10.6 10.2 10.0	28.0 33.8 31.2	40.6 45.2 44.0	49.0 52.0 50.6
	1,0	33.2 37.6 35.4	34.2 37.2 36.6	26.2 29.8 28.6	7.8 10.4 8.8	9.0 7.0 8.0	11.6 8.0 11.6
	1,1	35.6 17.8 26.6	36.0 21.2 28.2	43.2 26.0 35.0	44.0 23.4 33.6	35.2 20.6 27.2	27.6 18.0 21.8

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 10 (cont'd): Model Selection for ARFIMA(0,d,1) by GPH

		θ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.1	0,0	6.8 15.8 11.2	11.0 26.8 17.4	17.2 31.4 24.0	18.4 30.2 23.4	16.0 26.6 21.0	9.6 19.6 13.8
	0,1	29.0 29.0 29.2	17.0 13.6 16.0	11.6 12.6 13.2	28.6 32.8 32.0	38.0 40.6 39.6	46.4 50.0 50.2
	1,0	30.0 33.8 32.4	37.0 42.0 40.0	25.8 31.0 27.4	9.8 9.6 10.0	9.8 9.2 10.4	12.8 11.0 12.2
	1,1	34.2 21.4 27.2	35.0 17.6 26.6	45.4 25.0 35.4	43.2 27.4 34.6	36.2 23.6 29.0	31.2 19.4 23.8
-0.2	0,0	6.0 17.8 10.6	11.2 25.0 17.6	19.2 32.4 26.4	19.8 34.0 27.2	14.4 27.8 21.4	11.4 20.0 15.4
	0,1	29.6 28.8 30.0	18.4 15.6 17.6	11.2 11.4 10.6	27.4 33.6 30.8	40.2 44.2 43.0	48.2 52.2 50.8
	1,0	32.0 36.4 34.2	33.8 36.8 35.4	26.2 29.4 28.6	7.6 10.4 7.2	10.4 6.4 7.8	12.6 9.8 11.4
	1,1	32.4 17.0 25.2	36.6 22.6 29.4	43.4 26.8 34.4	45.2 22.0 34.8	35.0 21.6 27.8	27.8 18.0 22.4
-0.3	0,0	6.0 16.8 10.4	11.6 25.2 16.8	16.2 30.6 22.0	20.4 30.0 23.8	16.0 29.0 20.4	10.8 20.4 14.8
	0,1	29.6 29.4 31.2	15.6 14.6 16.2	12.2 12.0 13.0	27.6 33.0 31.2	37.0 41.2 39.6	46.0 49.0 48.8
	1,0	29.0 34.2 32.0	37.4 40.8 39.6	26.4 31.8 29.0	8.6 10.8 9.6	10.0 8.2 10.4	13.4 11.6 13.0
	1,1	35.4 19.6 26.4	35.4 19.4 27.4	45.2 25.6 36.0	43.4 26.2 35.4	37.0 21.6 29.6	29.8 19.0 23.4

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 11: Model Selection for ARFIMA(1,d,0) by APER

		ϕ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
0.3	0,0	12.4 28.2 18.4	27.0 48.8 37.0	42.6 67.4 56.0	33.8 56.8 46.2	17.0 31.4 21.0	7.6 14.4 10.4
	0,1	33.0 26.2 30.6	25.0 15.6 21.0	16.0 8.8 13.4	28.4 21.4 25.2	34.4 32.8 34.0	26.4 27.2 26.8
	1,0	49.0 45.0 48.6	37.8 34.8 37.2	27.2 21.6 24.0	21.0 16.4 18.6	36.4 30.6 36.0	50.8 53.2 52.8
	1,1	5.6 0.6 2.4	10.2 0.8 4.8	14.2 2.2 6.6	16.8 5.4 10.0	12.2 5.2 9.0	15.2 5.2 10.0
0.2	0,0	14.8 31.4 21.4	25.0 50.0 37.0	40.4 63.0 51.2	31.6 53.8 42.4	14.6 30.2 21.6	7.4 14.2 9.8
	0,1	30.0 24.6 27.2	23.6 14.8 19.8	15.8 10.2 13.4	29.4 24.2 27.2	34.8 33.6 35.0	27.0 28.0 27.8
	1,0	47.0 42.6 46.6	36.6 32.4 35.8	24.8 21.8 24.2	18.2 14.8 17.4	35.2 30.0 33.4	47.8 50.4 50.4
	1,1	8.2 1.4 4.8	14.8 2.8 7.4	19.0 5.0 11.2	20.8 7.2 13.0	15.4 6.2 10.0	17.8 7.4 12.0
0.1	0,0	15.6 32.4 23.6	25.4 48.4 34.6	37.8 59.0 48.2	28.6 49.8 39.0	15.6 29.6 20.6	7.2 14.4 10.6
	0,1	27.8 22.6 26.0	21.2 13.2 17.8	15.2 11.0 12.8	30.8 26.4 28.6	36.0 35.4 36.6	27.6 29.2 29.0
	1,0	44.6 41.8 42.2	36.2 33.0 35.8	24.8 22.8 24.0	16.6 14.2 16.0	30.2 27.2 30.2	45.0 47.2 46.8
	1,1	12.0 3.2 8.2	17.2 5.4 11.8	22.2 7.2 15.0	24.0 9.6 16.4	18.2 7.8 12.6	20.2 9.2 13.6

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 11 (cont'd): Model Selection for ARFIMA(1,d,0) by APER

		φ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.1	0,0	0.4 2.4 0.8	6.6 18.0 10.6	32.0 55.8 43.2	65.4 85.6 77.0	32.0 54.2 41.4	7.6 16.8 12.0
	0,1	22.2 22.0 22.6	27.2 23.6 26.6	21.0 12.0 16.2	10.8 4.6 7.8	18.4 13.4 16.6	13.6 13.0 13.4
	1,0	73.6 75.4 75.6	63.2 58.2 61.6	42.0 31.8 39.0	18.4 9.6 13.8	43.2 32.0 39.8	66.8 68.6 68.4
	1,1	3.8 0.2 1.0	3.0 0.2 1.2	5.0 0.4 1.6	5.4 0.2 1.4	6.4 4.0 2.2	12.0 1.6 6.2
-0.2	0,0	3.0 7.0 4.6	14.6 30.2 20.4	44.8 66.0 54.0	54.2 76.2 67.6	22.8 43.2 32.4	7.2 15.0 10.8
	0,1	29.6 28.0 29.0	28.8 22.4 27.0	16.4 8.8 13.8	18.6 10.0 13.4	25.8 20.8 23.8	19.8 18.8 19.6
	1,0	65.0 64.8 65.8	53.6 47.2 51.2	34.4 24.8 31.0	21.0 13.2 17.0	44.8 35.2 40.8	62.0 64.6 65.0
	1,1	2.4 0.2 0.6	3.0 0.2 1.4	4.4 0.4 1.2	6.2 0.6 2.0	6.6 0.8 3.0	11.0 1.6 4.6
-0.3	0,0	5.4 14.2 8.6	22.0 40.8 30.0	47.6 69.8 57.6	45.2 69.0 57.2	17.4 38.0 28.4	7.6 15.8 10.6
	0,1	32.8 28.6 31.6	28.4 20.6 25.8	15.2 7.2 12.6	24.0 15.8 19.6	30.0 27.2 29.0	23.0 22.8 23.2
	1,0	59.2 56.8 58.8	45.4 38.4 42.4	30.4 22.6 28.0	21.6 14.2 17.6	43.2 32.8 38.8	58.6 59.2 60.6
	1,1	2.6 0.4 1.0	4.2 0.2 1.8	6.8 0.4 1.8	9.2 1.0 5.6	9.4 2.0 3.8	10.8 2.2 5.6

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 12: Model Selection for ARFIMA(0,d,1) by APER

		θ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
0.3	0,0	0.6	9.6	32.0	48.0	30.0	17.0
		3.6	21.8	48.4	69.4	52.0	28.6
		1.2	14.2	39.6	59.0	41.2	21.6
	0,1	53.2	38.2	17.2	17.0	35.2	48.2
		53.4	31.4	10.4	12.8	29.2	48.4
		54.4	36.4	15.2	16.2	33.4	48.8
	1,0	38.4	44.4	38.6	15.6	19.8	25.8
		41.0	45.0	37.8	12.8	15.2	21.2
		41.0	45.6	38.4	14.4	17.2	25.0
	1,1	7.8	7.8	12.2	19.4	15.0	9.0
		2.0	1.8	3.4	5.0	3.6	1.8
		3.4	3.8	6.8	10.4	8.2	4.6
0.2	0,0	3.8	16.4	35.6	43.4	28.0	13.4
		9.4	26.6	51.4	63.8	44.8	23.8
		6.0	20.6	43.0	54.0	36.0	17.2
	0,1	50.2	32.4	15.6	22.4	36.4	51.0
		51.6	28.4	9.0	17.4	34.2	50.6
		52.6	31.6	13.0	19.0	37.6	51.2
	1,0	35.0	44.2	35.8	16.8	22.0	24.8
		38.2	44.0	38.2	12.8	14.8	20.8
		27.0	45.4	37.8	15.0	18.2	24.0
	1,1	11.0	7.0	13.0	17.4	13.6	10.8
		0.8	1.0	1.4	6.0	6.2	4.8
		4.4	2.4	6.2	12.0	8.2	7.6
0.1	0,0	6.0	21.4	38.2	32.0	21.6	13.0
		18.2	40.0	60.4	53.8	34.2	20.6
		11.6	31.0	49.6	41.4	27.4	16.6
	0,1	51.8	31.4	14.0	28.6	42.2	56.0
		49.0	23.0	9.6	26.0	45.2	59.4
		51.8	28.4	13.0	28.8	44.4	58.2
	1,0	29.6	33.8	26.6	13.8	17.6	18.4
		31.0	34.8	25.6	11.4	14.4	16.2
		31.0	34.6	27.2	14.2	15.4	17.2
	1,1	12.6	13.4	21.2	25.6	18.6	12.6
		1.8	2.2	4.4	8.8	6.2	3.8
		5.6	6.0	10.2	15.6	12.8	8.0

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 12 (cont'd): Model Selection for ARFIMA(0,d,1) by APER

		θ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.1	0,0	7.2 23.6 13.6	18.0 38.2 26.2	28.6 48.6 38.6	29.8 45.6 37.8	19.6 31.8 26.4	10.0 19.6 13.8
	0,1	40.6 39.4 41.4	25.2 20.8 23.6	15.4 12.8 14.2	30.0 32.0 31.2	44.2 45.8 44.8	54.8 56.6 57.6
	1,0	29.8 32.0 31.0	35.8 35.8 36.4	28.6 28.8 28.4	12.2 11.2 11.8	14.8 12.8 14.2	19.6 15.4 17.8
	1,1	22.4 5.0 14.0	21.0 5.2 13.8	27.4 9.8 18.8	28.0 11.2 19.2	21.4 9.6 14.6	15.6 8.4 10.8
-0.2	0,0	7.6 26.0 13.6	20.0 41.2 29.4	28.8 48.0 37.4	23.0 39.2 31.8	16.6 27.2 20.6	9.4 19.2 14.2
	0,1	40.0 39.4 42.8	23.2 18.4 20.6	12.8 12.4 13.2	29.8 35.8 34.2	47.2 51.2 49.6	59.6 62.6 61.4
	1,0	24.2 25.8 25.4	27.6 28.2 28.6	21.8 23.0 22.0	12.0 10.2 11.6	12.6 11.0 12.2	14.0 10.8 12.4
	1,1	28.2 8.8 18.2	29.2 12.2 21.4	36.6 16.6 27.4	35.2 14.8 22.4	23.6 10.6 17.6	17.0 7.4 12.0
-0.3	0,0	4.6 22.8 11.6	14.0 23.6 21.0	25.8 42.6 32.8	23.8 39.4 31.4	16.4 26.8 19.8	8.8 17.6 12.0
	0,1	37.4 34.8 38.2	23.6 20.0 22.8	14.4 14.0 14.6	31.8 34.4 32.4	44.8 47.6 47.6	57.6 60.4 61.0
	1,0	27.2 30.0 29.4	31.8 33.8 33.6	24.2 26.2 25.0	11.4 10.6 11.4	14.8 12.4 14.4	15.0 11.6 14.2
	1,1	30.8 12.4 20.8	30.6 12.6 22.6	35.6 17.2 27.6	33.0 15.6 24.8	24.0 13.2 18.2	18.6 10.4 12.8

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 13: Model Selection for ARFIMA(1,d,0) by R/S

		ϕ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
0.3	0,0	0.0 0.0 0.0	0.2 0.6 0.2	4.2 15.4 9.0	61.6 90.2 76.8	69.0 92.6 83.6	32.2 60.8 44.8
	0,1	7.2 7.2 7.2	12.8 13.2 13.2	15.4 14.4 15.0	2.4 0.2 1.4	1.2 0.6 1.0	0.4 0.0 0.2
	1,0	83.8 91.8 88.8	74.6 84.0 79.6	58.6 66.2 64.6	11.2 6.6 10.0	15.2 5.6 10.6	42.4 35.6 44.4
	1,1	9.0 1.0 4.0	12.4 2.2 7.0	21.8 4.0 11.4	24.8 3.0 11.8	14.6 1.2 4.8	25.0 3.6 10.6
0.2	0,0	0.0 0.8 0.0	2.8 9.6 4.2	18.6 42.6 28.2	79.6 96.6 89.4	53.0 77.8 66.0	10.8 31.0 18.6
	0,1	18.2 18.0 18.4	23.0 21.0 22.4	22.4 17.0 21.2	3.0 0.8 2.2	6.2 2.8 4.2	5.2 4.0 4.4
	1,0	76.6 80.2 78.4	66.8 68.0 70.0	49.4 39.4 46.8	9.8 2.6 7.4	33.0 18.8 27.0	64.6 63.0 68.4
	1,1	5.2 1.0 3.2	7.4 1.4 3.4	9.6 1.0 3.8	7.6 0.0 1.0	7.8 0.6 2.8	19.4 2.0 8.6
0.1	0,0	0.6 4.6 1.8	12.6 30.4 20.4	46.6 74.0 58.4	69.0 91.2 81.2	24.6 53.6 38.6	1.4 11.0 4.4
	0,1	32.2 31.0 31.8	35.6 28.4 32.8	23.0 12.2 19.6	11.4 3.8 7.6	17.4 11.0 14.8	13.6 11.8 13.0
	1,0	64.4 64.4 65.2	48.2 40.6 45.2	26.2 13.2 20.6	15.6 5.0 10.4	51.4 34.6 43.6	74.0 75.4 77.2
	1,1	1.8 0.0 1.2	3.6 0.6 1.6	4.2 0.6 1.4	4.0 0.0 0.8	6.6 0.8 3.0	11.0 1.8 5.4

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 13 (cont'd): Model Selection for ARFIMA(1,d,0) by R/S

		ϕ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.1	0,0	17.8 43.6 27.8	56.6 84.6 73.2	75.4 97.8 90.4	11.8 43.2 25.8	0.6 7.8 2.2	0.0 0.0 0.0
	0,1	46.2 33.2 42.4	25.2 10.0 17.8	9.8 1.6 4.4	43.4 36.2 41.8	47.8 47.6 49.8	33.6 34.4 33.8
	1,0	31.2 22.6 27.6	11.0 4.4 7.2	2.2 0.2 1.2	27.4 19.2 24.4	44.8 43.2 44.6	64.0 65.0 64.4
	1,1	4.8 0.6 2.2	7.2 1.0 1.8	12.6 0.4 4.0	17.4 1.4 8.0	6.8 1.4 3.4	2.4 0.6 1.8
-0.2	0,0	42.2 75.2 57.2	68.0 97.6 87.0	42.2 91.0 70.6	0.4 13.6 4.4	0.0 0.0 0.0	0.0 0.0 0.0
	0,1	38.2 19.0 31.6	10.0 1.6 5.2	10.6 3.8 8.2	55.6 63.6 61.0	67.2 72.8 71.0	53.2 55.0 53.8
	1,0	10.8 5.4 8.6	1.2 0.0 0.8	2.0 0.4 0.6	16.6 15.4 16.8	24.0 24.4 24.2	42.4 43.8 43.6
	1,1	8.8 0.4 2.6	20.8 0.8 7.0	45.2 4.8 20.6	27.4 7.4 17.8	8.8 2.8 4.8	4.4 1.2 2.6
-0.3	0,0	49.6 90.6 75.2	27.2 86.6 54.8	8.0 56.0 22.2	0.0 0.8 0.2	0.0 0.0 0.0	0.0 0.0 0.0
	0,1	20.4 7.0 13.4	5.6 2.2 3.8	12.4 15.8 15.0	56.6 72.4 64.8	70.0 80.4 75.8	61.4 66.4 65.6
	1,0	2.6 1.2 1.8	0.8 0.0 0.4	1.4 0.8 1.0	6.8 7.2 7.2	14.2 15.0 14.8	30.2 32.6 32.4
	1,1	27.4 1.2 9.6	66.4 11.2 41.0	78.2 27.4 61.8	36.6 19.6 27.8	15.8 4.6 9.4	8.4 1.0 2.0

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 14: Model Selection for ARFIMA(0,d,1) by R/S

		θ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
0.3	0,0	0.0 0.0 0.0	0.0 0.6 0.2	4.2 15.4 8.6	63.2 90.2 80.6	75.0 96.0 86.0	43.0 71.0 57.0
	0,1	46.2 47.0 46.8	30.0 30.4 30.2	17.8 16.4 17.4	3.2 0.6 3.6	2.6 0.8 1.8	14.4 6.0 10.4
	1,0	50.2 52.4 52.0	63.0 68.0 66.8	59.0 65.0 64.8	12.0 7.0 9.0	14.0 3.2 9.0	14.4 6.0 10.4
	1,1	3.6 0.6 1.2	7.0 1.0 2.8	19.0 3.2 9.2	21.6 2.2 6.8	8.4 0.0 3.2	38.2 22.6 30.8
0.2	0,0	0.0 1.0 0.2	2.6 9.2 4.0	18.2 42.6 27.8	80.6 96.8 90.0	58.0 81.8 72.0	4.4 0.4 1.8
	0,1	59.6 61.0 61.2	41.6 40.0 42.0	25.2 19.2 23.6	3.6 0.8 2.6	10.8 5.4 6.8	17.6 42.8 28.0
	1,0	37.2 37.8 37.8	50.4 50.4 51.4	47.2 37.4 44.4	9.0 2.4 6.8	28.4 12.8 20.0	29.4 20.2 25.4
	1,1	3.2 0.2 0.8	5.4 0.4 2.6	9.4 0.8 4.2	6.8 0.0 0.6	2.8 0.0 1.2	50.2 36.8 45.8
0.1	0,0	0.2 3.4 1.0	10.2 28.4 18.2	46.6 74.0 57.4	69.6 91.8 81.6	28.6 58.0 43.4	2.8 16.8 8.2
	0,1	75.4 77.2 78.0	55.6 46.6 52.6	24.8 13.4 22.2	13.2 4.0 8.8	32.8 20.6 26.4	55.4 47.2 52.4
	1,0	19.0 18.6 19.0	29.2 24.2 26.8	24.0 12.0 18.6	13.8 4.2 9.0	33.6 21.0 28.4	38.0 35.6 37.4
	1,1	5.4 0.8 2.0	5.0 0.8 2.4	4.6 0.6 1.8	3.4 0.0 0.6	5.0 0.4 1.8	3.8 0.4 2.0

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 14 (cont'd): Model Selection for ARFIMA(0,d,1) by R/S

		θ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.1	0,0	12.4	51.2	75.0	11.6	0.6	0.0
		33.8	81.4	97.8	43.0	8.2	0.4
		20.6	67.4	90.0	25.4	3.0	0.0
	0,1	63.8	31.6	10.0	47.8	64.2	76.4
		59.0	15.0	1.6	39.4	68.2	82.2
		66.2	25.2	4.4	45.8	67.8	80.6
	1,0	3.2	4.4	2.0	21.8	22.4	15.2
		3.0	2.2	0.2	15.6	22.0	15.8
		3.2	3.2	1.0	19.8	22.4	15.6
	1,1	20.6	12.8	13.0	18.8	12.8	8.4
		4.2	1.4	0.4	2.0	1.6	1.6
		10.0	4.2	4.6	9.0	6.8	3.8
-0.2	0,0	27.0	55.2	41.0	0.4	0.0	0.0
		63.0	96.0	90.4	13.8	0.2	0.0
		44.6	79.4	69.6	4.6	0.0	0.0
	0,1	42.6	11.6	10.0	56.4	71.4	81.8
		32.4	1.6	3.6	65.2	84.8	90.2
		41.6	6.6	7.8	63.2	78.8	87.2
	1,0	1.4	0.8	1.6	13.0	10.2	7.2
		0.4	0.0	0.4	11.8	10.2	7.2
		1.0	0.4	0.6	12.6	10.2	7.2
	1,1	29.0	32.4	47.4	30.2	18.4	11.0
		4.2	2.4	5.6	9.2	8.4	2.6
		12.8	13.6	22.0	19.6	11.0	5.6
-0.3	0,0	27.6	20.6	7.4	0.0	0.0	0.0
		80.6	83.6	54.8	0.8	0.0	0.0
		55.4	44.2	21.0	0.2	0.0	0.0
	0,1	17.6	4.2	11.0	55.0	70.2	79.6
		10.8	1.4	13.8	72.8	86.0	91.8
		18.2	3.4	13.8	64.2	77.6	87.6
	1,0	0.6	0.2	0.6	5.0	4.6	3.2
		0.4	0.0	0.4	5.4	4.8	3.2
		0.6	0.0	0.6	5.2	4.6	3.2
	1,1	54.2	75.0	81.0	40.0	25.2	17.2
		8.2	15.0	31.0	21.0	9.2	5.0
		25.8	52.4	64.6	30.4	17.8	9.2

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 15: Model Selection for ARFIMA(1,d,0) by ML

		ϕ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
0.3	0,0	2.0 17.0 7.0	11.0 46.0 25.0	38.0 82.0 65.0	49.0 70.0 60.0	18.0 40.0 28.0	2.0 19.0 7.0
	0,1	21.0 21.0 20.0	23.0 19.0 20.0	18.0 5.0 12.0	9.0 5.0 9.0	18.0 12.0 14.0	12.0 8.0 10.0
	1,0	59.0 60.0 58.0	50.0 35.0 47.0	30.0 13.0 19.0	27.0 18.0 26.0	50.0 46.0 53.0	70.0 69.0 77.0
	1,1	18.0 2.0 15.0	16.0 0.0 8.0	14.0 0.0 4.0	15.0 3.0 5.0	14.0 2.0 5.0	16.0 4.0 6.0
0.2	0,0	2.0 23.0 6.0	11.0 53.0 31.0	41.0 79.0 59.0	35.0 79.0 53.0	14.0 38.0 24.0	3.0 18.0 7.0
	0,1	22.0 19.0 22.0	24.0 15.0 21.0	15.0 10.0 13.0	14.0 3.0 11.0	16.0 9.0 13.0	10.0 9.0 10.0
	1,0	66.0 56.0 67.0	51.0 32.0 41.0	28.0 11.0 20.0	36.0 14.0 29.0	55.0 51.0 57.0	73.0 74.0 79.0
	1,1	10.0 2.0 5.0	14.0 0.0 7.0	16.0 0.0 8.0	15.0 4.0 7.0	15.0 2.0 6.0	14.0 2.0 4.0
0.1	0,0	4.0 25.0 13.0	16.0 64.0 36.0	42.0 83.0 67.0	42.0 78.0 59.0	10.0 40.0 28.0	3.0 18.0 11.0
	0,1	27.0 27.0 29.0	31.0 16.0 28.0	22.0 7.0 13.0	14.0 2.0 8.0	19.0 9.0 12.0	19.0 15.0 18.0
	1,0	50.0 47.0 51.0	37.0 20.0 32.0	20.0 10.0 16.0	31.0 20.0 30.0	57.0 49.0 57.0	67.0 67.0 68.0
	1,1	19.0 1.0 7.0	16.0 0.0 4.0	16.0 0.0 4.0	13.0 0.0 3.0	14.0 2.0 3.0	11.0 0.0 3.0

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 15 (cont'd): Model Selection for ARFIMA(1,d,0) by ML

		ϕ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.1	0,0	3.0	17.0	46.0	41.0	15.0	6.0
		31.0	57.0	83.0	79.0	28.0	21.0
		10.0	40.0	70.0	58.0	20.0	8.0
	0,1	18.0	16.0	14.0	12.0	11.0	19.0
		19.0	12.0	5.0	6.0	10.0	15.0
		22.0	13.0	8.0	11.0	12.0	19.0
	1,0	56.0	47.0	23.0	35.0	64.0	66.0
		50.0	31.0	11.0	13.0	61.0	63.0
		63.0	36.0	18.0	26.0	63.0	69.0
	1,1	23.0	20.0	17.0	12.0	10.0	9.0
		0.0	0.0	1.0	2.0	1.0	1.0
		5.0	11.0	4.0	5.0	5.0	4.0
-0.2	0,0	3.0	18.0	50.0	67.0	22.0	5.0
		26.0	57.0	85.0	93.0	61.0	19.0
		15.0	31.0	64.0	82.0	44.0	11.0
	0,1	24.0	31.0	16.0	6.0	17.0	9.0
		16.0	15.0	6.0	1.0	7.0	7.0
		23.0	25.0	13.0	3.0	13.0	9.0
	1,0	59.0	43.0	23.0	17.0	51.0	75.0
		56.0	28.0	9.0	6.0	31.0	73.0
		58.0	43.0	19.0	14.0	41.0	76.0
	1,1	14.0	8.0	11.0	10.0	10.0	11.0
		2.0	0.0	0.0	0.0	1.0	1.0
		4.0	1.0	14.0	1.0	2.0	4.0
-0.3	0,0	7.0	27.0	58.0	60.0	27.0	3.0
		37.0	70.0	90.0	92.0	63.0	19.0
		20.0	47.0	76.0	82.0	44.0	8.0
	0,1	27.0	24.0	15.0	13.0	23.0	16.0
		19.0	12.0	6.0	5.0	10.0	12.0
		24.0	21.0	9.0	7.0	18.0	15.0
	1,0	59.0	39.0	14.0	20.0	45.0	76.0
		44.0	18.0	4.0	3.0	27.0	69.0
		55.0	31.0	11.0	11.0	38.0	77.0
	1,1	7.0	10.0	13.0	7.0	5.0	5.0
		0.0	0.0	0.0	0.0	0.0	0.0
		1.0	1.0	4.0	0.0	0.0	0.0

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 16: Model Selection for ARFIMA(0,d,1) by ML

d	Model	θ = 0.3	0.2	0.1	-0.1	-0.2	-0.3
0.3	0,0	0.0 13.0 4.0	11.0 43.0 23.0	38.0 80.0 62.0	50.0 74.0 60.0	27.0 44.0 34.0	15.0 28.0 19.0
	0,1	30.0 27.0 29.0	16.0 10.0 13.0	10.0 0.0 6.0	14.0 6.0 14.0	26.0 20.0 23.0	40.0 33.0 38.0
	1,0	44.0 51.0 49.0	53.0 46.0 54.0	34.0 19.0 27.0	22.0 16.0 21.0	40.0 33.0 37.0	40.0 38.0 39.0
	1,1	26.0 9.0 18.0	20.0 1.0 10.0	18.0 1.0 5.0	14.0 4.0 5.0	7.0 3.0 6.0	5.0 1.0 4.0
0.2	0,0	1.0 7.0 2.0	8.0 43.0 22.0	41.0 78.0 59.0	40.0 80.0 52.0	17.0 49.0 30.0	14.0 31.0 22.0
	0,1	22.0 18.0 21.0	7.0 2.0 4.0	9.0 5.0 9.0	13.0 3.0 10.0	31.0 22.0 26.0	41.0 36.0 39.0
	1,0	39.0 65.0 57.0	65.0 52.0 62.0	35.0 16.0 25.0	32.0 14.0 30.0	40.0 28.0 39.0	39.0 32.0 35.0
	1,1	38.0 10.0 20.0	20.0 3.0 12.0	15.0 1.0 7.0	15.0 3.0 8.0	12.0 1.0 5.0	6.0 1.0 4.0
0.1	0,0	2.0 13.0 5.0	13.0 48.0 27.0	40.0 82.0 65.0	40.0 77.0 57.0	21.0 54.0 37.0	13.0 36.0 24.0
	0,1	14.0 13.0 13.0	19.0 9.0 16.0	11.0 3.0 9.0	10.0 2.0 4.0	37.0 25.0 31.0	50.0 38.0 46.0
	1,0	29.0 55.0 45.0	40.0 36.0 39.0	29.0 15.0 19.0	36.0 21.0 35.0	32.0 21.0 29.0	29.0 26.0 29.0
	1,1	55.0 19.0 37.0	28.0 7.0 18.0	20.0 0.0 7.0	14.0 0.0 4.0	10.0 0.0 3.0	8.0 0.0 1.0

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 16 (cont'd): Model Selection for ARFIMA(0,d,1) by ML

		θ					
d	Model	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.1	0,0	0.0	9.0	44.0	37.0	15.0	16.0
		13.0	52.0	84.0	76.0	40.0	38.0
		4.0	35.0	65.0	58.0	28.0	30.0
	0,1	16.0	16.0	10.0	13.0	33.0	39.0
		14.0	7.0	1.0	5.0	25.0	29.0
		14.0	9.0	9.0	10.0	29.0	31.0
	1,0	41.0	48.0	27.0	38.0	42.0	39.0
		58.0	36.0	14.0	18.0	34.0	32.0
		51.0	40.0	21.0	29.0	40.0	37.0
	1,1	43.0	27.0	19.0	11.0	10.0	6.0
		15.0	5.0	1.0	1.0	1.0	1.0
		31.0	16.0	5.0	2.0	3.0	2.0
-0.2	0,0	0.0	15.0	48.0	67.0	38.0	18.0
		13.0	49.0	81.0	93.0	70.0	45.0
		3.0	24.0	63.0	84.0	57.0	26.0
	0,1	28.0	14.0	12.0	7.0	28.0	38.0
		22.0	6.0	7.0	1.0	11.0	29.0
		27.0	10.0	12.0	2.0	19.0	38.0
	1,0	33.0	55.0	28.0	14.0	30.0	38.0
		50.0	44.0	12.0	6.0	19.0	26.0
		40.0	63.0	21.0	13.0	23.0	34.0
	1,1	39.0	16.0	12.0	12.0	4.0	6.0
		16.0	1.0	0.0	0.0	0.0	0.0
		300	3.0	4.0	1.0	1.0	2.0
-0.3	0,0	1.0	19.0	57.0	63.0	40.0	12.0
		16.0	58.0	88.0	93.0	74.0	49.0
		8.0	34.0	76.0	85.0	59.0	25.0
	0,1	32.0	19.0	12.0	14.0	27.0	55.0
		28.0	10.0	5.0	4.0	11.0	33.0
		28.0	15.0	11.0	7.0	19.0	49.0
	1,0	31.0	45.0	19.0	15.0	31.0	29.0
		44.0	28.0	6.0	3.0	15.0	18.0
		37.0	45.0	10.0	8.0	22.0	26.0
	1,1	36.0	17.0	12.0	8.0	2.0	4.0
		12.0	4.0	1.0	0.0	0.0	0.0
		27.0	6.0	3.0	0.0	0.0	0.0

Note: The selection criteria used are AIC, SC, and HQ, respectively. Figures refer to the percent of occassions each model was selected.

Table 17: Bias in d for ML Estimation of ARFIMA(0,d,0) when DGP = ARFIMA(1,d,0)

	d					
ϕ	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.3	-.116	-.095	-.105	-.149	-.158	-.194
-0.2	-.076	-.068	-.070	-.067	-.114	-.138
-0.1	-.058	-.021	-.034	-.046	-.079	-.079
0.1	.036	.052	.056	.052	.052	.054
0.2	.101	.124	.125	.127	.129	.131
0.3	.155	.197	.204	.211	.214	.215

Table 18: Bias in d for ML Estimation of ARFIMA(1,d,0) when DGP = ARFIMA(0,d,1)

	d					
θ	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.3	-.025	-.013	.004	-.006	.018	.044
-0.2	-.015	-.016	-.010	-.009	-.008	.016
-0.1	-.028	-.017	-.005	-.018	-.040	-.008
0.1	-.039	-.028	-.014	-.025	-.031	-.011
0.2	-.011	.002	.007	.005	.001	.017
0.3	.032	.047	.051	.052	.049	.063

Table 19: Bias in d for ML Estimation of ARFIMA(0,d,1) when DGP = ARFIMA(1,d,0)

	d					
ϕ	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.3	-.124	-.113	-.113	-.116	-.121	-.100
-0.2	-.086	-.075	-.073	-.076	-.082	-.060
-0.1	-.066	-.055	-.051	-.054	-.061	-.036
0.1	-.119	-.102	-.072	-.091	-.099	-.060
0.2	-.150	-.182	-.116	-.124	-.154	-.095
0.3	-.187	-.189	-.131	-.153	-.153	-.112

Table 20: Bias in d for ML Estimation of ARFIMA(1,d,1) when DGP = ARFIMA(1,d,0)

	d					
ϕ	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.3	-.048	-.039	-.017	-.028	-.046	-.004
-0.2	-.043	-.031	-.026	-.027	-.044	-.007
-0.1	-.072	-.075	-.055	-.047	-.053	-.027
0.1	-.206	-.203	-.139	-.147	-.183	-.108
0.2	-.217	-.208	-.152	-.167	-.200	-.124
0.3	-.211	-.208	-.174	-.190	-.229	-.122

Table 21: Bias in d for ML Estimation of ARFIMA(1,d,1) when DGP = ARFIMA(0,d,1)

	d					
θ	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.3	-.160	-.157	-.127	-.138	-.150	-.097
-0.2	-.122	-.121	-.147	-.071	-.089	-.029
-0.1	-.148	-.119	-.099	-.094	-.103	-.063
0.1	-.199	-.195	-.136	-.157	-.177	-.110
0.2	-.162	-.163	-.123	-.147	-.167	-.103
0.3	-.121	-.146	-.109	-.137	-.140	-.094

Table 22: Expected Bias in d for ML Estimation of ARFIMA(1,d,0) Model (AIC)

	d					
φ	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.3	-.064	-.057	-.073	-.096	-.076	-.032
-0.2	-.056	-.046	-.056	-.048	-.060	-.029
-0.1	-.060	-.045	-.042	-.043	-.055	-.030
0.1	-.121	-.090	-.025	-.032	-.078	-.052
0.2	-.137	-.104	.007	.046	-.083	-.071
0.3	-.145	-.075	.063	.066	-.051	-.083

Table 23: Expected Bias in d for ML Estimation of ARFIMA(0,d,1) Model (AIC)

	d					
θ	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.3	-.069	-.055	-.068	-.115	-.071	-.029
-0.2	-.062	-.042	-.061	-.051	-.054	-.025
-0.1	-.096	-.049	-.037	-.040	-.059	-.029
0.1	-.109	-.066	-.008	-.011	-.034	-.011
0.2	-.079	-.013	.038	.057	.030	.014
0.3	-.045	.022	.089	.103	.076	.027

Table 24: Expected Bias in d for ML Estimation of ARFIMA(1,d,0) Model (SC)

	d					
φ	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.3	-.065	-.050	-.058	-.068	-.067	-.064
-0.2	-.064	-.049	-.057	-.043	-.089	-.099
-0.1	-.056	-.026	-.036	-.045	-.077	-.075
0.1	.011	.021	.034	.029	.030	.043
0.2	-.025	-.001	.044	.031	.015	.067
0.3	-.109	-.084	-.031	-.033	-.051	.020

Table 25: Expected Bias in d for ML Estimation of ARFIMA(0,d,1) Model (SC)

	d					
θ	0.3	0.2	0.1	-0.1	-0.2	-0.3
-0.3	-.066	-.049	-.052	-.085	-.099	-.107
-0.2	-.050	-.048	-.051	-.046	-.095	-.110
-0.1	-.056	-.030	-.028	-.038	-.079	-.077
0.1	.016	.031	.041	.036	.033	.042
0.2	.027	.043	.045	.051	.053	.066
0.3	.012	.022	.026	.029	.020	.040