Ad Valorem and Specific Taxation in a Model of Vertically Related Oligopolies

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Abstract

This paper looks at producers' taxation in a partial equilibrium model of vertically related oligopolies, where a downstream industry produces a final good using the output of an upstream industry as an input. Both ad valorem and specific taxes are considered and formulae expressing their effects on prices and profits are derived, showing how these depend on factors such as demand conditions, technology and market structure. Conditions for taxation to cause price overshifting and to raise profits are also given.

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1 Introduction

This paper examines the impact of ad valorem and specific taxes levied on producers in a model of vertically related oligopolies, where a downstream industry produces a final good using the output of an upstream industry as an input. The analysis of tax incidence focuses on the impact on producers' net prices and profits and conditions under which taxes are overshifted and raise profits are given. In accordance with previous studies, see Seade (1985) and Stern (1987), which consider only the downstream stage of production, tax incidence in the downstream industry turns out to be governed by final demand conditions and downstream market structure. The effects of taxation on the intermediate good price and upstream.

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profits are shown to depend on upstream and downstream market structure, final demand conditions and input substitution in downstream production.

Katz and Rosen (1983), Seade (1985) and Stern (1987) examine the effects of taxing producers in the homogeneous-product conjectural-variation oligopoly framework. The basic model assumes a fixed number of identical firms and constant returns to scale. With perfect competition price overshifting never occurs and therefore also the possibility of profits raising with taxation is ruled out. When the market is monopolized taxation may increase producer's net price; nevertheless a positive effect on profits can never occur. Their main finding is that in oligopoly both the net price and profits may increase with taxation.

Stern (1987) condiders also a model with free entry where in equilibrium profits are zero and the focus is on the effects of taxation upon output per firm, price and the number of firms. The impact of entry for tax incidence is also analyzed by Besley (1989). They show that taxation may induce entry and that price overshifting is more likely in the free entry case than with firms fixed in number. Myles (1987) explores the effects of taxation in an oligopolistic industry with free entry when taxes are levied in a related competitive industry, the interaction between the two markets arising via consumers' tastes.

All the models referred to above consist of a single oligopolistic industry facing a given market demand. Tax incidence in vertically related industries has been analysed by Myles (1989), who considers in a general equilibrium framework[1] the two polar cases of downstream (upstream) monopoly and upstream (downstream) competitive sector. He then focuses on tax reforms, where the government aims at improving welfare by introducing taxes (and subsides) on the final and intermediate goods while raising no revenue. Konishi (1990) examines a model in which a number of competitive industries supplies inputs to a free entry Cournot oligopoly producing a final good, showing that market performance can be improved by taxing the intermediate inputs.

All the papers referred to above focus on specific taxation. The recognition that ad valorem and specific taxes are not equivalent under monopoly dates back to Suits and Musgrave (1953), who show that if the same revenue is raised with a specific and an ad valorem tax, the former will bring about a higher price. Kay and Keen (1983) look at the optimal balance between the two kind of taxes in a model of perfect competition with endogenous quality and in a model of monopolistic competition with product variety. They show that prices are affected more by specific than ad valorem taxes, while ad valorem taxes are more powerful

[1]Myles's model is general equilibrium because it takes account of the impact of monopoly profits on final demand.

in controlling product quality and variety. Diericks, Matutes and Neven (1988) compare specific and ad valorem taxation in a model of Cournot oligopoly where firms differ in productive efficiency showing that ad valorem taxes are more likely to improve aggregate productive efficiency by raising market shares of low cost firms.

The paper is organized as follows. Section 2 describes the model and derives equilibrium prices and profits. Comparative statics begins with section 3, which illustrates the effects of taxation on the final and intermediate good prices. Subsection 3.1 gives the conditions for price overshifting. To illustrate the role of market structure, final demand conditions and downstream technology for tax incidence, subsection 3.2 examines the results for three typical final demand curves, namely isoelastic, exponential and linear demand. The formulae expressing the impact of taxation on profits and the conditions for profitable tax increases are contained in section 4. Conclusions are given in Section 5.

2 Equilibrium prices and profits

The model consists of two vertically related industries, where a downstream industry produces a final good using two inputs: labour and an intermediate good produced by an upstream sector. Labour is the only input in upstream production. In both industries product is homogeneous, technology is constant returns to scale and no entry / exit occurs so that the number of firms is fixed.[2] Each oligopolist has a conjecture about the way the other firms in the same industry will change their output levels in response to changes in its own output. These conjectural variations capture the degree of competition among oligopolists belonging to the same industry and this approach has the merit of encompassing a wide range of market structures as particular cases, including the two polar cases of monopoly and competitive pricing. Symmetry is also assumed: oligopolists belonging to the same industry have identical cost functions and conjectures.

Competition between industries is modelled assuming that downstream oligopolists act as perfect competitors in the market for the intermediate good. This means that upstream oligopolists move first by setting the price for the intermediate good and downstream oligopolists move second by setting the price of the final good taking the price of input as given. This is what Waterson (1980a) refers to as

[2]Quoting Katz and Rosen (1983, p. 10): "One may think of the model in two ways. First, it can be viewed as a short run analysis of a market in which capital stocks are fixed. Second, it can be viewed as a long run analysis of a market in which existing firms can adjust the levels of all productive inputs but sufficiently high barriers exist to preclude entry of new firms".

arms-length pricing. Technically the model is solved in three steps. First, downstream profits are maximized for given price of the intermediate good. Second, the derived demand facing the upstream industry is obtained and finally upstream profits are maximized.

The market inverse demand function for the final good is

$$q = q(mx) \tag{1}$$

where m is the number of downstream firms, x the output of each firm and q its price; mx is total output. It is assumed that $q(mx) \in C^3$, $q(0) > 0$ and $q'(mx) < 0$.

Each firm is assumed to maximize profits:

$$\pi^d = [(1-t_v)q(mx) - t_s - C(p,w)]x \tag{2}$$

where p is the price of the intermediate good, w the wage rate and $C(p,w) \in C^3$ is the average (and marginal) cost function, which is assumed to be nondecreasing, linearly homogeneous and concave in input prices. Two kind of taxes are levied on downstream producers: a specific tax t_s and an ad valorem tax at a tax inclusive rate $t_v < 1$. The cost function embodies the assumption that downstream producers are price takers in the input markets. The assumption of arms-length pricing implies that in the market for the intermediate good there is no market power on the demand side. The labour market is assumed to be competitive and the supply for labour to be perfectly elastic, so that the wage rate is given.

Each oligopolist conjectures that changes in its own output will cause the other firms to respond by changing outputs levels as well. Assuming linear responses the conjectural variation is constant and is defined as:

$$\frac{\partial(m-1)x}{\partial x} = v^d, \qquad -1 \leq v^d \leq m-1 \tag{3}$$

$v^d = -1$ corresponds to Bertrand behaviour or marginal cost pricing; Cournot conjectures give $v^d = 0$; when oligopolists collude $v^d = m-1$; if the industry is monopolized then $v^d = 0$ and $m = 1$.

The first order condition for a maximum of (2) is that *perceived* marginal profits are zero:

$$(1-t_v)[q(mx) + xq'(mx)(1+v^d)] - t_s - C(p,w) = 0 \tag{4}$$

This can be written as:

$$q(mx)\left[1 - \frac{\gamma^d}{\epsilon^d(mx)}\right] - \frac{C(p,w)+t_s}{1-t_v} = 0 \tag{5}$$

where $\gamma^d \equiv (1 + v^d)/m$, thus $0 \leq \gamma^d \leq 1$. $\epsilon^d(mx) \equiv -q(mx)/[mx\, q'(mx)]$ is the market elasticity of demand, while each oligopolist perceives the elasticity to be $\epsilon^d(mx)/\gamma^d$.

The second order condition is:

$$E^d(mx) < \frac{2}{\gamma^d} \tag{6}$$

where $E^d(mx) \equiv -mx\, q''(mx)/q'(mx)$ is Seade's (1985) elasticity of the slope of inverse demand.

Eq. (5) implicitly defines the equilibrium output mx. The condition for this to be positive, Stern's (1987, eq. 5) existence condition, is:

$$\begin{aligned} \epsilon^d(mx) &> \gamma^d \\ t_v &< 1 \\ t_s &> -C(p,w) \end{aligned} \tag{7}$$

This requires the perceived elasticity to be greater than one; market elasticity can instead be less than one if $\gamma^d < 1$.

Following Seade (1985, eq. 14), a stability condition can be the following:

$$E^d(mx) < \frac{1}{\gamma^d} + 1 \tag{8}$$

This ensures stability in response to common (symmetric) disturbances to the equilibrium. In other words, were all firms forced to expand (reduce) output away from the equilibrium by the same amount $\delta > 0$ ($\delta < 0$), then the stability condition (8) would ensure that perceived marginal profits become negative (positive), thus giving the oligopolists an incentive to reduce (increase) output. Notice that $1/\gamma^d + 1 \leq 2/\gamma^d$, thus the stability condition implies the second order condition.

The comparative statics below will make extensive use of the equivalent stability condition:

$$F^d(q) > 1 - \frac{\epsilon^d(q)}{\gamma^d}, \qquad \text{where} \quad F^d(q) = \frac{q\epsilon^d_q(q)}{\epsilon^d(q)} \tag{9}$$

see Stern (1987, eq. 6); $\epsilon^d(q) \equiv -q\,\chi'(q)/\chi(q)$ is the elasticity of the direct demand $mx = \chi(q)$, ϵ^d_q is the derivative of ϵ^d with respect to q and $F^d(q)$ is the price elasticity of the elasticity of direct demand. The relation between (8) and (9) comes from $\epsilon^d(q)E^d(mx) = 1 + \epsilon^d(q) - F^d(q)$, see appendix A.

In terms of the final good price the first order condition (5) can be written as:

$$q\left[1-\frac{\gamma^d}{\epsilon^d(q)}\right]-\frac{C(p,w)+t_s}{1-t_v}\equiv f(q,p,w,t_v,t_s)=0 \tag{10}$$

This is an implicit function of q. If the stability condition (9) is met, then the partial derivative of (10) with respect to q is positive (see (12) below), therefore the conditions of the implicit function theorem apply and the equilibrium price can be defined as:

$$q=\phi(p,w,t_v,t_s) \tag{11}$$

Letting subscripts denote partial derivatives (so f_q is the partial derivative of $f(.)$ with respect to q):

$$\begin{aligned}
f_q &= 1-\frac{\gamma^d}{\epsilon^d(q)}[1-F^d(q)]>0\\
\phi_p &= -\frac{f_p}{f_q}=\frac{C_p(p,w)}{1-t_v}\frac{1}{f_q}>0\\
\phi_{tv} &= -\frac{f_{tv}}{f_q}=\frac{C(p,w)+t_s}{(1-t_v)^2}\frac{1}{f_q}=\frac{q}{1-t_v}\left[1-\frac{\gamma^d}{\epsilon^d(q)}\right]\frac{1}{f_q}>0\\
\phi_{ts} &= -\frac{f_{ts}}{f_q}=\frac{1}{1-t_v}\frac{1}{f_q}>0
\end{aligned} \tag{12}$$

where $f_q>0$ from the stability condition (9).

The next step is to determine the equilibrium price for the intermediate good. Consider n identical upstream firms. Assuming that downstream oligopolists are the only customers of upstream firms the the market demand facing the upstream industry is given by the sum of downstream conditional input demands for the intermediate good. By Shephard's lemma the (direct) derived demand for the intermediate good is equal to $ny=C_p(p,w)\chi(q)$, where y is the output of each upstream firm. After substituting for (11) this becomes:

$$ny=C_p(p,w)\chi[\phi(p,w,t_v,t_s)]\equiv\psi(p,w,t_v,t_s) \tag{13}$$

Derived demand is negatively sloped if:

$$\psi_p(p,w,t_v,t_s)=C_{pp}\chi+C_p\chi'\phi_p<0 \tag{14}$$

A sufficient condition for ψ_p to be negative and finite for each $p>0$ is that inputs are not perfect substitutes in downstream production. Then $C_{pp}\leq 0$ (and finite), $C_p>0$, $\phi_p>0$; also $\chi'=1/q'<0$ from (1). If $\psi_p<0$ then (13) has an inverse, the inverse derived demand:

$$p = p(ny, w, t_v, t_s) \tag{15}$$

Let i_s be the specific tax and i_v the ad valorem tax levied on upstream producers. Profits of a representative oligopolist are:

$$\pi^u = [(1 - i_v)p(ny, w, t_v, t_s) - i_s - wa_{Ly}]y \tag{16}$$

where wa_{Ly} is the constant average (and marginal) cost of labour, which is assumed to be the only input. This is far from being restrictive: in this partial equilibrium setting the wage rate is given and so would be the price of other inputs entering the cost function of upstream producers. In other words, adding an input would not alter the results because, with fixed input prices, input substitution never occurs and so average and marginal costs are constant.

The first order condition for a maximum of (16) can be written as:

$$p\left[1 - \frac{\gamma^u}{\epsilon^u(p, w, t_v, t_s)}\right] - \frac{wa_{Ly} + i_s}{1 - i_v} \equiv g(p, w, t_v, t_s, i_v, i_s) = 0 \tag{17}$$

where

$$\epsilon^u(p, w, t_v, t_s) \equiv -\frac{p\psi_p}{\psi} = \frac{wC_w(p, w)}{C(p, w)}\sigma(p, w) + \frac{pC_p(p, w)}{C(p, w) + t_s}\epsilon^d(q)\omega(q) \tag{18}$$

and

$$\omega(q) \equiv \frac{1 - \frac{\gamma^d}{\epsilon^d(q)}}{1 - \frac{\gamma^d}{\epsilon^d(q)}[1 - F^d(q)]} > 0 \tag{19}$$

$\epsilon^u(.)$ is the price elasticity of the direct derived demand (13);[3] $\gamma^u \equiv (1 + v^u)/n$ where v^u is the conjectural variation of upstream oligopolists which is defined as $v^u = \partial[(n-1)y]/\partial y$, with $-1 \leq v^u \leq n - 1$ and constant; thus $0 \leq \gamma^u \leq 1$. $\sigma(p, w)$ is the elasticity of input substitution between labour and the intermediate good in downstream production, $q = \phi(p, w, t_v, t_s)$ from eq. (11) and $\omega(q)$ is positive by the existence and stability conditions (7) and (9). The elasticity (18) is derived in appendix B.

The existence and stability conditions are respectively:

[3]The elasticity of derived demand under profit maximizing monopoly has been first presented by Yeung (1972); Waterson (1980b) extends it to the constant returns, homogeneous-product conjectural-variation oligopoly model. Assuming isoelastic final demand they show that the expression for the elasticity of derived demand is identical to the corresponding expression under perfect competition. This point is taken up by de Meza (1982) who shows that with a general final demand curve market structure does influence the elasticity of derived demand. To see this in terms of eq. (18) notice that downstream market structure, represented by γ^d, influences $\omega(q)$ and q, see eqs. (19) and (10). If final demand is of constant elasticity then ϵ^d is constant, $F^d = 0$ and $\omega(q) \equiv 1$, thus (18) is independent of γ^d.

$$\epsilon^u > \gamma^u, \qquad i_s > -wa_{Ly}, \qquad i_v < 1 \tag{20}$$

$$F^u > 1 - \frac{\epsilon^u}{\gamma^u} \qquad \text{where} \qquad F^u \equiv \frac{p\epsilon^u_p}{\epsilon^u} \tag{21}$$

F^u is the price elasticity of the elasticity of derived demand (13); ϵ^u_p is the derivative of ϵ^u with respect to p.

When the stability condition (21) is met $g_p > 0$, thus the conditions of the implicit function theorem apply and eq. (17) defines the equilibrium price for the intermediate good:

$$p = \xi(w, t_v, t_s, i_v, i_s) \tag{22}$$

The equilibrium price for the final good is determined by inserting (22) into (11). Substituting for the equilibrium prices into the respective demand functions gives the equilibrium quantities. Finally, plugging equilibrium prices and quantities into (2) and (16) gives the equilibrium profits of downstream and upstream oligopolists respectively.

This framework can now be employed to carry out the analysis of tax incidence. Eq. (17) determines the effect of taxes on the intermediate good price; the impact of taxation on the final good price is governed by eqs. (17) and (10). Katz and Rosen (1983), Seade (1985) and Stern (1987) consider only one stage of production which in the context of the present model corresponds to the downstream sector and therefore their analysis is limited to the comparative statics of eq. (10) with p exogenous. The original feature of this model is the introduction of the upstream oligopoly setting the price of the intermediate good which makes p endogenously determined by eq. (17).

3 The impact of taxation on prices

Table 1 shows in the first column the partial derivatives expressing the impact of taxation on the final and intermediate good prices, together with the *indicator shifting coefficients* (second column) and the conditions for price overshifting (third column) that will be described shortly.

The partial derivatives with respect to the taxes levied on upstream producers are all positive: i_v and i_s raise upstream costs, thus the price of the intermediate good raises as well; this in turn represents a cost shift for downstream oligopolists (provided that $C_p > 0$) who increase the price of the final good.

More difficult is to assess the impact of the taxes levied on downstream producers. Consider first the effect on the price for the intermediate good. Of course,

Table 1: The impact of taxation on prices

$f_q = 1 - \frac{\gamma^d}{\epsilon^d}(1 - F^d) > 0, \qquad F^d = \frac{q\epsilon^d_q}{\epsilon^d}$		
$\epsilon^u = \frac{wC_w(p,w)}{C(p,w)}\sigma(p,w) + \frac{pC_p(p,w)}{C(p,w)+t_s}\epsilon^d(q)\omega(q), \quad \omega(q) = \frac{1 - \frac{\gamma^d}{\epsilon^d(q)}}{1 - \frac{\gamma^d}{\epsilon^d(q)}[1 - F^d(q)]}$		
$g_p = 1 - \frac{\gamma^u}{\epsilon^u}(1 - F^u) > 0, \qquad F^u = \frac{p\epsilon^u_p}{\epsilon^u}$		
$\frac{\partial p}{\partial t_v} = -\gamma^u \frac{p\epsilon^u_{tv}}{(\epsilon^u)^2}\frac{1}{g_p}$		$\frac{\partial p}{\partial t_v} > 0$ iff $F^d < 0$[a]
$\frac{\partial p}{\partial t_s} = -\gamma^u \frac{p\epsilon^u_{ts}}{(\epsilon^u)^2}\frac{1}{g_p}$		if $F^d \leq 0$ then $\frac{\partial p}{\partial t_s} > 0$[a]
$\frac{\partial p}{\partial i_v} = \frac{1}{1-i_v} p s^u_{iv}$	$s^u_{iv} = \left(1 - \frac{\gamma^u}{\epsilon^u}\right)\frac{1}{g_p}$	$s^u_{iv} > 1$ iff $F^u < 0$
$\frac{\partial p}{\partial i_s} = \frac{1}{1-i_v} s^u_{is}$	$s^u_{is} = \frac{1}{g_p}$	$s^u_{is} > 1$ iff $F^u < 1$
$\frac{\partial q}{\partial t_v} = \frac{1}{1-t_v}\left(q + C_p\frac{\partial p}{\partial t_v}\right) s^d_{tv}$	$s^d_{tv} = \frac{q\left(1 - \frac{\gamma^d}{\epsilon^d}\right) + C_p\frac{\partial p}{\partial tv}}{q + C_p\frac{\partial p}{\partial tv}}\frac{1}{f_q}$	$s^d_{tv} > 1$ iff $F^d < 0$[b]
$\frac{\partial q}{\partial t_s} = \frac{1}{1-t_v}\left(1 + C_p\frac{\partial p}{\partial t_s}\right) s^d_{ts}$	$s^d_{ts} = \frac{1}{f_q}$	$s^d_{ts} > 1$ iff $F^d < 1$
$\frac{\partial q}{\partial i_v} = \frac{1}{1-t_v}\frac{1}{1-i_v} pC_p s^u_{iv} s^d_{iv}$	$s^d_{iv} = \frac{1}{f_q}$	$s^d_{iv} > 1$ iff $F^d < 1$
$\frac{\partial q}{\partial i_s} = \frac{1}{1-t_v}\frac{1}{1-i_v} C_p s^u_{is} s^d_{is}$	$s^d_{is} = \frac{1}{f_q}$	$s^d_{is} > 1$ iff $F^d < 1$

(a) assuming that $\mid F^d\omega \mid > \mid q\omega_q \mid$
(b) see appendix C

if upstream producers price at marginal cost ($\gamma^u = 0$) then downstream taxation does not affect p. When upstream oligopolists price above marginal cost ($\gamma^u \neq 0$) then $\partial p/\partial t_v \gtrless 0$ iff $\epsilon^u_{tv} \lesseqgtr 0$ and $\partial p/\partial t_s \gtrless 0$ iff $\epsilon^u_{ts} \lesseqgtr 0$. In words, a tax on downstream oligopolists raises p if and only if it lowers the elasticity of derived demand. The expressions for ϵ^u_{tv} and ϵ^u_{ts} are

$$\epsilon^u_{tv} = \frac{pC_p}{C+t_s}(F^d\omega + q\omega_q)\frac{\epsilon^d}{q}\phi_{tv} \tag{23}$$

$$\epsilon^u_{ts} = -\frac{pC_p}{(C+t_s)^2}\epsilon^d\omega + \frac{pC_p}{C+t_s}(F^d\omega + q\omega_q)\frac{\epsilon^d}{q}\phi_{ts} \tag{24}$$

ϵ^u_{tv} takes the sign of $F^d\omega + q\omega_q$: ω and q are positive but F^d and ω_q may take positive and negative values. Notice that if final demand is of constant elasticity then $F^d = 0$, which implies $\omega \equiv 1$ and $\omega_q = 0$, see eq. (19), thus $\epsilon^u_{tv} = 0$ and t_v does not affect p. In general, a relation between the signs of F^d and ω_q cannot be found; also, there is no economic interpretation for ω_q, which contains the derivative of F^d and therefore the third derivative of the demand function $\chi(q)$. However, one may assume that $\mid F^d\omega \mid > \mid q\omega_q \mid$, so that $\epsilon^u_{tv} \gtrless 0$ iff $F^d \gtrless 0$, which implies the condition $\partial p/\partial t_v > 0$ iff $F^d < 0$ shown in table 1. This conjecture will be confirmed in section 3.2, where some typical demand functions are considered. Turning to the specific tax t_s, notice that if final demand is isoelastic ($F^d = 0$, $\omega_q = 0$) then $\epsilon^u_{ts} < 0$ and t_s raises p. Assuming $\mid F^d\omega \mid > \mid q\omega_q \mid$ implies that if $F^d < 0$ then $\epsilon^u_{ts} < 0$ and $\partial p/\partial t_s > 0$, which is only a sufficient condition because the first term in (24) is negative.

Consider now the impact of t_v and t_s on the price for the final good:

$$\frac{\partial q}{\partial t_v} = \phi_{tv} + \phi_p\frac{\partial p}{\partial t_v} = \frac{1}{1-t_v}\left[q\left(1 - \frac{\gamma^d}{\epsilon^d}\right) + C_p\frac{\partial p}{\partial t_v}\right]\frac{1}{f_q} \tag{25}$$

$$\frac{\partial q}{\partial t_s} = \phi_{ts} + \phi_p\frac{\partial p}{\partial t_s} = \frac{1}{1-t_v}\left(1 + C_p\frac{\partial p}{\partial t_s}\right)s^d_{ts} \tag{26}$$

In both derivatives the first term, which is positive, expresses the impact of increased tax liabilities and the second the effect that goes through the change in the price of the intermediate good. As explained above, if $\gamma^d = 0$ then the second term is zero, thus (25) and (26) are positive. In general, since $\partial p/\partial t_v$ and $\partial p/\partial t_s$ may take negative values, it cannot be excluded that (25) and (26) are negative as well. However, this seems a remote possibility and one may reasonably assume that $q(1 - \gamma^d/\epsilon^d) + C_p(\partial p/\partial t_v) > 0$ and $1 + C_p(\partial p/\partial t_s) > 0$.

3.1 The indicator shifting coefficients

The derivatives in the first column of table 1 express the impact of taxation on producers' *gross* prices q and p. From the point of view of producers what really matters is the difference between the impact on the gross price and the tax induced cost shifts, which gives the effect on the *net* price. For each unit of intermediate good produced and sold an upstream oligopolist receives p from his customer and pays $i_s + i_v p$ to the government and wa_{Ly} to the input supplier. The price p is the *gross* price, or revenue per unit of output. Let $G^u(p, w, i_s, i_v) \equiv i_s + i_v p + wa_{Ly}$ be total outlays per unit of output, where $p = \xi(w, t_v, t_s, i_v, i_s)$. The producers' net price is defined as the difference between the gross price p and total outlays G^u, which gives profits per unit of output. A tax is said to cause *price overshifting* if it increases producers' net price.

Following Seade (1985) define the *shifting coefficient* of a tax $\tau \in T = \{i_v, i_s\}$ as the ratio between the increase in the gross price p and total outlays G^u

$$S^u_\tau \equiv \frac{dp}{dG^u} = \frac{\partial p/\partial \tau}{i_v \frac{\partial p}{\partial \tau} + \frac{\partial G^u}{\partial \tau}}, \qquad \tau \in T = \{i_v, i_s\} \tag{27}$$

so that price overshifting occurs if and only if $S^u_\tau > 1$. Applying the following monotonically increasing transformation to the shifting coefficients (27) one obtains the *indicator shifting coefficients* which are reported in table 1

$$s^u_\tau \equiv \frac{(1 - i_v)S^u_\tau}{1 - i_v S^u_\tau} \equiv (1 - i_v)\frac{\partial p/\partial \tau}{\partial G^u \partial \tau}, \qquad \tau \in T = \{i_v, i_s\} \tag{28}$$

Given that $i_v < 1$, the property of these coefficients is that $s^u_\tau \gtreqless 1$ if and only if $S^u_\tau \gtreqless 1$; also, if $i_v = 0$ then $s^u_\tau = S^u_\tau$. The indicator shifting coefficients are convenient because they fit into the derivatives in the first column of table 1 and because they give the conditions for price overshifting shown in the third column.

If upstream oligopolists price at marginal cost ($\gamma^u = 0$), then taxes do not affect the net price and $s^u_{iv} = s^u_{is} = 1$. If $0 < \gamma^u \leq 1$ then overshifting may occur; note that the stability condition (21) does not rule out this possibility. Specific taxation i_s causes overshifting if and only if $F^u < 1$, while ad valorem taxation i_v if and only if $F^u < 0$. Specific taxation is therefore more likely to cause overshifting than ad valorem taxation. Moreover, the existence condition (20) implies that $S^u_{is} > S^u_{iv}$, the shifting coefficient of specific taxation is always greater than the corresponding coefficient of ad valorem taxes.

Turning now to downstream oligopolists, let $G^d(q, p, w, t_s, t_v) \equiv t_s + t_v q + C(p, w)$ be total outlays per unit of output of each downstream producer, where $p = \xi(w, t_v, t_s, i_v, i_s)$ and $q = \phi(p, w, t_v, t_s)$. Again, price overshifting occurs when a tax causes the gross price q to increase more than total outlays. For any tax $\tau \in T = \{t_v, t_s, i_v, i_s\}$ the corresponding shifting coefficient is defined as

$$S_\tau^d \equiv \frac{dq}{dG^d} = \frac{\partial q/\partial \tau}{t_v \frac{\partial q}{\partial \tau} + C_p \frac{\partial p}{\partial \tau} + \frac{\partial G^d}{\partial \tau}}, \qquad \tau \in T = \{t_v, t_s, i_v, i_s\} \tag{29}$$

so that price overshifting occurs if and only if $S_\tau^d > 1$. The indicator shifting coefficient is

$$s_\tau^d \equiv \frac{(1-t_v)S_\tau^d}{1 - t_v S_\tau^d} \equiv (1-t_v)\frac{\partial q/\partial \tau}{C_p \frac{\partial p}{\partial \tau} + \frac{\partial G^u}{\partial \tau}}, \qquad \tau \in T \tag{30}$$

Again, $t_v < 1$ implies that $s_\tau^d \gtreqless 1$ if and only if $S_\tau^d \gtreqless 1$; if $t_v = 0$ then $s_\tau^u = S_\tau^u$.

If downstream oligopolists price at marginal cost ($\gamma^d = 0$) then $s_\tau^d = 1$ for any $\tau \in T$. If $0 < \gamma^d \leq 1$ the conditions for price overshifting in the downstream industry depend entirely on final demand conditions, expressed by the price elasticity of the elasticity of demand. Table 1 shows that the ad valorem tax t_v causes overshifting if and only if $F^d < 0$. This result is not immediate, the proof and the assumptions introduced to derive it are given in appendix C. The taxes t_s, i_v and i_s raise downstream unit profits if and only if $F^d < 1$. Notice also that the existence condition (7) implies $S_{ts}^d > S_{tv}^d$.

3.2 The determinants of tax incidence

It has been shown that tax incidence is governed by the price elasticities of final and derived demand. These factors are now examined in more detail.

Consider first the final good price. The conditions for overshifting shown in table 1 are $s_{tv}^d > 1$ iff $F^d < 0$ and $s_\tau^d > 1$ iff $F^d < 1$, $\tau = t_s, i_v, i_s$, where

$$F^d(q) = 1 + \epsilon^d(q) - E^d(q), \qquad F^d \equiv \frac{q\epsilon_q^d}{\epsilon^d}, \quad E^d \equiv -\frac{q\chi''}{\chi'} \tag{31}$$

E^d is the elasticity of the slope of (direct) demand, see appendix A. From (31) one notice that if final demand is concave or linear ($\chi'' \leq 0$) then overshifting never occurs because $E^d \leq 0$ and $F^d > 1$. Overshifting may occur only with convex demand functions ($\chi'' > 0$). As an illustration tables 2 and 3 show the results for three different final demand curves, namely isoelastic (ISO), exponential (EXP), which are strongly convex, and linear (LIN) demand. ISO has the property that $F^d = 0$, thus $s_{tv}^d = 1$ and $s_\tau^d > 1$, $\tau = t_s, i_v, i_s$; with EXP $F^d = 1$, hence $s_{tv}^d < 1$ and $s_\tau^d = 1$; finally, with LIN all taxes are undershifted.

Tax incidence on the intermediate good price is determined by the partial derivatives of the elasticity of derived demand (18):

Table 2

	$mx = \chi(q)$	$\epsilon^d(q)$	$F^d(q)$	elasticity of derived demand ϵ^u
ISO	$mx = (bq)^{-\bar{\epsilon}}$	$\bar{\epsilon}$	0	$\epsilon^u(p,w,t_s) = \frac{wC_w}{C}\sigma + \frac{pC_p}{C+t_s}\bar{\epsilon}$
EXP	$mx = A\exp^{-bq}$	bq	1	$\epsilon^u(p,w,t_v) = \frac{wC_w}{C}\sigma + \frac{pC_p}{C+t_s}(\epsilon^d - \gamma^d) \quad = \frac{wC_w}{C}\sigma + pC_p\frac{b}{1-t_v}$
LIN	$mx = A - bq$	$\frac{q}{A/b - q}$	$\frac{A/b}{A/b - q} > 1$	$\epsilon^u(p,w,t_v,t_s) = \frac{wC_w}{C}\sigma + \frac{pC_p}{C+t_s}\frac{\epsilon^d - \gamma^d}{1+\gamma^d} \quad = \frac{wC_w}{C}\sigma + \frac{pC_p}{(1-t_v)A/b - C - t_s}$

	ϵ^u_p	ϵ^u_{tv}	ϵ^u_{ts}
ISO	$(1-\sigma)(\bar{\epsilon}-\sigma)\frac{wC_wC_p}{C^2} \qquad \forall t_v, \; t_s = 0$	0	–
EXP	$(1-\sigma)(\epsilon^d - \gamma^d - \sigma)\frac{wC_wC_p}{C^2} + \frac{pC_p^2}{C}\frac{b}{1-t_v}$	+	0
LIN	$(1-\sigma)\left(\frac{\epsilon^d - \gamma^d}{1+\gamma^d} - \sigma\right)\frac{wC_wC_p}{C^2} + \frac{pC_p^2}{C}\frac{\epsilon^d_q}{(1-t_v)(1+\gamma^d)^2}$	+	+

Table 3

	$\frac{\partial p}{\partial t_v}$	$\frac{\partial p}{\partial t_s}$	$s^d_{tv} > 1$ iff $F^d < 0$	$s^d_\tau > 1$ iff $F^d < 1$ $\tau = t_s, i_v, i_s$	$s^u_{iv} > 1$ iff $F^u < 0$	$s^u_{is} > 1$ iff $F^u < 1$
ISO	0	+	$s^d_{tv} = 1$	$s^d_\tau > 1$	$s^u_{iv} > 1$ iff $1 < \sigma < \bar{\epsilon}$ or $\bar{\epsilon} < \sigma < 1$	if $\bar{\epsilon} \geq \sigma - 2$ then $s^u_{is} > 1$
EXP	–	0	$s^d_{tv} < 1$	$s^d_\tau = 1$	if $\sigma \leq 1$ and $\epsilon^d \geq \sigma + \gamma^d$ or $\sigma \geq 1$ and $\epsilon^d \leq \sigma + \gamma^d$ then $s^u_{iv} < 1$	if $\epsilon^d \geq \sigma - 2 - \gamma^d$ then $s^u_{is} > 1$
LIN	–	–	$s^d_{tv} < 1$	$s^d_\tau < 1$	if $\sigma \leq 1$ and $\epsilon^d \geq (1+\gamma^d)\sigma + \gamma^d$ or $\sigma \geq 1$ and $\epsilon^d \leq (1+\gamma^d)\sigma + \gamma^d$ then $s^u_{iv} < 1$	

$$\epsilon_p^u = (1-\sigma)(\epsilon^d\omega - \sigma)\frac{wC_wC_p}{C^2} + \frac{pC_p}{C}(F^d\omega + q\omega_q)\frac{\epsilon^d}{q}\phi_p \tag{32}$$

which has been computed assuming that σ, the elasticity of substitution between labour and the intermediate good in downstream production, is constant. Hence the downstream cost function is assumed to be a CES with constant returns to scale. When $\sigma = 0$ technology is Leontief, $\sigma = 1$ gives the Cobb- Douglas case, if $\sigma \to \infty$ inputs are perfect substitutes. The derivatives ϵ_{tv}^u and ϵ_{ts}^u are given in (23) and (24). The elasticity of derived demand ϵ^u and the equilibrium price for the intermediate good p depend on final demand conditions (expressed by ϵ^d and F^d), downstream taxes (t_v, t_s), input substitution in downstream production (σ) and downstream market structure (γ^d). All these factors, together with upstream market structure (γ^u), determine the impact of taxation on p. With a general final demand curve though, it is difficult to identify the role played by each factor, thus the three special cases will be examined.

Notice first, see table 2, that ϵ^u and therefore p are independent of γ^d for all the three demand curves considered: tax incidence on p is not affected by downstream market structure.[4] Second, with ISO ϵ^u and p are independent of t_v while t_s raises p, with EXP ϵ^u and p are independent of t_s while t_v lowers p, with LIN both t_v and t_s decrease p, which confirms that the *ad hoc* assumption introduced on p. 10 seems reasonable.

Finally, the impact of upstream taxation. Table 1 shows that $s_{iv}^u > 1$ iff $F^u < 0$; from the definition of F^u this condition is equivalent to $\epsilon_p^u < 0$. With ISO $\epsilon_p^u < 0$ iff $(1-\sigma)(\bar{\epsilon} - \sigma) < 0$, thus overshifting occurs iff $1 < \sigma < \bar{\epsilon}$ or $\bar{\epsilon} < \sigma < 1$. With EXP and LIN ϵ_p^u contains a term (the second) which is positive, thus only sufficient conditions for undershifting ($s_{iv}^u < 1$) can be found. Turning to the specific tax i_s, the overshifting condition is $s_{is}^u > 1$ iff $F^u < 1$ which can be written $p\epsilon_p^u < \epsilon^u$. After some algebra one finds that

$$\text{ISO} \qquad s_{is}^u > 1 \quad \text{iff} \quad \sigma(\sigma - \bar{\epsilon} - 2) < \frac{wC_w}{pC_p}\sigma + \frac{pC_p}{wC_w}\bar{\epsilon}$$

$$\text{EXP} \qquad s_{is}^u > 1 \quad \text{iff} \quad \sigma(\sigma - \epsilon^d - \gamma^d - 2) < \frac{wC_w}{pC_p}\sigma$$

Both terms on the right hand side are positive, thus a sufficient condition for overshifting is that the term on the left hand side is negative or zero. Corresponding conditions for LIN cannot be found.

[4] The result that with linear final demand the elasticity of derived demand is independent of downstream market structure is also noticed by de Meza (1982).

4 The impact of taxation on profits

Taxation causes (gross) prices p and q to increase and this, since final and derived demands are downward sloping, causes equilibrium outputs to fall. If producers' net price (profit per unit of output) falls then also profits must fall. But if taxation causes price overshifting (an increase in profits per unit of output) then the possibility of a positive impact on profits arises.

Considering only one stage of production, Seade (1985) and Stern (1987) have shown that taxation has always a negative impact on profits in the two polar cases of monopoly and perfect competition.[5] Profitable tax increases arise as a distinct possibility when the market is oligopolistic. Assuming constant returns to scale, Seade (1985) finds that specific taxation increases profits when the elasticity of the slope of inverse final demand is greater than two. The same condition is expressed by Stern (1987) as $F > 1 - \epsilon$, where ϵ is the elasticity and F the price elasticity of the elasticity of final demand.

4.1 Downstream profits

Table 4 shows the derivatives expressing the impact of taxation on profits. If downstream producers price at marginal cost ($\gamma^d = 0$) then all the derivatives are zero, because profits are zero in equilibrium. If oligopolists collude, or if the industry is monopolized ($\gamma^d = 1$) then all the derivatives are negative and taxation reduces profits. When the downstream market is oligopolistic ($0 < \gamma^d < 1$) ad valorem taxation t_v raises profits if $F^d < \gamma^d - \epsilon^d < 0$, which is only a sufficient condition, the proof is given in appendix D. With ISO, EXP and LIN t_v never causes price overshifting, thus profits must fall. The taxes t_s, i_v and i_s raise downstream profits if and only if $F^d < 1 - \epsilon^d$. Thus with EXP and LIN ($F^d > 1$) profits always fall, while with ISO ($F^d = 0$), which causes price overshifting, profits increase if and only if $\bar{\epsilon} < 1$.

4.2 Upstream profits

Again, if $\gamma^u = 0$ then all the derivatives are zero and if $\gamma^u = 1$ taxation reduces profits. When the upstream market is oligopolistic ($0 < \gamma^u < 1$) then the taxes i_v and i_s may increase profits, the corresponding necessary and sufficient conditions are shown in table 4. Difficult to assess is the impact of downstream taxes t_v

[5] Taxation decreases profits in perfect competition provided that equilibrium profits are positive, for which decreasing returns to scale are required. If constant returns to scale are assumed, then equilibrium profits are zero and taxes have no effect on profits.

Table 4: The impact of taxation on profits

Downstream profits	$0 < \gamma^d < 1$
$\frac{\partial(m\pi^d)}{\partial t_v} = [(1-\gamma^d)s^d_{tv} - 1]\left(q + C_p\frac{\partial p}{\partial t_v}\right) mx$	if $F^d < \gamma^d - \epsilon^d$ then $\frac{\partial(m\pi^d)}{\partial t_v} > 0$
$\frac{\partial(m\pi^d)}{\partial t_s} = [(1-\gamma^d)s^d_{ts} - 1]\left(1 + C_p\frac{\partial p}{\partial t_s}\right) mx$	$\frac{\partial(m\pi^d)}{\partial t_s} > 0$ iff $F^d < 1 - \epsilon^d$ assuming $1 + C_p(\partial p/\partial t_s) > 0$
$\frac{\partial(m\pi^d)}{\partial i_v} = [(1-\gamma^d)s^d_{iv} - 1]\frac{1}{1-i_v}s^u_{iv}pC_pmx$	$\frac{\partial(m\pi^d)}{\partial i_v} > 0$ iff $F^d < 1 - \epsilon^d$
$\frac{\partial(m\pi^d)}{\partial i_s} = [(1-\gamma^d)s^d_{is} - 1]\frac{1}{1-i_v}s^u_{is}C_pmx$	$\frac{\partial(m\pi^d)}{\partial i_s} > 0$ iff $F^d < 1 - \epsilon^d$
Upstream profits	$0 < \gamma^u < 1$
$\frac{\partial(n\pi^u)}{\partial t_v} = (1-i_v)\left[(1-\gamma^u)\frac{\partial p}{\partial t_v} + -\frac{p}{1-t_v}\frac{\gamma^u}{\epsilon^u}(\epsilon^d - \gamma^d)\frac{1}{f_q}\right] C_pmx$	if $\frac{\partial(n\pi^u)}{\partial t_v} > 0$ then $\frac{\partial p}{\partial t_v} > 0$ if $\frac{\partial p}{\partial t_v} \leq 0$ then $\frac{\partial(n\pi^u)}{\partial t_v} < 0$
$\frac{\partial(n\pi^u)}{\partial t_s} = (1-i_v)\left[(1-\gamma^u)\frac{\partial p}{\partial t_s} + -\frac{p}{1-t_v}\frac{\gamma^u}{\epsilon^u}\frac{\epsilon^d}{q}s^d_{ts}\right] C_pmx$	if $\frac{\partial(n\pi^u)}{\partial t_s} > 0$ then $\frac{\partial p}{\partial t_s} > 0$ if $\frac{\partial p}{\partial t_s} \leq 0$ then $\frac{\partial(n\pi^u)}{\partial t_s} < 0$
$\frac{\partial(n\pi^u)}{\partial i_v} = [(1-\gamma^u)s^u_{iv} - 1]pC_pmx$	$\frac{\partial(n\pi^u)}{\partial i_v} > 0$ iff $F^u < \gamma^u - \epsilon^u$
$\frac{\partial(n\pi^u)}{\partial i_s} = [(1-\gamma^u)s^u_{is} - 1]C_pmx$	$\frac{\partial(n\pi^u)}{\partial i_s} > 0$ iff $F^u < 1 - \epsilon^u$

The stability conditions require $F^d > 1 - \epsilon^d/\gamma^d$ and $F^u > 1 - \epsilon^u/\gamma^u$.
The existence conditions require $\epsilon^d > \gamma^d$ and $\epsilon^u > \gamma^u$.

and t_s on upstream profits. The derivatives in table 4 contain a negative term, the second, which represents the fall in profits that arises from output falling: downstream taxation increases q and reduces downstream output mx, which in turn causes a fall in upstream production. The first term can instead take positive and negative values, depending on whether taxation raises or decreases p. Thus the net impact is undetermined. With EXP and LIN $\partial p/\partial t_v < 0$ and $\partial p/\partial t_s \leq 0$, thus upstream profits fall; with ISO t_v reduces profits (because $\partial p/\partial t_v = 0$) while t_s may be profitable (because $\partial p/\partial t_s > 0$).

5 Conclusions

The impact of ad valorem and specific taxes in a model of vertically related oligopolies has been examined. Following Seade (1985) the analysis of tax incidence has focused on finding the conditions for taxes to increase producers' net price and profits.

It has been shown that tax incidence on downstream price and profits is governed by the price elasticity of the elasticity of final demand and downstream market structure. Specific downstream taxation is more likely to cause price overshifting and to raise profits than ad valorem downstream taxation. Also, ad valorem and specific taxes on upstream producers are equivalent, in terms of incidence, to the specific tax on downstream production.

Tax incidence on upstream price and profits has been shown to depend on the price elasticity of the elasticity of derived demand, which in turn depends on upstream and downstream market structure, final demand conditions and input substitution in downstream production. Taxes on downstream producers may raise or lower the price of the intermediate good, but the ad valorem tax is more likely to reduce it. Specific taxes on upstream producers are more likely to raise upstream unit and total profits than upstream ad valorem taxation.

An area of research in which this framework could be profitably employed is the analysis of tax reform and optimal tax design in vertically related industries.

A The elasticity of the slope of inverse demand and the price elasticity of the elasticity of direct demand

Let $mx = \chi(q) \in C^2$ be direct demand, with $\chi'(q) < 0$ for each $q > 0$. The inverse demand is defined as $q = q(mx)$. To ease the notation let $m = 1$. Then $x_0 \equiv \chi[q(x_0)]$ and $q_0 \equiv q[\chi(q_0)]$. Two properties concerning the derivatives of direct and inverse demands which will be used below are the following:

$$q'(x_0)\chi'(q_0) = 1 \tag{33}$$

$$q''(x_0) = -\frac{\chi''(q_0)}{[\chi'(q_0)]^3} \tag{34}$$

for each (q_0, x_0) such that $x_0 = \chi(q_0)$.

The elasticity of demand can be expressed either as a function of quantity or as a function of price:

$$\epsilon(q_0) \equiv -\frac{\chi'(q_0)q_0}{\chi(q_0)} = -\frac{q(x_0)}{x_0 q'(x_0)} \equiv \epsilon(x_0) \tag{35}$$

The elasticity of the slope of inverse demand is defined as

$$E(x_0) \equiv -q''(x_0)\frac{x_0}{q'(x_0)} \tag{36}$$

and the elasticity of the slope of direct demand is

$$E(q_0) \equiv -\chi''(q_0)\frac{q_0}{\chi'(q_0)} \tag{37}$$

Using (33), (34) and (35) the relation between $E(x_0)$ and $E(q_0)$ turns out to be

$$E(x_0) = -q''\frac{x_0}{q'} = \frac{\chi''}{(\chi')^3}\chi\chi' = \frac{\chi'' q_0}{\chi'}\frac{\chi}{\chi' q_0} = \frac{E(q_0)}{\epsilon(q_0)} \tag{38}$$

The price elasticity of the elasticity of direct demand is

$$F(q_0) \equiv \frac{d}{dq}\epsilon(q_0)\frac{q_0}{\epsilon(q_0)} = \left(\frac{\epsilon\chi''}{\chi'} + \frac{\epsilon}{q_0} - \frac{\epsilon\chi'}{\chi}\right)\frac{q_0}{\epsilon} = -E(q_0) + 1 + \epsilon(q_0) \tag{39}$$

Finally, using (38)

$$F(q_0) = -\epsilon(q_0)E(x_0) + 1 + \epsilon(q_0) \tag{40}$$

which is the result given on p. 5.

B The elasticity of derived demand

The elasticity of the derived demand (13) is defined by

$$\epsilon^u(p, w, t_v, t_s) \equiv -\frac{p\psi_p}{\psi} = -\frac{pC_{pp}}{C_p} - p\frac{\chi'}{\chi}\phi_p \tag{41}$$

which makes use of (14).

The elasticity of substitution between labour and the intermediate good in downstream production is defined as

$$\sigma = -\frac{d\log(y/L^d)}{d\log(p/w)} = -\frac{pC_{pp}C}{wC_wC_p} \tag{42}$$

where L^d and y are respectively the conditional input demands for labour and the intermediate good. Thus the first term in (41) may be written

$$-\frac{pC_{pp}}{C_p} = \frac{wC_w}{C}\sigma \tag{43}$$

Substituting for ϕ_p from (12) the second term in (41) becomes

$$-p\frac{\chi'}{\chi}\phi_p = -\frac{\chi' q}{\chi}\frac{pC_p}{q}\frac{1}{1-t_v}\frac{1}{f_q} \tag{44}$$

Substituting for q from (10)

$$\frac{pC_p}{C+t_s}\epsilon^d(q)\left[1-\frac{\gamma^d}{\epsilon^d(q)}\right]\frac{1}{f_q} \tag{45}$$

Finally, to obtain the elasticity of derived demand (18) define

$$\left[1-\frac{\gamma^d}{\epsilon^d(q)}\right]\frac{1}{f_q} \equiv \omega(q) \tag{46}$$

C Ad valorem downstream taxation and price overshifting

The tax t_v causes q to overshift if and only if the corresponding indicator shifting coefficient is greater than one

$$s^d_{tv} \equiv \frac{q\left(1-\frac{\gamma^d}{\epsilon^d}\right) + C_p\frac{\partial p}{\partial t_v}}{q + C_p\frac{\partial p}{\partial t_v}}\frac{1}{f_q} > 1 \tag{47}$$

Assuming that $q(1-\gamma^d/\epsilon^d) + C_p(\partial p/\partial t_v) > 0$, so that $\partial q/\partial t_v > 0$, this condition may be written

$$s_{tv}^d > 1 \quad \text{iff} \quad C_p \frac{\partial p}{\partial t_v} > \left(q + C_p \frac{\partial p}{\partial t_v} \right) F^d \tag{48}$$

The term in brakets is positive. Assuming that $| F^d \omega | > | q\omega_q |$, see on p. 10, then $\partial p/\partial t_v \gtreqless 0$ iff $F^d \lesseqgtr 0$, which gives $s_{tv}^d > 1$ iff $F^d < 0$.

D Ad valorem downstream taxation and profitable tax increases

The tax t_v causes downstream profits to raise if and only if $s_{tv}^d(1-\gamma^d) > 1$ which may be written

$$q(F^d - \gamma^d + \epsilon^d) + C_p \frac{\partial p}{\partial t_v}(F^d - 1 + \epsilon^d) < 0 \tag{49}$$

Price overshifting is a necessary condition for profits to increase; from appendix C $s_{tv}^d > 1$ iff $F^d < 0$, $\partial p/\partial t_v > 0$. Noticing that if $F^d + \gamma^d - \epsilon^d < 0$ then $F^d - 1 + \epsilon^d < 0$, it follows that $F^d < \gamma^d - \epsilon^d$ is a sufficient condition for the unequality (49) to hold.

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