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ABSTRACT

Technical and allocative efficiency is estimated assuming a sequentially planned production process. Results show that the single equation approach is not useful in this context. Technical efficiency is not affected by using temporally disaggregated data. With regard to allocative efficiency the results for the single and the multi-stage models both show a wide divergence from profit maximisation. The virtue of the temporally disaggregated model lies in the information it yields about the importance of the interaction of inputs within and between stages. We use this information to explain the surprising result that farmers over utilise fertiliser.

1 INTRODUCTION

The degree, not to mention the notion, of efficiency of farmers in developing countries has, over time, been the focus of much debate and research. The definition of technical and allocative efficiency by Farrell (1957) has given impetus to renewed research, both theoretical and applied[1]*. In particular the extension of Farrell's work by Aigner, Lovell and Schmidt (1977) and Meeusen and Van den Broeck (1977) who independently introduced the concept of the stochastic frontier production function proved to be a breakthrough. Their methodology became widely applied with the contribution by Jondrow et al (1982) who suggested a way of calculating individual TE estimates. Consequently this approach to measure TE has become very popular indeed. A further extension by Schmidt and Lovell (1980) showed how one may estimate TE and AE simultaneously*[2]*.*

The stochastic frontier production function has been widely applied to agricultural data in developing countries. Applied work examining both TE and AE is rare however. All applied work uses annual data, i.e. it assumes that the inputs are applied in a single stage. A more realistic model would allow for the timing of inputs to be of relevance. As early as 1966 Zellner, Kmenta and Drèze (page 795) wrote that: "..an appropriate approach to this problem (of temporal aggregation)[3] *would involve analysing the intrayear sequential decision-making process to ascertain what implications it has for the annual data."*

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[1]For surveys of the literature on frontier production functions see Føsund, Lovell and Schmidt (1980), Schmidt (1985-6) and Bauer (1990). Technical efficiency is defined as the shortfall from the maximum output that a farmer may achieve. This maximum output is defined by the production technology, the input set and the sample of farmers. Allocative efficiency refers to the optimal input choice. The ratio of marginal value product should be equal to the marginal cost for inputs to be allocated efficiently. We abbreviate technical and allocative efficiency as TE and AE and use these abbreviations henceforth.

[2]Schmidt and Lovell (1980) use cost-minimisation as their behavioural assumption. Kumbhakar (1987) modelled the case for the profit-maximising agent. He allowed for farmers to observe TE, which means that the error component capturing TE will appear in the input demand equations which implies that the inputs can not be considered exogenous.

[3]Part in parentheses added by this author.

The issue of the importance of the timing of intermediate inputs was initially addressed using experimental data. Later Antle (1983) and Antle and Hatchett (1986) suggested ways of modelling and estimating a sequential-decision making process using survey data.

In this paper Antle and Hatchett's methodology is adapted to analyse farmer efficiency in the Indian village of Palanpur. The model is presented in part 2. In particular we are interested in determining how estimates of TE and AE change, how the estimation of the parameters of the production function are altered, and what additional information is generated when placing the analysis within a sequentially planned framework. The data set and the manipulation thereof is discussed in part 3. In part 4 the error structure and the estimation is explained and the results are given in part 5. Part 6 concludes the paper.

2 THE MODEL

The production process involving, say, wheat can be represented as consisting of a number (denoted by M) of stages, written as:

$$\text{(1)} \qquad \begin{aligned} &\textit{Stage 1:} \quad Y_1 = f_1\left(X_1, Z\right)e^{\varepsilon_1} \\ &\qquad\qquad \vdots \\ &\textit{Stage M:} \quad Y_M = f_M\left(Y_{M-1}, X_M\right)e^{\varepsilon_M} \end{aligned}$$

where: Y is the output that occurs in each stage; X is a variable input, one applied in each stage[4]; Z is the fixed input, appearing only in stage 1; the subscripts denote that stage, m = 1, ..., M[5]; and where the error term ε *is to be defined later.*

Only output in the last stage is actually observable. Output in the initial stages could be some physical plant growth that can be observed by the farmer. It could also

[4]In this general exposition we assume that there is one variable input in each stage.
[5]Subscripts to denote the farms are omitted in order to simplify the exposition.

be some unobserved, assumed, growth that is taking place out of eyesight. We assume that based on input usage up to the end of a stage, the weather conditions and past experience, the farmer can make some inferences with respect to the state that the plant is in, even if there is no physical growth.

As output of stages 1 $\rightarrow$ M-1 are not observed they are substituted into the production function of stage M, obtaining what Antle and Hatchett (1986) call the composite production function:

$$(2) \qquad Y_M = f_M\left(f_{M-1}\left(\cdots\left(f_1\left(X_1, Z\right)e^{\varepsilon_1}, \ldots\right), X_{M-1}\right)e^{\varepsilon_{M-1}}, X_M\right)e^{\varepsilon_M}$$

The farmer is assumed to maximise median profits[6] at each stage in a sequential manner. The error term ε , or rather its component parts as defined in part 4, are not observed by the farmer. Output and input prices are either known with certainty or are statistically independent of the production function disturbance. In each stage, the variable input X_m is chosen, based on the decision rules for X_n^*, where n = m+1, ..., M[7]. The problem is written as:

$$(3) \qquad \underset{X_m}{Max}\ \Pi = P \cdot Y_M^m - C_m \cdot X_m - \sum_{n=m+1}^{M} C_n \cdot X_n^{*,m}$$

where P is the expected price of the output; C is the expected price of the variable input; and were the superscript on the output variable Y_M^m is explained below equation (5). To solve (3) for X_m^0 one needs to derive the planned inputs, X_n^*. The first step is to solve:

$$(4) \qquad \underset{X_M}{Max}\ \Pi = P \cdot Y_M - C_M \cdot X_M$$

for X_M^*, termed the planned input for the last stage, M. This expression is a function of

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[6]As in Kumbhakar (1987).

[7]Variables written with a * or ^{0}, such as X^* or X^0, denote planned and optimal inputs respectively.

X_{M-1}, ..., X_1, P, C_M, and the parameters of the composite production function, β . We use X^*_M in Y_M, equation (2), to obtain:

$$(5) \qquad Y_M^{M-1} = h_{M-1}\left(Z, X_1, \ldots, X_{M-1}, P, C_M, \beta\right)$$

The superscript on Y in (5) reminds us that output is written in terms of variable inputs M-1 → 1 (as well as the other arguments). The expression derived in (5) is in turn used to solve for X^*_{M-1} and then Y_M^{M-2}. Continuing in this manner the planned input levels are solved for recursively. The resulting first order conditions are written as:

$$\frac{P\partial Y_M\left(Z, X_1 \ldots, X_M, P, \beta\right)}{\partial X_M} - C_M = w_M$$

$$(6) \qquad \frac{P\partial Y_M^{M-1}\left(Z, X_1, \ldots, X_{M-1}, P, C_M, \beta\right)}{\partial X_{M-1}} - C_M \frac{\partial X_M^{*,M-1}\left(Z, X_1, \ldots, X_{M-1}, P, C_M, \beta\right)}{\partial X_{M-1}} - C_{M-1} = w_{M-1}$$

$$\vdots$$

$$\frac{P\partial Y_M^1\left(Z, X_1, \ldots, P, C_M, C_{M-1}, \ldots, C_2, \beta\right)}{\partial X_1} - C_2 \frac{\partial X_2^{*,1}(\ldots)}{\partial X_1} \ldots\ldots - C_M \frac{\partial X_M^{*,1}(\ldots)}{\partial X_1} - C_1 = w_1$$

Where the w's reflect deviation from profit maximisation, hence are a measure of deviation from AE. This deviation from the optimum may occur due to farmer error, and/or a difference between expected and actual prices. Moreover they will also reflect some degree of measurement error as well as random noise. Equations (2) and (6) together comprise the system that is estimable by FIML, once a specific functional form has been assumed for the production function and an appropriate error structure is defined.

3 THE DATA

The methodology outlined in part 2 is applied to data on 47 wheat plots in the Indian village of Palanpur. This data was collected by Professors Bliss and Stern during their stay in that village in 1974/75[8]*. Since dates of the application of inputs are recorded a chronological order of the activities (inputs) can be established. At first inspection one might suggest that each application of an activity, such as a single ploughing or irrigation, could be taken as one stage. Unfortunately, while farms generally undertake the same activities they often vary in terms of the number of repetitions of these activities. It is therefore more useful to consider, for example, all ploughings as one stage. In this manner we divided the production process into four stages:*

Stage 1: *The number of ploughings. This would include the pre-irrigation (i.e. that irrigation applied prior to sowing).*

Stage 2: *The sowing of the seed, including the seed-bed preparation.*

Stage 3: *The amount of fertiliser applied (excluding that fertiliser applied at the time of sowing), the number of irrigations used at the time of fertiliser application, and any weeding carried out. Farmers did not all follow the same procedure and we did not distinguish between the various ways of timing of the inputs in this stage.*

Stage 4: *The remaining number of irrigations.*

A list of the variables is given in table 1 (subscripts denote the stage of application). Of importance is our treatment of the labour variable and the treatment of those variables which some farmers did not apply. The aggregate labour variable does not include labour used for ploughing and irrigation.[9] *Rather the ploughing and*

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[8]An account of their observations and experiences in Palanpur is given in: Bliss, C. and N. Stern, *Palanpur: The Economy of an Indian Village,* Clarendon Press, Oxford 1982.

[9]And it is this definition of labour which is disaggregated for use in the sequential-decision making model.

irrigation variables are assumed to reflect the effect of the combined inputs. With regard to the observations which were zero we chose to add a small constant to these[10]*.*

Table 1

List and description of variables used

Symbol	*Description*
Y	*Wheat output, Kg.*
PW	*Price of wheat, Rupees per Kg., a constant*
A	*Acreage*
P	*Number of ploughings*
I_1, I_3, I_4	*Number of irrigations*
L_2, L_3	*Standard man hours, (man = 1, woman = 1, child = 0.5)*
F	*Fertiliser, Kg.*
C_P	*Rupees per ploughing*
C_{I_1}, C_{I_3}, C_{I_4}	*Rupees per irrigation*
C_{L_2}, C_{L_3}	*Rupees per standard man hour*
C_F	*Rupees per Kg. of fertiliser*

Our choice was motivated by the fact that on the one hand the actual observation was zero so a very small number was intuitively appealing. On the other hand we found that the model estimated without the zero observations (in Croppenstedt (1993)) yielded virtually identical results. Finally in terms of significance tests this method worked better than adding larger constants.[11]

4 THE ESTIMATION

4.1 Technical Efficiency

To estimate TE we could simply use equation (2).[12] *The error term* ε *, on*

[10]The problem of zero observations in the context of a logarithmic specification is a common one. See Johnson and Rausser (1971) for the various possible ways of dealing with this issue.

[11]For more detail see Croppenstedt (1993).

[12]Single equation estimation of the production function is popular and justified by invoking Zellner et al's (1966) expected profit maximising scenario.

equation (2) is composed of the terms v and u, as first suggested by Aigner, Lovell and Schmidt (1977). The v is a symmetric component that captures exogenous shocks, such as unexpected weather patterns, or supply side shocks. It is assumed distributed as $N(0,\sigma_v^2)$. *The term u captures technical efficiency as the deviation from the frontier that defines maximal output, given the inputs. The u's are assumed to be distributed as* $|N(0,\sigma_u^2)|$, *i.e. the absolute value of a normal variable. The density of (v-u) is given by*[13]*:*

$$(7) \qquad f(v-u)=\frac{2}{\sigma}\frac{1}{\sqrt{2\cdot\pi}}\exp\left[-\frac{1}{2\cdot\sigma^2}(v-u)^2\right]\{1-F[(v-u)\lambda/\sigma]\}$$

where $\sigma=\sigma_v^2+\sigma_u^2$, $\lambda=\sigma_u/\sigma_v$, *and F is the cumulative distribution function of the standard normal distribution.*

Using equation (7) we can write the log-likelihood function:

$$(8) \qquad \ln(L)=N\cdot\ln\left(\sqrt{2/\pi}\right)-N\cdot\ln(\sigma)+\sum_{i=1}^{N}\ln\left[1-F\left(\varepsilon_i\lambda/\sigma\right)\right]-\left(2/\sigma^2\right)\cdot\sum_{i=1}^{N}\varepsilon_i^2$$

where i denotes the ith farm, with i = 1, ..., N. Applying maximum likelihood techniques to (8) we can estimate the parameters of the stochastic frontier production function as well as λ *and* σ *. As an alternative one may estimate the composite frontier production function directly by OLS and correct the constant term by adding to it the negative of the estimated bias,* $\sqrt{2/\pi}\cdot\sigma_u$[14]*. We use this technique as Monte Carlo evidence by Olson et al (1980) shows that the COLS estimator is more (MSE) efficient for sample sizes of 200 and below, and that COLS is also preferable when* λ *is less than 3.162. It is also straightforward to apply.*

Individual TE estimates are derived by using the mean of the conditional distribution of u for a given ε *(Jondrow et al (1982)). Hence:*

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[13]First derived by Weinstein (1964).

[14]The so-called COLS method was first proposed by Richmond (1974).

$$(9) \qquad E(u|\varepsilon) = \sigma_* \left[\frac{f(\varepsilon\lambda / \sigma)}{1 - F(\varepsilon\lambda / \sigma)} - \left(\frac{\varepsilon\lambda}{\sigma} \right) \right]$$

$$\text{where } \sigma_*^2 = \sigma_u^2 \sigma_v^2 / \sigma^2$$

The above methodology for estimating TE is applied to two frontier production functions, one with temporally aggregated data and one with temporally disaggregated data.

Model A: The Single Equation Sequential Model

The choice of production technology is limited by tractability[15]*. For the Cobb-Douglas function we can obtain a composite production function after substitution (of the single-stage functions) and explicit solutions to the first order conditions are possible. Therefore we substitute a Cobb-Douglas production function into (2) with the data temporally disaggregated as described in part 3.*

$$(10) \qquad \begin{aligned} \ln(Y_4) &= \beta_0 + \beta_A \ln(A) + \beta_P \ln(P) + \beta_{I_1} \ln(I_1) + \beta_{L_2} \ln(L_2) + \\ &\quad \beta_{L_3} \ln(L_3) + \beta_{I_3} \ln(I_3) + \beta_F \ln(F) + \beta_{I_4} \ln(I_4) + \varepsilon \end{aligned}$$

where $\varepsilon = v - u$*. We note that the error term is an aggregate disturbance made up of the single-stage disturbances (see equation (2)). Estimation of residuals and the structural parameters of the single-stage functions would only be possible if intermediate outputs were observable. This first model is estimated by the COLS method as described above. Results will suggest whether or not this single equation approach is a reasonable approximation to the full model.*

[15]See Antle and Hatchett (1986) for a discussion of this point.

Model B: The Single Equation Instantaneous Model

To compare the sequential model to the instantaneous one we also estimate the following stochastic frontier production function:

$$\text{(11)} \quad \ln(Y) = \alpha_0 + \alpha_A \ln(A) + \alpha_F \ln(F) + \alpha_I \ln(I) + \alpha_P \ln(P) + \alpha_L \ln(L) + \varepsilon$$

where the Y, A, F, P, and $\varepsilon = v - u$ *are as in equation (10) and I and L are aggregated over the various stages.*

4.2 Technical and Allocative Efficiency

We are also interested in AE. The point of using the frontier production function and the first order conditions is that this increases the efficiency of the parameter estimates (Schmidt, 1985-86). The error terms on the first order conditions follow a multivariate normal distribution, i.e. **w** ~ $N(0,\Sigma)$. *The density of* **w** *is given by:*

$$\text{(12)} \quad g(\mathbf{w}) = (2\pi)^{-(n-1)/2}|\Sigma|^{-n/2} \exp\left[-\frac{1}{2}\mathbf{w}'\,\Sigma^{-1}\mathbf{w}\right]$$

As (v-u) is assumed independent of **w** *the joint density of* **w** *and (v-u) is the product of the two densities. The resulting log-likelihood is shown in (13) below:*

$$\text{(13)} \quad \begin{aligned} \ln(L) &= N\cdot\ln(2) - N\cdot(M+1)\cdot\ln(2\cdot\pi)/2 - N\cdot\ln(\sigma^2)/2 - N\cdot\ln|\Sigma|/2 + \\ & N\cdot\ln(r) - (1/2)\sum_{i=1}^{N}\left[\mathbf{w}_i{}'\,\Sigma\mathbf{w}_i + \varepsilon_i^2\sigma^2/2\right] + \sum_{i=1}^{N}\ln\left[1 - F\left(\varepsilon_i\lambda/\sigma\right)\right] \end{aligned}$$

where Σ *is the covariance matrix of* **w**; *r is the Jacobian of the transformations of* ε *and the* $\mathbf{w}_i$*'s into the Y and X's. The covariance matrix* Σ *consists of the elements* σ_{mj}, *where at the maximum the following holds:*

$$(14) \quad \hat{\sigma}_{mi} = (1/N) \sum_{i=1}^{N} \hat{w}_{mi} \hat{w}_{mi}$$

where the $\hat{w}$*'s are the estimated residuals in (6). The subscripts refer to the mth stage and the ith farm (assuming only one input per stage). Hence the elements of* Σ *are expressed as a function of the* β *'s. Substituting this result into (13) we get the concentrated likelihood function, which, since this version is maximised only with respect to the* β *'s,* λ *, and* σ^2*, and not the elements of* Σ *, simplifies things somewhat. An estimate of the covariance matrix is formed according to (14).*

Model C: The Simultaneous Equation Sequential Model

Given our assumptions that: i) farmers do not observe v or u; ii) v is i.i.d $N(0,\sigma_v^2)$*; and iii) farmers maximise median profits we use the methodology outlined in part 2 to derive the six first order conditions, given below in (15a)*$\rightarrow$ *(15d):*

$$(15a) \quad w_{I_4} = \ln(PW) + \ln(\beta_{I_4}) + \beta_0 + \sum_i \beta_i \ln(X_i) - \ln(I_4) + \ln(C_{I_4})$$

where i = A, P, I_1, L_2, I_3, F, L_3, I_4

$$w_{L_3} = -\frac{1}{\phi_1}\ln(PW) - \frac{1}{\phi_1}\beta_0 - \frac{\beta_{I_4}}{\phi_1}\ln(\beta_{I_4}) + \ln(\beta_{L_3}) - \sum_i \frac{\beta_i}{\phi_1}\ln(X_i) - \ln(L_3) + \frac{\beta_{I_4}}{\phi_1}\ln(C_{I_4}) - \ln(C_{L_3})$$

(15b) $$w_{I_3} = -\frac{1}{\phi_1}\ln(PW) - \frac{1}{\phi_1}\beta_0 - \frac{\beta_{I_4}}{\phi_1}\ln(\beta_{I_4}) + \ln(\beta_{I_3}) - \sum_i \frac{\beta_i}{\phi_1}\ln(X_i) - \ln(I_3) + \frac{\beta_{I_4}}{\phi_1}\ln(C_{I_4}) - \ln(C_{I_3})$$

$$w_F = -\frac{1}{\phi_1}\ln(PW) - \frac{1}{\phi_1}\beta_0 - \frac{\beta_{I_4}}{\phi_1}\ln(\beta_{I_4}) + \ln(\beta_F) - \sum_i \frac{\beta_i}{\phi_1}\ln(X_i) - \ln(F) + \frac{\beta_{I_4}}{\phi_1}\ln(C_{I_4}) - \ln(C_F)$$

where i = A, P, I_1, L_2, I_3, F, L_3,

and $\phi_1 = \beta_{I_4} - 1$

(15c) $$w_{L_2} = -\frac{1}{\phi_2}\ln(PW) - \frac{1}{\phi_2}\beta_0 - \sum_j \frac{\beta_j}{\phi_2}\ln(\beta_j) + \ln(\beta_{L_2}) - \sum_i \frac{\beta_i}{\phi_2}\ln(X_i) - \ln(L_2) - \sum_j \frac{\beta_j}{\phi_2}\ln(C_j) - \ln(C_{L_2})$$

where i = A, P, I_1, L_2,

j = F, I_3, L_3, I_4,

and $\phi_2 = \beta_{I_4} + \beta_{L_3} + \beta_{I_3} + \beta_F - 1,$

(15d)

$$w_P = -\frac{1}{\phi_3}\ln(PW) - \frac{1}{\phi_3}\beta_0 - \sum_j \frac{\beta_j}{\phi_3}\ln(\beta_j) + \ln(\beta_P) - \sum_i \frac{\beta_i}{\phi_3}\ln(X_i) - \ln(P) + \sum_j \frac{\beta_j}{\phi_3}\ln(C_j) - \ln(C_P)$$

$$w_{I_1} = -\frac{1}{\phi_3}\ln(PW) - \frac{1}{\phi_3}\beta_0 - \sum_j \frac{\beta_j}{\phi_3}\ln(\beta_j) + \ln(\beta_{I_1}) - \sum_i \frac{\beta_i}{\phi_3}\ln(X_i) - \ln(I_1) + \sum_j \frac{\beta_j}{\phi_3}\ln(C_j) - \ln(C_{I_1})$$

where i = A, P, I_1,

j = L_2, F, I_3, L_3, I_4,

and $\phi_3 = \beta_{I_4} + \beta_{L_3} + \beta_{I_3} + \beta_F + \beta_{L_2} - 1$

which together with (10) comprise the full model. We note that the marginal product of a variable input in each stage depends on the planned inputs in later stages, as well as the level of inputs in previous stages. Using the error structure discussed above we apply FIML to derive the parameter estimates.

Model D: The Simultaneous Equation Instantaneous Model

Furthermore we estimate a model consisting of equation (11) and the profit maximising conditions for F, I, P, and L. As in model C, we add a composed error term to (11) and a vector of error terms, **w**, *to the first order conditions to capture AE.*

Models B and D are necessary in that they provide a set of results that can be compared to the sequentially planned model. This will help in evaluating the relevance of the latter.

3 Results

Table 2 gives the COLS and FIML estimates for models A $\rightarrow$ D. It is clear that they result in very different parameter estimates of A and B, particularly for the coefficient on fertiliser. A single equation using the temporally disaggregated data is not useful because it does not yield results approximating those obtained with model C. Comparing C and D we find that the parameter estimates are very close indeed. This is true if we sum the coefficients for the disaggregated variables in C.

Turning to TE we note that A and B give similar estimates of σ^2 and λ. We estimate average TE as 84% and 81%, and the range as 94 - 66% and 94 - 54% respectively[16]. *When using the full model estimates change substantially. In particular we find a larger variance and λ . This implies that factors that are within the farmers*

[16]TE is given by $e^{-u} = Y / Y^*$, where Y is actual and Y^* is the potential output. Thus TE is given as a percentage of the potential output that the farmer could have achieved.

Table 2

COLS and FIML estimates of the composite frontier production function parameters (t-ratios)[a]

VARIABLE	Model A	Model B	Model C	Model D
CONSTANT	6.1159 (-------)	5.5018 (------)	6.7517* (102.3000)	6.6489* (102.5606)
A	1.0011* (5.7374)	0.9833* (10.2482)	0.8832* (15.9370)	0.8749* (177379)
F	0.0005 (0.0053)	0.0253* (3.1787)	0.0288* (2.1670)	0.0368* (2.5844)
I_4	0.2645* (2.6468)	-------	0.0373* (2.9071)	-------
I_3	-0.0006 (-0.0051)	-------	0.0115 (1.6215)	-------
I_1	0.0074 (0.7494)	-------	0.0035 (0.2325)	-------
I	-------	0.3795* (2.7677)	-------	0.0600* (3.6781)
L_3	0.0481 (1.2718)	-------	0.0014 (0.2983)	-------
L_2	0.0624 (0.4371)	-------	0.0122* (3.3980)	-------
L	-------	0.1377 (1.4694)	-------	0.0154* (3.5972)
P	0.2439* (2.3278)	0.2454* (2.5094)	0.0566* (5.0493)	0.0560* (5.1102)
σ^2	0.0772 (-------)	0.0920 (-------)	0.1481* (2.5949)	0.1485* (2.6840)
λ	1.5926 (-------)	2.2417 (-------)	3.5549* (2.1655)	3.4858* (2.4695)

[a]*Calculated using the heteroskedastically consistent covariance matrix.*
**Statistically significant at least at the 5% level.*

control dominate the disturbance. The estimates of TE for C and D are virtually identical, with average TE at 76% and the range estimated as 95 - 38% for both models. There is no meaningful difference between the two models to report. This observation is supported by table 3 and the individual estimates, not reproduced here.[17]

Table 3

Frequency distributions of the technical efficiency estimates for models C and D (Cumulative frequency distribution)

FREQUENCY INTERVALS	MODEL C	MODEL D
100.00-95.00	1	1
94.99-90.00	6 (7)	7 (8)
89.99-85.00	7 (14)	6 (14)
84.99-80.00	10 (24)	9 (23)
79.99-75.00	4 (28)	4 (27)
74.99-70.00	5 (33)	6 (33)
69.99-65.00	5 (38)	4 (37)
64.99-60.00	2 (40)	3 (40)
59.99-55.00	3 (43)	3 (43)
54.99-50.00	1 (44)	1 (44)
49.99-45.00	1 (45)	1 (45)
44.99-40.00	1 (46)	1 (46)
39.99-35.00	1 (47)	1 (47)

[17]Evidence presented in Croppenstedt (1993) shows that Individual estimates are very close.

Table 4 reports the frequency distributions for the ratios of the estimated marginal value product (MVP) to marginal factor cost (MFC) ratio (denoted by R) for model C and D. With regard to Irrigation, model D suggests that about 45% of farmers over utilise this input. This changed in model C: for all three inputs 60% of the farmers over utilise (53% in stage 4, 47% in stage 3, and 79% in stage 1). In all cases most of the observations are to be found in the extreme regions. Labour, on the other hand is

Table 4

Frequency distribution of the ratios of the estimated MVP to MFC ratios

INTERVAL OF R	I_4	I_3	I_1	I	F Model C - D	L_3	L_2	L	P Model C - D
< 0.5000	10	16	33	16	11 -5	12	1	4	2 - 5
0.5001-0.7500	9	3	4	4	16 - 9	4	4	5	10 - 12
0.7501-0.9000	4	3	0	1	4 - 12	1	10	9	11 -6
0.9001-0.9500	1	0	0	0	1 - 0	0	2	2	1 - 3
0.9501-1.0000	1	0	0	0	1 - 2	0	3	1	3 - 3
1.0001-1.0500	0	0	0	1	2 - 2	1	3	1	1 - 4
1.0501-1.1000	0	0	0	0	1 - 0	1	4	3	4 - 0
1.1001-1.2500	5	0	0	1	1 - 3	4	11	10	4 - 3
1.2501-1.5000	7	12	0	10	1 - 4	4	7	4	4 - 4
1.5000	10	13	10	14	9 - 10	20	3	8	7 - 7
$0.95 \leq R \leq 1.05$	1	0	0	1	3 - 4	1	6	2	4 - 7

under utilised by most farmers in both models. The observations are noticeably less spread out. For labour in stage 2, 70% of farmers are within 25% efficient, and 13% are within 5% of AE. Finally while ploughing is over utilised by most farmers, 51% of farmers are within 25% of AE in model C.

It is clear from table 4 that the observations are well spread out and that in all cases the majority of farmers are less than 10% AE. However the means of the variables, given in table 5, would hide this divergence from efficient allocation. Indeed for both models the mean of the estimated R would suggest that some variables are allocated efficiently while others show only a relatively small divergence from the optimum. This is *less so in model C were the means for irrigation in stage 1, labour in stage 3 and fertiliser do suggest substantial divergence from profit maximisation.*

Table 5

Means of the estimated MVP to MFC ratios

Variable	*Model C*	*Model D*
F	*0.74**	*0.89**
I_4	*1.15*	-----
I_3	*1.01**	------
I_1	*0.41**	-----
I	-----	*1.18*
L_3	*1.51**	-----
L_2	*1.07*	-----
L	-----	*1.09*
P	*1.05*	*1.01*

**Excluding the values for the observations that were zero.*

We now turn to our findings pertaining to the allocation of fertiliser. The application of fertiliser at the time of sowing is important and we needed to separate it from fertiliser applied as a top dressing. We were not able to include a variable for fertiliser at the time of sowing[18]*. However, results in Croppenstedt (1993) show that using fertiliser at the time of sowing increased yield by 14%*[19]*. Given this information it is not convincing to argue that while 24 farmers applied no fertiliser at the time of sowing they chose to over utilise this input in stage 3*[20]*. Rather we believe that too little fertiliser was applied, but that the timing and interaction of inputs generated this odd result.*

With regard to the issue of timing we found that 36% of farmers irrigated 30 days or more after the date of sowing (for those farmers that used fertiliser) while 21 days is

[18] Including a dummy or the actual values with small constant added caused λ to explode. Using an aggregate fertiliser variable adds nothing to the analysis, i.e. accentuates our results, and would be quite misleading.

[19] We included a dummy variable for fertiliser at the time of sowing in the explanatory variables that we regressed on TE..

[20] The fact that 10 farms did not use a pre-irrigation and only 11 weeded although these activities are considered to raise yield substantially also suggests not to take the result of over utilisation of fertiliser at face value. A detailed account of recommended input levels is given in Bliss and Stern (1982, Ch. 7).

the recommended time for timely sowing. Moreover while fertiliser as a top dressing should be applied immediately after the first irrigation the average time was approximately 9 days later (excluding 2 extreme cases of 33 and 60 days and 7 cases of fertiliser applied before the first irrigation).

Turning to the interaction of inputs, we note that the recommended number of irrigations is 6 (excluding the pre-irrigation). Only 5 farmers had 5 irrigations and the average was 3.6[21]*. Irrigation in stage 4 was, on average, under utilised (excluding farmers not using fertiliser) and from the covariance matrix for C, below, we see that there exist a negative relationship between misallocation of* I_4 *and the stage 3 inputs*[22]*.*

Σ *for model C:*

	I_4	L_3	I_3	F	L_2	P	I_1
I_4	0.5095	−1.4898	−0.9874	−1.6632	−0.0001	0.0669	0.2718
L_3		17.4300	13.8690	19.0920	0.2828	−0.5157	2.4189
I_3			12.5610	16.4310	0.1925	−0.3500	2.2345
F				22.4440	0.2737	−0.5592	3.0321
L_2					0.0850	0.0254	0.1428
P						0.1708	−0.2661
I_1							14.7160

It is our conjecture that the counterintuitive result that fertiliser was over utilised is due to bad timing and the lack of complementary inputs in stage 4. This is possible if both factors work to lower the yield curve of fertiliser, i.e. implying a wasteful use of this input.

6 CONCLUSION

We have shown that when using temporally disaggregated data we need to estimate a simultaneous equation model since the production function alone does not yield meaningful results. With regard to TE the sequential model does not alter the

[21]We also note that the intervals between irrigations were neither regular nor of the recommended length in most cases.

[22]We note that I_3 was, on average, over utilised if we exclude farmers not using fertiliser.

results. However the estimation of AE, while substantiating the overall picture provided by the single stage simultaneous equation model, also generates more detail. This detail sheds light on how the mistakes made in allocating inputs is disaggregated. Moreover the temporal interaction of inputs is brought to the fore. This extra information is particularly important when considering fertiliser and its interaction with the other inputs and vindicates the use of the sequential model.

Our findings show that output could have been 20% higher and that there was a considerable range on TE estimates. This implies that with the same input levels substantial welfare gains could be achieved. The relevance of fertiliser applied at the time of sowing underlines the importance of extension services and probably credit availability for this purpose.

With regard to allocative efficiency it can be noted that we found little of this and again substantial increases in profits are possible. While the means hide the true picture, individual observations showed a wide divergence from the optimum. This is due to farmer error and factors not within their control. The fact that fertiliser was over utilised is surprising and it is suggested that the interaction and timing of inputs in stage 3 and 4 could provide an answer to this counterintuitive result.

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