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Death and the Keynesian Multiplier

By

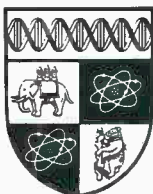
Neil Rankin\*

and

Domenico Scalera\*\*

No. 376

**WARWICK ECONOMIC RESEARCH PAPERS**



DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK  
COVENTRY

## Death and the Keynesian Multiplier

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### Abstract

The Blanchard-Yaari continuous-time model of overlapping generations is applied to a monetary economy with Keynesian unemployment. It is shown that a positive probability of death increases the short- and long-run multipliers of government spending on output. In the first example this is due to the government deficit being bond-financed, plus the failure of Ricardian equivalence which death implies. In the second, where the government budget remains balanced, it is due to the private accumulation of capital, plus a wealth effect on consumption which is implied by death.

JEL Classification No. E62

Keywords : Keynesian multiplier, death, overlapping generations.

July 1991

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.

## 1. Introduction

Keynes's famous remark that "in the long run, we are all dead", was not an observation designed to support the case for the Keynesian multiplier, but it could have been, as we show in this paper. Rather it was a retort to the classical quantity theory argument that the economy would return to full employment and a constant velocity of circulation in the long run<sup>1</sup>. In one sense, however, Keynes was wrong to dismiss the relevance of the long run: under rational expectations (which admittedly he may also have dismissed), the long run in general matters for the short run. We here provide a parable which shows the importance of death for Keynesian macroeconomics: not because it justifies our indifference towards the long run, but because the expectation of death itself directly impacts on the short run, in doing so strengthening the Keynesian multiplier. Whereas for classicals death is a drawback, making macroeconomic policy negatively effective, for Keynesians death is definitely an advantage.

Our framework is the Blanchard-Yaari (Blanchard (1985), Yaari (1965)) continuous-time model of overlapping generations, in which each individual faces an instantaneous probability of death,  $p$ . While this has been widely used to incorporate optimising agents into classical, Walrasian, macroeconomic models, it has been relatively under-exploited in the Keynesian context of sticky prices and non-market-clearing (Van der Ploeg's (1989) open-economy model constitutes an exception). Until recent work by Meijdam (1990, 1991), the most ambitious attempt at a dynamic non-market-clearing model was also attributable to Blanchard (Blanchard and Sachs (1982)); however, this assumes the traditional infinitely-lived representative agent. Like Meijdam, we marry Blanchard (1982) with Blanchard (1985), helping to fill this gap.

In the first model (section 2) we consider the effect of a debt-financed increase in government spending in a setting of Keynesian unemployment, where taxation depends on income and wealth. We show that when  $p$  is zero, the short- and long-run multipliers are unity, but that both multipliers rise above unity as  $p$  increases.  $p > 0$  implies that government bonds are "net wealth", i.e. that Ricardian equivalence does not hold, so that the rising bond stock has wealth effects which, under rational expectations about future incomes, have a

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<sup>1</sup> *Tract on Monetary Reform* (1923), Ch.3

positive impact in the short as well as in the long run. Although the nature of this result is not a surprise, our demonstration of it has a number of advantages. First, behaviour is explicitly derived from dynamic optimisation and rational expectations, in contrast to the traditional Keynesian literature on the dynamics of the government deficit (Blinder and Solow (1973), et al.). Second, the Blanchard-Yaari framework makes it possible to parameterise the expected lifetime of an individual, including infinity as a special case, unlike in the discrete-time model of Rankin (1985, 1986) based on the two-period overlapping-generations structure of Diamond (1965). Third, a phase diagram may be used to give a neat and explicit analysis of the dynamics, in contrast to the more complex models of Blanchard and Sachs (1982) and Meijdam (1990, 1991), which have to be simulated.

In the second model (section 3), Precious's (1985,1987) specification of investment subject to quantity constraints and adjustment costs is introduced. A positive probability of death is now shown to raise even the balanced-budget multiplier above unity in the long run. This is a surprising and little-known result, which does not arise from Ricardian non-equivalence since the stock of government debt is kept constant. Instead it comes from capital accumulation plus the presence of a "wealth effect" on consumption. In both models the long-run effects of introducing a positive probability of death on the fiscal multipliers are thus diametrically opposite to the effects in the classical market-clearing model (see e.g. Blanchard and Fischer (1989, Ch.2)), where zero multipliers for both bond-financed and tax-financed increases in government spending are converted into negative ones. Although the short run is harder to analyse in this model, we are able to establish some results for the case of  $p$  close to zero. In particular, the model suggests a resolution of the debate over whether anticipated fiscal policy is contractionary (Miller (1980), Blanchard (1981)), and one which is unfavourable to the contractionary thesis.

## **2. A Debt-Financed Fiscal Expansion**

We begin by abstracting from investment and capital accumulation, considering consumption and government spending as the two components of demand. As in Blanchard

(1985), the individual consumer has an instantaneous probability of death  $p$ , where  $0 \leq p < 1$ . Her expected remaining lifetime is thus  $1/p$ . The population is constant, normalised to one, so that each instant a flow  $p$  of consumers die and  $p$  more are born. Consumers obtain utility from consumption and from real money balances, which is discounted at the rate  $\theta$  to give the lifetime utility integral (mathematical details may be found in Appendix A). Since they maximise expected utility, however, their effective discount rate becomes  $\theta + p$ . Income is derived from labour sold in return for wage income, and also from a share in firms' profits. Since there is no utility of leisure, and since we focus on the Keynesian unemployment regime in which there is always excess supply in the labour market, labour sales are always rationed and income is always parametric to the consumer, with its decomposition between wages and profits being immaterial. Equal rationing of all workers is assumed, so that labour income is shared equally and does not vary with age.

The consumer has access to several asset markets. First she may hold money, which provides a utility flow representing its value as a medium of exchange, as in Sidrauski (1967) and many subsequent applications. Second, she may buy or sell bonds to smooth consumption over her lifetime, which pay an interest rate  $r$ . Third, as in Blanchard (1985), she may buy life insurance, in which all non-human wealth is forfeited to the insurance company upon death, in return for which  $\lambda p$  per  $\lambda$  of wealth is received from the company in the event of survival. Since consumers are large in number, with free entry to the insurance market actuarially fair insurance will be offered, in which insurance companies face no risk but make zero profits. Consumers, who obtain no utility from bequests, will choose to insure themselves completely.

Under these assumptions, the consumer's demands for consumption and real money balances may be derived as (see appendix A):

$$C = \frac{\theta+p}{1+\gamma} [H + W] \quad (1)$$

$$M = \frac{\gamma C}{r} \quad (2)$$

where  $\gamma$  is the coefficient on real balances in the logarithmic utility function.  $W$  is non-human wealth, being the sum of money and bond holdings:

$$W = M + B \quad (3)$$

$H$  is human wealth, a forward-looking variable which evolves according to:

$$\dot{H} = [r+p]H - [Y - T] \quad (4)$$

with  $T$  a lump-sum tax. Since consumers have identical and homothetic preferences, these demands hold in the aggregate as well as for the individual.

Firms have a purely background role until investment is introduced. Keynesian unemployment implies excess supply in the goods market, so that firms face a quantity constraint on sales, which determines their effective demand for labour via the production function. Output and employment thus move together. Since consumption demand does not depend on the distribution of income between wages and profits, the labour market has a residual role in the simple model and it is unnecessary to represent it formally.

The final type of agent is the government, whose budget constraint is given by:

$$\dot{B} + \dot{M} = rB + G - T \quad (5)$$

The government spends  $G$  on private output, and levies a lump-sum tax  $T$  on consumers. Any remaining deficit or surplus of spending plus interest payments over tax revenue is financed by issuing or withdrawing money or debt. In fact the greatest interest of the model is for fiscal policy, so we shall below consider only bond-financing of the deficit, setting  $\dot{M} = 0$  and making  $M$  exogenous.

To derive the macroeconomic equilibrium, we use the condition that under excess supply in the goods market, output is demand-determined. This yields the familiar "IS" equation:

$$Y = C + G \quad (6)$$

However, rather than work with output, it is easier to solve the model in terms of consumption. We can obtain a differential equation governing the equilibrium behaviour of consumption by differentiating the consumption function in (1) with respect to time, substituting out  $\dot{H}$  by (4) and  $\dot{W}$  by (5):

$$\dot{C} = \frac{\theta+p}{1+\gamma} [[r+p]H - [Y - T] + rB + G - T] \quad (7)$$

$$= \frac{\theta+p}{1+\gamma} [[r+p]H + rB - C] \quad \text{cancelling } T \text{ and using (6)}$$

$$= [r + p - \frac{\theta+p}{1+\gamma}]C - p\frac{\theta+p}{1+\gamma}B - [r + p]\frac{\theta+p}{1+\gamma}M \quad \text{substituting out } H \text{ using (1)} \quad (8)$$

To eliminate the interest rate from this expression, we now assume instantaneous clearing of the money market. If  $M$  is treated as exogenous, equation (2) already represents an "LM" condition. Using it,  $r$  may be substituted out of (8) as  $\gamma C/M$ , giving:

$$\dot{C} = \frac{\gamma C^2}{M} - \theta C - \eta[B + M] \quad (9)$$

where  $\eta \equiv p[\theta+p]/[1+\gamma]$ . Two properties of the model may be remarked on here. First, with  $M$  exogenous the interest rate is uniquely and proportionally related to  $C$ . This is very nearly what is postulated in the textbook IS-LM model, except that it is traditionally  $Y$  rather than  $C$  which is used as the "transactions" variable in the money demand function. Second, so long as  $p > 0$  there is a "wealth effect" on consumption given by the third right-hand term in (9). When  $p = 0$ , however,  $\eta = 0$  and the wealth effect disappears. This is central to the results of the paper.<sup>2</sup>

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<sup>2</sup> Even when  $p = 0$  there is, of course, a "wealth effect" on consumption inasmuch as the individual's consumption depends on her wealth, as (1) makes clear. What is referred to here is the additional information which aggregate wealth provides, over and above our knowledge of the current consumption level, in enabling us to predict future aggregate consumption. When  $p = 0$ , so that there is a single infinitely-lived consumer, then we only need to know current consumption in order to predict future consumption, as Hall (1978) first showed. When  $p > 0$  this is not the case, since future aggregate consumption consists partly of the consumption of individuals not yet born.



The second differential equation, which completes the definition of the macroeconomic equilibrium, is provided by the government budget constraint (5). As noted, we assume bond-financing of any deficit or surplus and so set  $\dot{M} = 0$ . We also now assume that taxation is dependent on the aggregate consumption and government debt levels, with propensities to tax  $t_C, t_B$ :

$$T = T_0 + t_C C + t_B B \quad (10)$$

Substituting this into (5) then gives:

$$\dot{B} = \gamma \frac{BC}{M} + G - T_0 - t_C C - t_B B \quad (11)$$

where  $r$  has again been substituted out as  $\gamma C/M$ .

(9) and (11) now fully define the dynamic behaviour of the model in terms of time paths for  $(B, C)$ .  $B$  is an asset stock and so its level at any instant in time is predetermined by the history of past government deficits.  $C$  is the outcome of forward-looking intertemporal choices made by individual consumers and so a non-predetermined or "jump" variable. Note that output moves exactly with  $C$  except when government spending is changed, by (6). As in all rational expectations models, to obtain an initial condition for  $C$  and so identify a unique solution path we need first to establish that the system (9) and (11) possesses the saddlepoint property, which then enables the initial value of  $C$  to be fixed such that the economy lies on the saddlepath given an arbitrary initial  $B_0$ . This is most conveniently done by drawing the phase diagram of the system, as in the figures below.

Setting  $\dot{C} = 0$  in (9) and  $\dot{B} = 0$  in (11) defines the stationary loci:

$$\dot{C} = 0: \quad B = \frac{\gamma}{\eta} \frac{C^2}{M} - \frac{\theta}{\eta} C - M \quad (12)$$

$$\dot{B} = 0: \quad C - t_B M/\gamma = \frac{t_B t_C [M/\gamma]^2 - [G - T_0] M/\gamma}{B - t_C M/\gamma} \quad (13)$$

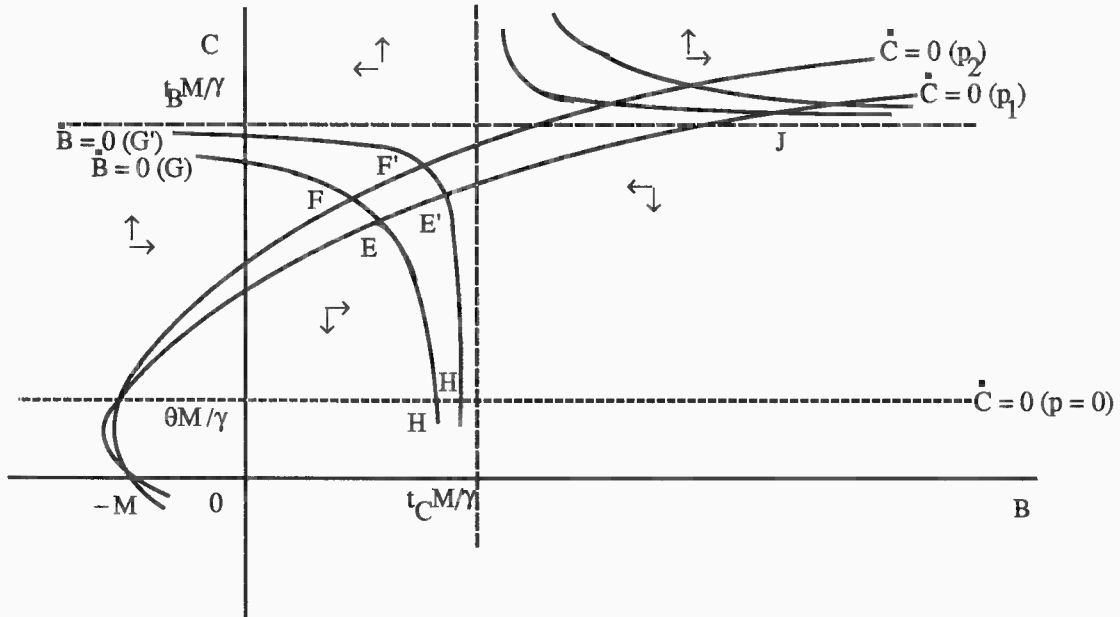


Figure 1

$\dot{C} = 0$  is clearly a parabola centred on  $C = \theta M/2\gamma$ , cutting the horizontal axis where  $B = -M$ . In the positive quadrant of the diagram it is therefore an upward-sloping locus. Consider what happens as  $p$ , and thus  $\eta$ , is increased. Neither the central axis nor the intersection with  $\dot{B} = 0$  is affected. Instead the arms of the parabola become more spread out, pivoting on the points  $(-M, 0)$ ,  $(-M, \theta M/\gamma)$ , as in Figure 1. Conversely, as  $p$  tends to zero the parabola closes up, and in the limit tends to a pair of horizontal lines,  $(C = \theta M/\gamma, C = 0)$ .  $\dot{B} = 0$  is clearly a rectangular hyperbola with vertical and horizontal asymptotes at  $B = t_C/\gamma$ ,  $C = t_B/\gamma$ . Whether it lies in the north-east and south-west, or in the north-west and south-east, quadrants, depends on whether the numerator in (13) is positive or negative, respectively. Consider what happens as  $G$  is increased. The positions of the asymptotes are unaffected. If the numerator in (13) starts out positive, then it falls towards zero, shrinking the hyperbola in towards its asymptotes. Beyond a certain level of  $G$  the numerator becomes negative, at which point the hyperbola re-emerges in the north-west and south-east quadrants, thence expanding away from the asymptotes. The directions of motion on either side of the stationary loci are easily established from (9) and (11).

Given the above properties of the stationary loci, three configurations of the phase diagram are possible. These are illustrated in Figures 2(a)-(c).

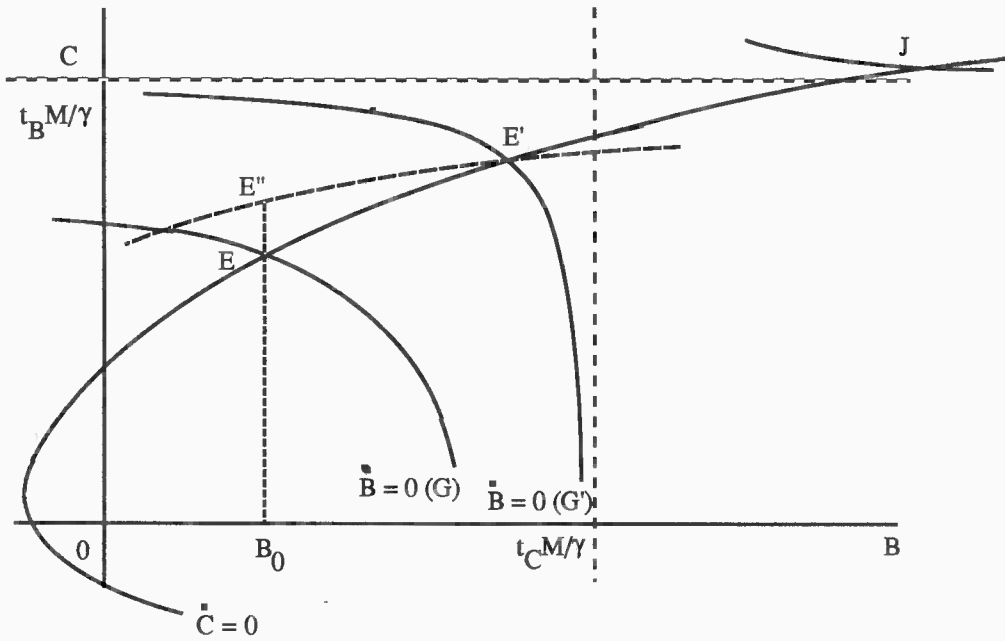


Figure 2(a)

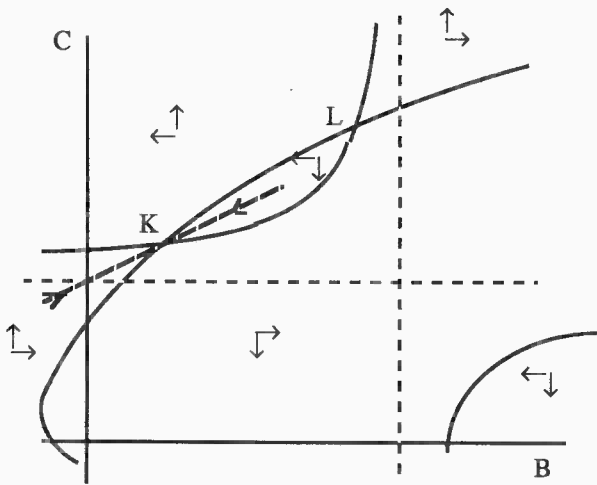


Figure 2(b)

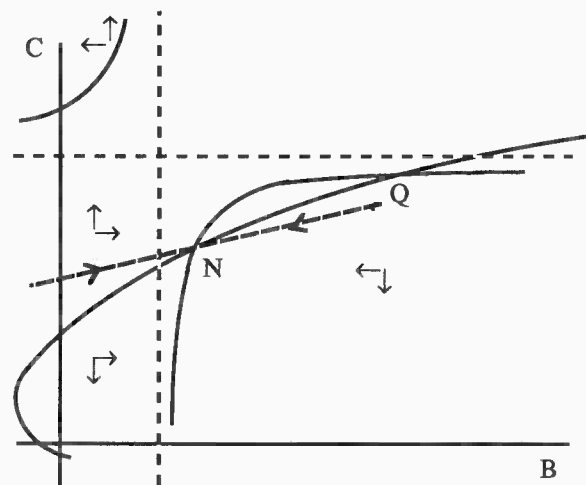


Figure 2(c)

Figure 2(a) (which is an enlargement of the same configuration as in Figure 1) arises for low values of  $G$ , causing  $\dot{B} = 0$  to lie in the north-east and south-west quadrants. There are two steady states, at  $E$  and  $J$ , but only  $E$  has the required saddlepoint property,  $J$  being globally unstable. The saddlepath is seen to be an upward-sloping locus, along which  $B$  and  $C$  (and thus  $Y$ ) rise or fall together. Figures 2(b) and 2(c) arise for a higher value of  $G$ , causing  $\dot{B} = 0$  to lie in the north-west and south-east quadrants. In 2(b) the intersections with  $\dot{C} = 0$  occur in

the north-west quadrant. Again there are two, at K and L, but only K has the saddlepoint property. In 2(c) the steady states lie in the south-east quadrant, with N being saddlepoint stable but Q not. The saddlepath is still upward-sloping in both.

Now suppose the economy is initially in a steady state when there is an unanticipated, permanent increase in  $G$ . The effect is qualitatively identical in all three cases, so we use Figure 2(a) for the illustration. The  $\dot{B} = 0$  hyperbola shrinks towards its asymptotes, shifting the steady state from  $E$  to  $E'$ . In the new steady state consumption is therefore higher, indicating that the long-run multiplier of spending on output  $dY^*/dG (= 1 + dC^*/dG$ , from (6)) must be greater than one. In the short run  $C$  jumps onto the new, higher saddlepath at  $E''$ , over time moving along it to  $E'$  as the initial government deficit causes a rise in the bond stock. Thus the impact multiplier,  $dY_0/dG (= 1 + dC_0/dG)$  also exceeds unity, but is smaller than the long-run multiplier. What enables convergence to a new steady state? The story here is a familiar one: see, for example, Blinder and Solow (1973). As the bond stock rises, with its consequent wealth effects on consumption demand and thus output, tax revenue rises, since it has been posited as an increasing function of both the bond stock and of consumption. This eventually restores budget balance and stops the bond stock rising. The new twist to the story is the effect of rational expectations in increasing the short run multiplier. The possibility that in a Keynesian world rational expectations can *increase* the effectiveness of policy, rather than reduce or destroy it as in a new classical world, was first shown by Neary and Stiglitz (1983) in a two-period model. The mechanism is that agents who base current demand on expected lifetime, or "permanent", income, not just on current income, will correctly anticipate the higher future income caused by the projected increase in bond wealth, and thus increase their spending immediately. This is what causes the immediate jump in consumption. By contrast under adaptive expectations of future income, the rise in current income due directly to the rise in government spending itself will, at the instant of the shock, have no significant effect on consumers' perceived "permanent" income<sup>3</sup>, so that the impact multiplier will simply be unity.<sup>4</sup>

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<sup>3</sup> Note that, in a continuous-time setting, the instantaneous rise in income at the moment of the shock is of measure zero in relation to a consumer's observed history of past incomes, and so will have a zero

The effect of  $p$  on the multipliers may be seen by considering the limiting case where  $p = 0$ , i.e. where there is no death. As noted,  $\dot{C} = 0$  then tends to a pair of horizontal lines. In Figure 1 the steady state is now at point H, and the associated saddlepath is the stationary locus  $C = \theta M/\gamma$  itself. In this case, because of the absence of a wealth effect on consumption, the differential equation for  $C$  is independent of that for  $B$ , as may be seen from (9), setting  $\eta = 0$ . Since (9) is an "unstable" equation ( $d\dot{C}/dC = \theta > 0$ , evaluating at the steady state), a non-divergent time path requires that  $C$  should jump immediately to its steady state value and stay there. Therefore, an increase in  $G$  which shifts the steady state from H to H' has no effect on the short- or long-run level of  $C$ , which in turn implies that the short- and long-run multipliers of spending on output are unity, as in the textbook case of the balanced-budget multiplier. The reason for this, of course, is that when  $p = 0$  government debt is no longer "net wealth", i.e.. "Ricardian equivalence" holds. Although over time the bond stock rises, and is expected to rise, as a result of the government deficit, consumers anticipate the higher future tax burden necessary to service the extra interest payments. When  $p = 0$ , currently-alive consumers live forever and so face all the tax increase themselves, rather than only a part of it as would happen if they expected to die and the burden to be shouldered by following generations. The present value of expected future taxes then offsets the increased current bond wealth, leaving consumers' perceived lifetime net wealth unchanged, and so their consumption demand unchanged. As  $p$  is raised above 0, the short- and long-run spending multipliers increase above one, because a smaller fraction of the anticipated higher future tax burden is expected to be borne by currently-alive consumers, and thus a higher percentage of the current market value of the bond stock is perceived as "net wealth". In the diagram, this is shown by the  $\dot{C} = 0$  locus, and so too the saddlepath, becoming steeper, so that a given shift in the  $\dot{B} = 0$  locus results in a bigger increase in the steady state and short-run values of  $C$ . ( $F \rightarrow F'$  rather than  $E \rightarrow E'$ ).

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instantaneous impact on permanent income and thus on consumption. Not until a finite interval of time has elapsed will permanent income and thus consumption demand respond.

<sup>4</sup> The reason why in "ad hoc" Keynesian models without rational expectations (e.g. Blinder and Solow (1973)) the impact multiplier would nevertheless usually be greater than unity is that current income and wealth are assumed to enter as separate arguments into the consumption function. This additional influence of income cannot be justified on microeconomic grounds (unless, for example, it is further implicitly assumed that consumers are credit rationed).

It is clear that instability is a possibility in this economy. This would occur if, in Figure 2(b) or 2(c) (which correspond to "high" values of  $G$ ), the  $\dot{C} = 0$  locus were to pass between the north-west and south-east parts of the  $\dot{B} = 0$  hyperbola, in which case no steady state would exist. The solution is to cut  $G$  (or raise  $T_0$ ), shrinking the hyperbola towards its asymptotes, until an intersection is achieved. In the traditional literature on the dynamics of the government deficit (Blinder and Solow (1973), et al.), considerable attention was focused on the possibility of instability under bond financing. One factor helpful in reducing it is to make taxation dependent on the bond stock, as suggested by Christ (1979). It is only this which saves our model from instability in the case  $p = 0$ : if  $t_B = 0$ , then there exists only an unstable steady state. At the same time a central source of instability in the traditional literature was the worry that a sufficiently strong wealth effect on money demand could cause a leftward shift in the LM curve sufficient to outweigh the rightward shift in the IS curve, resulting in a negative overall effect on output of a rise in the bond stock. Our microfoundations provide reassurance on this point: as already noted, the demand for money here can be written as a function of current consumption and of the interest rate alone, with no additional effect of wealth on money demand separate from that already operating through consumption.

The effects of changes in a variety of other exogenous variables can be analysed using the model. The effect of an exogenous increase in  $p$  itself on the long-run value of output depends on which of the cases (a)-(c) prevail. In (a) and (c) the upward shift in the  $\dot{C} = 0$  locus raises the steady-state value of  $C$ , but in (b) it lowers it. Other experiments with exogenous changes are left to the reader.

### 3. A Tax-Financed Fiscal Expansion

In the absence of investment and capital accumulation, a balanced-budget increase in government spending has a short- and long-run multiplier on output of unity, as in the simple textbook result. This can easily be seen from our earlier equation (9): if the government maintains its budget in balance at all times then the stocks of money and bonds are exogenous,

whence consumption must permanently equal the steady-state value implied by (9).<sup>5</sup> We now show that when investment and capital accumulation are introduced, then even under a policy of keeping the budget permanently in balance, a rise in government spending has a positive long-run effect on consumption (and thus a multiplier effect on output greater than one) provided there is a positive probability of death. When  $p = 0$ , this effect is absent.

The model of investment is taken from Precious's (1985, 1987) "accelerator" analysis of investment subject to quantity constraints and adjustment costs. Precious's model is a partial equilibrium one, so that our model can be seen as taking his and placing it in a general equilibrium setting. Firms are now endowed with a constant-returns production function over labour and capital:

$$Y = AK^\alpha L^{1-\alpha} \quad 0 < \alpha < 1 \quad (14)$$

For simplicity, capital does not depreciate. There are costs to firms of installing new capital which are convex in the level of investment:

$$\Psi = [1/2]\psi I^2 \quad \psi > 0 \quad (15)$$

The firm's problem is to maximise its value,

$$V(t) = \int_t^\infty [Y(s) - wL(s) - I(s) - \Psi(s)]R(s)ds \quad R(s) \equiv \exp\left(-\int_t^s r(u)du\right) \quad (16)$$

subject to  $\dot{K} = I$  and to  $Y(t) = \bar{Y}(t)$ , a quantity constraint on sales of output which arises from the assumption of Keynesian unemployment and thus excess supply in the goods market.<sup>6</sup>

The solution to the firm's problem is derived in appendix B. Adjustment costs are important in obtaining smooth adjustment of the capital stock in response to a shock. In their absence, the economy would jump immediately to the new steady state, implying an infinite

<sup>5</sup> Noting that (9) is locally unstable, and so invoking the "saddlepath" solution once more.

<sup>6</sup> By taking the constraint to be on  $Y$  rather than on  $Y-I$ , or on  $Y-I-\Psi$ , we are supposing that each firm is forced to buy its investment goods, and goods used in the installation of new capital, from other firms rather than providing them from its own production. This is somewhat arbitrary so long as consumption goods, investment goods and installation goods are homogeneous as they are here, but it provides a better approximation to the real world, where these goods are produced by different sectors.

level of investment in the instant after the shock. In their presence, investment depends on "q", the marginal value of a unit of installed capital. Since q is a "forward-looking" variable, this adds another order of dynamics to the model, in addition to the "backward-looking" dynamics due to capital accumulation itself. This can be expressed in terms of a differential equation for q or - more conveniently, since q and I bear a unique linear relationship, as shown in appendix B - for I:

$$\dot{I} = \left[\frac{1}{\Psi} + I\right]r - \frac{1}{\Psi} w \frac{\alpha}{1-\alpha} A^{1/(\alpha-1)} \left[\frac{Y}{K}\right]^{1/(1-\alpha)} \quad (17)$$

The addition of capital to the model means that consumers' wealth must now be augmented by the value of their ownership of firms. Since there is no uncertainty involved in holding shares or bonds (apart from that of death, which affects both equally and has already been accounted for), equity and bonds are perfect substitutes and must pay the same rate of return. However V does not simply equal K, since by assumption new capital cannot costlessly be converted into installed capital, from which it follows that installed capital will command a price ("average Q") in general different from new capital, or output. We thus need to add a further equation to the model, governing the evolution of V. This is obtained by differentiating V (as defined in (16)) with respect to time:

$$\dot{V} = rV - [Y - wL - I - \Psi] \quad (18)$$

The complete model is defined by the following four equations in (C,I,K,V):

$$\dot{C} = \frac{\gamma C^2}{M} - \theta C - \eta[M + B + V] \quad (19)$$

$$\dot{I} = \left[\frac{1}{\Psi} + I\right] \frac{\gamma C}{M} - \frac{1}{\Psi} w \frac{\alpha}{1-\alpha} A^{1/(\alpha-1)} \left[\frac{C + I + G + [1/2]\Psi I^2}{K}\right]^{1/(1-\alpha)} \quad (20)$$

$$\dot{K} = I \quad (21)$$

$$\dot{V} = \frac{\gamma CV}{M} - C - G + wA^{1/(\alpha-1)} [C + I + G + [1/2]\Psi I^2]^{1/(1-\alpha)} K^{\alpha/(\alpha-1)} \quad (22)$$



(19) differs from section 2 only by the addition of  $V$  to wealth. (20) has been obtained from (17) by substituting out  $r$  as  $\gamma C/M$  using the LM condition, and by substituting out  $Y$  using the IS equation, which is now  $Y = C + I + G + \Psi$ . (22) has been obtained from (18) by substituting out  $r$  as  $\gamma C/M$ , by replacing  $L$  by the effective demand function for labour  $A^{1/[\alpha-1]}Y^{1/[1-\alpha]}K^{\alpha/[\alpha-1]}$  (see appendix B), and by eliminating  $Y$  using the IS equation. We here assume the government balances its budget by setting  $T$  such that  $T = rB + G$  at all times, so that  $B$  as well as  $M$  is now exogenous. Of the four state variables  $(C, I, K, V)$ , only  $K$  is predetermined, so the necessary saddlepoint stability condition to define a unique solution in the neighbourhood of the steady state is that one of the eigenvalues of the system should have a negative real part, and that the other three should be positive (see, e.g., Begg (1982) Ch.3).

Consider first the long-run effect of an increase in government spending. The steady state of the model is defined by setting  $\dot{C} = \dot{I} = \dot{K} = \dot{V} = 0$  in (19)-(22). From  $\dot{C} = 0$  we obtain a relationship ("CC") between  $C$  and  $V$ :

$$CC: \quad V = \frac{\gamma}{\eta} \frac{C^2}{M} - \frac{\theta}{\eta} C - [M + B]$$

This is identical to (12) in section 2, and may be plotted as a parabola in  $(V, C)$ -space, as in Figure 3 below. Combining (20)-(22) to eliminate  $I$  and  $K$ , we obtain a second relationship ("VV") between  $C$  and  $V$ :

$$VV: \quad V = \frac{M}{\gamma} \frac{C + G}{C} \left[ 1 - \left( \frac{1-\alpha}{\alpha} \right)^{\alpha} w^{1-\alpha} \frac{1}{A} \left( \frac{\gamma C}{M} \right)^{\alpha} \right]$$

This is clearly decreasing in  $C$ . It implies a maximum value of  $C$ ,

$$\bar{C} \equiv A^{1/\alpha} w^{[\alpha-1]/\alpha} \frac{\alpha}{1-\alpha} \frac{M}{\gamma}, \quad (23)$$

consistent with steady-state  $V$  remaining positive. As  $C$  decreases from  $\bar{C}$ ,  $V$  tends to  $M/\gamma$  if  $G = 0$ , or to infinity if  $G > 0$ , as depicted in Figure 3:

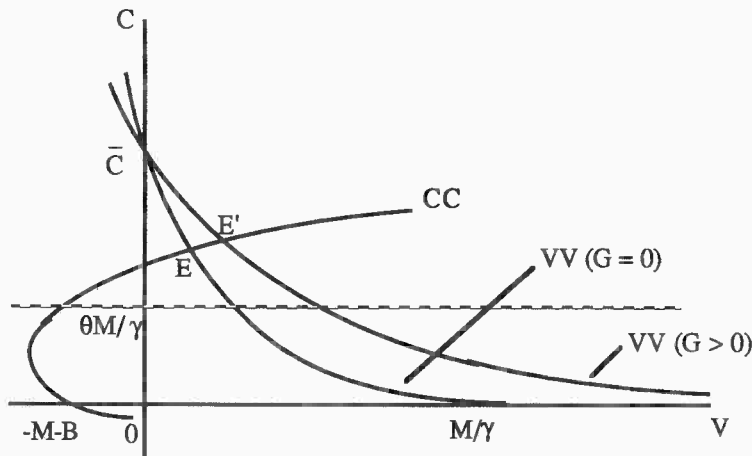


Figure 3

The effect of an increase in  $G$  is to shift  $VV$  to the right, pivoting it about the point  $(0, \bar{C})$ . The long-run values of  $C$  and  $V$  therefore increase, from  $E$  to  $E'$  in the diagram. Noting that investment, and thus also adjustment costs, are zero in the steady state (i.e.  $Y = C + G$  for steady states), it is clear from this that the long-run multiplier  $dY^*/dG$  exceeds unity, despite the government budget being kept permanently in balance. The source of this result is that an increase in  $G$  now stimulates investment, through an "accelerator" effect. Over time this results in a rise in the capital stock and also in the value of the firm. For  $p > 0$ , this increase in the firm's value has a "wealth effect" on consumption. If  $p = 0$ , on the other hand, then  $CC$  is a horizontal line at  $C = \theta M / \gamma$ , as seen in section 2. In this case Figure 3 shows that  $C$  is unaffected by a rise in  $G$ . This effect of death on the Keynesian multiplier is much less well documented in the literature. It does not arise from the failure of Ricardian equivalence - since this time the bond stock has not increased - but from the induced accumulation of private capital and from the "wealth effect" which the latter has on consumption when  $p > 0$ . Just as in the case of a debt-financed expansion, there is a contrast between the enhanced effectiveness of policy due to death in the Keynesian world, and the negative effect on output which arises due to death in the classical world. In the classical model, any issue of debt crowds out private capital and so reduces long-run output so long as  $p > 0$  (Blanchard (1985)); and a balanced-

budget fiscal expansion also reduces long-run output so long as  $p > 0$ .<sup>7</sup> In our Keynesian model, as seen, death in both cases increases the multiplier.

What about the short-run behaviour of the model? For the general case, we cannot draw a phase diagram since the dynamics are fourth-order. However, in the special case of  $p = 0$ , it can be observed that the system (19)-(22) separates.  $\eta = 0$ , so that  $\dot{C}$  depends only on  $C$ , and hence  $C$  stays permanently at its long-run value  $\theta M/\gamma$ , as noted previously. The equation for  $\dot{V}$  is redundant, since no other variables depend on  $V$ . This leaves (20) and (21) as an independent system in  $(I, K)$ . With  $C, G$  constant, the short-run behaviour of output will vary monotonically with  $I$ . Figure 4 depicts the dynamics of  $I$  and  $K$  for this case:

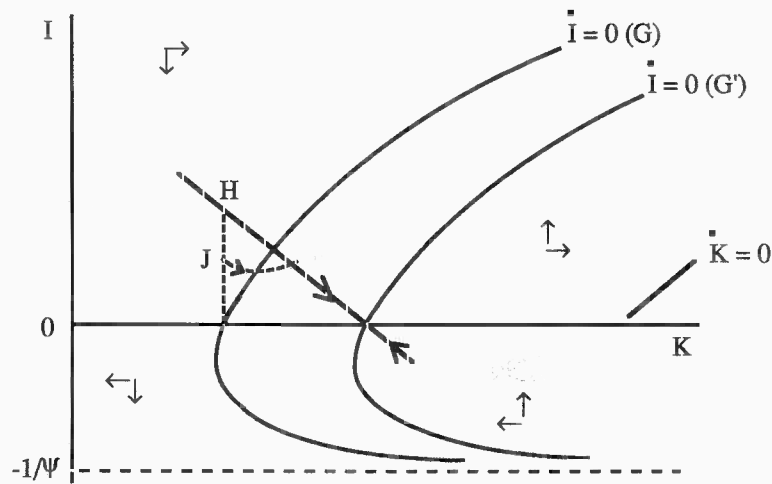


Figure 4

The stationary locus for  $K$  is clearly where  $I = 0$ , and thus coincides with the horizontal axis.

The stationary locus for  $I$  is plotted from:

$$\dot{I} = 0: \quad K = \left[ \frac{1}{\psi\theta} \frac{\alpha}{1-\alpha} wA^{1/(\alpha-1)} \right]^{1-\alpha} \frac{\theta M/\gamma + G + I + [1/2]\psi I^2}{[1/\psi + I]^{1-\alpha}} \quad (24)$$

The steady state is readily seen to have the saddlepoint property.

An increase in  $G$  shifts  $\dot{I} = 0$  to the right. If it is unanticipated and the economy starts in a steady state, then investment increases as the system jumps onto the new saddlepath at  $H$ . This

<sup>7</sup> This result is recorded, though not explicitly demonstrated, in Blanchard and Fischer (1989), p.132

shows that the impact multiplier  $dY/dG$  is greater than one, due to the short-run stimulus to investment. The long-run multiplier equals one because investment returns to zero in the long run, while consumption remains unchanged throughout. Thus, since there is a short-run balanced-budget multiplier which is greater than one even when  $p = 0$ , for values of  $p$  positive but close to zero, the same must hold true.<sup>8</sup>

A question debated in the context of "ad hoc" rational-expectations Keynesian models (see Miller (1980), Blanchard (1981)) is whether a pre-announced fiscal expansion can be contractionary. If investment is postulated to depend on the long-run rate of interest and there is some sluggishness in the adjustment of output, then the real interest rate may jump upwards on the announcement of a future fiscal expansion (since it is a forward-looking integral of all future short rates), depressing investment and causing a recession. Against this, however, must be set expectations of higher future profits due to higher future output, which should provide a stimulus to current investment. The net effect is unclear in the ad hoc framework. However our model, based on explicit optimisation, suggests an answer. In Figure 4 the effect of a preannounced fiscal expansion is still to cause investment to increase immediately, though by less (to J rather than H). I now jumps onto a divergent path such that, at the moment of implementation of the increase in  $G$ , it reaches the saddlepath associated with the new steady state. It is impossible for I to jump downwards, since no divergent path then exists to bring the economy onto the new saddlepath. This is the outcome when  $p = 0$ ; it follows that, at least for  $p$  positive but sufficiently small, the same type of behaviour will be observed.

## 5. Conclusions

The Blanchard-Yaari continuous-time model of overlapping generations has been used to show that, in a Keynesian world, increasing the probability of death increases the Keynesian multiplier. In general this arises because output depends on demand in the Keynesian setting,

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<sup>8</sup> Algebraic solutions for the case  $p > 0$  may be obtained by solving for the stable eigenvalue and eigenvector of the system (19)-(22). Since  $(C, I, V)$  are uniquely linked to  $K$  along the saddlepath, the adjustment to equilibrium following an initial shock must be monotonic. However the sign of the linkage between  $K$  and  $V$ , and thus between  $K$  and  $C$ , is indeterminable short of numerical experiments: it is this which governs whether  $C$  over- or undershoots its new long-run value.

and because the higher the probability of death the stronger the "wealth effect" on demand as assets (bonds or capital) accumulate following an increase in government spending.

In taking the price and wage to be permanently fixed, we are of course performing an analytical experiment rather than providing a "realistic" description of the very long run of an economy. If sluggish wage and price adjustment over time were permitted this would undeniably change the nature of the steady state, which would (though depending on the precise adjustment mechanism chosen) become a classical, full-employment one. However, it would also complicate the dynamics to the point of preventing any analytical treatment of out-of-steady-state behaviour. Blanchard and Sachs (1982) and Meijdam (1990, 1991), who include price adjustment, constitute examples of the alternative, simulation approach. Our model should be interpreted as a complement to these studies, an attempt to add some partial analytical insight to the dynamic forces at work in the more complete but more complex framework. For it to be meaningful, all that we require is that price adjustment is not extremely fast relative to the other dynamic processes at work: in this case our "long run" will capture some of the features of the "medium run" of a sluggish-price model.

## Appendix A

We here provide some further details of the consumer's optimisation problem; for a full discussion the reader is referred to Blanchard (1985), or to Blanchard and Fischer (1989, Ch.3).

The problem of an individual consumer (individuals' quantities are distinguished from aggregate ones by lower case letters) may be stated as:

$$\text{maximise } \int_t^{\infty} [\ln c(s) + \gamma \ln m(s)] \exp(-[\theta+p][s-t]) ds \quad (\text{A1})$$

$$\text{s.t. } \dot{a}(s) = [r(s) + p]a(s) + y(s) - x(s) - c(s) - r(s)m(s) \quad (\text{A2})$$

The consumer's maximand is expected utility discounted at rate  $\theta$ ; however, since the probability of living to exactly date  $s$  is  $p \cdot \exp(-p[s-t])$ , this conveniently simplifies to the form in (A1). In the budget constraint (A2), the consumer's non-human wealth,  $a$ , comprises bonds  $b$ , which pay a return  $r$ , and money  $m$ , which pays no interest. As noted, the consumer also takes out an insurance contract in which she forfeits  $a$  to the insurance company on death, in return for a payment  $p \cdot a$  for each instant she remains alive.  $y$  is the consumer's exogenous labour and profit income,  $x$  is the lump-sum tax paid to the government.

To solve this problem, we set up the Hamiltonian (see, e.g., Dixit (1990)), and hence write down the first-order conditions. Slightly rearranging, these yield:

$$\dot{c} = [r - \theta]c \quad (\text{A3})$$

$$m = \gamma c / r \quad (\text{A4})$$

(A3) is the familiar "Keynes-Ramsey" rule. Integrating the budget constraint from  $t$  to  $\infty$  (and applying the "No-Ponzi-Game" condition that the discounted value of terminal wealth tends to zero) gives the lifetime budget constraint:

$$a(t) + h(t) = \int_t^{\infty} [c(s) + r(s)m(s)] \exp\left(-\int_t^s [r(u)+p]du\right) ds \quad (\text{A5})$$

where  $h(t)$  is the discounted value of future disposable income, or "human wealth":

$$h(t) \equiv \int_t^{\infty} [y(s) - x(s)] \exp\left(-\int_t^s [r(u)+p]du\right) ds$$

Integrating (A3) from  $t$  to  $s$  we obtain:

$$c(t) = c(s) \exp\left(-\int_t^s [r(u)+p]du\right)$$

Substituting this and (A4) into (A5) and rearranging then gives:

$$c(t) = \frac{\theta + p}{1 + \gamma} [a(t) + h(t)]$$

Since this relationship of an individual's consumption to her wealth does not depend on her age, we can clearly sum over all individuals to get an identical relationship between the corresponding aggregate variables. This gives (1) in the paper.

## Appendix B

The firm's optimisation problem was given in equation (16) of the paper. To solve this, we set up the Hamiltonian (see Precious (1985)), defining  $q$  to be the costate variable on the capital stock and  $\lambda$  to be the Lagrange multiplier on the quantity constraint. From this, we obtain the first-order conditions:

$$w = -\lambda[1-\alpha]K^{\alpha}L^{-\alpha} \quad (\text{A5})$$

$$1 + \psi I = q \quad (\text{A6})$$

$$\dot{q} = [r + \delta]q + \lambda \alpha K^{\alpha-1} L^{1-\alpha} \quad (\text{A7})$$

(A6) shows that  $I$  depends uniquely on  $q$ ; hence the label "q-theory" of investment.

At any instant  $L$  is determined by the firm's effective demand for labour which, given the quantity constraint on output and the capital stock, is obtained by inverting the production function:

$$L = [\bar{Y}/A]^{1/[1-\alpha]} K^{\alpha/[\alpha-1]}$$

Using this and (A5) to eliminate  $L$  and  $\lambda$  from (A7), we have:

$$\dot{q} = [r + \delta]q - w \frac{\alpha}{1-\alpha} A^{1/[\alpha-1]} \left[ \frac{Y}{K} \right]^{1/[1-\alpha]}$$

This provides a differential equation describing the evolution of the "forward-looking" variable,  $q$ . Equivalently, it may be re-expressed in terms of  $I$  by using (A6) to substitute out  $q$  by  $I$ . This yields equation (17) in the paper.



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