STRUCTURAL BREAKS AND SEASONAL INTEGRATION

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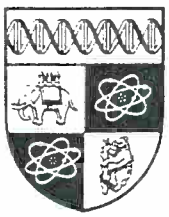

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Structural Breaks and Seasonal Integration

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Abstract

Perron (1989) investigated the effects of an exogenous change in either the level or growth rate of a series and found that the Augmented Dickey-Fuller unit root test, allowing for such a change under the alternative hypothesis, yielded a markedly more skewed test statistic. This paper looks at the effects on the HEGY tests of an exogenous change in the level or seasonal pattern of a series. The distribution of the test statistics associated with the HEGY test are more skewed. Applying these findings to Colombian money supply and GDP series as well as UK transportation expenditure, initial results suggesting the need for a unit and seasonal root are overturned.

Keywords: HEGY tests, unit roots, seasonal unit roots, structural break

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1. Introduction

Since Perron (1989), researchers have been particularly cautious about interpreting the results of unit root tests in the presence of apparent exogenous change in either the level or growth rate of a series. The presence of such "breaks" greatly reduces the power of the Augmented Dickey-Fuller (ADF) test to reject the null hypothesis of a unit root against the alternative of trend stationarity, if these "breaks" are not modelled. Using the model

$$Y_t = \mu_1 + (\mu_2 - \mu_1)DU_t + \beta t + \varepsilon_t, \quad t = 1,\ldots,T \tag{1}$$

where $\varepsilon_t \sim N(0,\sigma^2)$, Perron (1989) plotted the cumulative distribution of $\hat{\gamma}$ from the estimated equation

$$Y_t = \alpha + \gamma Y_{t-1} + \eta_t, \tag{2}$$

and showed that for μ_1=0.0, β=1.0 and $\mu_2 \geq 2.0$, $\hat{\gamma}$ is substantially biased towards unity. Using these results Perron (1989) found that, whereas Nelson and Plosser (1982) were not able to reject the null hypothesis of a unit root for 13 out of 14 US annual series, he rejected the null hypothesis for 10 out of 13 series[1] when allowing for a "break" in 1929.[2]

In this paper we use a similar approach to that of Perron (1989) to investigate the behaviour of the test of Hylleberg et. al. (1989) (HEGY) for both unit and seasonal unit roots, in the presence of an exogenous change in the level or seasonal pattern of the data. The evidence suggests that both unit root and seasonal root tests can be severely biased be these changes. New critical values for the HEGY tests are presented allowing a change in either the level and/or the seasonal pattern of the underlying series. Franses and McAleer (1994) address the issue of whether a change in the seasonal pattern of a series can affect the order of integration of that series. In particular, using Austrian GNP data they demonstrate that, a series which appears to be periodically integrated (see Osborn (1988)) is actually best described as an I(1) series with a change in the seasonal pattern.

The HEGY test is then applied to Colombian money supply and GDP data, both of which exhibit evidence of a break in the seasonal pattern, in order to calculate the order of integration of these two series allowing for a change in the seasonal pattern. Finally this procedure is used to determined the order of integration of real UK transportation expenditure

[1] Unemployment was not investigated by Perron as this series was found to be I(0).
[2] Perron's findings have been subsequently over-turned by Zivot and Andrews (1992) and more recently by Nunes et al. (1994).

data, bearing in mind the change in date for new car registration from January to August in 1977.

The outline of the paper is as follows. Section 2 looks at the behaviour of the HEGY test in the presence of breaks in both the overall mean and seasonal mean of trend stationary series. Section 3 applies these procedures to Colombian money supply and GDP data as well as UK transportation expenditure data and we determine the order of integration of these series. Finally, Section 4 presents some concluding remarks.

2. HEGY Tests

The HEGY tests, tests for the existence of both a unit root as well as a seasonal root in a series Y_t, by estimating the equation (3)

$$\tilde{Y}_{4t} = \pi_1 \tilde{Y}_{1t-1} + \pi_2 \tilde{Y}_{2t-1} + \pi_3 \tilde{Y}_{3t-2} + \pi_4 \tilde{Y}_{3t-1} + \upsilon_t, \quad t = 5,\ldots T,^3 \tag{3}$$

where

$$\tilde{Y}_{1t} = \tilde{Y}_t + \tilde{Y}_{t-1} + \tilde{Y}_{t-2} + \tilde{Y}_{t-3}$$

$$\tilde{Y}_{2t} = -\tilde{Y}_t + \tilde{Y}_{t-1} - \tilde{Y}_{t-2} + \tilde{Y}_{t-3}$$

$$\tilde{Y}_{3t} = -\tilde{Y}_t + \tilde{Y}_{t-2}$$

$$\tilde{Y}_{4t} = \tilde{Y}_t - \tilde{Y}_{t-4}$$

and $\tilde{Y}_t$ are the OLS residuals from the auxiliary regression

$$Y_t = \hat{\mu} + \sum_{j=2}^{4} \hat{\delta}_j D_{jt} + \hat{\beta} t + \tilde{Y}_t, \qquad t = 1,2,\ldots,T. \tag{4}$$

The methodology suggested by HEGY is to test for the existence of a unit root, by testing H_0: $\pi_1 = 0$ against the one-sided alternative H_1: $\pi_1 < 0$. To test for the existence of a seasonal unit root HEGY (p.) note that "there will be no seasonal unit roots if π_2 and either π_3 or π_4 are different from zero". Consequently, they recommend testing H_0: $\pi_2 = 0$ against the one-sided alternative H_1: $\pi_2 < 0$ and simultaneously testing the joint hypothesis H_0: π_3, $\pi_4 = 0$ against the alternative H_1: $\pi_3 < 0$, $\pi_4 \neq 0$. A null hypothesis of a seasonal unit root is only rejected when, both the t-test for π_2 and the joint F-test for π_3 and π_4 are rejected.

[3] The analysis is limited to a study of quarterly data.

Using this suggested methodology, we present power probabilities of the HEGY test to reject both a unit root and a seasonal root, when the underlying Data Generating Process (DGP) is an AutoRegressive (AR) model, of the form

$$Y_t = \sum_{i=1}^{4} \mu_i + \alpha_1 Y_{t-1} + \varepsilon_t \ . \tag{5}$$

The auxiliary regression (4) used to construct $\tilde{Y}_t$ in equation (3) is heavily over-parameterised by the inclusion of an intercept, seasonal dummies and trend.

The results are presented in Table 1 for the sample sizes considered by HEGY (T = 48, 100, 136, 200) and a range of values of $\alpha \leq 1.0$.[4] For $\alpha = 1.0$, and T = 200, the empirical size probabilities approach their theoretical p-values for the test of a unit root. However, the power of this test to reject the unit root null hypothesis when $1.0 > \alpha > 0.9$ is considerably less than that associated with the simple Dickey-Fuller test see, for example, Dickey and Fuller (1981). The power of this model to reject the null hypothesis of a seasonal unit root is always high and approaches 100% for T > 100.

Table 2 reports power probabilities when the underlying DGP is a Seasonal AR (SAR) model of the form

$$Y_t = \sum_{i=1}^{4} \mu_i + \alpha_4 Y_{t-4} + \varepsilon_t . \tag{6}$$

Using the auxiliary regression (4), for $\alpha = 1.0$ and $T \rightarrow \infty$, the empirical size probabilities again approach the theoretical p-values for the unit root null hypothesis. However, for the test of a seasonal unit root the empirical size probabilities are always too small, compared with their theoretical p-values, even as $T \rightarrow \infty$. Implying that the rule for rejecting the null hypothesis of a seasonal unit root, when both null hypotheses H_0: $\pi_2 \geq 0$ and H_0: $\pi_3, \pi_4 = 0$ are jointly rejected, is too stringent a hypothesis to impose. For $\alpha < 1.0$ power of the seasonal unit root is quite small, which is to be expected as the size probabilities are too small. The unit root null hypothesis is rejected far less than when the DGP was a simple AR model (see Table 1).

Perron (1989) investigated the performance of the Augmented Dickey-Fuller (ADF) test, when there was an exogenous change in the level or growth of a trend stationary process, at some point T_B. Following a similar approach, Table 3 reports the number of times the null

[4] All simulations are based upon 10,000 replications.

hypothesis of a unit root and a seasonal unit root is by the HEGY test rejected when the DGP is of the form

$$Y_t = \sum_{j=1}^{4} \delta_j D_{jt} + \sum_{j=1}^{4} \gamma_j D_{jt} DU_t + \beta t + \varepsilon_t,, \quad DU_t = \begin{cases} 0 \text{ if } t \le T/2 \\ 1 \text{ if } t > T/2 \end{cases}. \tag{7}$$

Consider firstly the case when $\delta_j = \mu_1 = 0.0\ \forall j$, $\beta = 1.0$ and $\gamma_j = \mu_2 = 0.0, 1.0, 2.0, 5.0, 10.0\ \forall j$ (this corresponds to case A in Perron (1989), that is, a change in the level of a series). The results confirm the finding of Perron (1989) (see Figure 4, p.1369), that as the "break" becomes increasingly large, so the ability of a unit root test to distinguish between stationarity and nonstationarity declines. As $T \rightarrow \infty$, for a given value of μ_2 ($\neq 0$), so the power of the unit root test to correctly reject the nonstationary null hypothesis increases. The power of the seasonal unit root to reject the null hypothesis is approximately 100% for $T \geq 100$. This result is unsurprising as a change in the level of the process will not affect the spectrum of the series at the seasonal frequency.

Figure 1 plots the power probabilities at the 5% significance level for T = 100, for the unit root test, when the break point (T_B) varies over the sample from T_B =1, ...T-1, for a variety of values for μ_2. The figure shows that the power probabilities follow a symmetric but flat W-shaped function with minima at T_B = T/4, 3T/4, and power increasing quickly at the extreme points, that is for $90 \leq T_B \leq 10$.

Table 4a reports the case when there is a change in the seasonal pattern of the series, that is, in equation (7) , $\delta_j = \mu_j = 0.0\ \forall j$, $\beta = 1.0$, $\gamma_j = 0.0$, j =1, 2, and $-\gamma_3 = \gamma_4 = 0.0, 1.0, 2.0, 5.0, 10.0$. The test of a unit root is unaffected as there is no actual change in the level of the series. However, the change in the seasonal pattern affects the spectrum at the seasonal frequency, and this adversely affects power performance of the HEGY test for a seasonal unit root, for example, for T = 100 and $\gamma_4 = 5.0$, power is 0.0% at the 5% significance level, compared with approximately 100% when there is no change in the seasonal pattern. Again increasing T, for a given size of break increases the power of the test.

Figure 2 reports the power probabilities at the 5% significance level for T = 100, for the seasonal unit root hypothesis, for all values of T_B and a range of values for γ_4. In contrast to Figure 1, the power function is a symmetric flat U-shaped curve with a minimum at T_B = T/2, and power only increasing at the end points, that is, $80 \leq T_B \leq 20$.

Table 4b reports the case when both the level and seasonal pattern of a series change, that is, $\delta_j = \mu_j = 0.0\ \forall j$, $\beta = 1.0$, $\gamma_j = 0.0$, j =1, 2, 3, and $\gamma_4 = 0.0, 1.0, 2.0, 5.0, 10.0$ in equation

(7). For both the unit root and seasonal unit root tests power falls as the size of the break increases. The power of the seasonal unit root tests to correctly reject the null hypothesis is slightly higher than in Table 4a. Directly comparison of Table 3 with Table 4b is not possible because in Table 4b the change in the level of the series is $\gamma_4/4$. Setting $\gamma_4 =$ 4.0, 8.0, the power of unit root test to reject the null hypothesis at the 5% significance level for T = 100, is 75.95% and 12.95%, respectively (compared with 94.19% and 56.14% for μ_2 = 1.0, 2.0 in Table 3).

Finally, Table 5 reports the power probabilities when the exogenous growth of the process changes at T/2. This corresponds to the case when the DGP is

$$Y_t = \sum_{j=1}^{4} \delta_j D_{jt} + \beta_1 t + (\beta_2 - \beta_1) DT_t + \varepsilon_t, \; DT_t = \begin{cases} 0 & \text{if } t \le T/2 \\ t - T/2 & \text{if } t > T/2 \end{cases}. \tag{8}$$

The unit root test is severely adversely affected, with power falling to zero for $\beta_2 < 0.975$. As a change in the trend does not affect the spectral density at the seasonal frequencies the seasonal unit roots tests have high power for $T \ge 100$, irrespective of the value for β_2.

2.1 HEGY Test and Structural Break

In this section, we present new critical values for the HEGY test given a change in the level or change in the seasonal pattern of a series. As a point of comparison with the existing critical values, Table 6 reports the critical values for π_1, π_2, π_3, π_4, and the joint F-test at the 1%, 2.5%, 5%, 10%, 90%, 95%, 97.5% and 99% significance levels when T = 1000.[5]

Three different cases are considered. Table 7 presents the critical values of the HEGY tests when there is an assumed change in the level of the process and there are no deterministic seasonal dummies included in the auxiliary regression (4). This case is similar to Perron's (1989) Case A. The critical values are sensitive to the position of the break and are presented for a range of known alternative break points, which are assumed to be some proportion, λ, of the sample size, T. The critical values in Table 9 are comparable with those in the third block in Table 6 (Intercept, No Seas. Dum. and Trend). Only those critical values corresponding to π_1 have changed to any substantive extent, this result is surprising as it has already been noted that a change in the level of a series does not affect the spectral density at

[5] This sample size is used because, following Perron (1989), we calculate the critical values of the HEGY tests allowing for a structural break using T = 1000. In this paper, the critical values in Tables 9, 10, and 11 are constructed using the regression method, rather than using finite sample representation of Weiner processes as is done by Perron (1989).

seasonal frequencies and therefore will not affect the critical values for tests of a seasonal unit root. When there is a change in the level of the process, and seasonal dummy variable are included in the auxiliary regression equation (4), the critical values for π_1 are almost identical to those reported in Table 7. The critical values for the seasonal unit root tests (π_2, π_3, π_4 and the F statistic) are unaffected by a break in the mean of the process and are therefore almost identical to those in the final block of Table 6 (Intercept, Seas. Dum. and Trend).

Finally, Table 8 reports the critical values, when there is an assumed change in the seasonal pattern and level of the data and seasonal dummies are included in the auxiliary regression equation (4). The critical values in Table 8 should again be compared to those in the last block of Table 6 (Intercept, Seas. Dum. and Trend). The critical values for both the unit root test (π_1) and the seasonal unit root tests (π_2 and π_3) have become substantially more leftward skewed compared with those when no change is permitted (Table 6), and there has been a marked increase in the critical values of the F-statistic.

3. Empirical Application

Figure 3 presents plots of seasonally unadjusted quarterly observations of the log of Colombian money supply, ln(MS), between 1970 and 1992[6]. It can be seen from the figure that the series exhibits an upward trend as well as a seasonal pattern, consisting of peaks during the fourth quarter; moreover, it is also possible to notice that after 1979.4 these peaks tend to be more pronounced, suggesting a change in the seasonal pattern of the series. According to Montenegro et al (1987), this change in the seasonal pattern of the series obeys to the fact that since the early eighties, the higher demand for currency, at the end of the year, is being satisfied by some of the components of the monetary base, instead of by a reduction in demand deposits. This change can be seen more clearly in Figure 4, which plots Δln(MS) between 1970 and 1992.

As a second illustration, Figure 5 exhibits plots of seasonally unadjusted quarterly observations of the log of Colombian GDP, ln(GDP), between 1975 and 1992. This series corresponds to that constructed by the Departamento Nacional de Planeación, based on the aggregation of the different expenditure components (i.e. private and public consumption,

[6]The source of this series is Banco de la República (1993).

investment, exports and imports)[7]. From the figure it can be seen that the GDP series exhibits an upward trend with a seasonal pattern consisting of peaks during the fourth quarter; furthermore, a closer inspection of the series also allows us to notice that after 1985.4 a peak in the second quarter is also present. Unfortunately, it is not possible to provide an explanation for such change in the seasonal pattern; therefore, we suggest that it may be due to a change in the seasonal pattern of one or some of the series involved in the construction of the GDP series. Again to emphasise the changing seasonal pattern Figure 6 plots Δln(GDP).

In the UK car registration moved from being January to August in 1977. This change necessarily shifted a portion of the demand for vehicles from quarter 1 to quarter 3. This change can be seen in Figures 7 and 8 which plot, from 1963 to 1994, the log of UK real expenditure for transportation, ln(Car) and its first difference, Δln(Cars).[8]

Table 9 presents the results from running a standard HEGY test where the auxiliary regression includes an intercept, seasonal dummies and a trend for all series. An augmented version of the simply HEGY test (equation (3)) is estimated with 2 extra lagged terms included for money supply, 2 extra terms included for GDP and 5 for UK transportation expenditure (the terms are of the form $\tilde{Y}_{4t-1},\ldots,\tilde{Y}_{4t-p}$). These lags are included to ensure the resultant regression did not exhibit any residual serial correlation or heteroscedasticity.[9] Using critical values corresponding to T = 100, none of the three series (ln(GDP), ln(MS), ln(Car)) rejects the hypothesis of either a unit root or a seasonal unit root at the 5% significance level, although ln(Cars) rejects the unit root hypothesis at the 10% significance level. Consequently, all series are believed to be I(1,1), where the first number reflects the order of integration at frequency zero, and the second number the order of integration at the seasonal frequency.

Allowing for a change in the seasonal pattern, define a change variable for GDP, money supply and car expenditure as:

$$DUGDP_t = \begin{cases} 0 \text{ if } t < 1985.4 \\ 1 \text{ if } t \geq 1986.1 \end{cases}, \quad DUMS_t = \begin{cases} 0 \text{ if } t < 1979.4 \\ 1 \text{ if } t \geq 1980.1 \end{cases}, \quad DUCAR_t = \begin{cases} 0 \text{ if } t < 1976.4 \\ 1 \text{ if } t \geq 1977.1 \end{cases}.$$

Interacting these dummy variables with the seasonal dummy variables, $\tilde{Y}_t$ is obtained as the residuals from the auxiliary equation (4), with the addition the 4 interaction dummy variables.

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[7] See Cubillos and Valderrama (1993) for a presentation of the methodology. The source of this series is Cubillos and Valderrama (1993) for the period 1980-1992, and worksheets of the Unidad de Análisis Macroeconómico at the Departamento Nacional de Planeación for the period 1975-1979.

[8] CSO series CCBJ; Monthly Digest of Statistics.

[9] For UK transportation expenditure the serial correlation test was only passed at the 4% significance level and the inclusion of extra lags did not improve this test statistic.

Applying the HEGY test to $\tilde{Y}_t$, yields the results presented in the bottom half of Table 9. Comparing the test statistics reported in this table with the critical values in Table 8, the results now suggest that ln(GDP), ln(MS), and ln(Cars) are I(1,0) at both the 5% and 10% significance levels.

Table 9: HEGY Tests for ln(MS), ln(GDP) and ln(Car)

Break Period	λ	p	Variable	π_1	π_2	π_3	π_4	F-test
No Break	0.0	2	ln(MS)	-1.723	-0.882	-2.171	-2.845	6.861
No Break	0.0	2	ln(GDP)	-2.305	-2.193	-2.696	-1.060	4.366
No Break	0.0	5	ln(Car)	-3.443	-1.071	-3.033	-0.367	4.665
1979.4	0.43	0	ln(MS)	-1.821	-3.781	-5.415	-5.648	47.97
1985.4	0.61	0	ln(GDP)	-2.250	-4.050	-6.683	-2.545	32.09
1977.1	0.44	0	ln(Car)	-2.523	-7.334	-5.221	-2.800	19.93

4. Concluding Remarks

This paper has shown that the HEGY tests for both unit roots and seasonal unit roots can be adversely affected by a change in either the level or seasonal pattern of a series. The unit root test is affected by a change in the level, although not by a change in the seasonal pattern of a series. In contrast, the seasonal root test is affected by a change in the seasonal pattern and remains unaffected by a change in the level of the process.

The position of the "break" can have a substantial effect on the power of the test statistic. For the seasonal unit root test a change in the seasonal pattern yields a U-shaped power curve with a minimum at T/2, whereas for the unit root test a change in the level yields a W-shaped power curve, with minima at T/3 and 2T/3.

Critical values for the HEGY test in the face of an exogenous change in the level and/or seasonal pattern are presented. Using these new critical values the Colombian money supply and GDP series, as well as the UK transportation expenditure series, all of which appear to be I(1,1) when no account is taken of the change in the seasonal pattern, are shown to be better I(1,0), if a change in the seasonal pattern is allowed.

Table 1
Power of HEGY Test: DGP is an AR(1)

		Pr{Reject Unit Root]				Pr[Reject Seasonal Unit Root]			
T	ρ	1%	2.5%	5%	10%	1%	2.5%	5%	10%
48	1.00	0.30	0.85	1.86	4.43	41.60	63.65	80.46	92.83
	0.95	0.26	0.93	2.20	5.15	40.01	63.57	80.44	92.09
	0.90	0.42	1.42	3.23	7.06	39.88	62.47	79.99	92.32
	0.80	0.94	2.59	5.61	11.66	38.80	61.49	78.73	91.59
100	1.00	0.55	1.52	3.49	7.31	99.89	100.00	100.00	100.00
	0.95	1.26	2.90	5.81	12.06	99.90	99.99	100.00	100.00
	0.90	2.75	6.20	11.76	22.08	99.86	99.99	100.00	100.00
	0.80	9.91	19.35	31.65	50.47	99.88	99.99	100.00	100.00
136	1.00	0.48	1.48	3.56	7.57	100.00	100.00	100.00	100.00
	0..95	1.29	3.68	7.92	16.23	100.00	100.00	100.00	100.00
	0.90	3.93	10.40	19.36	33.98	100.00	100.00	100.00	100.00
	0.80	23.13	41.89	59.16	77.17	100.00	100.00	100.00	100.00
200	1.00	0.89	2.12	4.23	8.85	100.00	100.00	100.00	100.00
	0.95	3.15	7.90	14.25	26.16	100.00	100.00	100.00	100.00
	0.90	15.43	29.34	43.51	63.40	100.00	100.00	100.00	100.00
	0.80	67.01	83.51	92.34	97.89	100.00	100.00	100.00	100.00

Table 2
Power of HEGY Test: DGP is a SAR(1)

		Pr{Reject Unit Root]				Pr[Reject Seasonal Unit Root]			
T	ρ_4	1%	2.5%	5%	10%	1%	2.5%	5%	10%
48	1.00	0.20	0.55	1.49	3.66	0.01	0.07	0.19	0.79
	0.95	0.27	0.71	1.67	4.18	0.03	0.14	0.36	1.05
	0.90	0.21	0.71	1.72	4.10	0.01	0.09	0.40	1.55
	0.80	0.26	0.96	2.33	5.35	0.03	0.19	0.74	2.48
100	1.00	0.63	1.57	3.36	7.61	0.01	0.10	0.24	0.76
	0.95	0.60	1.49	3.38	7.58	0.01	0.10	0.41	1.55
	0.90	0.99	2.22	4.24	9.32	0.07	0.28	0.67	2.37
	0.80	1.44	3.04	5.88	12.45	0.14	0.81	2.54	7.86
136	1.00	0.46	1.51	3.40	7.55	0.00	0.08	0.20	0.87
	0.95	0.68	1.72	3.86	8.38	0.01	0.18	0.56	2.01
	0.90	0.72	2.21	4.90	10.13	0.04	0.30	0.94	3.65
	0.80	1.86	4.67	9.51	18.93	0.36	2.05	5.56	15.77
200	1.00	0.88	2.09	4.18	9.11	0.01	0.06	0.24	0.82
	0.95	1.05	2.32	4.83	10.54	0.04	0.17	0.59	2.32
	0.90	1.45	3.42	6.73	14.03	0.08	0.67	2.13	6.86
	0.80	4.36	10.02	17.79	32.06	2.62	9.17	20.58	41.69

Table 3

Power of HEGY Test: Change in the level

		Pr[Rejecting Unit Root]				Pr[Rejecting Seasonal Root]			
T	μ_2	1%	2.5%	5%	10%	1%	2.5%	5%	10%
48	0.0	11.03	24.42	39.72	58.75	26.61	46.26	65.63	83.76
	1.0	6.02	13.97	25.19	41.53	23.85	43.75	63.16	81.67
	2.0	0.92	2.74	6.90	15.55	22.10	41.14	62.09	81.60
	5.0	0.00	0.00	0.02	0.09	27.90	49.87	69.41	86.38
	10.0	0.00	0.00	0.00	0.00	33.40	56.74	76.92	91.23
100	0.0	93.05	97.52	99.14	99.82	98.49	99.72	99.95	100.00
	1.0	74.34	86.80	94.19	98.45	97.87	99.67	99.92	99.99
	2.0	20.66	36.91	56.14	77.05	97.06	99.41	99.85	99.96
	5.0	0.00	0.00	0.01	0.39	97.95	99.71	99.98	100.00
	10.0	0.00	0.00	0.00	0.00	99.74	99.98	100.00	100.00
136	0.0	99.70	99.98	99.99	100.00	99.99	100.00	100.00	100.00
	1.0	95.40	99.01	99.71	99.97	99.96	100.00	100.00	100.00
	2.0	48.54	72.86	88.25	96.91	99.98	100.00	100.00	100.00
	5.0	0.00	0.01	0.14	2.12	99.99	100.00	100.00	100.00
	10.0	0.00	0.00	0.00	0.00	100.00	100.00	100.00	100.00
200	0.0	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	1.0	99.99	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	2.0	95.50	99.13	99.84	100.00	100.00	100.00	100.00	100.00
	5.0	0.00	0.24	2.06	15.31	100.00	100.00	100.00	100.00
	10.0	0.00	0.00	0.00	0.00	100.00	100.00	100.00	100.00

Table 4a: Power of HEGY Test: Change in the seasonal pattern (level unchanged)

		Pr[Rejecting a Unit Root]				Pr[Rejecting a Seasonal Unit Root]			
T	$-\gamma_3 = \gamma_4$	1%	2.5%	5%	10%	1%	2.5%	5%	10%
48	0.0	11.03	24.42	39.72	58.75	26.61	46.26	65.63	83.76
	1.0	6.93	16.48	29.67	48.80	10.58	22.73	38.32	58.79
	2.0	2.44	7.69	16.43	32.93	0.42	1.83	5.31	14.35
	5.0	1.82	6.26	15.36	32.77	0.00	0.00	0.00	0.00
	10.0	16.13	35.49	56.58	77.77	0.00	0.00	0.00	0.00
100	0.0	93.05	97.52	99.14	99.82	98.49	99.72	99.95	100.00
	1.0	87.97	95.54	98.55	99.74	83.70	94.64	98.30	99.72
	2.0	74.65	88.64	96.24	99.23	19.27	42.82	64.97	85.77
	5.0	64.56	83.38	94.04	98.89	0.00	0.00	0.00	0.01
	10.0	95.18	98.85	99.81	99.98	0.00	0.00	0.00	0.00
136	0.0	99.70	99.98	99.99	100.00	99.99	100.00	100.00	100.00
	1.0	99.25	99.92	100.00	100.00	98.86	99.89	100.00	100.00
	2.0	97.31	99.65	99.96	99.99	55.38	83.62	94.11	99.00
	5.0	94.29	99.20	99.93	100.00	0.00	0.00	0.00	0.14
	10.0	99.80	99.99	100.00	100.00	0.00	0.00	0.00	0.00
200	0.0	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	1.0	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	2.0	100.00	100.00	100.00	100.00	97.58	99.78	99.99	100.00
	5.0	100.00	100.00	100.00	100.00	0.00	0.01	0.17	3.90
	10.0	100.00	100.00	100.00	100.00	0.00	0.00	0.00	0.00

Table 4b: Power of HEGY Test: Change in the seasonal pattern

		Pr[Rejecting a Unit Root]				Pr[Rejecting a Seasonal Unit Root]			
T	γ_4	1%	2.5%	5%	10%	1%	2.5%	5%	10%
48	0.0	11.03	24.42	39.72	58.75	26.61	46.26	65.63	83.76
	1.0	9.51	19.97	34.14	53.60	19.21	36.97	55.59	75.07
	2.0	4.83	12.03	22.65	39.85	6.98	17.24	32.15	53.71
	5.0	0.17	1.01	3.38	9.42	0.01	0.04	0.13	1.30
	10.0	0.00	0.04	0.17	0.68	0.00	0.00	0.00	0.00
100	0.0	93.05	97.52	99.14	99.82	98.49	99.72	99.95	100.00
	1.0	90.16	96.36	98.89	99.86	95.42	98.97	99.88	99.98
	2.0	78.37	90.14	96.65	99.23	80.50	93.99	98.22	99.62
	5.0	18.73	36.16	57.12	80.46	0.83	5.60	17.56	45.71
	10.0	0.10	0.88	3.10	12.96	0.00	0.00	0.00	0.00
136	0.0	99.70	99.98	99.99	100.00	99.99	100.00	100.00	100.00
	1.0	99.40	99.94	100.00	100.00	99.88	100.00	100.00	100.00
	2.0	97.57	99.68	99.94	100.00	98.45	99.90	99.98	100.00
	5.0	51.08	77.84	92.14	98.32	7.88	31.58	57.72	86.30
	10.0	0.44	3.69	14.76	41.50	0.00	0.00	0.00	0.00
200	0.0	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	1.0	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	2.0	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	5.0	98.15	99.77	99.96	99.99	61.51	90.09	98.38	99.95
	10.0	12.82	39.33	68.75	93.60	0.00	0.00	0.01	0.84

Table 5

Power of HEGY Test: Change in the growth rate

		Pr[Rejecting Unit Root]				Pr[Rejecting Seasonal Root]			
T	β_2	1%	2.5%	5%	10%	1%	2.5%	5%	10%
48	1.00	11.03	24.42	39.72	58.75	26.61	46.26	65.63	83.76
	0.99	11.74	24.10	39.38	58.12	26.42	45.82	65.01	82.76
	0.975	9.99	21.66	36.21	54.29	24.85	45.32	64.80	83.23
	0.95	7.81	16.94	29.14	45.87	24.04	43.40	62.97	82.15
	0.90	2.00	5.13	11.15	20.86	18.79	38.07	57.41	77.53
100	1.00	93.05	97.52	99.14	99.82	98.49	99.72	99.95	100.00
	0.99	91.49	96.74	99.03	99.86	98.21	99.72	99.92	99.99
	0.975	81.11	90.99	96.08	98.94	97.90	99.59	99.87	99.97
	0.95	39.77	57.16	73.25	87.62	96.55	99.40	99.86	100.00
	0.90	0.38	1.50	3.94	11.75	95.10	99.33	99.89	99.98
136	1.00	99.70	99.98	99.99	100.00	99.99	100.00	100.00	100.00
	0.99	99.26	99.92	100.00	100.00	99.97	100.00	100.00	100.00
	0.975	94.02	98.47	99.58	99.87	99.99	100.00	100.00	100.00
	0.95	36.36	60.06	78.17	91.57	99.97	100.00	100.00	100.00
	0.90	0.01	0.07	0.36	2.51	99.95	99.97	100.00	100.00
200	1.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	0.99	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	0.975	99.39	99.92	100.00	100.00	100.00	100.00	100.00	100.00
	0.95	23.06	48.71	70.92	90.51	100.00	100.00	100.00	100.00
	0.90	0.00	0.00	0.00	0.01	100.00	100.00	100.00	100.00

Table 6
Critical Values for HEGY Test (T=1000)

Auxiliary Regression	Test	Fractiles							
		1%	2.5%	5%	10%	90%	95%	97.5%	99%
	π_1	-3.46	-3.13	-2.87	-2.57	-0.45	-0.11	0.18	0.58
Intercept	π_2	-2.57	-2.27	-1.98	-1.61	0.89	1.29	1.62	2.01
No Seas. Dum.	π_3	-2.63	-2.21	-1.90	-1.55	1.00	1.36	1.69	2.08
No Trend	π_4	-2.34	-1.94	-1.63	-1.27	1.27	1.61	1.94	2.32
	F	0.01	0.03	0.05	0.12	2.38	3.10	3.75	4.76
	π_1	-3.47	-3.13	-2.87	-2.57	-0.45	-0.12	0.18	0.57
Intercept	π_2	-3.45	-3.15	-2.87	-2.56	-0.44	-0.05	0.26	0.60
Seas. Dum.	π_3	-3.92	-3.62	-3.36	-3.07	-0.84	-0.51	-0.23	0.12
No Trend	π_4	-2.74	-2.28	-1.94	-1.51	1.51	1.92	2.26	2.67
	F	0.12	0.26	0.46	0.78	5.65	6.72	7.67	8.89
	π_1	-3.95	-3.63	-3.40	-3.12	-1.25	-0.96	-0.64	-0.34
Intercept	π_2	-2.58	-2.27	-1.98	-1.61	0.89	1.29	1.62	2.01
No Seas. Dum.	π_3	-2.63	-2.21	-1.90	-1.56	1.00	1.36	1.69	2.08
Trend	π_4	-2.34	-1.94	-1.63	-1.27	1.26	1.60	1.93	2.31
	F	0.01	0.03	0.05	0.12	2.38	3.10	3.76	4.76
	π_1	-3.95	-3.63	-3.41	-3.12	-1.25	-0.96	-0.65	-0.35
Intercept	π_2	-3.45	-3.14	-2.87	-2.56	-0.44	-0.06	0.26	0.59
Seas. Dum.	π_3	-3.93	-3.62	-3.36	-3.07	-0.84	-0.52	-0.23	0.12
Trend	π_4	-2.74	-2.28	-1.94	-1.51	1.51	1.92	2.27	2.67
	F	0.12	0.26	0.46	0.78	5.65	6.70	7.67	8.88

Table 7: Critical Values for HEGY Test: Change in level (no seasonals)

λ	Test	1%	2.5%	5%	10%	90%	95%	97.5%	99%
0.1		-4.11	-3.79	-3.51	-3.22	-1.30	-1.00	-0.71	-0.39
0.2		-4.21	-3.94	-3.68	-3.41	-1.50	-1.23	-0.96	-0.64
0.3		-4.20	-3.89	-3.66	-3.40	-1.72	-1.45	-1.19	-0.90
0.4		-4.25	-3.93	-3.65	-3.37	-1.67	-1.44	-1.23	-0.96
0.5	π_1	-4.19	-3.87	-3.60	-3.32	-1.43	-1.16	-0.92	-0.56
0.6		-4.22	-3.88	-3.64	-3.34	-1.28	-0.94	-0.63	-0.25
0.7		-4.21	-3.89	-3.63	-3.36	-1.50	-1.20	-0.91	-0.59
0.8		-4.29	-3.98	-3.69	-3.41	-1.76	-1.57	-1.38	-1.16
0.9		-4.21	-3.94	-3.69	-3.40	-1.79	-1.63	-1.50	-1.38
0.1		-2.66	-2.31	-2.01	-1.65	0.85	1.24	1.58	1.98
0.2		-2.63	-2.26	-1.97	-1.66	0.80	1.18	1.55	1.90
0.3		-2.61	-2.28	-1.99	-1.65	0.84	1.21	1.55	1.99
0.4		-2.66	-2.33	-2.01	-1.68	0.85	1.20	1.52	1.94
0.5	π_2	-2.62	-2.28	-1.99	-1.64	0.86	1.27	1.61	1.97
0.6		-2.66	-2.28	-2.01	-1.67	0.86	1.29	1.64	1.97
0.7		-2.62	-2.28	-2.00	-1.67	0.84	1.23	1.57	1.92
0.8		-2.66	-2.31	-2.01	-1.67	0.84	1.23	1.52	1.95
0.9		-2.55	-2.25	-1.99	-1.65	0.84	1.21	1.58	1.98
0.1		-2.68	-2.30	-1.95	-1.60	0.96	1.32	1.63	2.01
0.2		-2.65	-2.28	-1.99	-1.63	0.92	1.32	1.63	1.97
0.3		-2.67	-2.29	-2.00	-1.65	0.95	1.34	1.66	2.04
0.4		-2.60	-2.25	-1.98	-1.64	0.94	1.29	1.64	2.01
0.5	π_3	-2.58	-2.26	-1.96	-1.62	0.96	1.34	1.69	2.06
0.6		-2.62	-2.24	-1.95	-1.63	0.96	1.34	1.67	2.02
0.7		-2.67	-2.28	-2.01	-1.65	0.94	1.32	1.62	1.99
0.8		-2.69	-2.33	-2.04	-1.68	0.96	1.33	1.63	1.99
0.9		-2.56	-2.24	-1.93	-1.60	0.94	1.31	1.66	1.96
0.1		-2.33	-1.94	-1.66	-1.30	1.19	1.54	1.87	2.21
0.2		-2.32	-1.98	-1.67	-1.31	1.16	1.50	1.84	2.19
0.3		-2.36	-1.98	-1.66	-1.28	1.19	1.54	1.85	2.24
0.4		-2.34	-1.99	-1.67	-1.29	1.18	1.53	1.83	2.18
0.5	π_4	-2.34	-2.00	-1.68	-1.30	1.20	1.55	1.86	2.27
0.6		-2.35	-1.95	-1.66	-1.32	1.20	1.56	1.88	2.26
0.7		-2.32	-1.99	-1.68	-1.33	1.19	1.51	1.80	2.14
0.8		-2.38	-1.94	-1.64	-1.30	1.19	1.55	1.86	2.21
0.9		-2.36	-1.98	-1.66	-1.27	1.21	1.60	1.91	2.24
0.1		0.01	0.03	0.06	0.12	2.36	3.10	3.78	4.82
0.2		0.01	0.03	0.06	0.11	2.37	3.05	3.75	4.84
0.3		0.01	0.03	0.06	0.11	2.43	3.13	3.88	4.82
0.4		0.01	0.03	0.06	0.11	2.40	3.06	3.71	4.85
0.5	F-test	0.01	0.03	0.06	0.12	2.41	3.17	3.82	4.67
0.6		0.01	0.03	0.06	0.11	2.36	3.07	3.82	4.77
0.7		0.01	0.03	0.06	0.12	2.40	3.13	3.86	4.76
0.8		0.01	0.03	0.05	0.11	2.45	3.18	3.90	4.98
0.9		0.01	0.03	0.06	0.12	2.34	3.05	3.72	4.64

Table 8: Critical Values for HEGY Test: Change in seasonal pattern

λ	Test	1%	2.5%	5%	10%	90%	95%	97.5%	99%
0.1		-4.73	-4.27	-3.94	-3.59	-1.51	-1.22	-0.94	-0.63
0.2		-4.80	-4.41	-4.10	-3.77	-1.72	-1.43	-1.18	-0.90
0.3		-4.72	-4.36	-4.07	-3.75	-1.91	-1.64	-1.41	-1.09
0.4		-4.75	-4.36	-4.06	-3.72	-1.84	-1.64	-1.43	-1.16
0.5	π_1	-4.66	-4.31	-4.02	-3.69	-1.63	-1.34	-1.10	-0.79
0.6		-4.73	-4.36	-4.05	-3.71	-1.51	-1.16	-0.84	-0.47
0.7		-4.68	-4.35	-4.05	-3.72	-1.72	-1.41	-1.12	-0.77
0.8		-4.74	-4.42	-4.09	-3.76	-1.94	-1.74	-1.57	-1.36
0.9		-4.74	-4.36	-4.08	-3.75	-1.95	-1.77	-1.62	-1.48
0.1		-3.83	-3.43	-3.12	-2.77	-0.48	-0.15	0.14	0.46
0.2		-4.03	-3.68	-3.35	-3.00	-0.67	-0.34	-0.07	0.24
0.3		-4.11	-3.76	-3.45	-3.12	-0.85	-0.51	-0.21	0.17
0.4		-4.12	-3.78	-3.47	-3.17	-1.06	-0.74	-0.46	-0.10
0.5	π_2	-4.12	-3.80	-3.50	-3.19	-1.25	-0.97	-0.68	-0.36
0.6		-4.10	-3.78	-3.50	-3.21	-1.41	-1.18	-0.97	-0.66
0.7		-4.16	-3.77	-3.51	-3.20	-1.48	-1.30	-1.15	-0.97
0.8		-4.12	-3.75	-3.48	-3.17	-1.46	-1.31	-1.19	-1.07
0.9		-3.99	-3.62	-3.33	-3.04	-1.40	-1.26	-1.16	-1.04
0.1		-4.17	-3.83	-3.53	-3.15	-0.88	-0.57	-0.34	-0.02
0.2		-4.49	-4.17	-3.87	-3.50	-1.13	-0.83	-0.53	-0.21
0.3		-4.72	-4.35	-4.08	-3.75	-1.42	-1.08	-0.80	-0.45
0.4		-4.77	-4.43	-4.17	-3.88	-1.70	-1.36	-1.10	-0.77
0.5	π_3	-4.79	-4.47	-4.20	-3.89	-2.00	-1.69	-1.46	-1.14
0.6		-4.75	-4.47	-4.22	-3.94	-2.20	-1.97	-1.76	-1.48
0.7		-4.82	-4.53	-4.24	-3.94	-2.28	-2.09	-1.93	-1.75
0.8		-4.74	-4.40	-4.12	-3.85	-2.21	-2.05	-1.93	-1.81
0.9		-4.59	-4.25	-3.98	-3.67	-2.10	-1.95	-1.83	-1.71
0.1		-2.61	-2.20	-1.85	-1.44	1.58	1.97	2.33	2.73
0.2		-2.75	-2.31	-1.95	-1.54	1.65	2.04	2.40	2.78
0.3		-2.82	-2.36	-1.96	-1.52	1.70	2.14	2.54	3.00
0.4		-2.90	-2.45	-2.05	-1.55	1.73	2.21	2.62	3.02
0.5	π_4	-2.93	-2.43	-2.03	-1.55	1.75	2.18	2.55	3.03
0.6		-2.82	-2.39	-2.03	-1.56	1.73	2.22	2.65	3.06
0.7		-2.86	-2.39	-2.01	-1.57	1.63	2.08	2.46	2.94
0.8		-2.77	-2.29	-1.92	-1.49	1.63	2.07	2.43	2.85
0.9		-2.64	-2.24	-1.86	-1.44	1.58	2.01	2.37	2.82
0.1		0.15	0.27	0.45	0.76	5.97	7.27	8.52	10.20
0.2		0.23	0.44	0.68	1.08	7.26	8.66	10.00	11.69
0.3		0.37	0.66	0.99	1.46	8.17	9.48	10.79	12.61
0.4		0.62	0.98	1.42	1.99	8.69	10.08	11.42	13.04
0.5	F-test	1.00	1.44	1.92	2.52	8.80	10.19	11.47	13.29
0.6		1.44	1.92	2.35	2.90	9.00	10.24	11.49	13.24
0.7		1.82	2.23	2.58	3.05	8.94	10.26	11.56	13.14
0.8		1.91	2.18	2.47	2.90	8.38	9.65	10.96	12.49
0.9		1.72	1.97	2.27	2.64	7.76	8.94	10.15	11.65

Figure 1

Power of HEGY Unit Root Test as a Function of Break Period

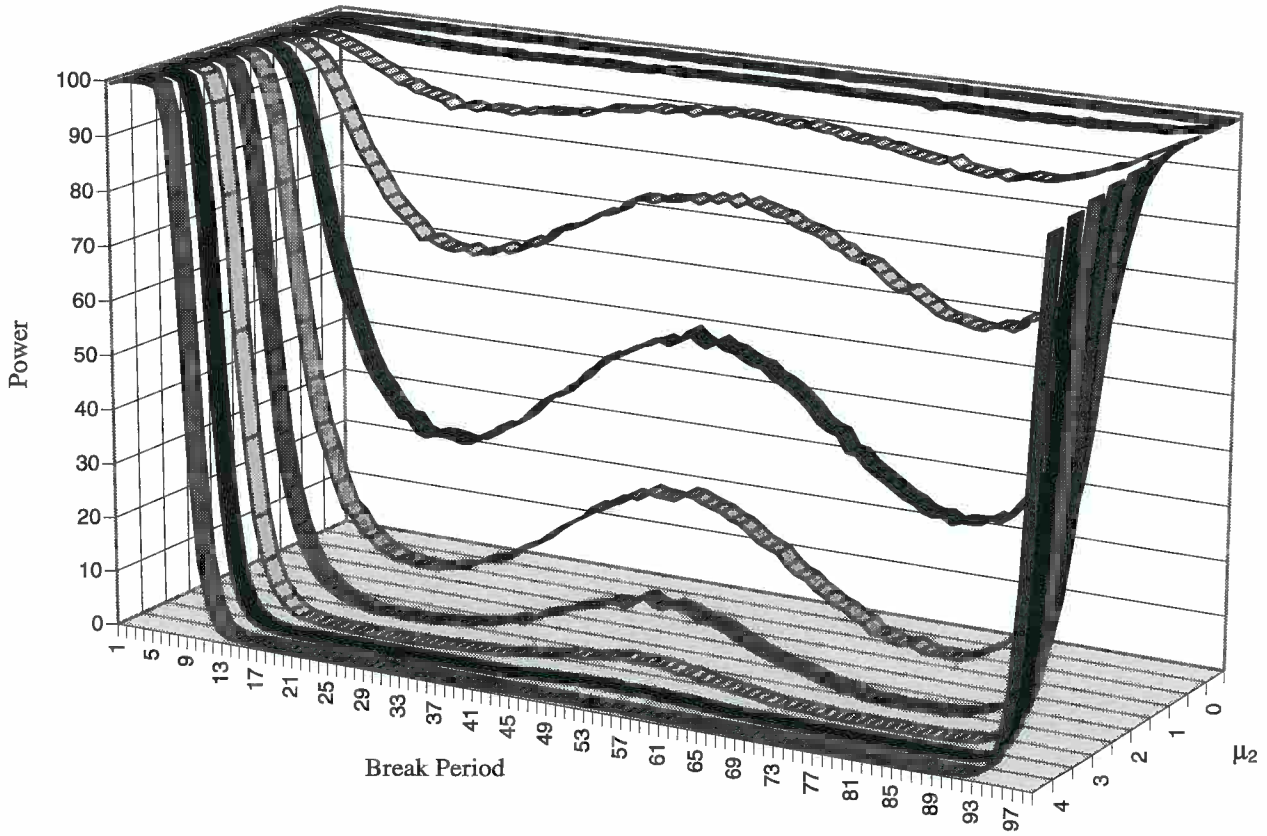


Figure 2

Power of Seasonal Unit Root Test as a Function of Break Period

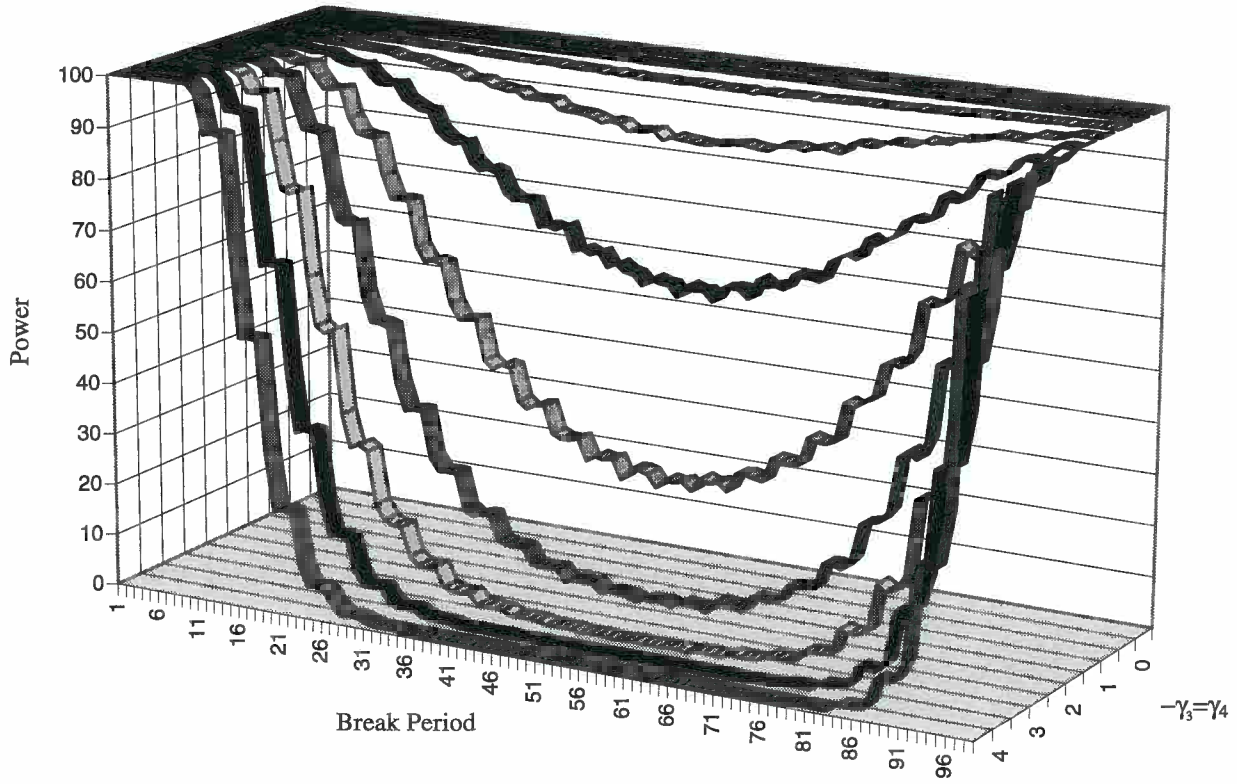


Figure 3

Log of Colombian Money Supply

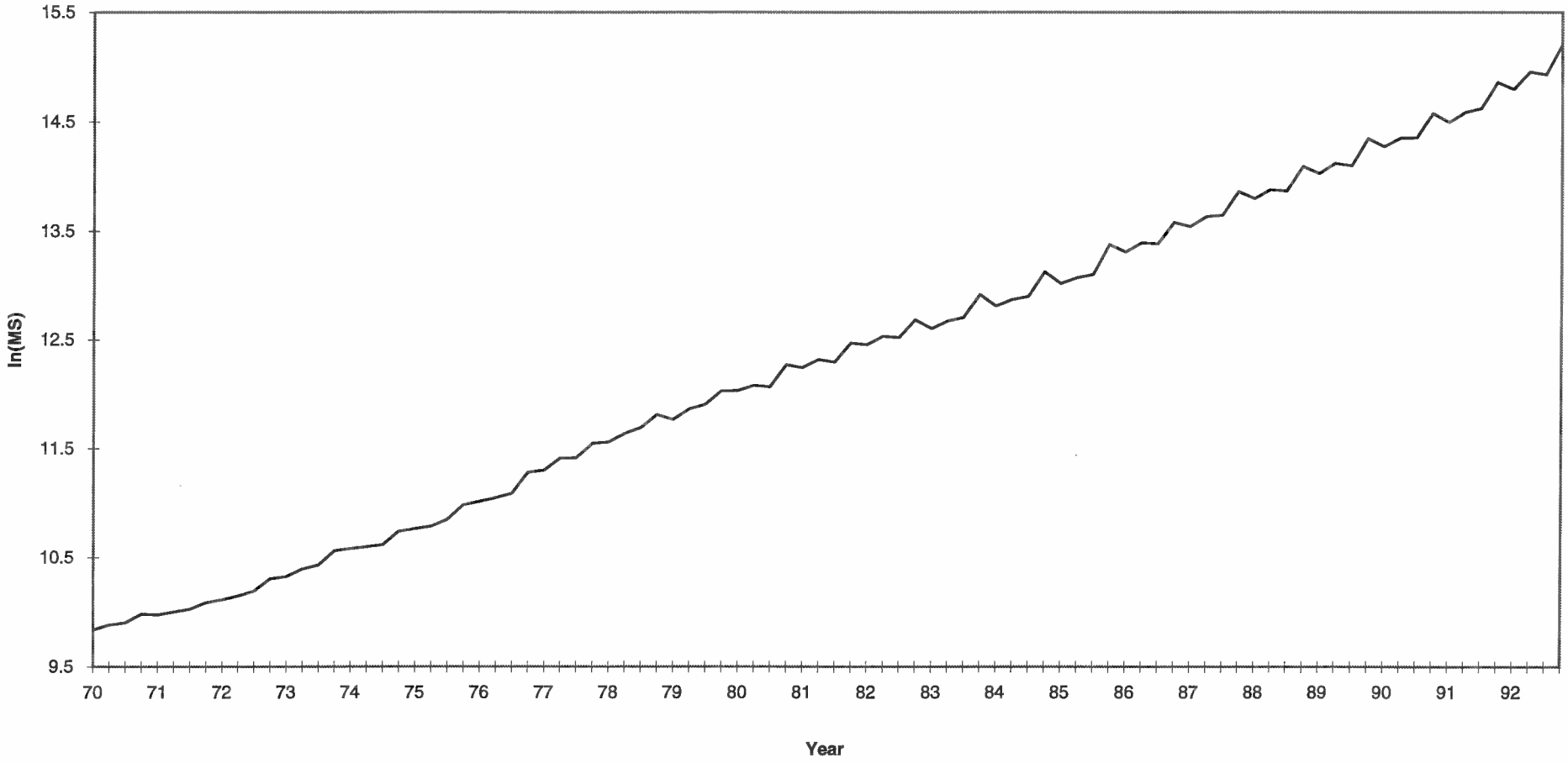


Figure 4

Growth in Colombian Money Supply

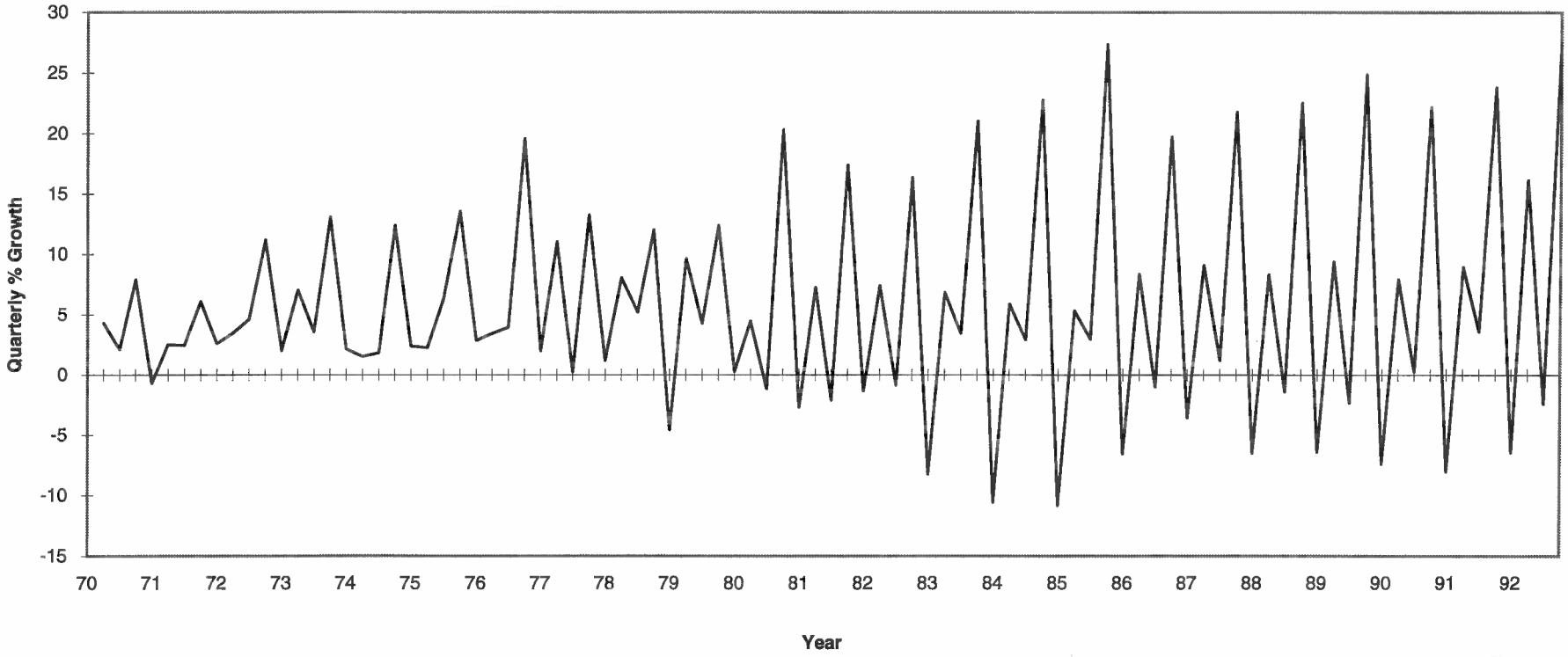


Figure 5

Log of Colombian Real GDP

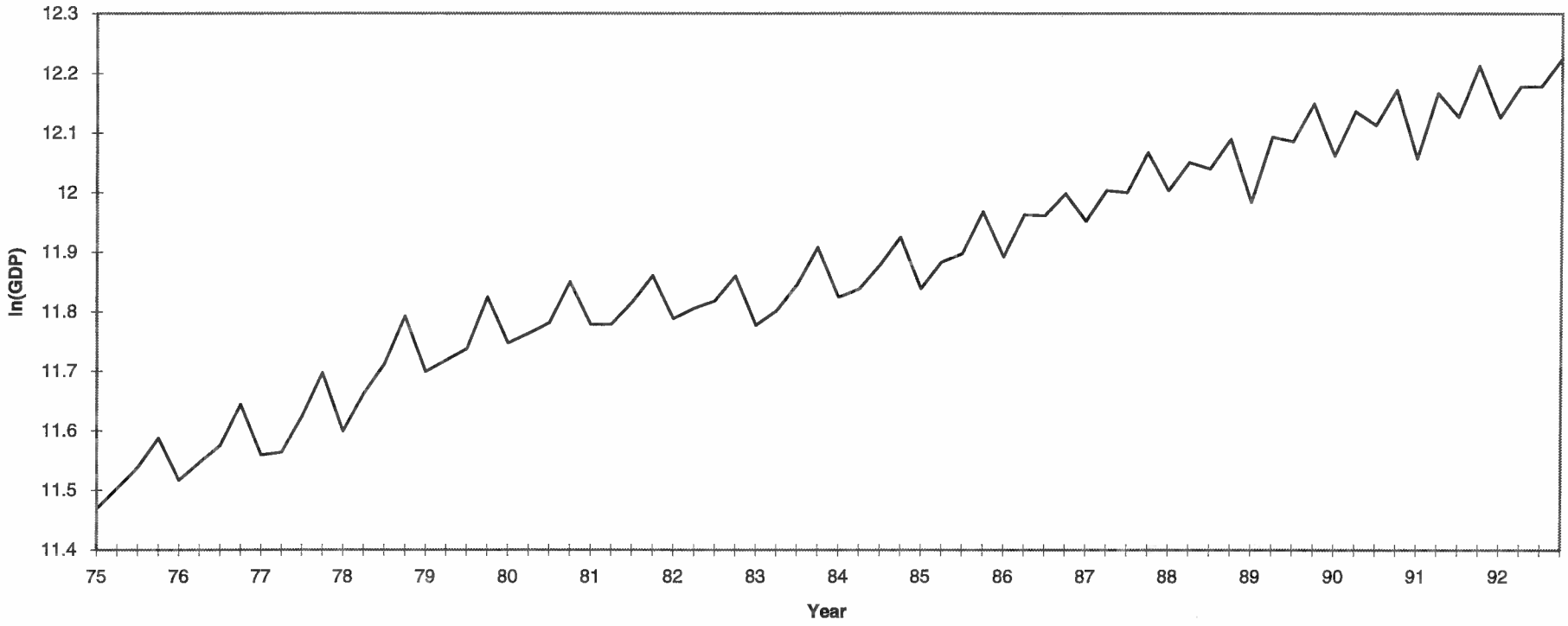


Figure 6

Growth in Colombian Real GDP

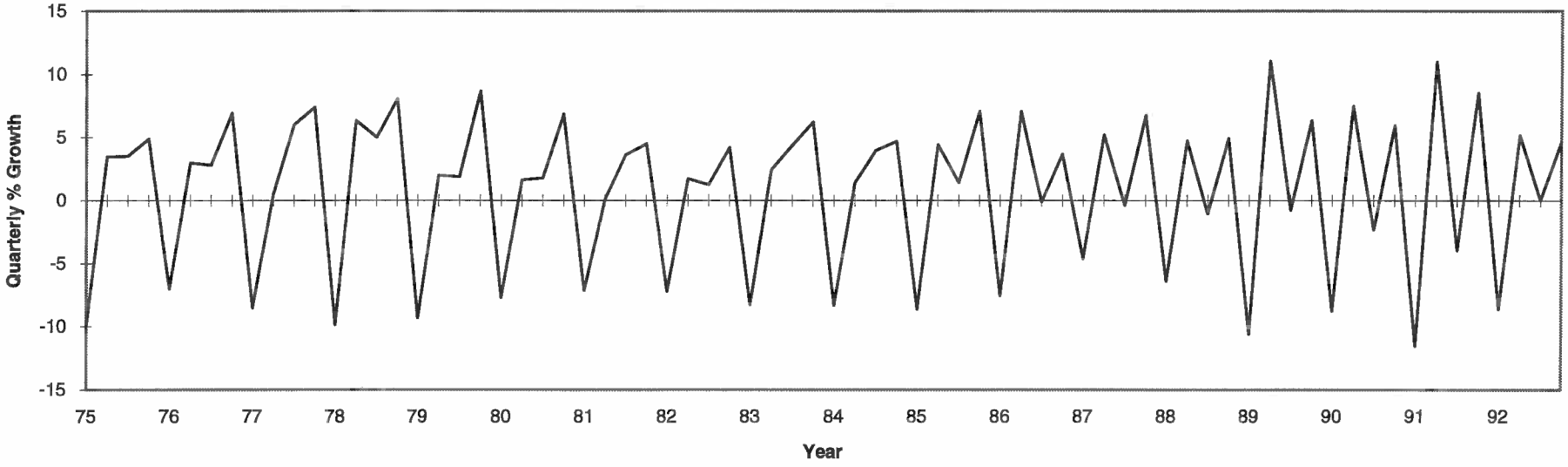


Figure 7

Log of Car Expenditure

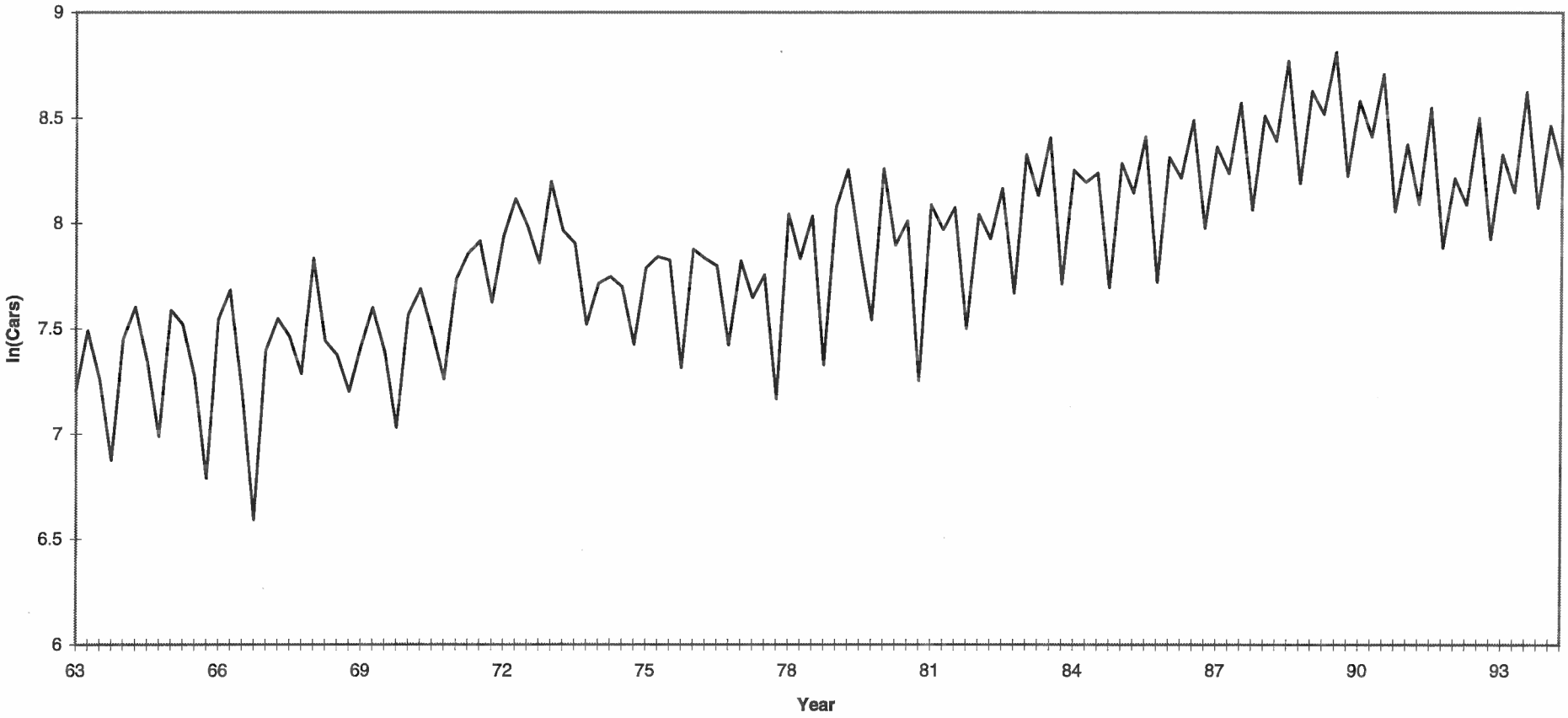
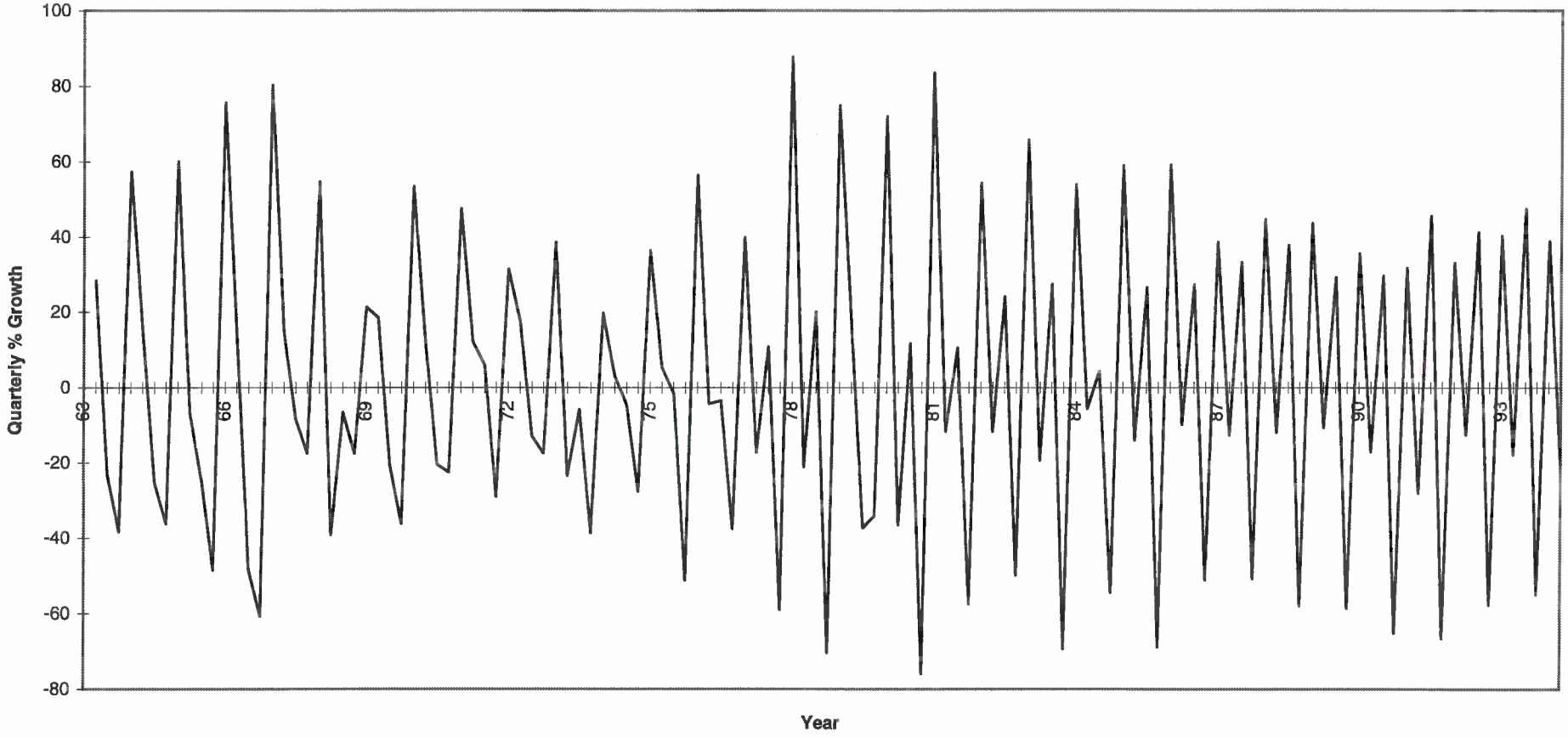


Figure 8

Growth in Car Expenditure



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