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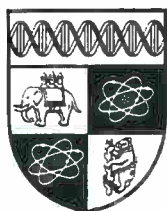
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**RISK, INSURANCE AND THE DEMAND FOR IRREPLACEABLE
COMMODITIES, THE CASE OF CHILDREN**

Clive Fraser

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RISK, INSURANCE AND THE DEMAND FOR IRREPLACEABLE
COMMODITIES, THE CASE OF CHILDREN

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RISK, INSURANCE AND THE DEMAND FOR IRREPLACEABLE COMMODITIES. THE CASE OF CHILDREN

By Clive D. Fraser, University of Warwick, February 1995*

ABSTRACT: Using a Beckerian model in which parents derive utility from per capita household consumption and the number of children, we examine how child mortality risk and insurance against this affect the demand for children. We show that child mortality risk increases the demand for children and, contrary to claims in the literature, optimal fair insurance against child mortality risk will not result in parents equalising marginal utilities of consumption across states of the world and can result in them opting for payouts in states where children die. This calls into question some of the criticisms levied against the operations of tort as an implicit insurance system.

Keywords: child mortality risk, insurance, tort

JEL Classifications : D1, D18, D81, K13

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ERRATA

1. On page 10, (15) should read:

$$(15) \quad \pi : \quad (1-p) \left[(1/(\bar{n}+1)) \tilde{U}_x^1 - (1/(\bar{n}+2)) \tilde{U}_x^0 \right] = 0$$

2. On page 12, lines 2 and 7, the expressions should be $\left. \frac{\partial CEU(\bar{n}, \pi)}{\partial \pi} \right|_{\pi=0, \bar{n}} > 0$.

3. On page 12, line 9, the expression should be

$$\left(\frac{1}{\bar{n}+1} \right) \delta \left(\frac{M+\bar{n}+1}{\bar{n}+1} \right)^{\delta-1} (\bar{n}+1)^\beta - \left(\frac{1}{\bar{n}+2} \right) \alpha \left(\frac{M+\bar{n}+2}{\bar{n}+2} \right)^{\alpha-1} (\bar{n}+2)^\beta > 0$$

4. On page 12, line 14, “(5)” should be “(18)”.

RISK, INSURANCE AND THE DEMAND FOR IRREPLACEABLE COMMODITIES.

THE CASE OF CHILDREN*. By Clive D. Fraser, Warwick University, February 1995.

Cook and Graham's (1977) seminal analysis of the demand for insurance and self-protection with irreplaceable commodities showed that consumers would not purchase actuarially fair insurance against the loss of irreplaceable commodities if, other things equal, this loss reduced the marginal utility of income as compared with the state without a loss. The reason for this finding is that, given the opportunity to purchase fair insurance, consumers would equate the marginal utility of income across states of the world. With the loss reducing the marginal utility as compared with the state without a loss, this equality across states can be achieved only if income in the loss state is less than in the no loss state. One example which they used repeatedly is that of the possible loss of loved ones such as a spouse or a child. Their finding has become part of the received wisdom in insurance economics and has influenced considerably the economic analysis of liability law. The consensus now is that, because consumers will not purchase insurance against possible irreplaceable losses which reduce the marginal utility of money, it is inefficient for them to receive compensation for such losses, usually referred to as "pain and suffering" losses. But, in practice, such compensation is widespread under tort and is the principal source of the so-called "liability crisis." [See, e.g., Cooter (1991), Priest (1991) and Shapiro (1991)].

This paper will consider the case of children as irreplaceable commodities. We show that Cook and Graham's conclusions only hold unambiguously if consumers' utility functions do not allow for a trade-off between the utility derived from children and the household's material standard of living. We construct a model along Beckerian lines in which parents derive utility from children as a consumption activity as well as the household's material standard of living, for which we use household per capita income as a surrogate. There is thus a trade-off between the number of children and the standard of living. We consider the impact of child mortality risk on the demand for children in the absence of insurance. We then show that parents with the opportunity to purchase fair insurance against the loss of a child would not equate the marginal utility of *consumption* across states of the world, although they might equate the household's marginal utility of *income* across states. We construct plausible utility functions for which

positive purchase of insurance against child mortality risk would be optimal. This, incidentally, accords with empirical evidence of widespread insurance purchase on children's lives in the past.

A model allowing explicitly for the utility from children also enables us to explore other interesting related issues. E.g., the relationship between children's mortality risk and the demand for children, though often alluded to, does not appear to have been analysed in an explicit expected utility maximising framework. A perhaps more important omission from the literature is examination of how the availability of insurance against children's mortality affects the demand for children, particularly given the suggestion that the tort system operates like an implicit insurance system in relation to liability for death or injury to children, especially in the USA.

We first considers the demand for children as a deterministic utility maximisation problem for a couple with a fixed income. The latter assumption is maintained throughout the paper. Next, we consider the demand for children when, for simplicity, it is assumed that there is a given risk of one and only one child dying, however many children the partners have, and there is no child life insurance. We show that introducing this mortality risk results in an increased demand for children. The possibility of purchasing actuarially fair insurance against child mortality risk is then analysed. As well as showing that parents might well purchase such insurance, we show that its availability can increase or decrease the demand for children.

I. The Model without Children's Mortality Risk

Suppose the two parents in a household each have utility function $U(x, n+2)$, where x = per capita consumption of the single composite market good and n is the number of children. We suppose U is strictly concave in both x and n and take its partial derivatives to satisfy

$$(A.1) \quad U_x > 0, U_n \begin{cases} > \\ < \end{cases} 0 \text{ as } n \begin{cases} < \\ > \end{cases} \bar{n}(x).$$

Although, on average, couples in the UK famously have approximately "2.4 children," children naturally come in discrete lumps. However, for analytical convenience we will take their number to be a continuous variable. The behaviour of the second partial in (A.1) then expresses the idea that there is an upper limit to the number of children beyond which further children are

unwanted. Typically, this upper limit will depend on the standard of living which can be enjoyed. Treating the number of children as continuous is not entirely unreasonable if the findings of our model are to be regarded as representing “average” behaviour and if what parents can do is to affect the *probability* of having additional children.

We will assume that the household’s income is fixed, irrespective of the number of children. Thus we will not consider explicitly household allocation of time. Hence the model is most applicable to a world without social security benefits for children - ‘child allowance’ - and in which the parents supply a fixed amount and quality of market labour, irrespective of the number of children they have. Letting M denote this fixed income, per capita consumption for a couple with n children is then given by

$$(1) \quad x = M / (n + 2)$$

The assumption of equal division of the household’s income between family members seems a plausible first approximation once we take into account child-rearing costs such as for child-minding, private education at all levels and transport to and from school. Such costs can more than outweigh the higher costs of parents’ food, recreation and suchlike which is usually assumed in “equivalence scales” (cf. Nelson, 1993).

Suppose for now that parents perceive no risks to either their own or their children’s life and limb. If we abstract from issues relating to the timing of children in order to concentrate simply on their desired number, we can take each parent’s problem to be the maximisation of its utility¹. I.e., using (A.1) and (1), the parents’ deterministic problem is

$$(2) \quad \underset{n}{Max.} U[M / (n + 2), n + 2]$$

To guarantee a solution to (2), we will assume:

$$(A.2) \quad U[M / (n + 2), n + 2] \text{ is strictly concave in } n.$$

Letting $*$ indicate optimal values and subscripts partial derivatives w.r.t. the indicated variables, (2) yields the following necessary and sufficient condition for the optimal choice of n :

$$(3) \quad \begin{cases} \frac{\partial U[M/(n^*+2), n^*+2]}{\partial n} = U_n[M/(n^*+2), n^*+2] - (M/(n^*+2)^2)U_x[M/(n^*+2), n^*+2] \leq 0 \\ n^* \frac{\partial U[M/(n^*+2), n^*+2]}{\partial n} = 0 (\text{complementary slackness}) \end{cases}$$

We will focus on cases where parents choose optimally to have a positive number of children. Thus, neglecting the integer problem, $n > 0$ satisfies

$$(4) \quad \frac{\partial U[M/(n^*+2), n^*+2]}{\partial n} = U_n[M/(n^*+2), n^*+2] - (M/(n^*+2)^2)U_x[M/(n^*+2), n^*+2] = 0$$

Equation (4) implicitly defines n^* as a function of M , $n^*(M)$. Thus, by standard comparative static techniques, it is easy to show that (suppressing inessential functional arguments)

$$(5) \quad n'*(M) = [U_x^* + (M/(n^*+2))U_{xx}^* - (n^*+2)U_{nx}^*] / D_1 \equiv N_1 / D_1$$

$$D_1 \equiv (M/(n^*+2))^2 U_{xx}^* - 2MU_{xn}^* + 2(n^*+2)U_n^* + (n^*+2)U_{nn}^*$$

As $D_1 < 0$ by the second-order condition for the maximisation, the satisfaction of which is ensured by the strict concavity of U the sign of $n'*(M)$ is opposite to N_1 's. Thus, even if U_{nx} is zero everywhere (as with an additively separable $U(x, n)$ or is single-signed, we cannot sign $n'*(M)$ unambiguously *a priori* without further restrictions on U . One might argue for presuming that $U_{nx} \geq 0$, i.e. Edgeworth-Pareto complementarity between private good consumption and children, on the grounds that being able to enjoy higher per capita consumption will enhance the pleasure to be obtained from the marginal child (perhaps because it allows for greater investment to augment the quality of the children.). Then, if the marginal utility of

consumption falls sufficiently rapidly as consumption rises (more precisely, if $-x^* U_{xx}^* / U_x^* \geq 1$) then $n^*(M) > 0$: children are superior goods.

II. The Demand for Children with Mortality Risk and Without Insurance

Suppose now each parent in the household again has an identical utility function, $U^i(x^i, \tilde{n}^i)$, where x^i is the household's per capita consumption of the composite private good in state ii and \tilde{n}^i is the household's size in that state. Thus, $\tilde{n}^i = n^i + 2$, where n^i is the number of children alive in state i , and $x^i = M / \tilde{n}^i$ in the absence of insurance purchase. Each U^i is again assumed to be strictly concave in its arguments, to be increasing in x . and, other things equal, to be increasing in \tilde{n}^i up to some level of \tilde{n}^i .

Let the possible states of the world be indexed $i = 0, 1, 2, \dots, n$, where n is the number of children the couple have prior to their exposure to any risk. Thus, the possible states are indexed by the number of children who die, with state "0" being the most favourable. An obvious reason why utility might be state-dependent as above is that the degree of bereavement affects the utility parents derive from private good consumption and their surviving children.

Suppose for now that there is a given probability, p , of one and only one child dying, irrespective of how many children a couple have². Then the probability of no child dying in the family is $(1 - p)$. The impact of this child mortality risk on the demand for children will be shown to depend on how bereavement affects the parents' utility function. It is easy to see that even if the state-dependent utility functions are identical, if $U_{nx} > 0$ then

$U_x^0(x, n+1) = U_x(x, n+1) > U_x(x, n) = U_x^1(x, n)$. This is the relationship between the state-dependent marginal utilities of consumption (equivalently, income in the single variable models in the literature - see Frech III and Calfee and Rubin) which is usually presumed and for which Viscusi and Evans (1990) provide empirical support.

Irrespective of whether or not the state-dependent utility functions are identical, we will make the following assumption:

(A.3) The 0-contingent utility function is the same as in the case without child mortality risk.

We now assume parents choose the optimal number of children (denoted n^{**}) to maximise their conditional expected utility (CEU)³. I.e., they solve the following problem:

$$(6) \quad \underset{n}{Max.} \left\{ CEU(n) \equiv (1-p)U^0[M/(n+2), n+2] + pU^1[M/(n+1), n+1] \right\}$$

An interior solution to this is characterised by the first-order condition:

$$(7) \quad \begin{aligned} & [M/(n^{**}+2)^2](1-p)U_x^0[M/(n^{**}+2), n^{**}+2] \\ & + [M/(n^{**}+1)^2]pU_x^1[M/(n^{**}+1), n^{**}+1] \\ & = (1-p)U_n^0[M/(n^{**}+2), n^{**}+2] + pU_n^1[M/(n^{**}+1), n^{**}+1] \end{aligned}$$

Equivalently, in a form also useful subsequently,

$$(7') \quad \begin{aligned} & (1-p)[[M/(n^{**}+2)^2]U_x^0[M/(n^{**}+2), n^{**}+2] - U_n^0[M/(n^{**}+2), n^{**}+2]] \\ & = -p[[M/(n^{**}+1)^2]U_x^1[M/(n^{**}+1), n^{**}+1] - U_n^1[M/(n^{**}+1), n^{**}+1]] \end{aligned}$$

Utilising (7'), we can examine the impact of child mortality risk on the demand for children. There will be two cases to consider: when utility is not state-dependent and when it is. In the latter case, we will make the following assumption:

$$(A.4) \quad U^i(x^i, \tilde{n}^i) = U(x^i, \tilde{n}^i, i), \quad U_{nx} \geq 0 \text{ and } U_{ix} \leq 0$$

If (A.4) is satisfied, the index i indicating the degree of bereavement parents feel is a parameter in the utility function and bereavement does not increase the marginal utility of consumption, other things equal. Our most clear-cut finding is then summarised in the following proposition.

Proposition 1. (i) If the utility functions are state-independent, increased child mortality risk will lead to an increased demand for children (i.e., $n^{**} > n^*$). (ii) If utility is state-dependent and satisfies (A.4), increased child mortality risk will increase the demand for children

if: (a) the utility function exhibits relative risk aversion greater than or equal to unity with respect to both n and x ; (b) other things equal, bereavement increases the total impact on utility from additional children (i.e., $\frac{\partial}{\partial i} \left[\frac{\partial U}{\partial n} \left(\frac{M}{n}, n, i \right) \right] > 0$).

Proof. For notational brevity, henceforth let $U^{0*} \equiv U[M / (n^* + 2), n^* + 2, 0]$ and $U^{1*} \equiv U[M / (n^* + 1), n^* + 1, 1]$, with their partials defined conformably. Define U^{0**} and U^{1**} and their partials analogously. (i) Clearly, with *state-independent* utility functions, $U^{0*} = U^{1*}[M / (n^* + 2), n^* + 2] = U^*[M / (n^* + 2), n^* + 2]$ say, and so on. Then, (7') states

$$(8) \quad (1-p)dU[M / (n^{**} + 2), n^{**} + 2] / dn = -pdU[M / (n^{**} + 1), n^{**} + 1] / dn$$

Thus, dU^{0**} / dn and dU^{1**} / dn have opposite signs at this optimum unless they are both equal to zero. With state-independent utility functions the latter can be ruled out because

$n^{**} + 1 \neq n^{**} + 2$ and U is strictly concave, by (A.2). Now, in this state-independent case, by (A.2) also, as $dU[M / (n^{**} + 2), n^{**} + 2] / dn$ and $dU[M / (n^{**} + 1), n^{**} + 1] / dn$ have opposite signs, it must be the case that $dU[M / (n^{**} + 2), n^{**} + 2] / dn < 0$ and $dU[M / (n^{**} + 1), n^{**} + 1] / dn > 0$. But, from (A.3) and (4), $dU[M / (n^* + 2), n^* + 2] / dn = 0$.

Thus, by the concavity of U in n , $n^{**} > n^*$. (ii) When utility is state-dependent and (A.4) applies, the optimum now satisfies

$$(9) \quad (1-p)dU[M / (n^{**} + 2), n^{**} + 2, 0] / dn = -pdU[M / (n^{**} + 1), n^{**} + 1, 1] / dn$$

Thus dU^{1**} / dn and dU^{0**} / dn are again of opposite signs. To establish the result, given (A.3), we need $dU^{1**} / dn > 0$. Now, each of dU^{1**} / dn and dU^{0**} / dn are of the form

$$(10) \quad dU^{i**} / dn = \left\{ n^{i**} U_n[x^{i**}, n^{i**}, i] - x^{i**} U_x[x^{i**}, n^{i**}, i] \right\} / n^{i**} \equiv N_1 / n^{i**}$$

The denominator n^{i**} in (10) is decreasing in i (because $n^{0**} = n^{**} + 2 > n^{1**} = n^{**} + 1$).

Thus, provided N_1 in (10) is increasing or constant in i , we will be done. Totally differentiating,

$$dN_1 = dx^{i**} [n^{**} U_{nx}^{i**} - U_x^{i**} - x^{i**} U_{xx}^{i**}] + dn^{i**} [n^{i**} U_{nn}^{i**} + U_n^{i**} - x^{i**} U_{nx}^{i**}] + di [n^{i**} U_{ni}^{i**} - x^{i**} U_{ni}^{i**}]$$

Now, as i increases from 0 to 1, $di = -dn = 1$ and $dx^{i**} = [M / (n^{**} + 1) - M / (n^{**} + 2)] = M / (n^{**} + 1)(n^{**} + 2)$. Substituting these into the expression for dN_1 evaluated at $i = 0$ yields

$$dN_1 = [x^{0**} + x^{1**}] U_{nx}^{0**} - \{M[x^{0**} U_{xx}^{0**} + U_x^{0**}] / (n^{**} + 2)(n^{**} + 1)\} \\ - [(n^{**} + 2) U_{nn}^{0**} + U_n^{0**}] + [(n^{**} + 2) U_{ni}^{0**} - x^{0**} U_{xi}^{0**}]$$

Given (A.4), the first term of dN_1 is non-negative while the last term is non-negative by (b) in the proposition. The two middle terms are non-negative by (a) in the proposition. Thus, under these conditions, $dN_1 \geq 0$ and hence $dU^{1**} / dn > dU^{0**} / dn$. But, as these last two expressions are of opposite signs, it follows that $dU^{0**} / dn < 0$. Thus, using (A.3) and an identical argument to that in part (i), it follows that $n^{**} > n^*$. Q.E.D.

Qualitatively, the effect of increasing p from $p = 0$ need not be the same as that from increasing a positive p . Nevertheless, similar arguments to those above allow us to obtain the effect on the optimal n of increasing a positive $p < 1$. We state the result as a corollary.

Corollary 1. When utility is state-independent, an increase in p increases n^{**} ; when utility is state-dependent and satisfies (A.4) and conditions (a) and (b) of Proposition 1, an increase in p increases n^{**} .

Proof. We can use the well known convexity of the indirect expected utility function to establish this result. Using the same notation for both state-dependent and state-independent utility, define the indirect expected utility function by

$$(11) \quad V(p) \equiv \underset{n}{Max} \{ (1-p)U[M / (n+2), n+2, 0] + pU[M / (n+1), n+1, 1] \}$$

where, in the state-independent case, $U_i = U_{in} = U_{ix} = 0$. Now, $V'(p) = U^{1**} - U^{0**}$ and

$$(12) \quad V''(p) = -\frac{\partial n^{**}}{\partial p} \left[\frac{(M / (n^{**} + 1)^2) U_x^{1**} - U_n^{1**}}{(1-p)} \right] = \left(\frac{1}{1-p} \right) \frac{\partial n^{**}}{\partial p} \frac{dU^{1**}}{dn} \geq 0$$

The last inequality follows from the convexity of $V(p)$ in p . Thus, $\partial n^{**} / \partial p$ and dU^{1**} / dn have the same sign. Now, Proposition 1 established the conditions for $dU^{1**} / dn > 0$. Under these same conditions, $\partial n^{**} / \partial p > 0$. Q.E.D.

Proposition 1 and its corollary establish formally a result which is part of development economics folklore: that increased child mortality risk leads to an increased demand for children and hence number of births⁴. In development economics the focus is somewhat different to our own. There, children are both a source of labour and a provider of social security for parents in their old age. What is interesting here is that without invoking any such motivation but, rather, simply assuming that children provide utility even though they reduce parents' consumption, we are able to show that the demand for them increases with their mortality risk.

It is also interesting to note that, perhaps surprisingly, it is less clear-cut that increased child mortality risk increases the demand for children when parents' preferences are state-dependent. The restrictions which have then to be imposed to get this outcome however are intuitive, provided we interpret "relative risk aversion w.r.t. n " as simply indicating the elasticity of the marginal utility from children w.r.t. the number of children.

It is trivial but tedious to show that assuming the same conditions on state-contingent utilities as in the deterministic problem suffices for $\partial n^{**} / \partial M > 0$ here so we omit the details.

III. Child Mortality Risk, Children's Life Insurance and the Demand for Children.

Suppose now that actuarially fair child mortality risk insurance is available. Initially, we maintain the assumption of a risk to one child only, however many the parents have, and consider possible insurance purchase on that basis. Subsequently, we examine when there are independent and identical risks to all children and insurance is available on a "joint life-first death" basis.

Assuming that there is no asymmetric information, parents purchasing insurance against the risk to one and only one child can pay a certain premium, denoted π , in return for a payoff, denoted s , paid on the death of a child. Premium and payoff are linked by the zero expected profit condition,

$$(13) \quad ps = \pi$$

This means that the *net* payment received in the bad “1” state is $\pi(1-p)/p$.⁵

Although there is restricted availability of such insurance in an explicit form, it could be argued that it exists in an implicit form in many instances. E.g., under tort, part of a paediatrician’s fees can be regarded as an implicit insurance premium and parents might well choose to pay the higher fees of the most reputable physicians on that basis. In any case, we wish to analyse the implications of the availability of such insurance in order to provide a benchmark against which we can assess certain claims which have been made in the literature.

With the possibility of purchasing this insurance, parents have a two-decision-variable problem of choosing n and π to maximise $CEU(n, \pi)$ given by, in our original notation:

$$(14) \quad \underset{n, \pi}{Max.} CEU(n, \pi) \equiv (1-p)U^0[(M-\pi)/(n+2), n+2] + pU^1[(pM+(1-p)\pi)/p(n+1), n+1]$$

Let \sim indicate the optimal values of magnitudes in this environment. Treating the sign of π as unrestricted for now, the solution to (14) satisfies the following two first-order conditions:

$$(15) \quad \pi : \quad (1-p)[(1/(\tilde{n}+1))\tilde{U}_x^1 - (1/(\tilde{n}+2))\tilde{U}_x^0] = 0$$

$$(16) \quad n : \quad (1-p)\left\{[(M-\pi)/(\tilde{n}+2)^2]\tilde{U}_x^0 - \tilde{U}_n^0\right\} = p\tilde{U}_n^1 - [(pM+(1-p)\tilde{\pi})/(\tilde{n}+1)^2]\tilde{U}_x^1$$

Equation (15) indicates the first important result of this section:

Proposition 2. Optimal fair child life insurance purchase leads to the 0-contingent marginal utility of consumption exceeding the 1-contingent one.

Proof. Rearranging (15), we see that

$$(17) \quad \tilde{U}_x^0 = [(\tilde{n} + 2) / (\tilde{n} + 1)]\tilde{U}_x^1$$

But, neglecting the integer issue, $\tilde{n} + 2 > \tilde{n} + 1$ for parents with $\tilde{n} > 0$ children. Q.E.D.

Many criticisms of the operations of the tort system as insurance are based on the belief that optimal actuarially fair insurance purchase results in parents equalising the marginal utility of consumption across states of the world. With either state-dependent or state-independent utilities, equalisation of marginal utilities would lead to $\tilde{x}^0 > \tilde{x}^1$ in our model, as in the conventional model with single argument utility functions, if the non-pecuniary loss reduces the marginal utility of money, other things equal. Economists have criticised large payouts to bereaved parents which result in $\tilde{x}^1 > \tilde{x}^0$ on that basis (e.g., cf. Frech III and Calfee and Rubin, and references therein). However, Proposition 2 shows that, in a model which incorporates the utility derived from children explicitly and allows for a trade-off between the number of children and the material standard of living, this argument cannot be supported. Marginal utilities of consumption are not equalised across states and we cannot say, without further restrictions on the utility function, whether $\tilde{x}^0 > \tilde{x}^1$ is optimal.

Proposition 2 shows that, with a typical family of four, the marginal utility of consumption in the no loss state will be 33% higher than in the loss state. This reveals that there is an incentive to transfer purchasing power to the loss state, as compared with the standard insurance model. This incentive arises because here the marginal value of an extra unit of money is greater with fewer people sharing it in the loss state.

This still does not tell us whether it would be optimal to purchase a positive amount of child mortality insurance. Because π is of unrestricted sign in deriving (15), the π satisfying (15) might be negative or zero, confirming the claim in the literature that parents would not choose to purchase such insurance.

For parents to wish to purchase insurance, it must be that conditional expected utility given in (14) is increasing in π at $\pi = 0$, i.e., $\left. \frac{\partial CEU(\tilde{n}, \pi)}{\partial \pi} \right|_{\pi=0} > 0$. It turns out that, for plausible utility functions, this will indeed be the case and, hence, parents would choose to purchase a positive amount of child mortality insurance. This can be verified by considering an example.

Example 1. Let the state-contingent utility functions be given by $U^0(x, \tilde{n}) = (x+1)^\alpha \tilde{n}^\beta$ and $U^1(x, \tilde{n}) = (x+1)^\delta \tilde{n}^\beta$, with $1 > \alpha, \beta, \gamma > 0$. Now, for the optimal π satisfying (15) to be positive, we must have $\left. \frac{\partial CEU(\tilde{n}, \pi)}{\partial \pi} \right|_{\pi=0} > 0$. I.e., given the state-contingent utility functions assumed, we require

$$\left(\frac{1}{\tilde{n}+1} \right) \delta \left(\frac{M+\tilde{n}+1}{\tilde{n}+1} \right)^{\delta-1} (\tilde{n}+1)^\beta (\tilde{n}+1)^\beta - \left(\frac{1}{\tilde{n}+2} \right) \alpha \left(\frac{M+\tilde{n}+2}{\tilde{n}+2} \right)^{\alpha-1} (\tilde{n}+2)^\beta > 0.$$

Equivalently, we require,

$$(18) \quad \left(\frac{\delta}{\alpha} \right) (\tilde{n}+2)^{\alpha-\beta} (\tilde{n}+1)^{\beta-\delta} (M+\tilde{n}+2)^{1-\alpha} (M+\tilde{n}+1)^{\delta-1} > 1.$$

Now, if $\alpha = \delta$, this requirement reduces to $\left(\frac{\tilde{n}+2}{\tilde{n}+1} \right)^{\alpha-\beta} \left(\frac{M+\tilde{n}+2}{M+\tilde{n}+1} \right)^{1-\alpha} > 1$. This can be seen to hold if $\alpha \geq \beta$. Thus, by continuity, provided $\alpha \geq \beta$, there will exist a range of parameter values satisfying $\alpha > \delta$ for which (5) remains true⁶.

Example 1, and many others which could be constructed, indicate that a willingness by parents to purchase insurance against child mortality risk is not implausible. Of course, this does not establish that such insurance policies would be purchased if they were marketed explicitly. Such purchases would depend on whether or not parents' true utility functions generated behaviour like that which we have analysed. Recognition must also be given to the fact that, in reality, there are severe problems of moral hazard and adverse selection associated with the availability of such insurance for general child mortality risk - i.e., for risks not associated with specific activities such as vaccination.

Becker and Murphy (1988, 4), Calfee and Winston (1993) and others point out that parents do not readily purchase insurance on their children's lives. But, however true this might be now, it certainly was not true in the relatively recent history of the life insurance industry. For example, Davenport and Gesell's (1940, vi) extensive survey for the U.S. found, *inter alia*, in the 78% of families with life insurance, 83% of the men, women and children in total were insured; 49.6% of all policies were for industrial insurance⁷, accounting for 64% of total premiums; 42.2% of these industrial premiums were for endowment policies and 55.8% of these were on the lives of children under 10 years of age; 24.8% of all industrial endowment policies were issued on the lives of children under 2 years old⁸. For the U.K., Garnett (1968, 87) and Geddes and Holbrook (1963, 236), indicate that the purchase of low value insurance on children's lives was widespread.

Such insurance purchases have been legally circumscribed in scale and scope at various times in both the U.K. and the U.S (see, e.g., Vance, 1951, 148, Geddes and Holbrook, 1963, 264, and Iwamy, 1980, 94-95)⁹. These restrictions are presumably at least partly in response to the adverse selection and moral hazard problems noted above. Nevertheless, given that laws are not usually enacted frivolously, the mere existence of such restrictions indicates a recognition that parents will purchase insurance on their children's lives, for whatever motives.

The above suggests that, contrary to the claims in the current literature but in accordance with the prediction of our model, parents will manifest a desire to purchase insurance against child mortality risk¹⁰. It also indicates that we cannot argue, *a priori*, that the tort system acting as an implicit insurance system promotes inefficiency by encouraging payouts in the "wrong" state of the world. Parents might optimally choose to receive such a payout and, of course, any $\pi > 0$ is sufficient to ensure $x^1 > x^0$ is optimal¹¹. A more legitimate criticism of the operation of tort in this context might be that it crowds out private insurance purchases that would otherwise occur.

As the theory and evidence both suggest that some parents might purchase life insurance to cover the death of a child, it is of some interest to know if and how the availability of such insurance might affect the demand for children. Typically, one would argue that insurance cover might encourage individuals to invest in a commodity from which they derive utility but which is subject to risk of loss, with the consequent loss of their investment. Here, although parents forego some consumption if they are successful in having surviving children, they expend nothing

beyond their grief (and then only if $U^0(x, n) > U^1(x, n)$ on losing a child. Thus, we need to be circumspect in applying this argument. We must also be circumspect in drawing conclusions from comparing the solutions to (6) and (14) because of well known difficulties in making deductions about the configuration of decision variables in multiple decision variable problems.

Inspection of (15) and (16), and particularly the comparison of (16), which characterises the optimal \tilde{n} , with (7), its counterpart for n^{**} proves uninformative. Thus, we choose the following alternative route to examine the impact of the availability of actuarially fair insurance. First, we consider the optimal choice of n with a mandatory fixed fair insurance policy with $\pi \geq 0$. Then we examine how the optimal n changes as the policy changes.

With a mandatory fixed fair insurance policy, n is again the parents' sole decision variable. Letting superscript \wedge index optimal values in this case, the optimal $n > 0$ now satisfies:

$$(19) \quad dCEU(n) / dn = (1-p)U_n^0[(M-\pi) / (\hat{n}+2), \hat{n}+2] + pU_n^1[(pM + (1-p)\pi) / p(\hat{n}+1), \hat{n}+1] \\ - (1-p)U_x^0(\cdot)(M-\pi) / (\hat{n}+2)^2 - pU_x^1(\cdot)[(pM + (1-p)\pi) / p(\hat{n}+1)^2] = 0$$

Consider the impact of $\Delta\pi > 0$ on this expression evaluated at $\pi = 0$ before n responds optimally.

$$(20) \quad \partial[dCEU(n) / dn] / \partial\pi|_{\pi=0} = (1-p) \left\{ \left[\frac{\hat{U}_x^0}{(\hat{n}+2)^2} + \frac{M\hat{U}_{xx}^0}{(\hat{n}+2)^3} \right] - \left[\frac{\hat{U}_x^1}{(\hat{n}+1)^2} + \frac{M\hat{U}_{xx}^1}{(\hat{n}+1)^3} \right] \right\} \\ + (1-p) \left[\frac{\hat{U}_{nx}^1}{(\hat{n}+1)} - \frac{\hat{U}_{nx}^0}{(\hat{n}+2)} \right]$$

Clearly, the sign of this expression can go either way. In general, the sign will depend on two forces. First, from the first RHS term, on the value of and the relationship between the state-contingent indices of relative risk aversion with respect to consumption (RRA; see Arrow, 1971, and Pratt, 1964); second, from the final term, on the impact of a small variation in the policy on the expected utility from the marginal child via its effect on the per capita consumption in the two states. An interesting but very special case is when both the state-dependent utilities are separable (which eliminates the last RHS term) and both exhibit constant relative risk aversion of unity.

Then $\partial[dCEU(n)/dn]/\partial\pi|_{\pi=0} = 0$ and the introduction of a small fair mandatory child life policy would not affect the demand for children. Were the policy not mandatory, the parents would find it optimal to neither insure nor gamble on the life of a child in this special case.

We can examine the impact of insurance more generally either by determining how a variation in a fair mandatory policy affects the first-order condition characterising the optimal choice of n at $\pi > 0$ or, equivalently, by studying the comparative static behaviour of n with respect to π . Adopting the former route, we obtain the following generalisation of (20):

(21)

$$\begin{aligned} \partial[dCEU(n)/dn]/\partial\pi = (1-p) & \left\{ \left[\frac{\hat{U}_x^0}{(\hat{n}+2)^2} + \frac{(M-\pi)\hat{U}_{xx}^0}{(\hat{n}+2)^3} \right] - \left[\frac{\hat{U}_x^1}{(\hat{n}+1)^2} + \frac{(pM+(1-p)\pi)\hat{U}_{xx}^1}{p(\hat{n}+1)^3} \right] \right\} \\ & + (1-p) \left[\frac{\hat{U}_{nx}^1}{(\hat{n}+1)} - \frac{\hat{U}_{nx}^0}{(\hat{n}+2)} \right] \end{aligned}$$

Rearranging the first term of this reveals how both the state-dependent degrees of RRA and whether or not the mandatory insurance policy is optimal impact on the demand for children. Here,

$$\begin{aligned} (22) \quad (1-p) & \left\{ \left[\frac{\hat{U}_x^0}{(\hat{n}+2)^2} + \frac{(M-\pi)\hat{U}_{xx}^0}{(\hat{n}+2)^3} \right] - \left[\frac{\hat{U}_x^1}{(\hat{n}+1)^2} + \frac{(pM+(1-p)\pi)\hat{U}_{xx}^1}{p(\hat{n}+1)^3} \right] \right\} \\ & = (1-p) \left(\frac{1}{(\hat{n}+1)} \right)^2 \hat{U}_x^1 \left[\left(\frac{(\hat{n}+1)}{(\hat{n}+2)} \right)^2 \frac{\hat{U}_x^0}{\hat{U}_x^1} - 1 \right] \\ & + (1-p) \left(\frac{1}{(\hat{n}+1)} \right)^2 \hat{U}_x^1 \left[\left(\frac{M-\pi}{\hat{n}+2} \right) \frac{\hat{U}_{xx}^0}{\hat{U}_x^0} \left(\frac{(\hat{n}+1)}{(\hat{n}+2)} \right)^2 \frac{\hat{U}_x^0}{\hat{U}_x^1} - \left(\frac{(pM+(1-p)\pi)}{p(\hat{n}+1)} \right) \frac{\hat{U}_{xx}^1}{\hat{U}_x^1} \right] \end{aligned}$$

Example 2. Suppose that the state-dependent utility functions are separable and exhibit a constant and equal degree of RRA, δ say. Then, if the parents are optimally insured against the

death of a child, an increase in the level of cover provided under a fair policy will decrease (increase) the demand for children if $\delta < (>)1$.

To verify this, note: if the conditions of example 2 are met, both (21) and (22) simplify to

$$(23) \quad \partial[dCEU(n)/dn]/\partial\pi = (1-p)\left(\frac{1}{(\hat{n}+1)}\right)^2 \hat{U}_x^1 \left[\left(\frac{(\hat{n}+1)}{(\hat{n}+2)}\right)^2 \frac{\hat{U}_x^0}{\hat{U}_x^1} - 1 \right] (1-\delta)$$

But, if parents are optimally insured, (15) holds and the coefficient of $(1-\delta)$ is negative. Thus the sign of $\partial[dCEU(n)/dn]/\partial\pi$ is opposite to that of $(1-\delta)$ and, given the concavity of $CEU(n)$ in n , verifies the claim. Indeed, (23) confirms that, here, the demand for children will tend to increase as the amount of insurance increases and as δ increases above the critical level.

Another interesting special case occurs when mandatory insurance results in parents being so over-insured that the first term on the RHS(23) equals zero. Then, if the utility functions are separable and assign the same sub-utility to any given per capita consumption, and if they satisfy Arrow's hypothesis on increasing RRA, $\partial[CEU(n)/dn]/\partial\pi > 0$. A move in the direction of optimal insurance would then decrease the demand for children.

Generally, $\left[\left(\frac{\hat{n}+1}{\hat{n}+2}\right)^2 \frac{\hat{U}_x^0}{\hat{U}_x^1} - 1 \right]$ in (23) goes from being negative when the parents are under- or optimally insured to being positive if they are sufficiently over-insured. How such an increase in insurance affects the $\left[\left(\frac{M-\pi}{\hat{n}+2}\right) \frac{\hat{U}_{xx}^0}{\hat{U}_x^0} \left(\frac{\hat{n}+1}{\hat{n}+2}\right)^2 \frac{\hat{U}_x^0}{\hat{U}_x^1} - \left(\frac{pM+(1-p)\pi}{p(\hat{n}+1)}\right) \frac{\hat{U}_{xx}^1}{\hat{U}_x^1} \right]$ term is ambiguous, however. If the state-dependent utilities exhibit constant (even if different) RRA, this latter term will decrease as the amount of insurance increases. If both exhibit increasing RRA, this would tend to check the rate of decrease in this latter term as π increases: $-\left(\frac{pM+(1-p)\pi}{p(\hat{n}+1)}\right) \frac{\hat{U}_{xx}^1}{\hat{U}_x^1}$ would be increasing while $\left(\frac{M-\pi}{\hat{n}+2}\right) \frac{\hat{U}_{xx}^0}{\hat{U}_x^0} \left(\frac{\hat{n}+1}{\hat{n}+2}\right)^2 \frac{\hat{U}_x^0}{\hat{U}_x^1}$ now could go either way. The balance between the state-dependent RRA indices is important as it captures the responsiveness of expected marginal utility to a shift in financial resources from one state to the other consequent on the change in the amount of insurance. Overall, this argument, albeit imprecise, shows that the

amount of insurance against child mortality which parents have is likely to be of importance in determining their demand for children.

As a final point, it is worth stressing that the above exercise examined the impact of variations in a mandatory life insurance policy. This is what it could be argued that parents are forced to have under the tort system because such a policy is then implicit in the prices of commodities and services. The latter are higher than they would otherwise be without tort liability¹². However, the analysis is also useful in indicating how parents might optimally behave were they free to purchase explicit fair insurance policies against child mortality. We have seen that, for some utility functions at least, parents would be prepared to purchase strictly positive amounts of such insurance. If it is the case that any positive amount of insurance always impacts on the expected utility derived from the marginal child in the same direction up to the optimal level of insurance (i.e, if $\partial[dCEU(n)/dn]/\partial\pi$ is single-signed for all $\pi \geq 0$ up to the optimal level), then we can deduce the impact which the opportunity to purchase an optimal amount of insurance will have on the demand for children as compared with the case when such insurance is unavailable. E.g., if $\partial[dCEU(n)/dn]/\partial\pi > 0$ everywhere up to the optimal π , it can be deduced that the availability of child mortality insurance will increase the demand for children.

IV. An Extension

Suppose now that *each* child in an household face an identical and independent once-for-all risk, p , of being killed at some pursuit. The environment otherwise remains as in section III except for two features. First, the household size is predetermined prior to any accident experience; second, we consider a more general insurance contract.

Suppose actuarially fair insurance is on a “joint-life first death basis.” I.e., the policy pays out on the first death of a child, irrespective of whom or how many are killed. This means that a premium of π can purchase *net* cover of $\pi(1-p)^n/[1-(1-p)^n]$. This is a more general specification of the insurance contract than that which is usually encountered in the literature. It is designed to accommodate the many possible states of nature involving the death of a child.

Parents considering the purchase of such insurance now face the following concave conditional expected utility maximisation problem:

(24)

$$\text{Max}_{\pi} \left\{ \sum_{i=1}^n {}^nC_i p^i (1-p)^{n-i} U^i \left[\frac{M(1-(1-p)^n) + (1-p)^n \pi}{(1-(1-p)^n)(n+2-i)}, n+2-i \right] + (1-p)^n U^0 \left[\frac{M-\pi}{n+2}, n+2 \right] \right\}$$

In (24), ${}^nC_i = \frac{n(n-1)(n-2)\dots(n-i+1)}{i(i-1)(i-2)\dots 1}$ is the binomial coefficient indicating the number of

ways of choosing i items from n .

Neglecting any non-negativity constraint on π and suppressing functional arguments for simplicity, the solution to (24) satisfies the first-order condition:

$$(25) \quad \partial CEU / \partial \pi = \sum_{i=1}^n {}^nC_i p^i (1-p)^{n-i} \frac{(1-p)^n}{(1-(1-p)^n)(n+2-i)} U_x^i - \frac{(1-p)^n}{n+2} U_x^0 = 0$$

In the usual two-state insurance problem, the marginal utility of money is equated across states. In the present problem, assuming for the moment that insurance purchase is positive, net income foregone in one state yields a payoff in all other states. By analogy with the two-state problem, a reasonable a priori expectation in this case might seem to be that the marginal utility of consumption in the “0” state would be equated to the expected marginal utility of consumption in the other states. We will show that, in line with Proposition 2 above, optimal insurance purchase now would not involve parents equating the marginal utility in the “0” state to the expected marginal utility of consumption in the other states. Instead, the marginal utility in the states with fatalities must, on average, be less than that in the state without a fatality:

Proposition 3. Given the opportunity to purchase “joint-life, first-death” insurance against child mortality risk on a fair basis, parents will not equalise the marginal utility of consumption in the state without a fatality to the expected marginal utility of consumption in the other states.

Proof. From (25), on rearrangement

$$\sum_{i=1}^n \frac{{}^nC_i p^i (1-p)^{n-i}}{(1-(1-p)^n)} \frac{(n+2)}{(n+2-i)} U_x^i = U_x^0$$

But, as $\left(\sum_{i=1}^n {}^nC_i p^i (1-p)^{n-i} \right) / (1 - (1-p)^n) = 1$, while $(n+2)/(n+2-i) > 1$, $\forall i > 0$, this implies

$$U_x^0 > \sum_{i=1}^n \frac{{}^nC_i p^i (1-p)^{n-i}}{(1 - (1-p)^n)} U_x^i \text{ Q.E.D.}$$

While this proposition tells us that the marginal utility in the states with fatalities must, on average, be less than that in the state without a fatality, to specify the relationship between the state-contingent marginal utilities more precisely requires further assumptions about the utility function. A particularly tractable assumption for this purpose is (A.4) from section II. Given (A.4), it is easy to show that $U_x^1 > U_x^2 > \dots > U_x^n$. Moreover, if $\pi \geq 0$, then $U_x^0 > U_x^1$ holds as well¹³. Thus, with optimal child mortality insurance, parents would then choose a situation where their marginal utility of consumption is decreasing in the number of children lost¹⁴.

The fact that our findings here and in the previous section are so at variance with those in the literature naturally prompts us to consider if and when the conventional results involving the equalisation of marginal utilities across states and its implications could be recovered. This would be possible if parents derived utility from the number of children and the household's aggregate rather than per capita income and if the risk to be insured is that of one and only one child dying. The inability to generate positive purchases of insurance on children lives in any circumstances under such a specification would seem to be a weakness rather than a strength of it because it sits uneasily with the empirical evidence which we reviewed in section III.

If parents' utility functions are the same as we have assumed hitherto, but they seek to maximise the *household's* expected utility and ascribe the same preferences to their children as they have themselves, then the conventional results would again be recovered if the risk is of one and only one child dying¹⁵. However, it is unclear why parents would ascribe the same utility function to their children as to themselves. Or, indeed, why they should be interested in maximising anything other than their own utility function which already subsumes the household's standard of living, hence that of their children. What is clear, however, is that the optimal behaviour regarding children's life insurance purchase is very sensitive to the

specification of the parent's utility function. Although we made the simplifying assumption of equal division of consumption within the household in our formulation, this is not essential for our results. All that we require is that utility depends on the material standard of living and that the latter is decreasing in family size. This would occur, e.g., if we use income divided by the number of "equivalent adults" in the household as the surrogate for the standard of living.

V. Conclusions

Children are costly. Nevertheless, successive generations of parents continue to have children despite the much diminished importance of a precautionary demand for children as guarantors of a tolerable old age. This suggests that the possession of children is a consumption rather than investment activity and that parents might require compensation for - and purchase insurance against - the loss of children. We have constructed a Beckerian model of the demand for children when parents derive utility from children and have to forego some household per capita private good consumption in order to have additional children. Children are found to be, plausibly, superior goods. We showed that introducing or increasing an exogenous child mortality risk would be expected to increase the demand for children

More importantly, our model enables us to show that, contrary to claims in the literature, when actuarially fair insurance against child mortality risk is available, parents will not equalise marginal utilities from consumption across states of the world but will choose a position where the marginal utility in the state without child mortality exceeds that in the state with. Thus the relationship between the state-dependent per capita consumption levels can go either way. We constructed a class of examples for which parents would choose to purchase insurance paying out in the child mortality state, again contrary to claims in the literature. Finally, more tentatively, we explored the impact of the availability of insurance on the demand for children. When the insurance is mandatory, such as that implicit in the pricing of hazardous commodities and services under the tort system, the impact on parents' demand for children will depend on the amount of insurance provided, in particular the extent to which they are under- or over-insured, and on the behaviour of the state-dependent degrees of relative risk aversion.

Footnotes

* I have benefited from discussions with Warwick University colleagues, notably Graham Aylott, Martin Judge and Carlo Perroni, and from a suggestion by Norman Ireland. A conversation with Jon Rowe (K.L. Plester Group, Ltd) proved most useful. The usual disclaimers apply.

1. We focus on the parents' utility function rather than the household's to avoid philosophical issues about the degree of parental sovereignty and the utility to be assigned either to the unborn or to extant children relative to their parents. Parents are essentially assumed to subsume children's interests within their own. However, this utility function could be rationalised as being appropriate in at least two ways. First, if all household members care about the same things and engage in a Nash bargain with equal bargaining strength. Second, if parents ascribe the same interests to all actual and potential family members and conceive of household utility as that which would emerge from a symmetric Nash bargaining game in which all had the same weight and zero reservation utility. We return to the issue of the appropriate utility function below.

2. We make this assumption to avoid the complication of the Binomial Theorem at this point. This would have to be employed if, instead, all children have a common but independent risk of dying, p . This extension will be pursued when we consider a particular form of insurance purchases in section IV.

3. See Luce and Krantz on conditional expected utility (CEU). The CEU concept has been applied extensively in contexts with risks to life and limb and other non-pecuniary losses. E.g., see Arrow (1974), Calfee and Rubin (1992), Cook and Graham (1977), Fraser (1984a,b; 1994, 1995), Friedman (1982), Jones-Lee (1989), Spence (1977), Viscusi (1979, 1980) and Viscusi and Evans (1990) among many.

4. Here, we have not stated why parents derive utility from children, merely that they do. This allows for a precautionary motive for children, or for any other.

5. Note that this description is flexible enough to cover the case where the parents gamble on the death of a child by opting to receive a payment in the state when a death does not occur and paying a premium when it does.

6. It is easy to verify that $U_x^0 \geq U_x^1$ at the same x in this example and that $U^0 > U^1$ can also be guaranteed simultaneously by choosing parameters suitably. We can construct similar examples

using separable state-dependent utilities which exhibit constant absolute risk aversion with respect to consumption gambles. Fraser (1994) examines the behaviour of constant absolute or relative risk averse state-dependent utilities and the requirements for them to give an unambiguous ranking of the marginal utilities of consumption which accords with empirical evidence.

7. Industrial insurance is a form of life insurance which is sold via weekly doorstep collection of premiums from households and is marketed primarily to low income families.

8. There are reasons to believe that Davenport and Gesell's findings understated the extent of life insurance coverage at the time. They confined their survey to Massachusetts because (pp. 2-3) insurance sales were relatively stringently regulated there and they felt that this would be reflected in less high pressure sales techniques (hence sales) than other states.

9. When and where limitations exist, they usually involve a restriction on the amount of insurance which can be taken out on the life of children below a certain age. See, e.g., Vance (1951, 148, 207) and Geddes and Holbrook (1963, 264). Indubitably, one reason for such limitations is the adverse incentive which insurance in excess of parents' pecuniary interest in their children might provide (cf. Vance, 1951, 208).

10. We stress again that while parents might not be able to purchase such insurance explicitly, nonetheless it is precisely what is available to them implicitly under tort. Thus, choice of a reputable, expensive paediatrician who is likely to be well-insured, rather than a less reputable and cheaper one, could be interpreted as purchase of such an implicit insurance policy. Exercising such a choice might simultaneously affect the child mortality risk, but we neglect this issue.

11. Note that the loss of any income from the death state, such as "child allowance" paid while children are alive, would reinforce the tendency to purchase child mortality insurance.

12. See Frech III and, especially, Calfee and Rubin for arguments along these lines.

13. We can verify these statements by comparing the $U_x \left[\frac{M(1 - (1 - p)^n) + \pi(1 - p)^n}{(1 - (1 - p)^n)(n + 2 - i)}, n + 2 - i, i \right]$,

$i = 1, 2, \dots, n$, and $U_x \left[\frac{M - \pi}{n + 2}, n + 2, 0 \right]$, using the concavity of U and (A.4).

14. We can use $U_x^0 > U_x^1$ (if $\pi = 0$) and $U_x^1 > U_x^2 > \dots > U_x^n$, given (A.4), to construct examples where insurance would be purchased against independent and identical mortality risks to each child along the lines of Example 1 above.

15. To see this, note that with a risk to one and only one child, expected household utility would be given by

$$(1-p)(n+2)U^0\left[\frac{M-\pi}{n+2}, n+2\right] + p(n+1)U^1\left[\frac{pM+(1-p)\pi}{p(n+1)}, n+1\right]$$

If we choose π to maximise this expression, we obtain the conventional result that $U_x^0 = U_x^1$.

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