Imperfect Competition with Intermediate Goods:

A Simulation Analysis of a Two-Sector Model.*

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Abstract: A two-sector model of imperfect competition with intermediate goods is developed and analysed by numerical simulation. It is shown how an objective notion of demand can be derived and employed in three concepts of equilibrium that differ in the possibilities for price-discrimination and collusion. The results indicate that there may be excessive use of labour relative to produced input in production, that price discrimination reduces both welfare and profits and that collusion between firms is beneficial to both the firms and the consumer. In addition, collusion may result in produced inputs being sold at a price less than marginal cost.

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1. Introduction

The analysis of this paper focuses upon a two sector general equilibrium model in which the outputs of imperfectly competitive industries are used both as intermediate inputs and for final consumption. It is shown how the objective demand function facing each industry can be constructed thus overcoming one of the difficulties raised by Hart (1975). The model is analysed by numerical simulation since the interdependences do not make it tractable for analytical investigation. As this work is primarily an exploratory one, the implications of a number of equilibrium concepts that differ in the possibilities for price discrimination and collusion are investigated. In addition, the simulations also consider the effect of the form of production technology and the demand relationship of the two goods upon the equilibrium outcomes.

The analysis of imperfectly competitive economies in which intermediate goods are employed has received little attention in the economics literature, a result, perhaps, of the pessimistic comments of Hart (1985) and Roberts and Sonnenschein (1977). A partial exception has been the work on vertical integration, for instance Panzar and Sibley (1989), but that work has been based firmly within a partial equilibrium framework. The intention here is to present a general equilibrium model with explicit profit and utility maximisation and a complete "circular flow" of income. The model has two produced goods with each good produced by a single firm. Each firm produces using as inputs labour and the output of the other firm. Although specialised, this model captures all the essential features of the situation.

One particular area of focus is the consequence of price discrimination between final and intermediate consumers. It is commonly observed that many goods are marketed with two distinct prices: one for final consumers of the good and a lower "trade" price for producers wishing to use the good as an input. The existing literature is at a loss to provide a convincing model of this phenomenon but it is clear that it will be the natural outcome in the presence of imperfect competition if producers are able to distinguish between the two classes of customer. Given that this practice occurs, it is natural to investigate its welfare consequences and to consider whether the model provides a justification for why trade prices are observed to be lower than final prices.

The results demonstrate that the model generates expected conclusions when no price discrimination occurs, although the extent to which the substitution of labour for produced input takes place is slightly surprising. In contrast, the results that emerge when price discrimination is allowed, and the relation of these to the no-discrimination case, are in some cases rather unexpected. That both firms can lose and the consumer can gain through price discrimination is contrary to expectations. The same is also true when collusion is permitted; two results of note being that both firms and the consumer can gain by collusion in the setting of trade prices and that trade prices may be below marginal cost.

The paper is organised as follows. Section 2 provides a formal description of the model, derives the objective demand functions and characterises the three equilibria. Section 3 describes the specification employed in the numerical simulations. Results of the simulations are given in section 4 and section 5 contains the conclusions.

2. Description of model

This section describes the structure of the model and the derivation of the aggregate demand functions. The three equilibrium concepts that are employed are also introduced.

The model has two firms, labelled 1 and 2, each producing a single good using labour and the product of the other firm. Two different prices are distinguished: intermediate, or "trade", prices denoted p_1, p_2 and final consumer prices q_1 and q_2. If there is no price discrimination then clearly $p_i = q_i$, i = 1, 2. Writing w for the wage rate, the cost function of firm i is given by

$$C^i = C^i(p_j, w, X^i), i, j = 1, 2, i \neq j, \tag{1}$$

where X^i is total production of firm i. Using Shephard's lemma intermediate good demand facing firm j, X^{jF}, is given by

$$X^{jF} = \frac{\partial C^i(p_j, w, X^i)}{\partial p_j} \equiv C^i_1(p_j, w, X^i), i, j = 1, 2, i \neq j. \tag{2}$$

The difficulty in providing a notion of objective intermediate demand is clear from (2): X^{jF} is not determined uniquely by the parameters p_j, w but is also conditional on X^i, which can only be determined by the equilibrium of the system. Hence X^{jF} given above cannot be employed directly in the description of firm i's objective function, a reflection of the circularity in the model. It will be shown below how a derived notion of intermediate demand can be developed that can be used in place of (2).

The demand for final consumption goods and the supply of labour are derived from the actions of a single, aggregate, utility-maximising consumer. Utility is represented by the function

$$U = U(X^{1C}, X^{2C}, L), \tag{3}$$

and the consumer's budget constraint is

$$\pi + wL = q_1X^{1C} + q_2X^{2C}, \tag{4}$$

where π is profit income, X^{iC} consumption of good i and L total labour supply. The budget constraint indicates that the consumer is the recipient of the firms' profits, so that

$$\pi = \pi^1 + \pi^2. \tag{5}$$

From (3) and (4), utility maximisation results in demands of the general form

$$X^{jC} = X^{jC}(q_1, q_2, w, \pi) \geq 0, j = 1, 2, \tag{6}$$

where q_1 and q_2 are the consumer prices. Note that given w and π this is uniquely defined by the choice variables of the firms.

To construct the objective aggregate demand functions it is first observed that the system must be consistent with demands equal to production. This implies

$$X^1 = X^{1C}(q_1, q_2, w, \pi) + C_1^2(p_1, w, X^2), \tag{7}$$

and

$$X^2 = X^{2C}(q_1, q_2, w, \pi) + C_1^1(p_2, w, X^1). \tag{8}$$

Substitution of (8) into (7) then yields

$$X^1 = X^{1C}(q_1,q_2,w, \pi) + C_1^2\left(p_1,w,\ X^{2C}(q_1,q_2,w, \pi) + C_1^1(p_2,w,X^1)\right) \tag{9}$$

If this equation can be solved for X^1, the solution will be of the form

$$X^1 = X^1(q_1, q_2, w, p_1, p_2, \pi). \tag{10}$$

Equation (10) represents the derived objective aggregate demand facing firm 1 incorporating the effects of input demand from firm 2. Repeating the construction, the demand facing firm 2 can be written

$$X^2 = X^2(q_1, q_2, w, p_1, p_2, \pi). \tag{11}$$

It follows from the above that objective intermediate demand can now be defined as

$$X^{jF} = X^j(q_1, q_2, w, p_1, p_2, \pi) - X^{jC}(q_1, q_2, w, \pi),\ j = 1, 2. \tag{12}$$

Intermediate demand expressed in this form is dependent upon the choice variables of the firms and can be used to replace (2).

To investigate the existence of a solution to (7) and (8), first assume

A1. For all q_1, q_2, w > 0, $X^{jC}(q_1, q_2, w, \pi) \geq 0$, j = 1,2.

A2. $C^i(p_j, w, 0) \geq 0$, i, j = 1,2.

Employing A1 and A2, the following lemma provides a sufficient condition for the existence of a solution.

Lemma.

There exists a non-negative solution to (7) and (8) if

$$\text{A3.} \quad 1 - C^1_{10}C^2_{10} > 0, \quad C^j_{10} \equiv \frac{\partial C^j_1}{\partial X^j} \geq 0, \; j = 1,2.$$

Proof.

Write (9) in the form

$$g^1(X^1) = X^{1C}(q_1,q_2,w, \pi) + C^2_1\big(p_1,w, X^{2C}(q_1,q_2,w, \pi) + C^1_1(p_2,w,X^1)\big).$$

It is clear that the solution occurs when $X^1 - g^1(X^1) = 0$. Now let $X^1 = 0$. From A1 and A2, for all p_1, p_2, q_1, q_2, w, π, $X^1 - g^1(0) \leq 0$; if it is zero for a particular set of parameter values then a non-negative solution has been found. Now take the case that $X^1 - g^1(0) < 0$. Using A3 $g^1(X^1)$ is a contraction mapping and thus has a unique fixed point which is clearly positive. Furthermore, it follows that this solution is continuously dependent on the parameters. The same construction can be applied to (8). •

The content of the restriction in A3 can be understood by noting that C^i_{10} represents marginal intermediate input use. The condition is therefore satisfied if the production of an extra unit of good j requires less than one unit of good i in the chosen units of measurement. Condition A3 is thus connected to the notion that the system is productive in the sense of being able to produce positive net outputs of both goods. In general, the restriction may hold for some price levels but not at others. In particular, if substitution can take place of intermediate good for labour, at low price levels input demand may be such that it is optimal to employ more than one unit of intermediate good per unit of output.

The form of intermediate demand in (12) is clearly an objective notion. It would have possible to define intermediate demand in a subjective manner following the practice adopted for final demand in Negishi (1961). There are two reasons why this approach was not adopted. Firstly, the choice of functional restrictions for subjective demand and the manner in which the subjective demands are formed by the firms are clearly open to discussion and a specific choice would be essentially arbitrary. Secondly if the firms' beliefs are not to be proved wrong it is natural to assume subjective and objective coincide at the equilibrium, the actual equilibrium is then invariant to the choice.

Using the derived demands, the description of the model is completed by specifying the profit functions

$$\pi^1 = q_1.X^{1C} + p_1.[X^1 - X^{1C}] - C^1, \tag{13}$$

and

$$\pi^2 = q_2.X^{2C} + p_2.[X^2 - X^{2C}] - C^2. \tag{14}$$

At this point it is possible to define the three equilibrium concepts that are analysed. The first assumes that there is no price discrimination between intermediate and final consumers. In this case, it is natural to define equilibrium as the Nash equilibrium in choice of prices with payoffs given by (13) and (14). Equilibrium therefore occurs when

$$q_1^* = \text{argmax}\left\{\pi^1 \mid q_2 = q_2^*\right\},$$

and

$$q_2^* = \text{argmax}\left\{\pi^2 \mid q_1 = q_1^*\right\},$$

with $q_i = p_i$, $i = 1, 2$. Computation of this equilibrium simply requires the simultaneous solution of the first-order conditions.

It is worth commenting on the treatment of income effects, via the entry of profit levels in the demand functions, in this equilibrium and the two that follow. One possibility would be to assume that the firms do not take account of the income effect when maximising, an assumption that is probably appropriate for a large economy. More attractive for the present small economy are the following two possibilities. Either the firms are completely objective and take account of how their actions affect both their profit level and the profit level of the other firm or they take account of the effect of their profit level but treat the other firm's level as fixed (thus forming a "Nash" conjecture on the price and profit of the other firm). However, the linear specification of the model used in the simulation obviates the need to settle on one of these options. If the model were to be approached analytically, a choice would need to be made.

The second equilibrium concept permits price discrimination. For this purpose the model is interpreted as a two - stage game in which the firms first set producer prices and then consumer prices are determined given the known producer prices. Equilibrium is then defined as the perfect equilibrium of this two stage game. The institutional arrangement behind this choice of form is that the firms enter longer-term contracts with their fellow producers and these are settled before consumer prices are determined.

Analytically, the equilibrium is found by obtaining the first-order conditions for optimal choice of q_1 and q_2 from (13) and (14) conditional on p_1 and p_2. Solving these will provide solutions

$$q_1 = q_1(p_1, p_2, w), \tag{15}$$

and

$$q_2 = q_2(p_1, p_2, w). \tag{16}$$

Substituting (15) and (16) into the definitions of profit then determine profits in terms of trade prices alone. Solving for the resulting Nash equilibrium gives the choice of trade prices and (15), (16) the consumer prices.

The final equilibrium concept is a variant of the second in that it permits price discrimination but also has an element of collusion. As before, it is assumed that the firms compete in their choice of final prices conditional upon previously selected trade prices. However, trade prices are selected collusively to maximise joint profits given the behaviour of final prices at the second stage. Optimal trade prices therefore solve

$$\max_{\{p_1,p_2\}} \pi^1 + \pi^2 \text{ subject to } q_i = q_i(p_1, p_2, w),\ i = 1, 2.$$

The motivation behind this form of equilibrium is that collusion over final prices is generally prevented by legislation but is rather less well regulated with respect to trade prices. In addition, collusion over final prices is subject to regulation as it is widely viewed to be harmful to the interests of the purchaser but, in direct contrast, collusion over trade prices will usually be welcomed by the "trade" purchaser.

To close this section some comments on the choice of model are offered. The model can be seen as an extension to an imperfectly competitive environment of the standard two-sector competitive model, discussed in detail in Atkinson and Stiglitz (1980). It improves upon the model of Harberger (1962) by including explicit profit maximising behaviour rather than imposing mark-up pricing. Although the assumption of profit maximisation has been subject to some criticism in this form of model, as it is not in the interests of the consumers who are assumed to be the final owners of the firms, it would still appear to be the most natural assumption and a defence of this approach has been offered by Gabszewicz and Vial (1972). It still seems worth noting that if the aggregate consumer is composed of many "small" consumers, each with little direct control over the firms, then profit maximisation seems natural. Even if the single consumer is literally interpreted as the sole owner of the firm (an interpretation I feel should be avoided), it can still be argued that they may have a dichotomous personality and when acting as the owner of the firm myopically seeks to maximise the firm's profits regardless of how this eventually affects their welfare.

It is a natural consequence of having only two sectors that the interactions and linkages between the two firms will be exaggerated compared to any larger model. In defence of this, it does have the benefit of highlighting the causes and effects involved in the model. Finally, with respect to the choice of equilibrium concepts, the first two reflect the obvious possibility, or otherwise, of price discrimination. The third is rather a hybrid but its consequences will be shown to be rather interesting and may go some way to explaining the observed relation between trade and producer prices. Other equilibria could also be defined but the intention here is to illustrate possibilities rather than to be encyclopaedic.

3. Simulation specification

Although conceptually simple, the above model is only analytically tractable in a limited number of special cases. It therefore appears more valuable to investigate the model via numerical simulation. In constructing the simulation two general factors are captured: the elasticity of substitution between labour and produced goods as inputs and the complementarity/substitutability on the final goods markets.

To allow the degree of substitutability to vary in production, the cost function for firm i is chosen to be of the C.E.S. form

$$C^i\left(p_j,\ w,\ X^i\right) = K_i\left[(m_i)^{1/1-\rho_i} p_j^{\rho_i/\rho_i-1} + (1-m_i)^{1/1-\rho_i} w^{\rho_i/\rho_i-1}\right]^{\rho_i-1/\rho_i} X^i \equiv C^i(p_j,\ w).X^i. \quad (17)$$

ρ_i and m_i are the parameters that define the underlying production function. By varying ρ_i it is possible to vary the elasticity of substitution between the intermediate input and labour. When ρ_i is 1, the technology is linear with an infinite elasticity of substitution. It is Cobb-Douglas, with unit elasticity, at $\rho_i = 0$ and cost function

$$C^i\left(p_j, w, X^i\right) = K_i\left[p_j^{m_i} w^{(1-m_i)}\right] X^i.$$

At the other extreme, costs are Leontief as $\rho_i \to -\infty$, so the elasticity of substitution is zero, with cost function

$$C^i\left(p_j, w, X^i\right) = K_i[p_j + w]X^i.$$

To satisfy the restriction on productivity of the system, K_i, the input of intermediate good per unit of output in the Leontief case, must be less than 1. It follows from (17) that the intermediate demand facing j is

$$X^{jF} = K_i\left[(m_i)^{1/1-\rho_i} p_j^{\rho_i/\rho_i-1} + (1-m_i)^{1/1-\rho_i} w^{\rho_i/\rho_i-1}\right]^{-1/\rho_i} (m_i)^{1/1-\rho_i} (p_j)^{1/\rho_i-1} X^i \equiv c^i(p_j, w).X^i. \quad (18)$$

The assumed form of utility function is

$$U = \alpha_1 X^{1C} - \frac{\beta_1 X^{1C^2}}{2} + \alpha_2 X^{2C} - \frac{\beta_2 X^{2C^2}}{2} + \delta X^{1C} X^{2C} - L, \quad (19)$$

where α_i, β_i, $i = 1, 2$ are positive constants. By varying the value of δ, it is possible to investigate the consequences of alternative substitutability and complementarity relations between the two goods. It should also be noted that additivity in labour supply eliminates income effects.

From (19), consumer demand is determined by the linear demand functions

$$X^{1C} = a_1 - b_1 q_1 + d_1 q_2, \quad (20)$$

and

$$X^{2C} = a_2 + b_2 q_1 - d_2 q_2, \quad (21)$$

where

$$a_1 = \frac{\alpha_2\delta+\beta_2\alpha_1}{\beta_1\beta_2-\delta\delta}, \quad b_1 = \left(\frac{1}{w}\right)\frac{\beta_2}{\beta_1\beta_2-\delta\delta}, \quad d_1 = \left(\frac{1}{w}\right)\frac{-\delta}{\beta_1\beta_2-\delta\delta},$$

$$a_2 = \frac{\alpha_1\delta+\beta_1\alpha_2}{\beta_1\beta_2-\delta\delta}, \quad b_2 = \left(\frac{1}{w}\right)\frac{-\delta}{\beta_1\beta_2-\delta\delta}, \quad d_2 = \left(\frac{1}{w}\right)\frac{\beta_1}{\beta_1\beta_2-\delta\delta}.$$

It is clear from these that the two goods are gross substitutes if $\delta < 0$ and gross complements if $\delta > 0$. In addition, the parameters must be restricted so that the inequality $\beta_1\beta_2-\delta\delta > 0$ is satisfied.

While the utility function above is fairly flexible, the linear structure of demand is more restrictive than would be ideal. However, although the linearity is not required for the analysis of the no-discrimination model, it is one of the few forms that permits explicit solution of the second stage of the two-stage models.

Combining the specifications of intermediate demand and solving as in (9) and (10), the objective aggregate demands are found to be

$$X^1 = \left(\frac{a_1+c^2a_2}{1-c^1c^2}\right) - \left(\frac{b_1-c^2b_2}{1-c^1c^2}\right).q_1 + \left(\frac{d_1-c^2d_2}{1-c^1c^2}\right).q_2, \tag{22}$$

and

$$X^2 = \left(\frac{a_2+c^1a_1}{1-c^1c^2}\right) + \left(\frac{b_2-c^1b_1}{1-c^1c^2}\right).q_1 - \left(\frac{d_2-c^1d_1}{1-c^1c^2}\right).q_2. \tag{23}$$

Note that these demands are dependent upon trade prices via the terms c^1 and c^2 from the first derivatives of the cost functions.

The characterisation of equilibrium for the no-discrimination model can be calculated by substituting from (19) - (23) into (12) and (13), differentiating with respect to the choice variables and solving the resulting simultaneous equations.

For the two-stage, price discrimination model, with trade prices taken as given the optimal values of q_1 and q_2 can be found to take the form

$$q_j = h_j + j_j(p_1) + k_j(p_2),\ j = 1, 2. \tag{24}$$

Substitution of these forms into the profit functions then allows the optimal trade prices to be calculated by an iterative procedure. In the case of Leontief costs, equations (24) are affine functions of the producer prices and a closed form solution can be derived.

The collusive equilibrium can be calculated by substituting the functions (24) into the definition of profits and then choosing producer prices to maximise the sum of profits.

4. Simulation results

The simulations reported below impose one further restriction on the model: only symmetric equilibria are considered. This restriction greatly simplifies the computation. Nine tables of results are given. The first three relate to the no-discrimination case and the remainder to the model with discrimination, with the no-collusion results being presented first. In each case the tables are distinguished by the value of δ, which captures the substitutability/complementarity relation. For each value of δ equilibrium prices and quantities are given for a range of values of ρ from 0.99, representing an elasticity of substitution close to infinity, to -10000 which is almost a zero elasticity of substitution.

In all tables X^C, X^F and X represent respectively final, intermediate and total consumption of each good. As the equilibria are symmetric, the prices and quantities are the same for both goods. L is total labour use and profit is that of a single firm.

The remaining parameter values, which are constant throughout, are as follows:

$$a_1 = a_2 = 2000,\ b_1 = b_2 = 0.8,\ m_1 = m_2 = 0.5,\ K_1 = K_2 = 0.5,\ w = 1.$$

It should be noted that as the model possesses standard homogeneity properties, the choice of value for w represents a normalisation procedure for the nominal variables.

Table 1. δ = -.4 (Gross Substitutes), no discrimination.

ρ	p	X^C	X^F	X	L	Profit	Utility
.99	667.3	1110.5	0	1110.5	2237	740000	2959967
.9	667.4	1110.5	0	1110.5	2398	739948	2959948
.8	667.5	1110.4	0	1110.4	2641	739882	2959395
.7	667.6	1110.3	5.8E-7	1110.3	2989	739764	2958936
.6	667.7	1110.2	1.5E-4	1110.2	3524	739579	2958234
.5	668.0	1110.0	4.9E-3	1110.0	4427	739266	2957053
.4	668.6	1109.5	0.05	1109.5	6076	738773	2954746
.3	670.3	1108.0	0.42	1108.4	9158	738196	2949701
.2	677.5	1102.0	2.08	1104.1	14438	739469	2936333
.1	700.9	1082.6	7.48	1090.1	21715	747925	2902234
.08	708.6	1076.2	9.28	1085.4	23262	750941	2891643
.05	722.0	1065.0	12.51	1077.5	25542	756159	2873387
.01	743.4	1047.2	17.86	1065.0	28397	764265	2844400
-.01	755.4	1037.2	20.99	1058.2	29700	768626	2828109
-.05	781.2	1015.7	28.11	1043.8	31981	777448	2792791
-.1	815.6	987.0	38.47	1025.5	34112	787941	2744885
-.2	883.7	930.2	62.85	993.1	35858	804132	2646703
-.5	1025.8	811.8	144.4	956.2	29376	818090	2427069
-.8	1092.6	756.2	215.8	971.9	21042	815666	2317479
-1	1118.6	734.5	254.7	989.2	17037	813093	2273575
-2	1184.1	679.9	374.4	1054.3	7922	801128	2157000
-5	1252.8	622.7	480.4	1103.0	3154	778499	2022255
-10	1288.3	593.1	518.8	1111.9	1989	763074	1948246
-100	1328.7	559.4	551.7	1111.2	1185	742704	1860945
-1000	1333.2	555.7	554.9	1110.6	1118	740255	1851030
-10000	1333.7	555.2	555.2	1110.4	1111	739981	1849925

Table 2. $\delta = 0$, no discrimination.

ρ	p	X^C	X^F	X	L	Profit	Utility
.99	1000.6	1249.2	0	1249.2	2516	1248742	3745983
.9	1000.6	1249.2	0	1249.2	2698	1248650	3745801
.8	1000.6	1249.2	0	1249.2	2971	1248514	3745528
.7	1000.7	1249.1	1.7E-7	1249.1	3362	1248318	3744887
.6	1000.8	1249.0	6.3E-5	1249.0	3965	1248017	3744034
.5	1001.0	1248.7	2.5E-3	1248.7	4985	1247506	3742514
.4	1001.5	1248.1	0.03	1248.2	6887	1246553	3739360
.3	1002.6	1246.7	0.27	1247.0	10624	1244680	3732868
.2	1007.8	1240.2	1.55	1241.8	17556	1241146	3712868
.1	1027.1	1216.1	6.28	1222.4	27908	1235128	3653424
.08	1033.9	1207.6	7.98	1215.6	30168	1233479	3633645
.05	1046.3	1192.1	11.10	1203.2	33500	1230570	3598070
.01	1066.6	1166.7	16.44	1183.2	37621	1225645	3540335
-.01	1078.2	1152.2	19.61	1171.9	39460	1222626	3507396
-.05	1103.4	1120.7	26.92	1147.7	42563	1215354	3435572
-.1	1136.8	1079.0	37.70	1116.7	45217	1203999	3339390
-.2	1201.0	998.7	63.22	1062.0	46576	1176211	3150423
-.5	1318.2	852.2	149.0	1001.3	35834	1105519	2792102
-.8	1361.9	797.6	226.2	1023.8	24926	1073823	2656610
-1	1376.4	779.5	269.3	1048.8	19979	1062914	2611925
-2	1409.2	738.5	406.4	1144.9	9112	1036138	2508582
-5	1447.9	690.1	532.4	1222.5	3581	997441	2375901
-10	1470.3	662.1	579.1	1241.3	2248	972398	2295524
-100	1496.9	628.9	620.2	1249.1	1334	940696	2197779
-1000	1500.0	625.0	624.1	1249.1	1257	936871	2186243
-10000	1500.3	624.6	624.5	1249.2	1249	936499	2185125

Table 3. $\delta = .4$ (Gross Complements), no discrimination.

ρ	p	X^C	X^F	X	L	Profit	Utility
.99	1200.5	1998.7	0	1998.7	4025	2397487	6392974
.9	1200.5	1998.7	0	1998.7	4317	2397340	6392682
.8	1200.5	1998.7	0	1998.7	4754	2397122	6392245
.7	1200.6	1998.5	1.5E-7	1998.5	5380	2396709	6391019
.6	1200.7	1998.2	6.3E-5	1998.2	6344	2396127	6389455
.5	1200.8	1998.0	2.8E-3	1998.0	7979	2395209	6387220
.4	1201.2	1997.0	0.04	1997.0	11051	2393271	6381745
.3	1202.1	1994.7	0.34	1995.1	17207	2389285	6370182
.2	1205.9	1985.2	2.04	1987.3	29043	2379491	6335470
.1	1221.5	1946.2	8.81	1955.1	47419	2353635	6222425
.08	1227.3	1931.7	11.30	1943.0	51490	2345092	6182847
.05	1237.8	1905.5	15.97	1921.5	57506	2329875	6112122
.01	1255.3	1861.7	24.06	1885.8	64913	2304598	5995642
-.01	1265.3	1836.7	28.92	1865.7	68190	2289945	5929350
-.05	1287.1	1782.2	40.22	1822.5	73628	2257120	5784806
-.1	1315.9	1710.2	57.00	1767.2	78085	2211476	5592933
-.2	1369.8	1575.5	97.06	1672.6	79790	2118225	5229330
-.5	1458.5	1353.7	235.2	1588.9	60495	1944197	4621449
-.8	1484.6	1288.5	364.5	1653.0	42143	1891835	4447764
-1	1491.4	1271.5	438.6	1710.1	33876	1879377	4405439
-2	1504.7	1238.2	681.2	1919.5	15612	1855389	4324083
-5	1527.2	1182.0	911.8	2093.8	6188	1802057	4162962
-10	1542.9	1142.7	999.5	2142.3	3896	1761201	4044752
-100	1562.9	1092.7	1077.8	2318.5	2318	1706700	3891041
-1000	1565.2	1087.0	1085.5	2172.5	2187	1700279	3873185
-10000	1565.5	1086.2	1086.1	2172.3	2173	1699437	3870851

Reviewing tables 1 - 3, it can be seen that these display generally the same pattern of results. The equilibrium price rises as the elasticity of substitution falls, explained by aggregate demand becoming less elastic, and this is accompanied by a fall in final demand. At high elasticities of substitution no intermediate input is used and production is entirely by labour alone. Labour use rises at first and then falls, it is highest around the Cobb-Douglas case of $\rho = 0$. At these values around zero, production is taking place at extreme points on the isoquants: labour is being substituted for intermediate input despite the low degree of substitutability.

For δ equal to 0 and 0.4, profits fall monotonically as ρ falls. The firms therefore prefer the high elasticity of substitution, because it prevents them being exploited by their rival, despite the reduction it causes in their own market power. As ρ falls the firms are able to capture each other's surplus, a process that ultimately reduces the welfare of both. In contrast, for $\delta = -0.4$ profit first falls then rises, with a peak at $\rho = -0.5$, and then falls again. The period of rising profit coincides with a rapidly rising price and substitution of intermediate input for labour.

Utility is highest in all cases when the elasticity of substitution is high, the consumer benefits from the firms being unable to exploit each other's demand for intermediate input. The consumer thus prefers a high elasticity of substitution. It is interesting to compare the reasoning behind this conclusion with the motivation for a similar result in Ireland (1989).

Finally, contrasting the tables, the prices are lowest when the goods are substitutes on the final market and highest when they are complements - the expected conclusion.

Table 4. $\delta = -.4$ (Gross Substitutes), price discrimination.

ρ	p	q	X^C	X^F	X	L	Profit	Utility
.99	16.4	667.3	1110.5	0	1110.5	2237	739994	2959976
.9	21.8	667.4	1110.5	0	1110.5	2399	739940	2959763
.8	34.5	667.5	1110.4	2.7E-5	1110.4	2641	739860	2959440
.7	64.7	667.6	1110.3	1.3E-3	1110.3	2989	739744	2958978
.6	163.0	667.7	1110.2	5.1E-3	1110.2	3522	739566	2958271
.5	692.5	668.0	1110.0	4.6E-3	1110.0	4427	739265	2957060
.4	8418.1	668.5	1109.5	8.9E-4	1109.5	6239	738663	2954632
.3	1448986	670.0	1108.3	8.7E-6	1108.3	11086	737046	2948149
.2*	8.7E9	677.2	1102.3	1.0E-8	1102.3	34703	729110	2916469
.1*	4.6E12	908.2	908.2	0	908.2	634408	474691	2011571
.08*	1.5E13	1539.4	383.8	0	383.8	937895	11894	420629
.05*	3.5E7	1898.7	84.4	1.0E-4	84.4	203539	58450	125445
.01	3.9E6	1878.4	101.3	0.01	101.3	161091	109747	231875
-.01	1229608	1860.2	116.0	0.07	116.1	152737	139452	295311
-.05	283351	1846.6	127.8	0.39	128.2	122082	174982	371015
-.1	95464	1836.2	136.5	1.38	137.9	92919	204218	435868
-.2	30147	1830.2	141.5	5.22	146.7	56507	230719	504571
-.5	9308.7	1827.9	143.4	22.79	166.2	20177	252049	611500
-.8	6289.3	1799.6	167.0	45.91	212.9	11838	294629	785441
-1	5429.7	1776.0	186.7	63.35	250.0	9337	326875	915801
-2	3921.8	1696.1	253.3	138.94	392.2	4382	427369	1379874
-5	3237.6	1649.8	291.9	225.05	516.9	1731	480631	1745264
-10	3075.5	1645.7	295.2	258.22	553.4	1072	485359	1845224
-100	2966.0	1651.1	290.7	286.73	577.4	621	479714	1909076
-1000	2957.1	1652.2	289.8	289.39	579.2	583	478519	1913615
-10000	2956.2	1652.3	289.7	289.66	579.4	580	478404	1914093

Table 5. $\delta = 0$, price discrimination.

ρ	p	q	X^C	X^F	X	L	Profit	Utility
.99	16.5	1000.5	1249.4	0	1249.4	2516	1248741	3746225
.9	21.6	1000.5	1249.3	0	1249.3	2699	1248650	3745951
.8	34.0	1000.6	1249.3	3.3E-5	1249.3	2971	1248514	3745542
.7	61.5	1000.7	1249.2	1.8E-3	1249.2	3362	1248318	3744959
.6	143.5	1000.8	1249.0	8.0E-3	1249.0	3961	1248018	3744070
.5	413.1	1001.0	1248.7	9.4E-3	1248.8	4976	1247511	3742547
.4	4030.2	1001.4	1248.2	3.4E-3	1248.2	6992	1246502	3739498
.3	169037	1002.5	1246.9	2.1E-4	1246.9	12330	1243827	3731447
.2*	5.1E8	1007.8	1240.3	2.4E-7	1240.3	38394	1230725	3692054
.1*	6.6E12	1183.6	1020.5	0	1020.5	722221	813370	2526691
.08*	4.5E13	1700.1	374.8	0	374.8	985412	-76813	401442
.05*	6.3E8	1949.3	63.4	4.9E-5	63.4	179008	34026	71262
.01	6.6E6	1943.2	71.0	9.2E-3	71.0	144493	65650	135364
-.01	2.1E6	1939.4	75.8	0.04	75.8	132265	80874	166487
-.05	471866	1932.0	84.9	0.22	85.2	111082	108562	223744
-.1	155969	1924.7	94.1	0.85	94.9	89143	136525	283394
-.2	48036	1916.2	104.7	3.65	108.4	58111	171601	365941
-.5	14209	1888.5	139.4	21.86	161.3	25644	250441	598598
-.8	9241.2	1834.9	206.4	56.45	262.8	18024	369660	977989
-1	7790.1	1797.9	252.5	85.45	338.0	15084	446503	1245470
-2	5195.3	1682.9	396.4	217.4	613.8	7530	663367	2146274
-5	4034.6	1612.9	483.8	373.0	856.9	2977	778890	2837127
-10	3775.4	1601.1	498.6	436.0	934.6	1844	797346	3037723
-100	3612.8	1599.7	500.4	493.5	993.9	1070	799896	3184154
-1000	3600.5	1600.1	499.8	499.1	998.9	1006	799284	3186481
-10000	3599.3	1600.2	499.8	499.7	1000.2	1000	799216	3197698

Table 6. $\delta = .4$ (Gross Complements), price discrimination.

ρ	p	q	X^C	X^F	X	L	Profit	Utility
.99	16.4	1200.4	1999.0	0	1999.0	4026	2397584	6393557
.9	20.8	1200.4	1999.0	0	1999.0	4318	2397408	6393089
.8	33.4	1200.5	1998.8	5.7E-5	1998.8	4754	2397147	6392391
.7	58.5	1200.5	1998.6	3.5E-3	1998.7	5379	2396771	6391398
.6	128.0	1200.6	1998.4	0.02	1998.4	6337	2396196	6389896
.5	401.5	1200.8	1998.0	0.02	1998.0	7952	2395225	6387323
.4	2391.1	1201.1	1997.2	0.01	1997.2	11141	2393307	6382174
.3	52567	1202.0	1995.1	1.7E-3	1995.1	19487	2388284	6368732
.2*	3.2E7	1206.0	1984.9	1.2E-5	1984.9	594589	2364149	6304266
.1*	1.3E13	1350.4	1624.0	0	1624.0	1180006	1546067	4261101
.08*	9.7E12	1685.1	787.3	8.0E-9	787.3	1862820	401726	1038377
.05*	1.0E9	1946.5	133.6	7.2E-5	133.6	427059	46575	100312
.01	1.0E7	1902.1	244.6	0.02	244.7	602833	163933	351904
-.01	3.3E6	1909.0	227.7	0.09	227.8	510936	179165	379404
-.05	763731	1887.2	282.1	0.62	282.7	497614	283558	601292
-.1	253674	1865.5	336.1	2.71	338.8	443647	405225	865751
-.2	78193	1837.2	406.9	13.4	420.3	319946	587679	1290686
-.5	22739	1784.1	539.9	83.5	623.4	134070	896094	2203982
-.8	14300	1724.3	689.3	187.6	876.9	76356	1150307	3123614
-1	11745	1686.4	784.0	264.5	1048.5	57340	1293412	3696933
-2	7044.6	1572.0	1070.0	586.4	1656.3	22482	1670690	5505351
-5	4957.4	1501.8	1245.5	960.3	2205.8	7931	1866524	6869000
-10	4521.7	1488.3	1279.2	1118.7	2398.0	4809	1901447	7286843
-100	4275.2	1484.1	1289.9	1272.2	2562.0	2764	1912843	7619758
-1000	4258.6	1484.1	1289.7	1287.9	2577.7	2597	1912792	7650307
-10000	4257.1	1484.1	1289.7	1289.5	2579.2	2581	1912762	7653282

The results presented in tables 4 - 6 also have a similar general pattern but there are a number of interesting differences when they are viewed closely. In all cases trade prices first rise, almost become unbounded and then fall, the asterisks indicate difficulties with achieving convergence. The reason for this behaviour is that these prices are determined by the resolution of two factors: the intermediate good demand and the effect on the final market price. Without the latter of these effects, trade prices would be unbounded for values of ρ less than or equal to zero. The fact they are bounded in the results above is a consequence of the second effect but the strength of this effect is also dependent on ρ; for positive values of ρ it is clearly very weak. The ranking of trade prices across the values of δ is of some interest. For high values of ρ they are lowest when the goods are complements but for high values the ranking is reversed and they are lowest for the substitutes case.

Final consumer prices at the highest elasticities of substitution are very similar to the no-discrimination case. This indicates that for these values of ρ, the trade prices are chosen only to achieve the optimal consumer prices and have no other purpose since no intermediate inputs are actually employed. The ranking of final consumer prices bears the same relation to δ as in the no-discrimination case for high elasticities but is reversed as the elasticity falls; for the Leontief case final prices are lower when the goods are complements and highest when they are substitutes. This is the converse of the no-discrimination result and contrary to the natural expectation.

Contrasting trade and final prices it can be seen that trade prices are not always lower than final prices. The strategic importance of the trade price eliminates the incentive to charge a low price in order to receive a lower-priced input in return. Trade prices are only lower at very high elasticities of substitution when little or no intermediate input is used.

Final consumption mirrors that for no-discrimination: falling and then rising. In contrast intermediate demand starts at zero, becomes slightly positive, falls to zero again and then rises, reaching a maximum either at or just before the Leontief case. Aggregate consumption mirrors final consumption. Labour use begins low, rises to a maximum and then falls to reach a minimum at the zero elasticity point. Around the Cobb-Douglas point, labour use is high and production is taking place on the tail of the relevant isoquant.

Profits fall, rise and then fall slightly again. When trade prices reach their maximum, profits are markedly reduced: even to a negative value in one case (this has been left in the table despite the question mark that hangs over its interpretation). The strategies of the firms are therefore having a "beggar thy neighbour" effect with the firms inflicting considerable harm on each other.

For all values of δ utility falls and then rises. As δ increases, the value of utility at $\rho = 0.99$ and $\rho = -10000$ becomes closer. For $\delta = -0.4$, utility is at a maximum when the elasticity of substitution is high, as for the no discrimination case, but for $\delta = 0.4$ it is highest in the zero elasticity case, in contrast to previous findings. The consumer is obviously better-off with extremes of substitutability since this reduces the firms' market power but precisely which extreme depends on the form of demand. For the complements case, the high elasticity eliminates market power on the intermediate market but for the substitutes case the dominating effect seems to be via the effect upon the final goods market so that a low elasticity is preferred.

Comparing across the no-discrimination and discrimination cases for given values of δ, it can be seen that the final market price is lower for the no-discrimination case for δ equal to -0.4 and 0. In contrast, the final market price is higher with no-discrimination when $\delta = 0.4$. Final consumption has the same pattern: it is higher for no-discrimination for the two lower values of δ but lower when the goods are substitutes. Consequently, the final market pricing and consumption depend on both the structure of demand and the institutional arrangement of the market.

Labour use at the extreme elasticities of substitution corresponds to relative rankings of final consumption but around $\rho = 0$ is always substantially higher in the price-discrimination case. This reflects the high levels of producer prices forcing production around the isoquants, with labour substituted for intermediate input despite technologies which do not encourage substitution.

With a minor number of exceptions, most noticeably close to the Leontief technology with the goods as substitutes, profits are greater when there is no price discrimination. Price discrimination is therefore harmful to the firms since it results in high levels of trade prices and the use of techniques that replace intermediate input with labour. In general the firms would actually welcome the elimination of price discrimination.

For all three values of δ, utility is greater for the no-discrimination case for high elasticities of substitution than for price discrimination. The ranking is reversed for low elasticities of substitution. Consequently, whether a utilitarian government should encourage or eliminate price discrimination is highly dependent upon the elasticity of substitution.

Table 7. $\delta = -.4$ (Gross Substitutes), collusion.

ρ	p	q	X^C	X^F	X	L	Profit	Utility
.99	1.01	667.3	1110.5	1110.5	2221.1	2221	740000	2962222
.9	1.03	667.3	1110.5	827.9	1938.4	2225	740003	3242462
.8	1.08	667.4	1110.5	760.5	1871.0	2235	740007	3280997
.7	1.14	667.4	1110.5	725.9	1836.4	2247	740012	3296505
.6	1.20	667.4	1110.5	715.9	1826.4	2258	740016	3300465
.5	1.29	667.4	1110.5	683.6	1794.1	2279	740021	3311558
.4	1.40	667.5	1110.4	657.4	1767.8	2304	740027	3318744
.3	1.52	667.5	1110.4	640.5	1750.9	2330	740034	3322479
.2	1.69	667.5	1110.4	614.7	1725.1	2369	740042	3326863
.1	1.93	667.6	1110.3	584.0	1694.3	2425	740051	3329988
.08	2.00	667.6	1110.3	574.8	1685.1	2442	740053	3330482
.05	2.12	667.7	1110.3	560.2	1670.5	2471	740057	3330844
.01	2.31	667.7	1110.2	538.4	1648.6	2520	740062	3330424
-.01	2.45	667.7	1110.2	525.3	1635.5	2551	740065	3329615
-.05	2.82	667.8	1110.1	491.9	1602.0	2641	740072	3325681
-.05043	2.82	667.8	1110.1	492.4	1602.6	2640	740073	3325764
	2680.5	730.7	1057.8	17.9	1075.6	65641		2848525
-.1	7570.3	852.3	956.4	18.8	975.2	126134	752087	2633416
-.2	5210.1	932.5	889.6	42.7	932.3	106957	776053	2579318
-.5	2383.1	983.7	846.9	143.0	989.9	51011	807605	2732702
-.8	1767.9	993.9	838.4	236.1	1074.5	30081	818261	2882527
-1	1586.1	996.6	836.2	288.1	1124.3	22951	821848	2957422
-2	1263.1	1000.0	833.3	458.8	1292.1	9918	828374	3155022
-5	1097.7	1000.6	832.8	642.6	1475.4	4128	831269	3285316
-10	1047.7	1000.6	832.9	728.6	1561.4	2742	831962	3317299
-100	1005.0	1000.5	832.9	821.5	1655.5	1759	832453	3331395
-1000	1000.9	1000.5	832.9	831.8	1664.7	1675	832496	3331654
-10000	1000.4	1000.4	833.0	832.8	1665.8	1667	832500	3331666

Table 8. $\delta = 0$, collusion.

ρ	p	q	X^C	X^F	X	L	Profit	Utility
.99	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
.9	1.000	1000.5	1249.4	1249.4	2489.8	2498.8	1248750	4997500
.8	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
.7	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
.6	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
.5	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
.4	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
.3	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
.2	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
.1	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
.08	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
.05	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
.01	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
-.01	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
-.05	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
-.1	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
-.2	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
-.5	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
-.8	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
-1	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
-2	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
-5	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
-10	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
-100	1.000	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
-1000	1.001	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500
-10000	1.005	1000.5	1249.4	1249.4	2498.8	2498.8	1248750	4997500

Table 9. $\delta = .4$ (Gross Complements), collusion.

ρ	p	q	X^C	X^F	X	L	Profit	Utility
.99	0.9995	1200.40	1999.0	2101.5	4100.5	3998.0	2397601	9672384
.9	0.9943	1200.40	1999.0	2116.8	4115.8	3998.3	2397601	9683262
.8	0.9887	1200.40	1999.0	2116.2	4115.2	3998.7	2397601	9682875
.7	0.9832	1200.40	1999.0	2115.7	4114.7	3999.0	2397601	9682477
.6	0.9779	1200.40	1999.0	2114.5	4113.5	3999.3	2397601	9681681
.5	0.9727	1200.40	1999.0	2113.6	4112.6	3999.6	2397601	9681026
.4	0.9674	1200.40	1999.0	2113.5	4112.5	3999.9	2397602	9680961
.3	0.9623	1200.40	1999.0	2113.0	4112.0	4000.2	2397602	9680560
.2	0.9573	1200.40	1999.0	2112.4	4111.4	4000.5	2397602	9680139
.1	0.9523	1200.40	1999.0	2112.0	4111.0	4000.7	2397602	9679881
.08	0.9513	1200.40	1999.0	2111.9	4111.0	4000.8	2397602	9679845
.05	0.9499	1200.40	1999.0	2111.6	4110.7	4000.9	2397602	9679627
.01	0.9479	1200.40	1999.0	2111.6	4110.6	4001.0	2397602	9679584
-.01	0.9470	1200.40	1999.0	2111.3	4110.3	4001.0	2397602	9679407
-.05	0.9449	1200.40	1999.0	2111.5	4110.5	4001.2	2397602	9679545
-.1	0.9427	1200.40	1999.0	2110.9	4109.9	4001.3	2397603	9679084
-.2	0.9379	1200.40	1999.0	2110.6	4109.6	4001.5	2397603	9678860
-.5	0.9241	1200.39	1999.0	2109.3	4108.3	4002.3	2397603	9677935
-.8	0.9108	1200.39	1999.0	2108.1	4107.1	4003.0	2397604	9677127
-1	0.9024	1200.39	1999.0	2107.2	4106.2	4003.4	2397604	9676451
-2	0.8627	1200.39	1999.0	2103.8	4102.8	4005.5	2397605	9674032
-5	0.7687	1200.38	1999.0	2094.8	4093.8	4009.9	2397609	9667530
-10	0.6585	1200.38	1999.0	2084.8	4083.9	4014.2	2397614	9660269
-100	0.2207	1200.35	1999.1	2039.9	4039.0	4019.3	2397641	9626610
-1000	0.0322	1200.34	1999.2	2010.3	4009.4	4006.8	2397660	9603514
-10000	0.0033	1200.33	1999.2	2001.2	4000.3	4000.1	2397666	9596273

The effect of introducing collusion can be seen from tables 7 - 9 to have quite a dramatic effect on the form of the results. In table 7, with the goods as gross substitutes, trade prices start low and then "leap" to a higher level of price. This does not reflect a discontinuity in the model but instead is explained by there being two local maxima at values of ρ around -.05, one at a low price and the other with a far higher price. Below -.05043 the low price maximum is the global maximum, the two give the same level of profit at -.05043 and above this the high price becomes the global maximum. This behaviour would appear to arise from the two conflicting aims of preferring a low priced input but at the same time competing on the final goods market. When substitution of labour for produced input is easy, the first option will raise more profit. The converse is true when substitution becomes more difficult.

Final prices rise slowly and then jump and continue to rise. They are lower than the price discrimination case for all values of ρ and lower than the no-discrimination level except at the jump. Corresponding to this pattern, final consumption falls, drops at the jump and then falls again. However, it is higher than for the other two equilibria except around the jump. Intermediate input use is also higher than in previous cases, due to its lower price. This is mirrored by the use of labour which has a far lower range than it had without discrimination. Collusion thus results in less extreme combinations of labour and produced input in production.

Profits are far higher with collusion than for the other two cases; the expected conclusion. However, what is most surprising is that utility is also higher, with the exception that the no-discrimination case has slightly higher utility around the price jump, so that the collusion is in the interest of both the firms and the consumers. This can mostly be explained by the more effective use of intermediate input in production and the move away from labour intensive techniques.

Table 8 is especially interesting for its almost complete lack of variation. The explanation for this must lie in the fact that the firms are not interacting at all on the final goods market and the collusive setting of trade prices is then aimed solely at aiding their independent profit maximisation at the final stage. A trade price of 1 indicates that the collusion results in marginal cost pricing of intermediate inputs hence the only inefficiency in this case is arising through the monopoly pricing on the final market. The equality of $2X^F$ and L indicates that production takes with equal quantities of labour and intermediate input so that this case has none of the extreme substitution of labour for produced input seen in previous tables.

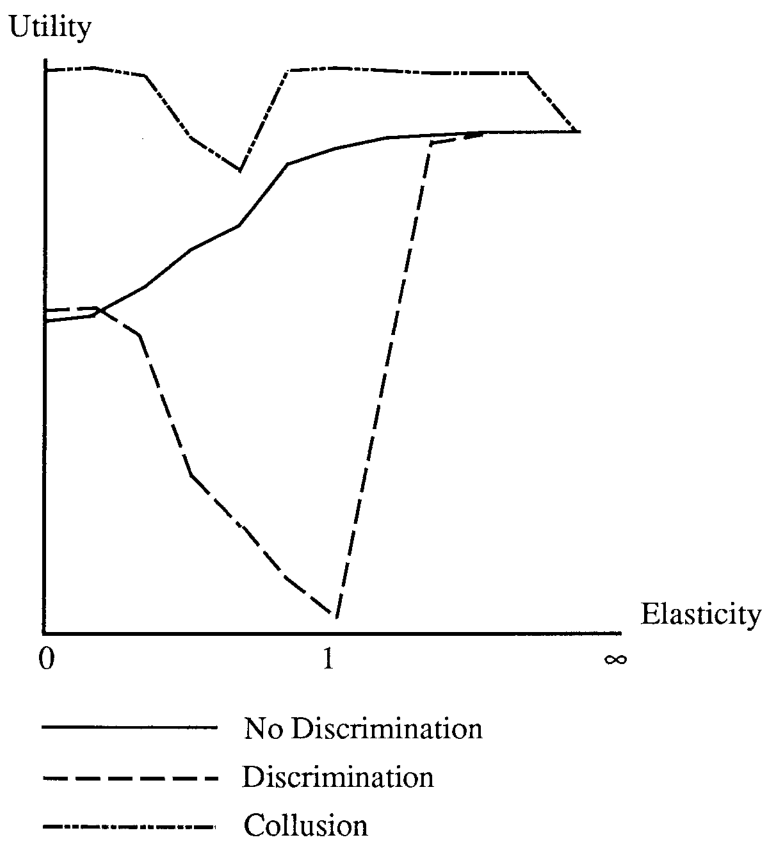
Final consumption, profit and utility are all higher than for the corresponding tables for the two other equilibrium concepts. As in table 7, the collusion is benefiting both the firms and the consumer by removing the excessive use of labour, the lack of interaction on the final market having the consequence of reducing trade prices to marginal cost.

The final table, involving gross complements and collusion, continues the natural progression apparent in tables 7 and 8. The effect of the complementarity on the final goods market is to reduce the trade price below marginal cost. For high elasticities of substitution it is only a little below marginal cost, so that labour and produced input are used fairly equally in production, but falls substantially below cost as the Leontief case is approached. Final prices remain almost constant at a level well below that reported in the corresponding tables for the equilibria with no collusion. The low trade and final prices results in high final consumption, intermediate use and total production. Labour use is also correspondingly high.

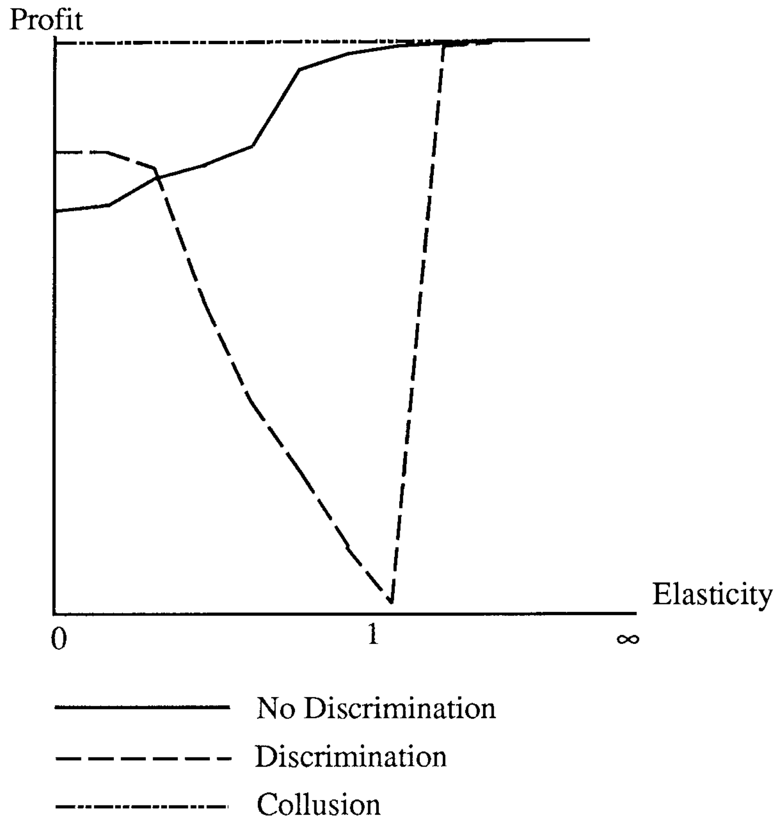
Despite the loss taken on the sales of intermediate input, profits are high and rise as the elasticity of substitution falls. Utility follows a undulating path but its range of variation is small and it remains at a level well in excess of that attained in tables 3 and 6. As in the previous two tables, collusion is beneficial for both the firms and the consumer.

Returning to the question raised in the introduction of the observed relation between final and producer prices, only in tables 8 and 9 are trade prices everywhere less than consumer prices. Interpreted literally, this result suggests that the explanation for trade prices being lower is that firms are colluding and consider their products to be either complements or entirely independent in respect of final demand. This may be too exact an application of the model, but these elements are likely to form the basis of any explanation.

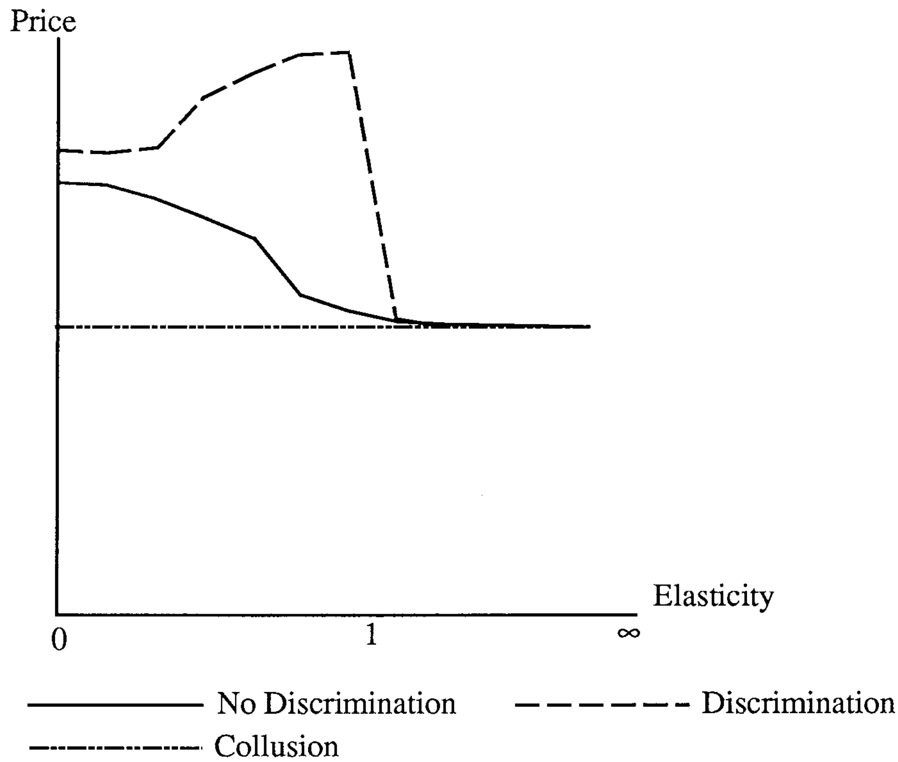
Finally the graphs below provide a qualitative illustration of the typical equilibrium values of utility, profit and final consumer prices.



Graph 1. Utility for Gross Substitutes



Graph 2. Profit for Gross Complements



Graph 3. Consumer Price for no interaction

5. Conclusions

As the numerical results have been discussed at length in the previous section, the conclusions will constitute some broad reflections on the outcome of the analysis. The first point is it has proved possible to construct a consistent model of general equilibrium with imperfect competition and intermediate goods and, furthermore, to expose some special forms of this model to numerical analysis. The model has been the simplest conceivable but a number of extensions could be made, although I would hesitate to claim they are easy. The symmetry could be dropped but at the cost of introducing computing difficulties. Retaining symmetry it would not be too difficult to increase the number of firms in each industry provided a switch was made to Cournot quantity-setting. Also possible would be an increase in the number of sectors.

Extensions aside, the present analysis has highlighted several features. It is clear that the form of market organisation is of critical importance for the equilibrium that emerges. A thorough knowledge of institutional features is therefore required to predict the behaviour of such markets. The application of objectivity throughout and the small scale of the model has in most cases made the equilibrium values highly sensitive to the value of the elasticity of substitution, rather more so than would have been initially suspected. This is particularly true of the price-discrimination model, although I would suggest that some of the very high values of trade prices should be treated as a reflection of the particular special case analysed. However, the important lesson to be drawn from this case is that price discrimination may be mutually harmful, a result that may not be new in the trade literature but may be surprising in the context of imperfect competition. In addition, collusion has been seen to be beneficial both to the firms and to the consumer and to lead to prices that fall below marginal cost.

Finally, the model has also shown that there may also be excessive substitution of labour for produced input. This has the effect of reducing the total output of society to the detriment of all concerned and represents a welfare loss additional to that typically identified in the analysis of monopoly.

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