AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search
http://ageconsearch.umn.edu
aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

Credibility and the value of information transmission in a model of monetary policy and inflation.

Tamer Başar and Mark Salmon†

No. 289

WARWICK ECONOMIC RESEARCH PAPERS

DEPARTMENT OF ECONOMICS

UNIVERSITY OF WARWICK
COVENTRY

Credibility and the value of information transmission in a model of monetary policy and inflation.

Tamer Başar and Mark Salmon[†]

No. 289

May 1987
Revised : January 1988
September 1988

[†]Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, USA. and Department of Economics, University of Warwick, Coventry, CV4 7AL, England, respectively.

This paper is circulated for discusion purposes only and its contents should be considered preliminary.

Abstract

In this paper we solve for the optimal (Stackelberg) policy in a model of credibility and monetary policy developed by Cuckierman and Meltzer(1986) . Unlike the (Nash) solution provided by Cuckierman and Meltzer the dynamic optimisation problem facing the monetary authority in this case is <u>not</u> of a linear quadratic form and certainty equivalence does not apply. The learning behaviour of the private sector (regarding the policy maker's preferences) becomes intimately linked with the choice of the optimal policy and cannot be separated as in the certainty equivalent case. Once the dual effect of the optimal Stackelberg policy is recognised the monetary authority has an additional channel of influence to consider beyond that taken into account by sub-optimal certainty equivalent ,Nash, policy rules. Unlike Nash behaviour the Stackelberg solution implies no inflationary bias but it lacks credibility. The learning behaviour of the private sector does not sufficiently inhibit the incentive of the monetary authority to cheat in this model despite the fact that this learning is explicitly recognised in the Stackelberg solution.

1: Introduction

The Lucas Critique (Lucas ,1976) provided a fundamental reassessment of the traditional theory of economic policy in the face of rational expectations held by the private sector. Although often expressed differently the implication of the critique in the present context is that failing to take account of the endogeneity of the private sector's expectations in any policy optimisation exercise naturally leads to the adoption of time inconsistent policies. A number of papers, including Kydland and Prescott (1977) and more recently Cohen and Michel (1988), Miller and Salmon (1985) and Whitman (1986), have subsequently discussed how time consistent policies may be constructed in models with rational expectations.

The essential contribution of Kydland and Prescott was to note that such time consistent solutions are inevitably suboptimal from the point of view of the policy maker since they are computed under the additional restrictions of time consistency. Miller and Salmon (1985) and Lucas and Sargent(1981) emphasised that the problem of policy optimisation with rational expectations was formally equivalent to the structure of a dynamic Stackelberg game. The *ex ante* optimal (open loop Stackelberg) policy in such problems would be time inconsistent since at each instant there would be an incentive for the policy maker to renege on the (announced) *ex ante* optimal policy and implement the *ex post* optimal action.The usual resolution of this problem has been to construct time consistent solutions to the policy problem that achieve their objective by forcing the fundamentally asymmetric structure between the policy maker and private sector to become symmetric so that a (time consistent) Nash equilibrium may be computed. However such symmetric solutions are invariably globally inferior to the optimal asymmetric solution *for the policy maker*. In other words the imposition of the Nash structure achieves time consistency but does not neccessarily remove all credibility issues since the symmetric Nash solution will not, in general, be preferred by the leader. In effect the imposition of the symmetric solution solves a different policy problem from the one originally posed and ,unless some explicit mechanism can be established that implies the removal of the asymmetry, such Nash solutions would only seem to be weakly justified.

In addition such non-cooperative "Nash" solutions to the policy problem are , as mentioned above, naturally inefficient and as Barro and Gordon (1983) demonstrated in their study of monetary policy lead to a non zero inflationary bias while the *ex ante* optimal (committment or open loop Stackelberg) strategy delivers a zero inflationary bias. Since this latter policy is itself not credible without some external credible constraint ,possibly legislation, the question of whether it may be sustained through the evolution of the policy maker's reputation has been considered by Barro and Gordon (1983), Backus and Driffill(1985) and Barro(1986) amongst others. Such reputational mechanisms are attractive because they retain the asymmetry inherent in the original problem structure.

Cuckierman and Meltzer (1986) extended the Barro-Gordon analysis to a dynamic model of monetary policy and inflation that incorporates asymmetric information between the private sector and the monetary authority. Their principle interest lay in determining the role played by the policymaker's credibility in the design of policy and the scope for exploiting the private sector's uncertainty as to the policymaker's preferences. As Cuckierman and Meltzer emphasise the optimal policy of the monetary authority will not in general be to select the current money supply so as to simply minimise the current component of their intertemporal loss. Today's policy choice transmits information to the private sector that will affect their expectations regarding the future course of monetary policy, and the monetary authority must therefore take account of its influence on the private sector's learning and expectation formation when constructing its optimal policy.Cuckierman and Meltzer develop a Nash solution to this policy problem in which each side takes the other's reaction ,the policy rule and rational expectation respectively,as given. However this Nash solution not only generates a suboptimal policy (for the monetary authority) but one that ignores critical aspects of the information transmission issue. In particular, as we show below, the optimal policy may affect not only the information set of the private sector but also the manner in which the information is used when forming rational expectations. The Nash solution imposes certainty equivalence on the policy problem which necessarily ignores this latter

channel of policy influence[1].

In what follows we develop the (optimal) Stackelberg solution which retains the natural asymmetry inherent to the problem. In this case we show how the optimal policy is determined from a non quadratic optimisation problem for which certainty equivalence does not apply. The dual control aspect of the optimal policy is reflected in the recognition by the monetary authority that it may simultaneously influence the weight the private sector places on its most recent innovation when updating its rational expectation of the policy maker's preferences as well as the innovation itself. The Stackelberg solution , which implies a zero inflationary bias ,is however not fully strategically credible in finite horizons as there will invariably be an incentive for the policy maker to renege on its policy and cheat the private sector. The learning behaviour of the private sector is non-strategic in this model (which is hardly surprising given that the private sector is essentially passive) and does not provide a sufficient disincentive to prevent the monetary authority cheating. We then offer the Stackelberg solution developed below as a basis on which perhaps more refined reputational devices may be placed in order to sustain the optimal policy for the policy maker. However we note the beauty of the dual control formulation in that the policy maker's reputation and credibility are endogenous to the optimisation problem and no further potentially *ad hoc* extraneous device need be constructed to capture the repuational mechanism. The fact that the learning behaviour of the private sector does not restrict the monetary authority sufficiently is determined by the model's structure and assumptions.

We proceed in stages towards an understanding of the optimal policy. Solutions to nonfinite state dual control problems are well recognised to be typically infeasible and approximations , such as the imposition of certainty equivalence, are invariably required in order to derive computationally tractible , albeit suboptimal, decision rules. First we discuss the model and alternative solution concepts and then derive a Nash solution for a two period version of the problem set by Cuckierman and Meltzer. Then we compare this certainty equivalent

[1]Cripps (1988) considers the impact of learning on the Nash solution in a model similar to Cuckierman and Meltzer's ,however this passive learning does not recognise the role of active learning as employed in the Stackelberg solution below.

solution with the Stackelberg solution under the restriction of certainty equivalence, before providing the optimal (unrestricted) Stackelberg solution for the two period problem. The Stackelberg solution for a full N stage problem is highly involved mathematically and is not presented here, see Başar(1988). The essential economic issues raised by information transfer are captured in the two period problem. Finally we consider the question of the informational and strategic credibility of these solutions.

2: The model and different equilibrium concepts

The basic model we adopt is that used by Cuckierman and Meltzer (1987) but instead of taking an infinite horizon we assume that the monetary authority faces a finite horizon policy optimization problem with objective function of the form,

$$J = E\left\{ \sum_{i=0}^{N} \beta^i (e_i x_i - \tfrac{1}{2}(m_i^p)^2 \right\} \tag{1}$$

where the policy instrument is the planned rate of monetary growth, m_i^p. The private sector is only able to observe m_i, the actual rate of monetary growth which results after m_i^p has been affected by the random shock ψ_i associated with imperfect monetary control;

$$m_i = m_i^p + \psi_i \qquad \psi_i \sim N(0, \sigma_\psi^2) \quad i=0,1,.. \tag{2}$$

As indicated, the shocks $\{\psi_i\}$ are taken to be zero mean Gaussian random variables, which are serially uncorrelated and have a fixed variance of σ_ψ^2.

The private sector is assumed to act as a "passive" decision maker which simply forms conditional expectations of the actual rate of monetary growth, given the observed history on past growth;

$$I_i = \{m_{i-1}, m_{i-2}, \ldots\ldots\ldots m_0\} \tag{3}$$

That is , letting d_i denote the forecast of the private sector at time i, and δ_i , the mechanism by which rational expectations are generated, we have

$$d_i = \delta_i(I_i) = E[m_i|I_i] \quad , \quad i=0,1,2... \tag{4}$$

The monetary surprise , e_i, is given by

$$e_i = m_i - \delta_i(I_i) = m_i^p - E[m_i^p|I_i] + \psi_i \tag{5}$$

where the last relation follows from (2) and the assumption that $\{\psi_i\}$ forms an independent sequence.

The variable x_i in (1) is the preference parameter of the monetary authority which trades off the benefit from stimulating the economy through the monetary surprise, with the loss from increased inflation in period i. This basic preference parameter of the monetary authority which is unknown to the private sector, is stochastic and assumed to change over time with both permanent and transitory components, leading to what is effectively the state equation for the policy optimization problem;

$$x_i = \rho x_{i-1} + A(1-\rho) + v_i \tag{6}$$

with $v_i \sim N(0,\sigma_v^2) \quad i=1,2,....$

$x_0 \sim N(\bar{x}_0, \sigma_{x_0}^2)$

Here again , $\{v_i\}$ is a sequence of zero mean serially uncorrelated Gaussian random variables with fixed variance σ_v^2, which are also independent of the shocks $\{\psi_i\}$. The random variable x_0, is also an independent Gaussian random variable, with mean $\bar{x}_0$ and variance $\sigma_{x_0}^2$, representing the prior beliefs of the private sector. The monetary authority constructs its optimal policy based on a knowledge of its

current and past preferences as well as I_i. So in general we seek a sequence of policy rules $\{\gamma_i\}$ of the form,

$$m_i^p = \gamma_i(\eta_i) \qquad , \eta_i = \{x_{i-1}, x_{i-2}, \ldots\ldots\ldots x_0, I_i\} \tag{7}$$

Note the asymmetric form of the information structure between the monetary authority and the private sector, which enables the monetary authority to solve the private sector's prediction problem, which it will in fact do as part of the exercise when designing its optimal monetary policy.

A important feature of the formulation above is the presence of asymmetry not only in the information structure but also in the way the decision makers affect the decision process. The private sector's only role is to form conditional expectations, which however depend critically on the policy rules of the monetary authority. To indicate this dependence explicitly we introduce the notation

$$\delta = f(\gamma) \tag{8}$$

where $\delta = \{\delta_i\}_{i=0}^N$, $\gamma = \{\gamma_i\}_{i=0}^N$. Hence in compact notation, the policy optimisation problem faced by the monetary authority is

$$\max_{\gamma} J(\gamma, f(\gamma)) = J(\gamma^*, f(\gamma^*)) \tag{9}$$

where the function J is defined by (1), and the maximisation is over all possible sequences γ. The policy optimisation problem (9) captures the general form of all similar problems where the cost function ,J, does not necessarily have to be in the form (1) but where f is always the conditional expectation operator which determines the forecast rule for the passive player. Under the adopted behavioural assumptions the maximum in (9) is the best performance the policy maker can expect to achieve in this problem, and it cannot be dominated by any other solution, given commitment to the rule γ^*.

Cuckierman and Meltzer (1987) instead obtained a different type of solution for the problem. Their interest has been on the characterisation of a policy , $\hat{\gamma}$ for the monetary authority with the property that

$$\max_{\gamma} J(\gamma, f(\hat{\gamma})) = J(\hat{\gamma}, f(\hat{\gamma})) \tag{10}$$

There may exist multiple $\hat{\gamma}$'s that satisfy the above relationship (10), and it may be difficult (if not impossible) to obtain the entire set of such solutions. This, in turn, makes it impossible to determine the "best" policy within this set and even if such a solution exists it will in general be different from γ^* and hence lead to an inferior overall performance for the policy maker.

To place the two types of solutions, provided by (9) and (10) into better perspective, it is helpful to pose the problem as a dynamic game (although strictly it is a single decision maker optimisation problem), where the forecast rules δ_i are now regarded as strategic variables. Toward this end consider the two person nonzero-sum dynamic game with the objective functions J_1 and J_2 ,

$$J_1(\gamma, \delta) = J \tag{11a}$$

$$J_2(\gamma, \delta) = E\left[\sum_{i=0}^{N} (\delta_i(I_i) - m_i)^2 c_i\right] \tag{11b}$$

where γ is the composite policy rule of player 1 (the monetary authority) who strives to maximise J_1, and δ is the composite decision rule of player 2 (the private sector) who wishes to minimise J_2, with the c_i's taken as positive weighting terms . If we adopt the Stackelberg solution concept for this problem, where player 1 is the leader and player 2 is the follower, then for each policy rule γ of the leader, the unique optimal response of the follower will be given by (4), thus making the optimisation problem (9) precisely the one faced by the leader in this Stackelberg game. This holds since for every fixed γ,

$$\min_{\delta} E\left[\sum_{i=0}^{N-1}(\delta_i(I_i)-m_i)^2\right]=\sum_{i=0}^{N-1}\min_{\delta_i} E\left[(\delta_i(I_i)-m_i)^2\right]=\sum_{i=0}^{N-1} E\left[(E[m_i|I_i]-m_i)^2\right]$$

Hence, the optimal solution γ^* defined by (9) is the leader's Stackelberg policy in this nonzero-sum dynamic game.

If, however, we adopt the Nash equilibrium solution concept for the two person game, then a corresponding solution will be the pair $(\hat{\gamma},\hat{\delta})$ satisfying,

$$\max_{\gamma} J_1(\gamma,\hat{\delta}) = J_1(\hat{\gamma},\hat{\delta}) \tag{12a}$$

$$\min_{\delta} J_2(\hat{\gamma},\delta) = J_2(\hat{\gamma},\hat{\delta}) \tag{12b}$$

and the $\hat{\gamma}$ here is precisely the one determined by (10), since $\hat{\delta} = f(\hat{\gamma})$, where f is that used in (10). Hence, solutions generated by (10) , such as that derived by Cuckierman and Meltzer, are the Nash equilibrium policies for Player 1 in the non zero-sum game constructed above , and they will not lead to a better performance for player 1 than that provided by the Stackelberg policy γ^*.

To appreciate further the difference between the two different solutions, let us consider the single period version of the problem formulated above, where we drop the indices for convenience. Then , $J_1 = J$ is given by

$$J = E\{(m - \tilde{d})x - \tfrac{1}{2}(m^P)^2\} \tag{13}$$

where $\tilde{d} = \delta$ and where under the <u>Stackelberg approach</u>, the rational response of the private sector is taken into account as $\tilde{d}$ is calculated from (12b) to be the conditional expectation ,E[m], and hence,

$$J(\gamma, f(\gamma)) = E\{(m - E[m])x - \tfrac{1}{2}(m^P)^2\} \tag{14a}$$
$$= E\{(m^P - E[m^P])x - \tfrac{1}{2}(m^P)^2\}$$
$$= E\{m^P x - m^P E[x] - \tfrac{1}{2}(m^P)^2\} \tag{14b}$$

whose unique maximum is achieved by

$$m^P = \gamma^*(x) = x - E[x] \tag{15a}$$

yielding the value

$$\max J(\gamma, f(\gamma)) = \tfrac{1}{2}E\{(x - E[x])^2\} = \tfrac{1}{2}\mathrm{var}(x) \tag{15b}$$

Under the <u>Nash equilibrium</u> solution, however with $\tilde{d}$ taken as a fixed parameter, the unique solution to (12a) (i.e. what maximises (13) over $m^P = \gamma(x)$) is

$$m^P = \hat{\gamma}(x) = x \tag{16a}$$

which is independent of $\tilde{d}$). Hence, the minimising solution to (11b) with N=0 and γ taken as in (16a), is uniquely given by

$$\hat{\delta} = \hat{d} = E[\hat{\gamma}(x)] = E[x] \tag{16b}$$

yielding the value

$$J(\hat{\gamma}, \hat{\delta}) = \mathrm{var}(x) - \tfrac{1}{2}E[x^2]$$
$$= \tfrac{1}{2}\mathrm{var}(x) + \tfrac{1}{2}\mathrm{var}(x) - \tfrac{1}{2}E[x^2] \tag{16c}$$

which is strictly smaller than (15b) since $\mathrm{var}(x) < E[x^2]$.

It is also important to recognise that even in terms of the pair (J_1, J_2) the unique Stackelberg solution strictly dominates the unique Nash equilibrium solution since,

$$\tfrac{1}{2}\mathrm{var}(x) = J_1(\gamma^*, \delta^*) > J_1(\hat{\gamma}, \hat{\delta}) = \mathrm{var}(x) - \tfrac{1}{2}E[x^2]$$
$$J_2(\gamma^*, \delta^*) = J_2(\hat{\gamma}, \hat{\delta}) = \mathrm{var}(x) + \sigma_\psi^2$$

The other point to be emphasised in the derivation of the Stackelberg solution is the equivalence between the maximisation of (14a) and that of (14b) under the unconditional expectation operator. This follows from the nestedness property of conditional expectations as it can be seen that

$$E[E\{m_i^p \mid I_i\}x_i] = E[E\{E\{m_i^p \mid I_i\}x_i \mid I_i\}] = E[E\{m_i^p \mid I_i\}E\{x_i|I_i\}]$$
$$= E[\ m_i^p\ E\{x_i|I_i\}] \qquad (17)$$

This implies that the leader's optimal Stackelberg policy γ^* is also a Stackelberg policy for him when the original objective function (13) is replaced by

$$J_1 = E\{(x-d)m^p - ½(m^p)^2\} \qquad (18a)$$

and with the follower's cost function (to be minimised) taken as

$$J_2 = E\{(x-d)^2\} \qquad (18b)$$

where now the follower's decision variable is d. The equivalence of these Stackelberg solutions does not carry over to the Nash solutions for the two different games ,since one is a single decision maker problem and the other a genuine dynamic game between two players.

3: The "Nash" solution for a two-period problem

Consider the two-period version of the dynamic game problem of section 2, with the cost functions for the two players given by (11a) and (11b), with N=1. To determine a Nash equilibrium we need to find strategies $\hat{\gamma}=(\hat{\gamma}_0, \hat{\gamma}_1)$ for the government and forecast functions $\hat{\delta} = (\hat{\delta}_0, \hat{\delta}_1)$ for the private sector which satisfy the inequalities

$$J_1(\hat{\gamma},\hat{\delta}) \geq J_1(\gamma,\hat{\delta}) \quad \text{for all permisible } \gamma \tag{19}$$
$$J_2(\hat{\gamma},\hat{\delta}) \leq J_2(\hat{\gamma},\delta) \quad \text{for all permisible } \delta$$

The following theorem provides such a solution for this two period.

<u>Theorem: (Nash)</u>

(i) A Nash equilibrium solution for the two-period problem formulated above is given by;

$$\hat{\gamma}_1(x_1) = x_1 \quad ; \quad \hat{\gamma}_0(x_0) = Mx_0 + k \tag{20a}$$

$$\hat{\delta}_1(m_0) = E[x_1 | m_0] = \rho\bar{x}_0 + \rho \frac{M\sigma^2_{x_0}}{M^2\sigma^2_{x_0} + \sigma^2_\psi}(m_0 - M\bar{x}_0 - k) + A(1-\rho) \tag{20b}$$

$$\hat{\delta}_0 = E[m_0] = M\bar{x}_0 + k \tag{20c}$$

where M is a real solution to the third order polynomial equation;

$$M = 1 - \beta\rho^2 \frac{M\sigma^2_{x_0}}{(M^2\sigma^2_{x_0} + \sigma^2_\psi)} \tag{21a}$$

and k is given by, $k = -\dfrac{A(1-\rho)\rho\beta M\sigma^2_{x_0}}{M^2\sigma^2_{x_0} + \sigma^2_\psi}$ (21b)

The Nash values are;

$$J_1^{Nash} = \frac{\beta}{2}E[x_1^2] - \beta\rho^2\bar{x}_0^2 - \beta\rho\bar{x}_0(A(1-\rho)) - \beta\rho^2\frac{M^2\sigma_{x_0}^4}{M^2\sigma_{x_0}^2 + \sigma_\psi^2} \ldots\ldots$$

$$\ldots\ldots - \frac{1}{2}M^2(\bar{x}_0^2 + \sigma_{x_0}^2) + M\sigma_{x_0}^2 \tag{22}$$

$$J_2^{Nash} = \underbrace{\left(\frac{\rho^2\sigma_\psi^4\sigma_{x_0}^2}{(M^2\sigma_{x_0}^2 + \sigma_\psi^2)^2} + \sigma_\psi^2 + \sigma_v^2\right)}_{E[e_1^2]} c_1 + \underbrace{(M^2\sigma_{x_0}^2 + \sigma_\psi^2)}_{E[e_0^2]} c_0 \tag{23}$$

(ii) The Nash equilibrium above is <u>unique</u> if either δ_1 or γ_0 is restricted to the class of general affine mappings and (21a) admits a unique real solution.

<u>Proof:</u> (i) For any γ_0, γ_1 and arbitrary positive c_1, c_2 the following pair minimise J_2;

$$\delta_1 = E[\gamma_1(.)|\gamma_0(.)+\psi_0] \quad , \quad \delta_0 = E[\gamma_0(.)],$$

and $\hat{\delta}_1$ and $\hat{\delta}_0$ given in the theorem are indeed the true conditional expectations , for the given $\hat{\gamma}_0, \hat{\gamma}_1$. This verifies the $2^{nd.}$ Nash inequality.

To verify the first Nash inequality, first note that regardless of the choices for $\delta_1, \delta_0, \gamma_0$ the unique choice for γ_1 that maximises J_1 is $\hat{\gamma}_1(x_1)=x_1$. Hence all that we need to show is that

$$\hat{\gamma}_0 = \underset{\gamma_0}{\operatorname{argmax}}\ J_1(\gamma_0, \hat{\gamma}_1, \hat{\delta}_0, \hat{\delta}_1)$$

Let us first evaluate $J_1(\gamma_0, \hat{\gamma}_1, \hat{\delta}_0, \hat{\delta}_1) \equiv F(\gamma_0)$,

$$F(\gamma_0) = E\{\beta(x_1+\psi_1-\hat{\delta}_1(m_0))x_1 - \tfrac{1}{2}x_1^2\beta + (m_0-\hat{\delta}_0)x_0 - \tfrac{1}{2}(m_0^p)^2\}$$

$$= E\left\{\{m_0^p x_0 - \tfrac{1}{2}(m_0^p)^2 - \frac{\rho\beta M\sigma_{x_0}^2}{M^2\sigma_{x_0}^2+\sigma_\psi^2} m_0^p x_1\right\} + S \tag{24}$$

where $m_0^p = \gamma_0(x_0)$, and S is the collection of all terms that do not depend on m_0^p.

Since $F(\gamma_0)$ is quadratic and strictly concave in m_0^p, it admits a unique maximum, given by

$$x_0 - m_0^p - \frac{\rho\beta M\sigma_{x_0}^2}{M^2\sigma_{x_0}^2+\sigma_\psi^2} E[x_1 | x_0] = 0 \tag{25}$$

which implies, since $E[x_1|x_0] = \rho x_0 + A(1-\rho)$,

$$\hat{\gamma}_0(x_0) = \left(1 - \frac{\rho^2\beta M\sigma_{x_0}^2}{M^2\sigma_{x_0}^2+\sigma_\psi^2}\right)x_0 - \frac{A(1-\rho)\rho\beta M\sigma_{x_0}^2}{M^2\sigma_{x_0}^2+\sigma_\psi^2} \equiv Mx_0 + k \tag{26}$$

where M satisfies the 3rd order polynomial equation given by (21a).

ii) For a proof of uniqueness, we first note that (as also pointed out in the proof of (i) above) the policy $\hat{\gamma}_1$ in (20a) is a universally optimal decision rule, regardless of the choices for γ_0, δ_1, and δ_0. Now substituting this policy into J_1 ,we arrive at the "reduced" welfare function

$$J_1^*(\gamma_0, \delta_0, \delta_1) = E\left\{\tfrac{1}{2}\beta x_1^2 - d_1 x_1 \beta + (m_0^p - d_0)x_0 - \tfrac{1}{2}(m_0^p)^2\right\}$$

$d_1=\delta_1(m_0)$; $m_0^p=\gamma(x_0)$; $d_0=\delta_0$ a constant.

J_2, on the other hand ,is equivalently expressed as

$$J_2^*(\gamma_0, \delta_0, \delta_1) = E\left\{c_1(d_1 - x_1)^2 + c_0(d_0 - m_0^p)^2 + (c_1 + c_0)\sigma_\psi^2\right\}$$

Hence the Nash inequalities (19) become equivalent to

$$J_1^*(\hat{\gamma}_0, \hat{\delta}_0, \hat{\delta}_1) \geq J_1^*(\gamma_0, \hat{\delta}_0, \hat{\delta}_1) \quad \text{for all permissible } \gamma_0$$

$$J_2^*(\hat{\gamma}_0, \hat{\delta}_0, \hat{\delta}_1) \leq J_2^*(\hat{\gamma}_0, \delta_0, \delta_1) \quad \text{for all permissible } \delta_0, \delta_1$$

Now if γ_0 is restricted to be affine , say $\gamma_0(x_0)=Mx_0+k$, the δ_0 and δ_1 minimising J_2^* will be given by

$$\begin{aligned}\delta_1(m_0) &= E[x_1|m_0] = \rho E[x_0|m_0] + A(1-\rho)\\ &\equiv N(m_0 - M\bar{x}_0 - k) + A(1-\rho)\\ \delta_0 &= E[m_0^p] = M\bar{x}_0 + k\end{aligned}$$

where $N = \dfrac{\rho M \sigma_{x_0}^2}{M^2\sigma_{x_0}^2 + \sigma_\psi^2}$,

since the underlying statistics are Gaussian. This then says that as long as γ_0 belongs to an affine class, δ_1 can be taken to belong to an affine class also, without any loss of generality. Hence now taking $\delta_1=Nm_0+ n$, where N and n are arbitrary constants, we can maximise J_1^* over γ_0 (in this case not necessarily over the affine class) to obtain the <u>unique</u> solution,

$$\begin{aligned}\gamma_0(x_0) &= -x_0 - \beta N E[x_1|x_0] \\ &= (1-\beta\rho N)x_0 - \beta N A(1-\rho) \\ &= Mx_0 + k\end{aligned}$$

Then , a Nash equilibrium exists under the restriction that γ_0 is affine, if and only if the following consistency condition holds;

$$M = 1 - \beta\rho N \equiv 1 - \frac{\beta\rho^2 M\sigma^2_{x_0}}{M^2\sigma^2_{x_0} + \sigma^2_{\psi}} \tag{27}$$

which is identical with (21a). One can further show that k is given by (21b). Hence, the solution is unique provided that (21a) has a single real solution. In case of three real roots, the problem admits three (and only three) Nash equilibria.

A similar analysis shows that if, instead, δ_1, is restricted to be affine, the Nash equilibrium is again unique and given by (20), if (21a) admits a single real solution.

QED

Notice that in this Nash solution, in determining the optimal policy parameter M, the reaction of the private sector is <u>not</u> taken into account , unlike the Stackelberg case to be studied below. In effect the Nash solution is determined by the mutual consistency of two single player , certainty equivalent, optimization problems where the reaction of the other player is taken as <u>given</u>. Hence we refer to this Nash solution as <u>individually certainty equivalent</u> since the optimal decisions for the two players do not reflect the potential for dual control action.

The two period Nash solution presented above may be extended to the N period case (see Başar and Salmon (1988b)) and then to a stationary solution that coincides with the solution to the infinite horizon problem considered by Cuckierman and Meltzer. The advantage of taking a finite horizon lies in the use of the Kalman filter to model

the private sector's learning and expectation formation process which allows the possibility to monitor the evolution of credibility both in steady state and during the important transient phase of adjustment to the staedy state. Whereas Cuckierman and Meltzer ,by adopting an infinite horizon and a Wiener filter to solve the private sector's prediction problem, are restricted to only consider credibility in steady state . As stressed above, in either case, in this Nash equilibrium the level of credibility at any time is determined by structural parameters of the model .

The distribution of monetary growth and hence the extent of inflationary bias also evolves over time. The stationary distribution given in Cuckierman and Meltzer implies an inflationary bias given by the nonzero mean ,

$$E[m_i] = \bar{m}^P = B_0 A, \tag{28a}$$

with variance $V[m_i] = B^2(1-\rho^2)^{-1}\sigma_v^2 + \sigma_\psi^2$. (28b)

where $B_0 = \dfrac{1-\beta\rho}{1-\beta\lambda}$ and $B = \dfrac{1-\beta\rho^2}{1-\beta\rho\lambda}$ (29)

and λ measures the speed of response to new information in the Weiner predictor;

$$m_{i|i-1} = \sum_{j=0}^{\infty} \lambda^j [(1-\rho)\bar{m}^P + (\rho-\lambda)m_{i-1-j}] \tag{30}$$

where $\bar{m}^P = \dfrac{1-\beta\rho}{1-\beta\lambda}A$ and $0<\lambda\leq 1$.

The time varying distribution in the two period Nash solution above implies

$$E[m_0] = M\bar{x}_0 + k, \quad V[m_0] = M^2\sigma_{x_0}^2 + \sigma_\psi^2 , \tag{31a}$$

and

$$E[m_1]=A, \qquad V[m_1]=(1-\rho^2)^{-1}\sigma_v^2+\sigma_\psi^2. \tag{31b}$$

In Başar and Salmon (1988b) we investigate how the distribution of monetary growth in a finite horizon converges to the stationary solution given in (28) above .

4: The myopic Stackelberg solution

We now turn to the Stackelberg formulation of the decision problem and assume for the moment that the monetary authority acts myopically in that it only recognises that it affects the information set available to the private sector in the following period through its decision today . Under this assumption the optimisation problem (19) becomes one with a Linear Quadratic Gaussian (LQG) form which essentially decomposes the intertemporal problem into that of determining a sequence of static decisions. Consider the basic problem;

$$\max_{\{m_i^p\}} \quad J = \sum_{i=0}^{N} \beta^i c(x_i, I_i, m_i^p)\} \tag{32}$$

subject to

$$x_i = \rho x_{i-1} + A(1-\rho) + v_i \qquad , \; v_i \sim N(0,\sigma_v^2) \; , \; i=1,2,\ldots.$$

$$m_i = m_i^p + \psi_i \qquad , \; \psi_i \sim N(0,\sigma_\psi^2) \quad i=0,1,\ldots$$

where $$c(x,I,m^p) = m^p(x - E[x \mid I]) - \tfrac{1}{2}(m^p)^2. \tag{33}$$

If we ignore the intertemporal links we obtain for each i,

$$\arg\max_{m_i^p} \quad c(x_i, I_i, m_i^p) = (x_i - E[x_i \mid I_i]) \tag{34}$$

$$m_i^p = \gamma_i(\eta_i)$$

The myopic policy in this certainty equivalent problem is then simply given for each period as a linear function of the forecast error,

$$m_i^P = x_i - E[x_i \mid I_i] \equiv x_i - \hat{x}_{i|i-1} \tag{35}$$

where $\hat{x}_{i|i-1}$, the optimal prediction (rational expectation) by the private sector of the monetary authority's preferences , will be generated in this sequential decision problem by the Kalman filter;

$$\hat{x}_{i|i-1} = \rho\ \hat{x}_{i-1|\,i-1} + A(1-\rho) \quad , \qquad \hat{x}_{0|\,-1} = E[x_0] = \bar{x}_0 \quad (36i)\,(36)$$

$$\hat{x}_{i|i} = \hat{x}_{i|i-1} + \frac{\sigma^2_{i|i-1}}{(\sigma^2_{i|i-1} + \sigma^2_{\psi})}(m_i - m_{i|i-1}) \tag{36ii}$$

$$\sigma^2_{i+1|i} = \rho^2\ \sigma^2_{i|i} + \sigma^2_v \tag{36iii}$$

$$\sigma^2_{i|i} = \frac{\sigma^2_{i|i-1}\sigma^2_{\psi}}{(\sigma^2_{i|i-1} + \sigma^2_{\psi})}; \qquad \sigma^2_{0|-1} = \mathrm{var}\ (x_0) \tag{36iv}$$

Notice also that

$$m_{i|i-1} = E\{m_i^P \mid I_i\} = E\{x_i - E[x_i \mid I_i]\ |I_i\} = 0 \quad , \tag{37}$$

so given the myopic policy and the structure of the model the best conditional forecast that the private sector can make of the next period's monetary growth is zero hence every observation on monetary growth represents an observation on the innovation process driving the Kalman Filter. This follows directly from the form of the policy rule under the myopic assumption . Under the rational expectations assumption the private sector knows the form of the policy rule adopted by the government but given that it doesn't know the preference parameter x_i the rational expectation is substituted which

leads to the zero expected value for monetary growth. Notice that this policy may be rationalised if we consider the class of policy functions that are linear in the expectational error, in other words,

$$m_i^p = L(x_i - E[x_i \mid I_i]) \equiv L(x_i - \hat{x}_{i|i-1}) \tag{38}$$

where the parameter L is to be chosen. The objective function (33) is maximised (in this myopic case) by setting L=1 , where the benefit that accrues from surprise inflation is twice the cost that follows from the induced inflation. This net benefit is greatest when L=1 and decreases for all other choices of L , becoming negative as L exceeds 2. Alternatively given the model structure and objective function there is no incentive to change the monetary growth rate if the private sector are able to perfectly predict the monetary authorities' true preferences. Clearly , given the objective function , when no surprises are possible the best the monetary authorities can do is to set planned monetary growth equal to zero.

Substituting this myopic policy back into the cost function we find

$$\max J \geq E\left\{\sum_{i=0}^{N} \tfrac{1}{2}\beta^i (x_i - E[x_i \mid I_i])^2\right\} = \sum_{i=0}^{N} \tfrac{1}{2}\beta^i \sigma^2_{i|i-1} \tag{39}$$

In other words we know that the optimal (nonmyopic) policy must be at least able to better this valuation given by the certainty equivalent or myopic approximation.

Comparing this myopic certainty equivalent solution with that derived under the Nash assumption in the previous section we can see that ,as in the commitment (or rules) solution of Barro and Gordon, there is no inflationary bias whereas in the Nash (or discretionary) solution the bias is given by (28). The distribution of monetary growth in this case implies

$$E[m_i]=0 \tag{40}$$

and $$V[m_i]=\sigma_x^2 + \sigma_\psi^2$$

We shall consider the strategic credibility of such a zero inflationary bias policy later in section 7 but as may be expected it will not be sustainable without some form of commitment. The interesting question , to which we now turn, is whether the fully optimal dual control solution that explicitly recognises the interaction between the choice of policy and the learning behaviour of the private sector will endogenously provide this commitment.

5: The optimal Stackelberg policy for the two stage problem.

In the previous section we found a lower bound for the optimal cost and so we know that the optimal policy will satisfy

$$\max_{\gamma_0 \cdots \gamma_N} J \geq E\left\{\sum_{i=0}^{N} \tfrac{1}{2}\beta^i (x_i - \hat{x}_{i|i-1})^2\right\} = \sum_{i=0}^{N} \tfrac{1}{2}\beta^i \sigma^2_{i|i-1} \equiv \underline{J}_{max} \tag{41}$$

The question now becomes whether we can find a sequence of policies $\{\gamma_0, \ldots, \gamma_N\}$ which maximises J and whether max J > $\underline{J}_{max}$. We approach this problem in this section by solving for the optimal policy in a two period problem in which the dual effect is now formally recognised.

Consider the problem of maximising the objective function

$$J = E\left\{\sum_{i=0}^{1} \beta^i (x_i - E[x_i | I_i]) m_i^p - \tfrac{1}{2}\beta^i (m_i^p)^2\right\} \tag{42}$$

For the last period ,i=1 , there will be no issue of information transmission and so the optimal policy will be given by the myopic solution developed in section 4. So the solution will be

$$m_1^p = \gamma_1(\eta_1) = x_1 - E[x_1 \mid I_1] \tag{43}$$

Now we need to consider the optimal policy for period 0 taking into account the full effect of the information transmission to period 1. The cost function for the two period problem can then be written , having substituted the optimal policy rule for the final period, as

$$\max_{m_0^p,\, m_1^p} J_0 = \max_{m_0^p} E\left\{\tfrac{1}{2}\beta(x_1 - E[x_1 \mid I_1])^2 + (x_0 - E[x_0 \mid I_0])m_0^p - \tfrac{1}{2}(m_0^p)^2\right\} \tag{44}$$

But, given that

$$I_1 = m_0 = m_0^p + \psi_0 ,$$

we may rewrite the innovation in period 1 as

$$x_1 - E[x_1 \mid I_1] = \rho\ (x_0 - E[x_0 - E[x_0 \mid m_0^p + \psi_0\ ]\ ) + v_0 \tag{45}$$

so the cost function becomes

$$\begin{aligned}\max_{m_0^p,\, m_1^p} J_0 = \tfrac{1}{2}\beta\sigma_v^2 + \max_{m_0^p} E\{\tfrac{1}{2}\rho^2\beta(x_0 - E[x_0 \mid m_0^p + \varphi_0])^2 + \ldots \\ \ldots (x_0 - E[x_0 \mid I_0])m_0^p - \tfrac{1}{2}(m_0^p)^2\end{aligned} \tag{46}$$

which may be written in terms of the yet unkown policy rule γ_0

$$\max_{m_0^p,\, m_1^p} J_0 = \tfrac{1}{2}\beta\sigma_v^2 + \max_{\gamma_0} F(\gamma_0) \tag{47}$$

where $F(\gamma_0)$ is the maximand on the right hand side of (46). The difficulty in solving this problem lies as discussed earlier in that the optimal policy in the initial period , m_0^p , is part of the conditioning information when the private sector's expectation of monetary growth (or the preference parameter) is taken in period 1. The way we solve the problem below is to simultaneously solve for the optimal predictor for the private sector in period 1 , $\delta(I)$, and the optimal policy rule for the government in period 0 , say γ. Since we know that the private sector's forecast function will depend on the government's policy rule and the government's optimal policy rule will depend on the private sector's forecast function we need to examine the fixed points of the mappings $\delta[\gamma]$ and $\gamma[\delta]$ where in addition it should be stressed that in this Stackelberg solution δ is a <u>different</u> mapping for each γ and vice versa.

Substituting the unknown predictor function into the objective we first define

$$G(\delta,\gamma) = E\ \{\ \tfrac{1}{2}\rho^2\beta\ (\delta(I_1)-x_0)^2 + (x_0 - E\ [x_0|I_0])m_0^p - \tfrac{1}{2}(m_0^p)^2\ \} \quad (48)$$

$$m_0^p = \gamma(x_0, I_0) \quad , \quad I_1 = m_0^p + \psi_0 \quad (49)$$

noting that the information set I_0 will be empty. The " policy problem" facing the private sector is to minimise its prediction error through its choice of δ and the monetary authority recognising this will solve the following problem in order to determine its optimal γ, chosen so as to <u>maximise</u> the prediction error.

$$\max_{\gamma} F(\gamma) = \max_{\gamma} \min_{\delta(\gamma)} G(\delta, \gamma) \quad (50)$$

We next show that the function G in fact admits a unique saddle point, that is there exists a unique pair of policies (δ^*, γ^*) such that

$$G(\delta^*, \gamma^*) = \max_{\gamma} \min_{\delta} G(\delta, \gamma) = \min_{\delta} \max_{\gamma} G(\delta, \gamma) \quad (51)$$

or alternatively,

$$G(\delta^*,\gamma) \leq G(\delta^*,\gamma^*) \leq G(\delta,\gamma^*) \tag{52}$$

Clearly given any such saddle-point pair (δ^*,γ^*) we have from (50) that

$$F(\gamma^*) = \max_{\gamma} F(\gamma) \tag{53}$$

and furthermore γ^* is the unique maximising solution above if (δ^*,γ^*) is unique as a saddle point solution.

Before presenting the main result of this section , we first introduce some notation.

Let $L = L_0$ be a real solution to the polynomial equation

$$1 - L = \frac{L\,\sigma^2_{x_0}\sigma^2_{\psi}\rho^2\beta}{(L^2\sigma^2_{x_0} + \sigma^2_{\psi})^2} \equiv g(L) \tag{54}$$

and let K_0 be given by

$$K_0 = \frac{L_0\,\sigma^2_{x_0}}{(L_0^2\sigma^2_{x_0} + \sigma^2_{\psi})} \equiv \Delta(L_0) \tag{55}$$

Futhermore introduce the function

$$\Gamma(K) \equiv \frac{(1 - K\rho^2\beta)}{(1 - K^2\rho^2\beta)} \tag{56}$$

for $\qquad K^2 \neq 1 / \rho^2\beta$,

and the condition $\qquad L_0(1-L_0)\sigma^2_{x_0} < \sigma^2_{\psi}$ (57)

Theorem: (Stackelberg)

(i) The polynomial equation (54) is identical with $L=\Gamma(\Delta(L))$, and admits a maximising real solution L_0 , with $0 < L_0 < 1$.

(ii) If L_0 satisfies (57), the game G admits the unique saddle point solution

$$\delta^*(I_1) = \bar{x}_0 + K_0 I_1 \equiv \bar{x}_0 + \Delta(L_0) I_1 \tag{58}$$

$$\gamma^*(x_0) = L_0(x_0 - \bar{x}_0) \equiv \Gamma(K_0)(x_0 - E[x_0 | I_0])$$

where γ^* also provides the unique solution of (53).

(iii) Condition (57) can equivalently be written as

$$1-\Delta(L_0)^2\rho^2\beta > 0 \Leftrightarrow 1 - K_0^2\rho^2\beta > 0 \tag{59}$$

Proof:

(i) Existence of L_0 follows from the simple observation that since (54) is a 5th order polynomial it will admit one, three or five real solutions. Furthermore, all these solutions will lie in the open interval (0,1) since g(L) is nonnegative and is zero only if L=0. If the polynomial has more than one real root, let L_0 be the one that provides the largest value of $F(\gamma)$ defined in (46). If there is more than one such solution then L_0 could be taken to be any one of them which we refer to as a maximising solution of (54). The fact that (54) is identical with the equation $\Gamma(\Delta(L))=L$ follows from the substitution of (55) into (56).

(ii) Here we verify the pair of inequalities (52). The right hand side follows since $G(\delta,\gamma)$, given by (48) , is minimised for any γ by the conditional mean of x_0 (given I_1), and when $\gamma = \gamma^*$, this conditional mean is linear in I_1 as given. For the left hand side inequality, note that $G(\delta^*,\gamma)$ is a quadratic function of γ , with the coefficient of the quadratic term being

$$-\frac{1}{2}\left(1-\frac{L_0^2\sigma_{x_0}^2\sigma_{x_0}^2\rho^2\beta}{(L_0^2\sigma_{x_0}^2+\sigma_\psi^2)^2}\right)\equiv\alpha \tag{60}$$

The condition $\alpha < 0$ directly implies that $G(\delta^*,\gamma)$ is a strictly concave function of γ ,and being quadratic , it admits a unique solution which is

$$\gamma(x_0) = \Gamma(\Delta(L_0))(x - \bar{x}_0)$$

and by (i)

$$\gamma(x_0) = L_0(x - \bar{x}_0) \ .$$

This verifies the left hand side of the inequality (52), under the condition $\alpha < 0$. Using the fact that L_0 satisfies (54), α can be simplified to

$$\alpha = \tfrac{1}{2}\ (\ L_0(1-L_0)(\sigma_{x_0}^2/\sigma_\psi^2)\ -1)$$

and hence the concavity condition is indeed equivalent to (57). Note that under this condition , $G(\delta^*,\gamma)$ admits a <u>unique</u> maximum , and using the interchangeability property of multiple saddle point equilibria [Basar and Olsder, (1982)] it readily follows that (58) is indeed the unique saddle point solution of G under (57), which also means that the maximising solution alluded to in part(i) will have to be unique under condition (57).

(iii) This follows readily by noting that

$$\alpha = -\tfrac{1}{2}\ (1-\Delta(L_0)^2\rho^2\beta) \quad ,$$

and hence the condition $\alpha < 0$ is equivalent to (59).

QED

The condition (57) of the Theorem is given in terms of the solution of (54) , and this depends on the parameters of the problem only implicitly. A more explicit dependence purely on the parameters $\sigma^2_{x_0}$, σ^2_ψ , ρ and β can be seen in the condition

$$\sigma^2_{x_0}\rho^2\beta < 4\sigma^2_\psi \tag{61}$$

which implies (57). To see this implication , note that in view of (59), condition (57) is equivalent to

$$\frac{L_0^2\sigma^2_{x_0}\sigma^2_{x_0}\rho^2\beta}{(L_0^2\sigma^2_{x_0}+\sigma^2_\psi)^2} < 1 \tag{62}$$

but since

$$\max_{L_0} \frac{L_0^2\sigma^2_{x_0}\sigma^2_{x_0}\rho^2\beta}{(L_0^2\sigma^2_{x_0}+\sigma^2_\psi)^2} = \frac{\sigma^2_{x_0}\rho^2\beta}{4\sigma^2_\psi}$$

the preceding inequality is always satisfied under (61).

Condition (61), or the less restrictive (57), are sufficient for the linear solution γ^* given in the Theorem to be overall maximising , but there is no indication that it is also necessary. In fact , it is quite plausible that the result is valid for all values of the parameters defining the problem. Non-satisfaction of (57) simply means that the auxilary game G does not admit a saddle point (that is the upper value is strictly larger than the lower value) ; however this does not rule out the possibility that the maximising solution for $F(\gamma)$ is still linear.

If we restrict the monetary authority to affine policies at the outset , say of the special form

$$\gamma(x_0) = L(x_0 - \bar{x}_0) , \tag{63}$$

then

$$E[\ x_0 - \bar{x}_0 \mid m_0 = L(x_0 - \bar{x}_0) + \psi_0\ ] = \frac{L\sigma^2_{x_0} m_0}{(L^2\sigma^2_{x_0} + \sigma^2_\psi)} \qquad (64)$$

and substituting this into $F(\gamma)$ we obtain

$$F(\gamma = L_0(x_0 - \bar{x}_0)) \equiv \tilde{F}(L)$$

$$= \tfrac{1}{2}(\sigma^2_v + \rho^2\sigma^2_{x_0}) - \tfrac{1}{2}\frac{\rho^2\beta L^2(\sigma^2_{x_0})^2}{(L^2 + \sigma^2_\psi)} - \tfrac{1}{2}L^2\sigma^2_{x_0} + L\sigma^2_{x_0}$$

as the function to be maximised over the scalar L. Being continuous and bounded above , $\tilde{F}$ admits a maximum , and differentiating it with respect to L and setting the derivative equal to zero we obtain the equation

$$1 - L = \frac{L\,\sigma^2_{x_0}\sigma^2_\psi\rho^2\beta}{(L^2\sigma^2_{x_0} + \sigma^2_\psi)^2}$$

which is (54). Hence the affine policy γ^* given in the theorem is optimal for all values of the parameters if the search is restricted to the linear class.

6: Policy Comparison with the myopic solution

The essential difference between the dual solution and certainty equivalent solution presented in section 4 lies in that although the policy is still linear in the innovation or forecast error of the private sector the optimal policy parameter L_0 now lies in the range (0,1). Moreover the value of L_0 has been chosen taking into account the feedback from the effect on the private sector's expectation formation process in period 0. In particular the effect of L_0 on the Kalman gain, $K_0=\Delta(L_0)$ determines the weight attached to the most recent information and as we shall see in the next section, this parameter determines the policymaker's credibility. Thus the monetary authority has the ability to directly affect its own credibility with its choice of monetary growth rate by , in effect, modifying the way in which the private sector forms its rational expectation. Moreover this can be achieved without recourse to an additional policy instrument ,for instance σ_ψ^2 ,as in the Cuckierman and Meltzer's (section 6) analysis of ambiguity.

We notice that as in the myopic certainty equivalent solution there is no inflationary bias induced by the optimal policy but the variance of monetary growth is increased over that in the certainty equivalent case. This point and those that follow will be made more apparent if we consider briefly the steady state solution of this optimisation problem .

Consider the following policy ;

$$m_i^p = L_i(x_i - \hat{x}_{i|i-1}) \tag{65}$$

where

$$\hat{x}_{i|i-1} = \rho\, \hat{x}_{i-1|i-1} + A(1-\rho) \quad , \qquad \text{(i)} \tag{66}$$

$$\hat{x}_{i|i} = \hat{x}_{i|i-1} + \frac{L_i\sigma^2_{i|i-1}}{(L_i^2\sigma^2_{i|i-1} + \sigma^2_\psi)}(m_i - m_{i|i-1}) \qquad \text{(ii)}$$

$$\sigma^2_{i+1|i} = \rho^2 \sigma^2_{i|i} + \sigma^2_v \qquad \text{(iii)}$$

$$\sigma^2_{i|i} = \frac{\sigma^2_{i|i-1} \sigma^2_\psi}{(L_i^2 \sigma^2_{i|i-1} + \sigma^2_\psi)} \quad ; \qquad \text{(iv)}$$

and $\{L_i\}$ maximises

$$\tilde{J}(L) = \sum_{i=0}^{N} \beta^i (L_i - \tfrac{1}{2}L_i^2) \sigma^2_{i|i-1} \qquad (67)$$

subject to (66iii) and (66iv) above.

This cost function follows from substituting the linear feedback rule (65) into the original objective function turning the problem into one of maximising with respect to the scalar L_i at each point of time. The steady state solution (which would in a finite horizon be suboptimal) is given by setting $L_i = L$, a constant for all i. This steady state solution has a constant error variance $\bar{\sigma}$ given by;

$$\bar{\sigma}^2 = \frac{\rho^2 \bar{\sigma}^2 \sigma^2_\psi}{(L^2 \bar{\sigma}^2 + \sigma^2_\psi)} + \sigma^2_v \qquad (68)$$

and the optimal value of L can be quite easily shown to lie in the interval(0,1). [see Basar(1988) for a verification of this property and for a rigorous justification on the existence of a` solution to the maximisation problem(67) for both finite and infinite N]

This steady state solution allows us to determine that the derivative of $\bar{\sigma}^2$ with respect to L^2 is negative, when L is considered a

variable, and so one effect of the optimal policy in the dual problem having a feedback coefficient of less than one is to increase the variability of monetary growth when compared with the myopic solution.

It is ambiguous whether the steady state gain Kalman gain will be an increasing or decreasing function of L, when L is again considered a variable, but it can be seen from the cost function (67) above that the government selects that policy parameter , L , and the implied level of credibility , that optimally trades off the effects on the two components $(L_i - \frac{1}{2}L_i^2)$ and $\sigma^2_{i|i-1}$. The first can be seen to be maximised when L =1 as in the myopic solution and the second, being a decreasing function of L , is maximised at zero. Hence there will be an optimal policy and corresponding level of credibility that trades of the benefits from surprises and the costs of inflation that reflects the government's ability to control the extent of surprise.

7: Credibility

There are two aspects to the question of credibility that we are forced to consider in any problem where the dual effect of policy is present. The first is what may be called <u>informational credibility</u> (*i-credibility)* and is simply a function of the implications of the imperfect information held by the private sector regarding the government's preferences. Central to this is of course the learning behaviour of the private sector and how the government may interfere with this learning. The second aspect of credibility that we call <u>strategic credibility</u> (*s-credibility)* below turns on the sustainability of the solution. This issue of essentially the time consistency of the solution would of course remain without any uncertainty regarding the policy maker's preferences. Nash solutions are generally *s-credible* being time consistent if not sub-game perfect whereas Stackelberg solutions on the other hand are generally only credible with some form of commitment. The question of interest is whether the government's optimal policy when the dual effect is recognised may affect its informational credibility in such a way so as to enhance its strategic credibility. The ability of the government to manipulate its informational credibility could in principle sustain an otherwise non *s-credible* policy. Since the interaction between these two aspects of

credibility is not trivial we shall discuss each in isolation to clarify the issues involved. We first consider the question of informational credibility and draw comparisons with the discussion of credibility in Cuckierman and Meltzer.

Cuckierman and Meltzer define credibility to be the absolute value of the difference between the policymaker's plans and the public's beliefs about those plans ($| m_i^p - m_{i|i-1}^p |$) . However since the public cannot observe planned monetary growth rates directly , a more meaningful interpretation turns on the value of λ which measures the degree of sluggishness in the adjustment of expectations in the expression Cuckierman and Meltzer derive from the Weiner filter (30).The higher λ, the longer it takes the public to recognise a change in governmental preferences and the lower the government's credibility. Alternatively the higher λ, the longer the memory of the private sector and the less important are recent developments for the formation of current expectations. So "credibility depends on the speed with which the private sector learns ; actions that delay learning lower credibility", (CM. p.1108-9 and p.1122).

This interpretation of credibility as the speed of learning is reflected in the Kalman Filter approach to the private sector's expectation formation used in sections 3,4 and 5. We define full *i-credibility* to exist when the private sector has no incentive to update its expectation of the government's preference parameter , in other words when ($\hat{x}_{i|i} = \hat{x}_{i|i-1}$). The rate of private sector learning is governed by the behaviour of the Kalman gain ,K_i, given in the Nash solution at stage i by

$$K_i = \frac{M_i \sigma^2_{i|i-1}}{(M_i^2 \sigma^2_{i|i-1} + \sigma^2_{\psi})} \tag{69}$$

(where $\{M_i\}$ is the sequence of policy reaction coefficients that converge to the value B (given in (29)) found by Cuckierman and Meltzer)which determines the weight that the current innovation in monetary growth has in updating the public's current forecast of the preference parameter (see (20b) for instance). So a small Kalman gain corresponds to a high value of λ in Cuckierman and Meltzer's

analysis and these two characterisations of *i-credibility* are then inversely related . The exact relationship developed in Başar and Salmon(1988b) shows that $\lambda=\rho(1-KB)$ where K is the steady state Kalman gain.

Clearly if the innovation in monetary growth is zero there is no new information that the private sector can use to update its current expectation. So when actual money growth is equal to the private sector's expectation (which is zero) the monetary authority has full *i-credibility*. In general however the innovation will be non-zero (as $\sigma_\psi^2 > 0$) and *i-credibility* will be determined by the product of the Kalman gain and the innovation . Notice that given the assumptions of the model the monetary authority will never be completely *i-credible* as the private sector is unable to distinguish from (2) whether an observation on monetary growth is planned or a result of the monetary disturbance ,ψ . So for any given value of money growth, *i-credibility* will be determined by the value of the Kalman gain. The larger the Kalman gain the more impact current observations have on updating the estimate of the preference parameter.

If the Kalman gain were itself zero then once again the monetary authority would be perfectly *i-credible* since there would be no call for the private sector to update their estimate of the monetary authorities preferences even though observed monetary growth was non-zero. Notice once again, that given the assumptions of the model, in particular that $\sigma_v^2 \neq 0$ and $\rho < 1$,the Kalman gain will never be zero and hence the policy maker will never be perfectly informationally credible as the steady state value of the Kalman gain will be bounded between zero and one .

Under the optimal myopic policy ($M_1=1$ in (69)) the rate of learning or alternatively the rate of convergence of the Kalman gain (Government's *i-credibility*) is determined entirely exogenously by the underlying parameters of the model, $\rho, \sigma_\psi^2, \sigma_v^2$, although the discount factor , β plays no role. Starting from some prior distribution for the preference parameter with mean $\bar{x}_0$ and variance $\sigma_{x_0}^2$, presumably reflecting a low initial level of *i-credibility*, convergence to the steady state level of *i-credibility* will be relatively faster the higher is the variability in preferences (σ_v^2) and the lower the variance in the

monetary disturbances (σ_ψ^2) . If the ratio σ_v^2/σ_ψ^2 is large the steady state will be reached quickly as the level of uncertainty regarding preferences will be large compared with the accuracy of the observations on monetary growth so a new expectation will heavily dependent upon the new observations and less related to the previous expectation. The steady state level of *i-credibility* itself will be a decreasing function of σ_v^2 and an increasing function of σ_ψ^2. The more variability in preferences, the lower *i-credibility* and the more noise , the higher *i-credibility*. Finally considering the autoregressive parameter ρ, we note that if preferences were not systematically changing through time but were entirely random , $\rho=0$, then the monetary authorities' informational credibility would remain at its presumably low initial value for ever. As ρ increases leading to a greater persistence in preferences *i-credibility* increases. Some of these points are shown in figure 1 where *i-credibility* is measured by 1-K .

INSERT FIG 1 HERE

A crucial difference between the approach we have adopted to measuring informational credibility and that used by Cuckierman and Meltzer can now be seen from Figure 1 . Their interpretation of learning and hence credibility refers to how data is processed by the private sector after the steady state is reached. Whereas , through the use of the Kalman Filter , we are able to get a deeper understanding of how the mechanism by which *i-credibility* evolves is affected by the fundamental parameters of the model.

In particular the monetary authorities' *i-credibility* can be seen in general to pass through a transitory learning phase that may significantly affect the overall costs and policy in a finite horizon problem before reaching the steady state position described by the Cuckierman and Meltzer analysis. The relationship between σ_v^2 and σ_ψ^2 and the Kalman gain show us that the simple interpretation of Cuckierman and Meltzer's claim that actions that delay learning lower *i-credibility* may be misplaced. As can be seen from Figure 1 , it is perfectly possible for a situation to arise in which a longer learning

period is associated with a higher level of steady state *i-credibility*. This example serves to emphasise that they are only referring to "learning" as how information is processed in steady state.

The preceding discussion applies equally to the Nash and Stackelberg solutions presented in sections 4 and 5 except that the Kalman gain will be influenced by the different policy reaction parameters B or L respectively. However it is crucial to recognise that in the Nash case ,while the gain is a function of the policy parameter B the value of B has not been determined taking into account its effect on the gain and hence the government's *i-credibility*. In other words the value of B has been chosen on the assumption that the private sector's expectation would not change as B changed. On the other hand in the Stackelberg case the government explicitly takes into account this feedback when calculating the optimal reaction parameter L. Hence the monetary authority can actively manipulate it's *i-credibility* to it's advantage in the Stackelberg case but not in the Nash solution.

Turning to the question of strategic credibility, we first simply note that the Stackelberg solution is not *s-credible*. Considering the one period problem introduced in section 2, a "cheating solution", may be constructed that allows the government to renege on the policy that the private sector has taken as given when it formed its rational expectation. Since there is no information transfer issue in this single period problem the optimal Stackelberg policy is just the myopic policy derived in section 4, $m_i = x_i - x_{i|i-1}$, which induces an expectation on the part of the private sector of $\delta_i = E[m_i|I_i] = 0$. Having forced the private sector to adopt this expectation the optimal (cheating) policy for the government is to simply inflate by setting $m_i = x_i$. The resulting welfare value for the government, $\frac{1}{2}E[x^2]$, improves on the Stackelberg solution by exactly the same amount that the Stackelberg itself improves on the Nash solution. This demonstration of the incentive for the government to cheat applies equally to the final period of any finite horizon optimisation problem. Given cheating in the final period we may proceed backwards arather tedious analysis demonstrates a similar incentive to cheat in the earlier periods. In general therefore, the optimal Stackelberg solution with its zero inflationary bias, is as in the Barro and Gordon, analysis not *s-credible*. The question remains as to whether the existence of learning by the private sector and the recognition of the dual effect of the

optimal policy could serve as a disincentive to prevent such time inconsistent behaviour.

Notice that the expectation formation process of the private sector essentially corresponds to a threat strategy imposed on the leader. For instance if we take (66ii) as a typical expression for the updating of expectations we find

$$\hat{x}_{i|i} = \hat{x}_{i|i-1} + \frac{L_i \sigma^2_{i|i-1}}{(L_i^2 \sigma^2_{i|i-1} + \sigma^2_{\psi})} (m_i - m_{i|i-1})$$

So if the observed money supply deviates from its expected value then the private sector responds by adjusting its expectation of the government's preference parameter. This adjustment provides a threat that would act to reduce any potential surprise inflation in the following period. However as we have emphasised throughout the paper the government through its choice of L_i can directly control the impact of this threat by adjusting its credibility. By setting $L_i = 1$ in the cheating solution as in the optimal myopic solution of section 4 the government chooses its optimal level of credibility simultaneously with its optimal policy choice $m_i = x_i$ so as to minimise the effect of this policy on its subsequent level of credibility. Since the private sector has no independent strategic role in this model there is no effective strategic threat that it can employ to prevent the government cheating. The learning behaviour of the private sector is essentially nonstrategic.

Thus in this model the learning behaviour of the private sector does not provide a sufficient incentive to sustain the zero inflationary bias solution. However we would anticipate that in more general models the recognition of learning and strategic behaviour by the private sector within a dual policy framework may well provide an endogenous explanation for reputational forces that could sustain a zero inflationary bias policy.

8: Conclusions

In this paper we have provided what we believe to be the first closed form solution to a noncertainty equivalent policy problem with rational expectations. Quite generally we have stressed that the consideration of the optimal Stackelberg policy in dynamic models with rational expectations necessarily requires that the dual effect of the policy be taken into account . In particular the interaction of the policy with the learning process of the private sector must be fully recognised. In the case of the Cuckierman and Meltzer model of monetary policy such a policy delivers a zero inflationary bias mimicing the corresponding result for the static analysis by Barro and Gordon. We have introduced two separate notions of credibility , informational and strategic and have discussed their interaction. Within the context of the Cuckierman and Meltzer model we found that the recognition of the learning behaviour of the private sector within the optimal dual policy did not serve as a sufficient threat to prevent the government cheating. Therefore as in the Barro and Gordon analysis the optimal Stackelberg policy is not strategically credible. One explanation for this lies in the fact that the learning behaviour of the private sector is essentially nonstrategic in this model.

Acknowledgements

The second author wishes to acknowledge financial support for this research from the Ford Foundation and Alfred P.Sloan Foundation, administered by the CEPR. under its programme on Macroeconomic Interactions and Policy Design in Interdependent Economies is gratefully acknowledged. The collaboration between the two authors has also been made possible by a NATO collaborative research grant. We are grateful to Martin Cripps, John Driffill and John Vickers and a referee for helpful comments on an earlier version of this paper.

References:

Başar T., (1988): " Solutions to some nonstandard stochastic control problems with active learning", forthcoming in I.E.E.E Trans A.C.33, December, No. 12.

Başar T. and G.J. Olsder , (1982): " Dynamic Non-cooperative Game Theory", Academic Press.

Başar T. and Salmon M., (1988a): "On the convergence of beliefs and policy to a rational expectations equilibrium in a dual policy problem". forthcoming in Issues in Monetary Economics, W.Marois, A.Soubeyran and D.Laussel Eds. Springer Verlag.

Başar T. and Salmon M., (1988b), " On the evolution of credibility and inflation with disparate beliefs", mimeo , University of Warwick,

Cohen D. and P.Michel , (1988): "How should control theory be used to calculate a time consistent government policy?", Review of Economic Studies, Vol LV, 263-274.

Cripps M., (1988): " Learning rational expectations in a policy game", University of Warwick discussion paper, No. 297. June 1988.

Cuckierman A. and A. Meltzer, (1986): "A theory of ambiguity, credibility, and inflation under discretion and asymmetric information", Econometrica , Vol 54, No.5, (September), 1009-1128.

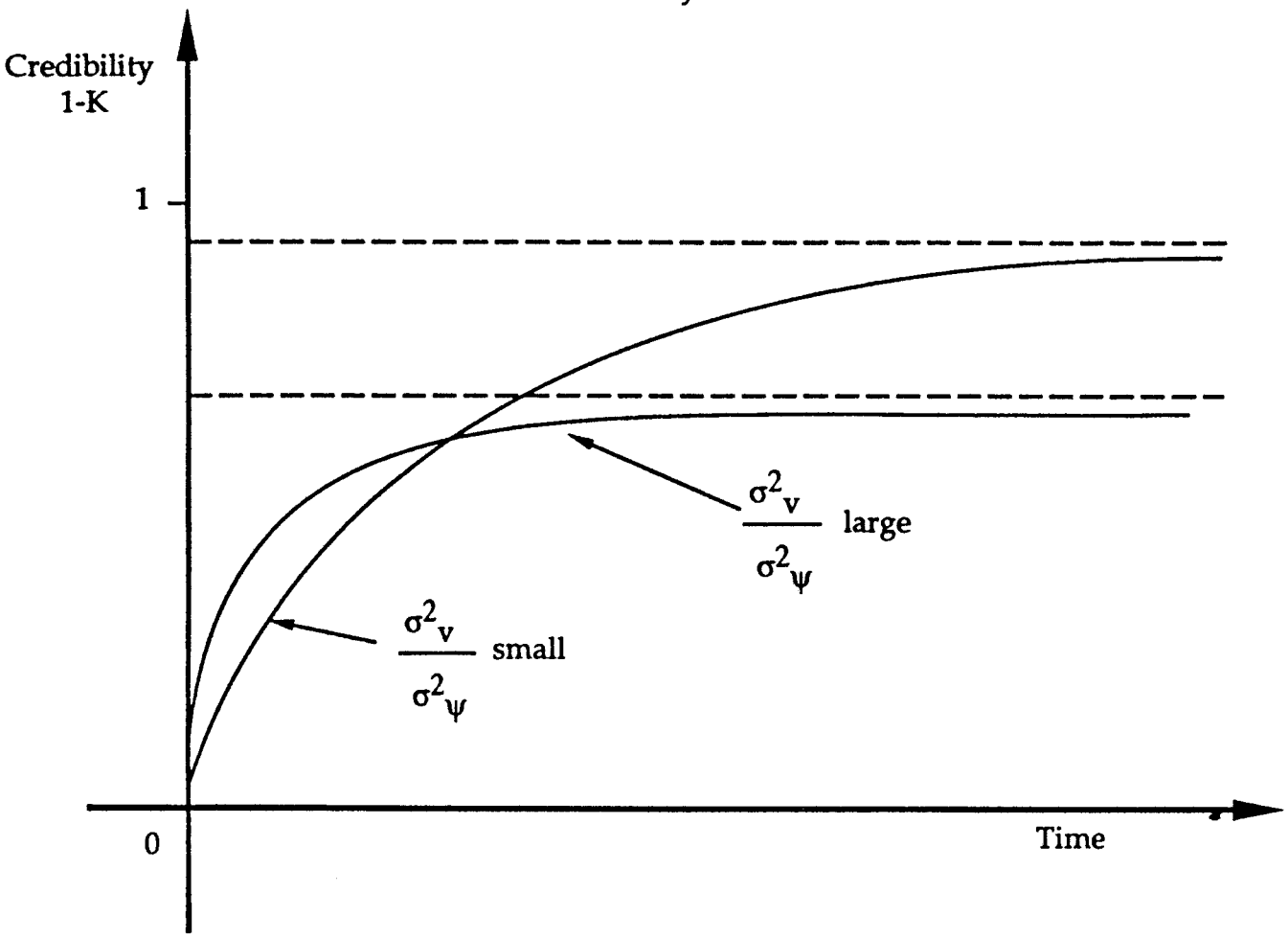
Kydland F. and E.Prescott, (1977): " Rules rather than discretion, the inconsistency of optimal plans", Journal of Political Economy, No. 3.

Lucas R.E. Jr., (1976), "Econometric policy evaluation: a critique", in The Phillips curve and Labor markets, edited by K.Brunner and A.H.Meltzer, Amsterdam: North Holland.

Lucas R.E. Jr. and T.Sargent, (1981), The introduction to Rational expectations and Econometric Practice, George Allen and Unwin.

Miller M.and M.Salmon , (1985): "Policy coordination and the time inconsistency of optimal policy in an open economy", Economic Journal, supplement, 124-135.

Whitman C. (1986): " Analytical policy design under rational expectations", Econometrica, vol 54, No.6.

Figure 1.

Convergence of credibility to its Steady State