THE STABILIZING PROPERTIES OF TARGET ZONES

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This paper is circulated for discussion purposes only and its contents should be considered preliminary.
Abstract

We examine the ability of a target zone to stabilize exchange rates in the presence of two stylised forms of market inefficiency - stochastic bubbles and "fads". We show how the usual saddlepath phase diagram is modified in the presence of bubbles (both where fundamentals are deterministic and where they are stochastic), and how a credible policy to defend a target zone prevents the emergence of any such bubbles. In the case of fads, we suppose that the source of shocks driving the exchange rate from its long-run equilibrium level is the fluctuating sentiment of "noise traders". The presence of "smart money" in the market exerts a stabilizing influence upon the exchange rate even in the absence of a target zone. In the presence of a fully credible target zone, where the authorities can take the necessary actions to check the swings in market sentiment, we use results on regulated Brownian motion to characterise the exchange rate trajectory, and demonstrate that the stabilizing influence of "smart money" is increased.
1. Introduction

Exchange rate targeting is an integral element of the "blueprint" proposals for policy coordination developed previously by two of the authors of this paper (Williamson and Miller 1987). Specifically, we proposed that a set of mutually consistent exchange rate trajectories calculated to reconcile internal and external balance in the medium term should be adopted as intermediate targets. Sterilized intervention and international differences in interest rates would be used to maintain exchange rates within relatively wide zones around the exchange rate targets.

To this "target zone" proposal (previously developed in Williamson (1985)), the blueprint added another set of intermediate targets, namely growth rates of domestic demand calculated to support the gradual elimination of any inherited inflation and the gradual restoration of internal and external balance. The average world level of interest rates and fiscal policy would be used to manage the growth of demand.

Two important purposes of these proposals are to secure international consistency of macroeconomic policies and to impose a measure of discipline on governments. These objectives seem to have a certain appeal to academic economists, though the second of them has not been embraced by governments or central bankers (Pohl 1987). But matters are very different so far as the specific focus on exchange rate targeting is concerned. The G-7 governments have adopted this with enthusiasm: while their rhetoric has supported efforts to construct a set of indicators to secure comprehensive policy coordination, their main practical efforts were first directed towards elimination of the dollar overvaluation, and then to the prevention of a dollar overshoot. In contrast, most academics regard exchange rate targeting as an intellectual error.
whose only conceivable justification is that it may secure some coordination in fiscal and monetary policies which affect the exchange rate. But in this view it would be better to focus directly on the policy coordination and to forget about the exchange rate targeting.

This paper is directed at the academics skeptical of the utility of exchange rate targeting rather than at the officials skeptical of the virtues of policy discipline. This does not imply that we believe disciplining markets to be a more important objective of our proposals than disciplining governments. The blueprint is directed at achieving both: we focus on only one objective in this paper in accordance with the principle of division of labour.

We start by setting out the issue to be analyzed in the remainder of the paper. The three main sections that follow analyze the impact of imposing a target zone in three different models: the standard Dornbusch model with Blanchard bubbles incorporated; a similar model with noise in the fundamentals; and a model where traders are divided into "sophisticated" investors who look at fundamentals and "noise traders" (or chartists) who do not. A short final section summarizes our conclusions.

2. **The Issue**

We believe that academic hostility to exchange rate targeting arises largely because the models used to analyze macroeconomic policy have treated exchange rates as an accurate reflection of economic fundamentals -- a reflection marred at most only by serially uncorrelated errors. However, the significance of economic fundamentals in determining exchange rates was thrown into question by the empirical work of Meese and Rogoff (1983), a paper written before the dollar rose to the extraordinary heights which triggered the turnaround in US
policy in 1985, and subsequently by Frankel and Froot (1986). Moreover, the accumulating evidence of excess volatility and serially correlated errors in financial markets in general -- not to mention the stock market crash of 1987 -- has called into question the efficient markets paradigm. In particular, it has been suggested that the failure of market efficiency may be due to the presence of "noise traders" who both increase market volatility and also make systematic errors about the underlying asset values, see for example De Long et al. (1987) and Campbell and Kyle (1988). Sophisticated investors or "smart money" must allow for the behaviour of such noise traders as well as economic fundamentals in forecasting market performance. So too should policymakers.

A prime example of the importance of the assumption that exchange rates are an accurate reflection of the fundamentals arises in the debate on the ability of a target-zone system to limit misalignments. This debate was initiated by Edison, Miller, and Williamson (1987), henceforth EMW, who performed historical simulations on the MCM model designed to test the ability of a system of target zones to limit exchange-rate misalignments over the period 1976-85. They concluded that the feedback rule from exchange rate misalignments to short-run interest rates by which they represented target zones "does indeed succeed in appreciably diminishing the magnitude of misalignments" (p. 205). Subsequent historical simulations on the GEM model reported by Currie and Wren-Lewis (1988) also came to the conclusion that target zones could have had a modest but significant effect in limiting misalignments.

In striking contrast, the historical simulations reported by Frenkel, Goldstein, and Masson (1988), henceforth FGM, show almost no ability of target zones or the blueprint to influence exchange rates, even though FGM conscientiously attempted to stay close to the spirit of the original proposals and adopted essentially the same feedback rule from exchange rate misalignments to
the short-run interest rate as that used by EMW. Real effective exchange rates simply could not be kept within target zones even as wide as ± 10 percent, because this would have required driving interest rates negative. They attributed the small impact of short-term interest rates on exchange rates to their use of MULTIMOD, in which exchange rates are anchored by perfect foresight. The relative success of EMW and Currie and Wren-Lewis in limiting exchange rate misalignments by the use of monetary policy is attributed to the failure of the MCM or GEM models to incorporate forward-looking expectations, and their findings are thus dismissed as subject to the Lucas Critique.

In contrast, EMW had argued that the modest ability of monetary policy to influence the real exchange rate found in their simulations probably understated its real power, for three reasons. First, the feedback rule used was fairly weak: interest rates could have been adjusted more than they were. Second, the model made no allowance for intervention. Third, "it neglects the impact that exchange rate targeting might have had in focusing market expectations and thus limiting misalignments caused by speculative bubbles" (EMW, p. 205).

The present paper develops the third argument. It shows how broadening the conventional rational expectations/efficient market models to allow for market inefficiencies serves to strengthen the case for focusing policy on exchange rates per se, rather than simply on the economic fundamentals that should affect them. It demonstrates that forward-looking expectations do not preclude the possibility of using monetary policy (including the announcement of targets, and intervention, as well as interest rates) to manage the exchange rate. On the contrary, credible exchange rate targeting could be expected, in the models we present, to limit misalignments more effectively than would be
suggested by perfect foresight simulations of models estimated over the period of free floating.

The models that we use to make these points doubtless lack something in terms of descriptive accuracy. They are used because they provide an analytically tractable way of showing that the claims of FGM are critically dependent on the absence of market inefficiencies such as those incorporated in these particular models. Rejection of our conclusions requires not a demonstration that these particular representations of market inefficiency fail to describe some period of history, but proof of the proposition that markets are efficient.

We in fact use three models of market inefficiency to establish our points. The first of these is the Blanchard (1979) model of a rational bubble. A Blanchard bubble is characterized by a progressively faster deviation of the asset price from its equilibrium trajectory; investors know that the bubble must ultimately burst and the price collapse back to equilibrium, but they are tempted nonetheless to continue holding the asset by the hope that the price will rise yet further, sufficiently fast to compensate them for the risk of its collapse, before the collapse occurs. We show in the next section that, if all market participants believe that the authorities will prevent the exchange rate moving outside a specified target zone by sterilised intervention, this must cause the bubble to burst when it hits the edge of the zone. But if collapse were certain at that time, it must occur earlier because the possibility of a capital gain in the previous period is zero. Extending the argument back, it can be shown that a credible target zone will prevent a Blanchard bubble ever starting.

The second model adds the existence of "noise" to the fundamentals. This produces Krugman's (1987) "bias in the band", where the presence of a band
and the credible promise of action at its margins tends to push the rate toward the middle of the band even without intervention or policy change occurring.

The third model introduces "fads", meaning autoregressive deviations of an asset price from the equilibrium trajectory associated with "fundamentals". The high dollar of 1981-86 might, for example, be regarded as in part the consequence of the bullish views of a group of unsophisticated investors. Our analysis shows how a target zone could help to reinforce the influence of informed speculators in limiting the impact of such unsophisticated investors in pushing rates away from equilibrium. This is a particular case of the general proposition that anything the public sector can do to inform the private sector of the rationale for its actions -- assuming that they do indeed have a rationale -- is likely to improve the performance of the system. Once again, an FGM-type analysis that takes it for granted that all speculators are well-informed -- even without any guidance from the public sector -- will underestimate the stabilizing power of a target zone.

3. **Blanchard Bubbles and Currency Bands in a Dornbusch Model**

To examine the impact of credible currency bands on the behaviour of the exchange rate in a setting with rational expectations, we use the log-linear model of Dornbusch (1976). The equations are given Appendix A (which also contains a list of the symbols used) but, as they are doubtless familiar, they are not discussed in detail in the text. It is surely true that the absence of expectations from the inflation process in the Dornbusch model enhances the power of interest rate policy to affect the real exchange rate, relative to MULTIMOD, for example -- how much is an interesting question for investigation. But, as we argue below, to end bubbles, it is probably direct foreign exchange market
intervention to check the exchange rate that is more important than the effects of monetary policy on interest rates.

In part (a) we first indicate the effect of including Blanchard bubbles on the usual phase diagram, before turning to the impact of target zones. In (b) the procedure is repeated, in circumstances where fundamentals are themselves subject to stochastic shocks.

(a) The Deterministic Case

As may readily be confirmed with reference to the Appendix, the formal model consists of four linear equations determining four variables. The first two equations are not dynamic, being the LM and IS curves which determine output and interest rates, conditional on current values for the price level and the exchange rate. The evolution of the latter over time is characterised respectively by a Phillips relation and the "no-profitable-arbitrage condition" -- that any anticipated changes in the exchange rate be matched by the international interest differential. (Foreign interest rates are assumed constant as is the domestic money stock, except at the edge of a currency band - see below.)

The essential feature of this model is the contrast between the dynamic behaviour of the domestic price level and that of the exchange rate. The domestic price level, \( p \) (in logs), is a "sticky" variable which changes relatively slowly over time, influenced by movements in current production above or below the non-inflationary level. By contrast, the nominal exchange rate, \( x \) (in logs), is a "forward looking" variable in no way influenced by its previous value. As indicated formally in the Appendix, its current value is the long run equilibrium value, "discounted" by the international interest differential. (Here the exchange rate is defined as the foreign currency value of the home currency, so a
future interest differential which is, on average, in favour of the domestic economy will be associated with an appreciation of the currency above equilibrium.

As there are only two dynamic processes involved, the system of four equations can be reduced to a couple of simultaneous, linear differential equations in the variables, \( p \) and \( x \), with a two dimensional phase diagram which possesses a "saddlepoint" structure. These saddlepoint dynamics are indicated by the solid lines in Figure 1. There SS indicates the unique stable path leading to equilibrium (defined to be at the origin); UU indicates the unstable path leading from equilibrium: and other paths which satisfy the equations of the model are shown to be diverging progressively away from SS towards UU as time proceeds.

Faced with this multiplicity of solutions, it is typically assumed in such saddlepoint models that, for any historically given value for the predetermined variable (here the price level), the forward-looking variable (here the exchange rate) adjusts precisely so as to position the system on the stable adjustment path, SS. All other paths are ruled out because, although they satisfy the necessary arbitrage condition, they also require the exchange rate to diverge ever further from equilibrium as time passes.

However, in the context of models with such forward-looking asset prices, Blanchard (1979) has proposed an alternative class of rational expectations solutions, which exhibit "bubble" instability, but do not involve the exchange rate following a divergent path for ever. A "Blanchard bubble" always has a positive probability of "bursting": and when it does the exchange rate reverts to the stable manifold and the system then moves progressively towards equilibrium. It is essential that such bubbles end with probability one—which Blanchard assures by assuming that the bursting is governed by a Poisson process.\(^a\) In this section
we first describe the nature of these "rational bubble" paths in the Dornbusch model; then we show that a currency band can effectively "kill off" all such deviant behaviour.

The characteristic feature of these bubble paths is, of course, the risk of a jump in the exchange rate when the bubble bursts. (If the speculation has been in favour of the domestic currency this will imply a downwards collapse; if the speculation has been adverse, then it is the foreign currency which collapses, giving a jump appreciation to the home currency.) To take account of this, the usual arbitrage condition needs to be modified as the expected movement of the rate along a bubble path must now equal the international interest differential together with a term which can be thought of buying an insurance premium against a crash (see equation (4)' in Appendix A and footnote b below for further discussion).

The effect that this modification has on the phase diagram, Figure 1, is easy to see. Firstly, there is no change to the stable path SS itself. But the other trajectories are affected. Specifically, the dotted lines in the Figure represent bubbles which have to diverge faster from the stable manifold than do the deterministic paths, as necessary to compensate for the possibility of a collapse. The unstable manifold of the original system UU is rotated to U'U': by the same logic, a steadily exploding Blanchard bubble also has to have faster appreciation in the exchange rate to compensate for the ever-present likelihood of collapse.c

Of course it is true for all these bubble paths that eventually, when the bubble bursts, the system reverts to the stable path of adjustment to equilibrium. But, as the exchange rate movements may nevertheless wreak considerable havoc on the way, it would be reassuring to know whether or not there exists a form of
market intervention which could identify the errant paths and bring them to an end. In fact, as we shall argue, the imposition of a fully credible currency band has an even stronger impact: no such bubbles can ever get started.

The reasoning is straightforward. If all market participants believe that, when the exchange rate hits the edge of the band, the authorities will defend the rate by sudden intervention (designed to produce a jump in the rate), this must cause the bubble to burst. But this is inconsistent with the existence of a bubble in the first place, as can be seen by the following recursive argument. If all know that collapse is certain to occur at time \( t \), all will wish to sell the currency at \( t - \epsilon \). But then collapse will occur at \( t - \epsilon \). Repeating this argument we find that "collapse" must occur at time zero, i.e. all such bubbles are "strangled at birth".

The effect of imposing a fully credible currency band in a Dornbusch model containing Blanchard bubbles is consequently easy to show, as the infinite range of possibilities suggested by the paths depicted in Figure 1 is reduced to the single stable trajectory in Figure 2. Between \( L \) and \( U \) the system lies on the stable path itself. When the price level is above these limits, the exchange rate lies on the upper edge of the currency band and is held there by a suitable adjustment in monetary policy designed to set the interest differential to zero; but this exerts a downward pressure which ensures stable convergence towards SS. (Similar arguments apply along the bottom edge.)

There is an important distinction to be drawn between the monetary policy intended to hold an equilibrium path at the edge of the band and that designed to deter bubbles. The latter, consisting as we have argued above of sudden intervention in the exchange market, is analogous to a so-called "punishment strategy" in game theory, and possesses the same characteristic that
in equilibrium, if the strategy is fully credible, it is never observed. The former, in contrast, is observed if, for whatever reason, the price level lies outside the range \([p_L, p_U]\). (For further discussion of the monetary policy changes required at the edges of the band see Miller and Weller (1989).)

It seems natural to ask how it might be that the price level is driven outside the range \([p_L, p_U]\). One answer is as a result of changes in exogenous factors (such as foreign prices or the money stock); another possibility is stochastic disturbances to the price level. But, as Paul Krugman (1987) has pointed out, in a stochastic context currency bands should surely affect the behaviour of the rate inside the currency band itself.

In the next section, therefore, we discuss how imposing currency bands causes a "bending" of the stable trajectory inside the band when the inflation process is stochastically disturbed by white noise errors. We also indicate how Blanchard bubbles would affect these stochastic solutions, and discuss the stabilizing effects of credible currency bands.

(b) Shocks to the Fundamentals

Let stochastic disturbances be introduced in the form of serially uncorrelated shocks to the Phillips curve. Then, even in the absence of bubbles, the arbitrage equation must be written in expected value terms, see Appendix A, section (b).

How to find the solutions to such a system is a matter of some technical difficulty (see Appendix C), but the details need not detain us here. Suffice it to say that, even without the presence of bubbles, the solutions to the resulting stochastic system now include an infinity of paths connected to the
equilibrium at the origin (in addition to the two paths SS and UU already encountered). These solutions are shown graphically in Figure 3.

What happens if Blanchard bubbles are now added to this stochastic system? Given our earlier analysis, it is easy to show that once again the bubbles modify the arbitrage condition and, while leaving SS unchanged, they distort the trajectories as necessary to cover the risk of a collapse, rotating the UU path clockwise towards the vertical axis as before. (These paths could be shown as dotted trajectories in Figure 3 but for the sake of clarity have been omitted.)

It is important to note that those features of rational bubbles which some have argued render them implausible in an otherwise deterministic model disappear in our stochastic model. These paths are considerably more well-behaved in that they can spend periods of time converging towards equilibrium in the presence of favourable shocks to fundamentals. There are many paths which will stay close to the "free float" path for long periods of time, so that when they burst the discontinuity in the exchange rate will not be large. So far we are unaware of any attempts to test for the existence of this type of bubble.

The reason for the more regular behaviour of such bubbles in a stochastic framework is clear enough. When a system is subject to random shocks which (as a consequence of feedback effects) generate a tendency to return to equilibrium, this acts in part to offset the divergent influence of the bubble. In addition, of course, a stochastic system in equilibrium can (with low probability) display highly divergent behaviour. Nevertheless, the appearance of such bubbles would represent an undesirable degree of instability.

Can we say what the effect of imposing a fully credible currency band would be? An exactly similar argument to that used in the deterministic
model establishes that no such bubble is sustainable. The unique exchange rate path is given by the S-shaped curve tangential to the top and bottom of the band, as shown in Figure 4. This is the solution first identified by Krugman (1987) in a model in which fundamentals were assumed to follow a random walk.

The benefits stemming from the imposition of a target zone are now twofold. Not only does it result in the demise of the bubbles, but it also generates what Krugman has labelled "the bias in the band", the bending of the exchange rate path away from the stable manifold towards the middle of the band; and this produces what he refers to as a "honeymoon effect", in which the presence of a credible band delays, perhaps quite significantly, the time at which intervention to hold the rate at the edge of the band is necessary.
There is increasing evidence of a number of phenomena, such as "excess volatility" in equity markets and the predictability of "excess returns", which cannot be satisfactorily explained within the framework of the efficient markets paradigm. This has naturally led to empirical and theoretical work designed to document and to explain such phenomena.

At a theoretical level, one of the standard criticisms levelled against any appeal to irrational behavior in asset markets is that any such behavior would be taken advantage of by sophisticated agents in the market; that this would lead to irrational agents incurring losses; and that as a consequence they would exit from the market. In other words, there exists a powerful selection mechanism which would tend to promote market efficiency by eliminating naive investors.

But it has recently been demonstrated that this argument is not correct. De Long et al. (1987) show that it is possible to have a situation in which sophisticated and "irrational" groups coexist in a market, and in equilibrium the irrational group earns a higher expected return. This comes about because the misperceptions of the irrational group increase the noise in the market and sophisticated investors rationally adjust to this by requiring a higher yield on the risky asset. In certain circumstances--in particular if the unsophisticated group are on average "bullish"--it is possible to show that they end up holding a portfolio earning a higher expected return than the sophisticated group. Of course, they are paying a price for this higher return in the form of a higher variance. But the force of the "natural selection" argument is considerably weakened. (The case, as De Long et al. point out, is not conclusive, because it may be important to avoid a high
probability of very low wealth—a probability which increases for the unsophisticated group.)

On the empirical side, Poterba and Summers (1987) have argued that standard tests of market efficiency have very low power in detecting autoregressive "fads". Campbell and Kyle (1988) have tested a general equilibrium model of asset prices which includes such fads and find that one possible explanation for the data is that noise trading increased price volatility in U.S. stocks over the period 1871-1976. De Bondt and Thaler (1985, 1987) present evidence to suggest that a trading strategy of buying stocks that have recently performed poorly, and going short in those which have done well, can earn excess returns. (See also Lehmann (1987)).

Since the autoregressive fads associated with noise trading can explain not only excess volatility but also prolonged periods of excess returns, and since there is some econometric support for these departures from efficient markets, we consider the effect of including such fads in a model of exchange rate determination.

Already, in Miller and Williamson (1987), a first step was taken by including a coloured noise term in the arbitrage equation of a Dornbusch style model—with the natural result that the exchange rate could deviate substantially (and for a prolonged time) from the level implied by fundamentals. We did not, however, consider how imposing wide target zones might affect the stochastic solutions discussed there—not because it was unimportant, but simply because at that time, we could not see how to analyze this nonlinear stochastic problem. (We essentially assumed that the edges of the target zone coincided!) As risk neutrality was postulated, this fad suffered from the criticism that risk neutral speculators would have been able to make profits in betting against it.
Thanks largely to the work of Paul Krugman we can now tackle the problem of imposing wide bands: and, by introducing risk aversion, we can avoid the criticism just discussed. (Also, to simplify the problem, we suppress all shocks other than those propagating the fad, and take domestic prices as fixed.) The formal specification is given in Appendix B. Here we outline the key features and then proceed diagrammatically.

In addition to the effect of (properly discounted) fundamentals, the exchange rate, \( x \) (in logs), is now assumed to be disturbed by an autoregressive "fad", denoted \( n \) (in logs) for "noise traders". This fad is expected to revert to zero (with an autoregressive coefficient of \(-\psi\) ), but it is constantly renewed by uncorrelated shocks with mean zero and variance \( \sigma \). The expected change in the exchange rate anticipated by the rational investors ("smart money") must take account both of the stochastic elements involved in the fad process (since they are assumed to be risk averse) and of the expected change in the fad itself.

The arbitrage condition relevant to the exchange rate now requires that the expected change in the rate should equal the international interest differential (as before) but now corrected by a risk term (assumed to be a scalar multiple of \( \sigma \)) and augmented by a term giving the expected decline of the fad (see equation (13) of Appendix B).

The model to be analysed consists of four equations. The first two are, as before, the IS and LM curves for the goods and money market; the third is the arbitrage condition governing the expected change in the exchange rate, modified as just described. But, as the price level is assumed to be fixed, there is no Phillips curve: instead the fourth equation is a dynamic description of how the fad evolves. Once again the system can be reduced to a couple of (stochastic) differential equations (see equation (17) in Appendix B). Since the noise process
is taken to be autonomous, the diagrams showing the stable and unstable paths $SS$ and $UU$ are rather simpler than for the Dornbusch model (where the evolution of the price level was endogenous). This can be seen with reference to Figure 5, where the exchange rate appears on the vertical axis, as before; but it is the fad, $n$, (rather than $p$) which appears on the horizontal axis.

The independence of the noise process from the rest of the economy means that the unstable path $UU$ now coincides with the vertical axis. As for the stable path $SS$, this has a positive slope which is flatter than the $45^\circ$ line. Both of these lines must pass through the point $E$, which lies below the deterministic equilibrium shown as the point $x = 0$ on the vertical axis. (The reason why the log of the exchange rate must be less than zero, even when the fad is actually zero, is that risk is still present, which means that the domestic interest rate must be higher than foreign interest rate; and this is achieved by a "devaluation" to $E$ where $x = x < 0$.)

In the absence of target zones it is assumed that the system will be stochastically "diffused" along the stable eigenvector, so the exchange rate will have an asymptotic normal distribution centred on $E$ (formally, $N(x, \sigma^2/2\psi)$). Note that for points to the right of the asymptotic expected value $E$ in Figure 5, the bullishness of noise traders has lifted the currency above equilibrium: as the high exchange rate moves the economy into a recession, it cuts the domestic interest rate and hence the asset value of the domestic currency. Consequently, in the model, the effect of the bullish psychology of noise traders is in part offset by the calculations of rational investors.

In contrast to what we observe in the fully rational model, it is possible for the currency to be strong despite the fact that the expected interest differential is in favour of the foreign currency. Sophisticated investors are
aware of this, and their short selling depresses the currency below the level it would have been driven to by noise trading alone.

At point A, for example, the effect of the noise traders on the value of the currency is given by the horizontal distance CE, which is, of course, the same as the vertical distance BC measured from the 45° line. Hence the implied contribution of rational investors ("smart money") must be the negative quantity AB—which corresponds to the (negative) risk-adjusted interest differential at point A scaled by the factor $1/\psi$.

Despite this damping factor, the activities of the noise traders can take the exchange rate anywhere along the line SS. And such meandering will have real consequences in a model (since own goods prices are fixed in domestic currency) where the exchange rate represents a relative price for traded goods--real effects which have, by construction, nothing to do with the fundamentals which have been held constant.

In such a situation it is reasonable to suppose that the authorities will be eager to limit the resulting distortions to trade. Specifically, let us assume that the monetary authorities are determined - by intervening in the exchange market, by changing interest rates or by whatever other means contribute to the same end - to try to stop any growth of bullish sentiment in the currency beyond a certain point; and to take the appropriate action (of opposite sign) if and when bearish sentiment has grown to the same extent. And let us assume that these actions will indeed be effective; and that the "smart money" knows this.

What we have described is an example of the regulation of a stochastic process. The relationship between the exchange rate and the noise process is characterised, as before, by a solution to the fundamental differential
equation (see Appendix C), together with appropriate boundary conditions. The shape of all solutions symmetric about the origin is shown in Figure 6. Note that, because the noise process is autonomous, they have a simpler pattern than in the Dornbusch example, simply bending away from the line SS, with no points of inflection anywhere but at the origin. In the figure, the upper bound on the fad is imposed at C, and the lower bound at C'. The appropriate boundary conditions for a problem of this kind are derived in a recent paper by Avinash Dixit (1988). They consist of the requirement that the solution trajectories have a slope of zero at the points where the process is checked or "regulated." 

By constructing the locus of points (the dashed line FF') where the solution paths reach a maximum or minimum, we can work out how the exchange rate will behave for any given bounds on the fad. In the figure, the upper and lower limits at C and C' (which act as reflecting barriers) will have the effect of restricting the range of variation of the exchange rate to the interval \([L, U]\). Within this range the rate will lie on the trajectory R'ER, buffeted now this way, now that by the vagaries of market sentiment.

It is important to emphasise the fact that the currency band is here derived from the limits imposed upon the noise process. In this respect the analysis is different from the case of the stochastic Dornbusch model, where the currency band was directly imposed, and the driving process was free to wander unchecked. Note that these limits on the gross amount of noise, if feasible and credible, increase the "stabilizing" effect of smart money. At C, for example, this has increased from BA to BR (where the extra effect, the bending of the trajectory, arises because the occurrence of a rebound from C opens up the possibility for sophisticated investors to make arbitrage profits if the trajectory is not stationary at C).
What this means for target zones in this simple model can be appreciated by considering a zone with upper and lower edges at R and R' (as shown), where these limits are to be defended by taking whatever action is necessary to stop the fad growing--here the only thing that can take the rate further away. The bands are narrower than the limits that are set for the cumulated noise (CC') for reasons we have just considered. But, since the outcome is to limit the sentiment of noise traders and not to prick a speculative bubble, the rate can wander about inside the range RR', tending toward the middle (the asymptotic equilibrium value), but being gently reflected from ceiling and floor as and when these are reached.

So far we have assumed that actions taken to limit the propagation of noise are both effective in practice and credible in prospect. Doubts about the willingness of the authorities to take action will weaken the effects that this has in inducing smart money to do some of the stabilizing. Suppose that "smart money" attaches only 50 percent credibility to the necessary steps being taken to hold the rate within a given band. Then the rate will follow the path Q'EQ. The point Q(Q') will lie halfway between G and H (G' and H'). The authorities will have to intervene earlier to hold the rate within a given range. However if by doing so they establish full credibility, the rate will jump on to R'ER.

5. Summary and Conclusions

Recent analysis of financial markets suggests that policies specifically focused on those markets might be an effective way of checking "noise" not associated with economic fundamentals. As a contribution to this debate, we have studied the stabilizing properties of currency bands or target zones when the foreign exchange market suffers from two stylized forms of inefficiency--"rational bubbles" and autoregressive "fads" attributable to noise.
traders. It was assumed implicitly in this analysis that the authorities choose the bands or zones "correctly", in the sense that the equilibrium rate is encompassed within the zone.

As for Blanchard bubbles, the deterrent effect of credible bands was shown to apply both in an (otherwise) deterministic context and also when shocks disturbed the fundamentals. In the latter case, however, target zones have an additional stabilizing effect. Paul Krugman has labelled this the bias in the band.

As the end of the dollar's dizzy rise in 1985 came not with a crash but with a prolonged decline, we also examined how a target zone might operate in the context of an autoregressive fad caused by noise traders. Assuming that policy could not prevent swings in sentiment from emerging, but that it could put limits to the gross size of such fads, results on such regulated stochastic processes suggest that the same sort of "beneficial bias" would emerge because the willingness of the monetary authorities to do this would encourage the smart money to play a stabilizing role.

An exercise such as this raises many questions. Not the least of these is how robust the findings reported by the researchers at the IMF are to such simple changes in the stochastic specifications as we have examined here. If exchange rates are in fact importantly influenced by market inefficiencies--of the stylized forms examined in this paper or of more general forms--this must cast some doubt on their conclusions about the inability of monetary policy to manage exchange rates. For if, as we believe to be the case, their simulations of target zones leave unchanged the residuals observed in the historical data base--residuals which may include bubbles, fads and forecasts which are sensitive to the exchange regime in force--then they are not immune to the Lucas Critique!
Appendix A

(a) The Deterministic Case

The equations of the Dornbusch model used here as follows:

(1) \( m - p = \kappa y - \lambda i \)

(2) \( y = -\gamma i - \eta(x + p - p^*) \)

(3) \( Dp = \phi y \quad \text{or} \quad p = \phi \int_{-}^{t} y(s) \, ds \)

(4) \( Dx = i^* - i \quad \text{or} \quad x = \int_{-}^{t} (i^* - i(s)) \, ds + \bar{x} \)

where the symbols used are defined as follows:

m the log of the domestic money stock,

p the log of the price of domestic final product,

y the log of the level of domestic final production, measured from its non-inflationary level,

X the log of the exchange rate, defined as the foreign currency price of domestic currency,

i (instantaneous) domestic nominal interest rate,

* denotes a variable in the "rest of the world",

D denotes differentiation with respect to time, \( Dx = dx/dt \).

- a bar above a variable denotes its long run equilibrium value.

The system described in equations (1) - (4) may be succinctly summarised as two simultaneous differential equations:

(5) \( \begin{bmatrix} \frac{Dp}{Dx} \end{bmatrix} = A \begin{bmatrix} p \\ x \end{bmatrix} \)
where \( p \) and \( x \) are now measured as deviations from long-run equilibrium. The matrix \( A \) takes the form

\[
A = \frac{1}{\Delta} \begin{bmatrix}
- \phi(\gamma + \lambda \eta) & - \phi \lambda \eta \\
\kappa \eta - 1 & \kappa \eta
\end{bmatrix}
\]

where \( \Delta = \kappa \gamma + \lambda \).

Since the system displays saddlepoint dynamics, there is a unique stable path converging to equilibrium. This is the linear path associated with the (negative) stable root \( (\rho_s) \), so

\( (6) \quad x = \theta_s p \)

where \( \theta_s = \frac{1 - \kappa \eta}{\kappa \eta - \rho_s \Delta} \).

To introduce Blanchard bubbles it is simplest to assume that the bubbles have a constant "death" probability \( \pi \). We can then rewrite (5) as:

\( (7) \quad \begin{bmatrix}
Dp \\
Dx + \pi (\theta_s p - x)
\end{bmatrix} = A \begin{bmatrix} p \\ x \end{bmatrix} \)

where the arbitrage equation is modified to take into account the fact that interest rates must adjust to compensate asset holders for the possibility that the bubble will burst\(^b\). The term \( \pi(x - \theta_s p) \) may be thought of as the cost of buying insurance against a collapse onto the stable manifold.
We compare the dynamics of the systems with and without such bubbles in Figure 1, see text.

(b) **Noise in the Fundamentals**

We modify the original model by adding a white noise disturbance term to the Phillips curve equation (3):

\[
(8) \quad dp = \phi y \, dt + \sigma dz
\]

and modifying the arbitrage equation:

\[
(9) \quad E(dx) = (i^* - i) \, dt
\]

assuming risk neutrality for simplicity. The nature of the solutions to this system, illustrated in Figure 3, is analyzed in detail in Miller and Weller (1988), (1989). A brief outline of the solution procedure is presented below in Appendix C.

In the absence of specified boundary conditions, we assume that the system follows a "free float" along the stable manifold of the deterministic system. Now introduce the possibility that Blanchard bubbles may appear. We rewrite (9) as

\[
(9') \quad E(dx) + \pi(\theta_s p - x) \, dt = (i^* - i) \, dt
\]
Appendix B

Let the (log of the) exchange rate be treated as the sum of two components—the integral of expected future (risk adjusted) interest differentials plus a "noise" term reflecting the psychology of noise traders, cf. Poterba and Summers (1987) and Campbell and Kyle (1988). Assuming that the noise component is autoregressive, we may write

\begin{equation}
    x = E_t \int_h^t (i(s) - i^* - \alpha \sigma) ds + n
\end{equation}

where

\begin{equation}
    dn = \psi ndt + \sigma dz
\end{equation}

and

- \( n \) is the coloured noise component
- \( \psi \) is the autoregressive coefficient
- \( \sigma \) is the instantaneous variance parameter of the noise
- \( z \) is a unit variance Brownian motion process
- \( \alpha \) is a risk-aversion parameter.

As time evolves, (10) implies that

\begin{equation}
    dx = (i^* - i + \alpha \sigma) dt + dn.
\end{equation}

Note that \( dx \) and \( dn \) involve white noise terms. By taking their expected values we get the desired arbitrage condition.
(13) \[ E(dx) - E(dn) = (i^* - i + \alpha \sigma)dt. \]

Now suppose that the foreign interest rate is determined exogenously but the domestic rate is determined by the interaction of the domestic money and goods markets, i.e. by equations (1) and (2). In order to simplify the exposition, we now take the price index to be constant. If the money supply is also constant, then setting \( p = m = 0 \) for convenience, we find from (1) and (2) that

(14) \[ i = \frac{\eta x}{k^1 \lambda + \gamma} = \bar{\eta} x, \]
i.e. the interest rate falls when the high exchange rate depresses the economy. By substituting this into equation (13), and combining this with the coloured noise equation (11), we obtain the simple two equation model to be analyzed, namely

(15) \[ dn = -\psi ndt + \sigma dz \]

(16) \[ E(dx) = (i^* - i + \alpha \sigma)dt + E(dn) = (i^* + \alpha \sigma)dt + \bar{\eta} x dt - \psi ndt. \]

The asymptotic expected value of the noise is zero by construction; but the risk associated with this noise means that domestic rates must stand above foreign rates (by \( \alpha \sigma \)) even when \( E(dx) = E(dn) = 0 \). As a consequence (assuming symmetric boundary conditions), the mean level of the exchange rate is negative, specifically \( \bar{x} = -(i^* + \alpha \sigma)/\bar{\eta} \), where \( \bar{\eta} \) is defined in (14) above.

By redefining the variable \( x \) as a deviation from equilibrium, the system can be written in homogeneous matrix fashion, as

(17) \[ \begin{bmatrix} dn \\ E(dx) \end{bmatrix} = \begin{bmatrix} -\psi & 0 \\ -\psi & \bar{\eta} \end{bmatrix} \begin{bmatrix} \bar{\eta} \\ x \end{bmatrix} dt + \begin{bmatrix} \sigma \\ 0 \end{bmatrix} dz. \]
Formally this is rather similar to the stochastic Dornbusch model used earlier in the paper. But because the first equation now describes an autonomous noise process (rather than the endogenous adjustment of prices as in the Dornbusch example), there is no feedback coefficient in the top right hand corner of the matrix. This simplifies the analysis, as may be seen with reference to Figure 5, see text.
Appendix C

Solving the stochastic model

The model can be written in a form analogous to (5), as

\[ \begin{bmatrix} dp \\ Edx \end{bmatrix} = A \begin{bmatrix} pdt \\ xdt \end{bmatrix} + \begin{bmatrix} odz \\ 0 \end{bmatrix} \]

We postulate first a deterministic functional relationship between \( x \) and \( p \); thus:

\( x = f(p) \).

Applying Ito's Lemma, we write

\[ dx = f'(p)dp + \frac{\sigma^2}{2} f''(p)dt \]

Taking expectations, and noting from (18) and (19) that

\[ Edp = (a_{11} p + a_{12} f(p))dt \]

we find that

\[ Edx = (a_{11} p + a_{12} f(p)) f'(p)dt + \frac{\sigma^2}{2} f''(p)dt \]

But from (18)

\[ Edx = (a_{21} p + a_{22} f(p))dt \]

Equating (22) and (23) we obtain the fundamental differential equation.
$$\frac{a^2}{2} f'' + (a_{11} p + a_{12} f) f' - (a_{21} p + a_{22} f) = 0$$

whose solutions characterise the equilibrium relationship between $x$ and $p$.

If we impose the initial condition $f(0) = 0$, we obtain solutions which satisfy the symmetry condition $f(x) = -f(-x)$. These are the appropriate ones to examine in the presence of symmetric boundary conditions.

The fundamental differential equation has no general closed form solutions, but we have derived a complete qualitative characterisation of the symmetric solutions (see Miller and Weller (1988, 1989)).
There have been a number of attempts to test for the existence of bubbles in asset markets (see, for example, Blanchard and Watson (1982), Flood and Hodrick (1986), West (1987), Smith (1987)) with mixed results. There is also some controversy surrounding the question of whether bubbles are truly consistent with rationality. Tirole (1982) argues that if agents have infinite horizons rational bubbles cannot occur (but see Gilles and Leroy (1987)). But in a deterministic overlapping generations model such bubbles are a possibility (Tirole (1985)).

The arbitrage equation is an approximation which in discrete time takes the form $(1 - \pi)x_{t+1} + \pi0_{t+1} - x_t = i_t^* - i_t$. On the left hand side is the expected "capital gain" (see Appendix A, equation (7)). (The logarithmic transformation preserves symmetry between domestic and foreign currencies and is a means of avoiding the "Siegel paradox", where strict arbitrage conditions for the two currencies are inconsistent.) We are grateful to Willem Buiter for pointing out an error in our original formulation.

The presence of negative bubbles in the diagram requires comment, since a number of writers have made the simple point that a non-negativity constraint upon prices must rule out a negative bubble. Since we work with logarithmic transformations of variables, there is no such constraint in our model. However, the approximation used in the reformulated arbitrage equation clearly becomes less satisfactory as variables diverge progressively further from their equilibrium values.

Well-organised, coordinated intervention has certainly delivered on occasion the necessary "punishment", witness the following quotation from Peter Norman in the Financial Times (10 January 1989):

"After frantic speculative activity in 1987, currency markets treated the stabilisation efforts of the G7 with notably more respect last year, after being caught in a costly central bank "bear trap" early in January. At that time, sudden, coordinated intervention to prop up the dollar when it had been oversold forced commercial banks to cover open positions at considerable cost".
De. Long et al. (1988) prove some results on the long run survival of noise traders taking this factor into account, but have to assume that noise traders have no effect upon prices.

It is true that in the mid-1980's interest rates in the US were high by historical standards but they were not high enough to explain the strength of the US dollar, see for example Krugman (1985).

Dixit's results are derived for a regulated Brownian motion process, but are also applicable to the autoregressive process in our model.

Note that, in a later paper (see this volume), FGM do report historical simulations "for which shocks to interest parity conditions were assumed to be absent". On the basis of these, indeed, they deny that their results are "strongly affected by changes in speculative behaviour in currency markets that might be associated with the exchange rate regime". But the simulations they report are for a regime of fixed exchange rates, and not for target zones.
References


(1989) "A Qualitative Analysis of Stochastic Saddlepaths and its Application to Exchange Rate Regimes", mimeo, University of Warwick.


